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Publisher's version / Version de l'éditeur:

Journal of the Acoustical Society of America, 47, 3(Part 1), pp. 667-675, 1970-07-01

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INFLUENCE OF DIFFUSIVITY ON THE TRANSMISSION LOSS OF A SINGLE-LEAF WALL

BY

A. DE BRUIJN

REPRINTED FROM
THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA,
VOL. 47, No. 3, (PART I), MARCH 1970
P. 667 - 675

RESEARCH PAPER NO. 444
OF THE
DIVISION OF BUILDING RESEARCH

OTTAWA

PRICE 25 CENTS JULY 1970 NRCC 11462

INFLUENCE DE LA DIFFUSIVITE SUR L'AFFAIBLISSEMENT DU SON A TRAVERS UN MUR A PANNEAUX SIMPLES

SOMMAIRE

La théorie de Cremer sur la transmission du son à travers un mur à panneaux simples est généralisée par une expression dans laquelle la pression acoustique est exprimée en fonction de la corrélation spatiale. Le coefficient de corrélation a été mesuré dans une chambre de réverbération immédiatement en face du mur sous essai. L'affaiblissement du son à travers un panneau simple de placoplâtre a été calculé en employant les mesures de diffusivité et a été comparé avec les données expérimentales. Il semble y avoir un bon accord entre les valeurs théoriques et expérimentales en particulier dans cette région de la fréquence qui est inférieure à la fréquence de coincidence. Enfin, quelques champs de sons quasi-diffus et fictifs ont été examinés aux fins d'étudier d'une façon plus approfondie l'influence du degré de diffusivité sur l'affaiblissement du son.

Reprinted from THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA, Vol. 47, No. 3, (Part 1), 667–675, March 1970

Influence of Diffusivity on the Transmission Loss of a Single-Leaf Wall

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Cremer's theory of sound transmission for a single-leaf wall is transformed into a more general formulation in which the exciting pressure is expressed in terms of the spatial cross-correlation. The correlation coefficient was measured in a reverberation room just in front of the wall under test. The sound-transmission loss of a single gypsum-board wall was calculated employing these diffusivity measurements and compared with experimental data. Agreement between theory and experiment is good, especially in the frequency range below the coincidence frequency. Finally, a few fictitious quasidiffuse sound fields are investigated in further study of the influence of the degree of diffusivity upon transmission loss.

INTRODUCTION

Sound-transmission loss through a single panel is not yet completely understood. Cremer's theory[1] regarding the influence of the various parameters on transmission loss has been the most informative to date. It assumes that a panel extends to infinity and that sound waves induce simple flexural waves in the panel. The first essential step in the analysis is the determination of the transmission loss for a wall excited by a plane wave incident under a certain angle θ. It may be seen that the transmission-loss factor is a rather complicated function of the angle of incidence. The transmission loss for a diffusively incident sound field is obtained by averaging transmission loss with respect to the angle of incidence with the appropriate weighting function.

The resulting expression is a complicated function and an exact integration has not been performed. Northwood[2] used a numerical integration method and obtained figures for a number of examples. The numerical data agreed reasonably well with experimental data if some correction was included.

Assumption of internal friction in the material is quite natural from a physical point of view. Assumption of a complex Young's modulus provides the easiest method of including the damping of the material, the imaginary part representing the damping factor. The influence of damping is very important above the coincidence frequency range. It is essential in this range to make a proper choice of values in order to match theoretical with experimental data. In practice, however, the panel is finite, and it might be expected that the treatment of the edges would have some bearing upon transmission loss. The model might therefore be too simple to predict the finer points.

Another important feature for better agreement between experiment and practice below coincidence is the introduction of additional suppositions concerning the incident sound field. As has been explained, the most appropriate model for a diffuse sound field is the superposition of plane waves travelling in all directions. In the middle of the reverberation chamber this mathematical model is correct, but near the walls one could expect divergences from the ideal case. One way out of this difficulty is the supposition of a lack of grazing incident waves, i.e., plane waves with angles of incidence more than, say, 85°. In other words, an integration with respect to the angle of incidence from $\theta=0°$ to $\theta=85°$ in Cremer's formulation yields much better results in matching theory and practice. Again, there is a weakness in Cremer's analysis: the lack of a good representation of the sound field exciting the panel.

Gershmann[3] and Cook[4] have indicated that the correlation function in space might be a powerful quantity for characterizing the measure of diffusivity. Cook obtained a simple formula for the cross-correlation coefficient by averaging the cross-correlation coefficient for a plane wave over all angles of incidence. In the middle of a reverberation room, the experimental data fit the theoretical curve[4,5] reasonably well. In field measurements, there may be deviations from ideal diffusivity. The correlation coefficient might be the most useful parameter to describe the sound field. Hence, Cook's formula will be replaced by one that takes such deviations into account.

It is essential to change to random vibration methods to acquire a better comprehension of the important features governing transmission loss. If energy from the incident sound field has been accepted by the plate, part of it will be radiated into the media around the plate and the remainder will be dissipated into the plate. Internal friction of the material and damping at the edges of the panel are responsible for the energy dissipation in the panel. The treatment of the edges (supported or clamped) and the connection of studs and joists are uncertain factors in the analysis. The influence of damping is in general small, but for prediction of the finer details of transmission loss it is the most difficult problem to handle. Radiation efficiency is rather awkward to predict, but it is important for successful application of the random vibration method. From a reciprocity argument, the radiation efficiency is associated with the response of the structure to the incident sound field. Hence, the transmission of noise through a plate is a problem combining response to a sound field and radiation of sound.

An important phenomenon, "coincidence," provides major complications for a better understanding of transmission loss. The wavenumber of the flexural vibration in the plate is about equal to the wavenumber of the fluid medium of the environment. In this frequency region, the difference between data obtained from Cremer's theory and experimental data is still quite large. The difficulty ensues from the improper definition of the radiation efficiency in this region, because the boundary condition at the edges seems to be important. Cremer's theory, which ignores the effect of edges and the size of the panel, yields inaccurate results in this case.

The purpose of this paper is twofold. The first part will be devoted to a random vibration approach to Cremer's problem, and an attempt will be made to prove that the random vibration method, applied to a finite plate, gives approximately the same results as Cremer's analysis. This indicates that even a finite plate can in some circumstances act like an infinite plate; this is confirmed by experiment. The second part presents an analysis of the influence of the degree of diffusivity upon transmission loss. Cremer's formulation will be transformed into a more general formulation which employs the cross-correlation coefficient in the representation of the exciting pressure. The correlation coefficient was measured in a reverberation room close to the wall under test in a number of positions in order to obtain an average coefficient. The average coefficient was then employed to calculate the transmission loss of a gypsum-board wall. Finally, to extend the analysis to a greater range of room conditions, quasidiffuse sound fields were investigated with respect to their influence upon transmission loss.

I. RANDOM VIBRATION THEORY OF TRANSMISSION LOSS

A. Basic Theory of Random Vibrations

Assume a rectangular elastic plate with dimensions a and b, area A, and thickness h. Sound is incident from one side and transmitted through the plate to the other side. The differential equation governing the vibration of the panel is given by

$$B\left(\frac{\partial^4 w}{\partial x^4}+\frac{2\partial^4 w}{\partial^2 x\partial^2 y}+\frac{\partial^4 w}{\partial y^4}\right)+\rho_p\frac{\partial^2 w}{\partial t^2}+\beta\frac{\partial w}{\partial t}=p(x,y,t), \quad (1)$$

where $w(x,y)$ represents deflection, B the bending stiffness, ρ_p the mass per unit area of the plate, and β the coefficient of damping, representing acoustic radiation load and a structural damping part, and $p(x,y,t)$ is the incident sound pressure or external force at time t and position $r(x,y)$.

The normal mode approach of generalized harmonic analysis will be used to determine the response of the plate to the randomly varying loading. Small damping is assumed, so that cross-coupling of the modes from this effect can be ignored.

Consider an entirely random pressure field acting on a panel with a general mode shape $\psi_{mn}(\mathbf{r})$. The generalized force is

$$l_{mn}(t)=\int_A \psi_{mn}(\mathbf{r})p(\mathbf{r},t)d\mathbf{r}, \quad (2)$$

the integration being taken over the entire surface A of the panel. The Fourier spectrum of $l_{mn}(t)$ becomes

$$L_{mn}(\omega)=\int_{-\infty}^{+\infty}\int_A \psi_{mn}(\mathbf{r})p(\mathbf{r},t)\exp(j\omega t)d\mathbf{r}dt. \quad (3)$$

The power density spectrum of the total displacement at a point $\mathbf{r}$, including all modes, becomes[6]

$$\varphi_{ww}(\mathbf{r};\omega)=\lim_{T\to\infty}\frac{1}{2T}\sum_{m,n}\sum_{r,s}\psi_{mn}(\mathbf{r})\psi_{rs}(\mathbf{r})\times\frac{L_{mn}(\omega)L_{rs}^*(\omega)}{Z_{mn}(\omega)Z_{rs}^*(\omega)}, \quad (4)$$

where

$$Z_{mn}(\omega)=A\rho_p[(\omega^2-\omega^2{}_{mn})+2j\delta_{mn}\omega\omega_{mn}],$$

δ_{mn} being the damping factor in the m, nth mode and ω_{mn} the circular frequency of the m,nth mode. Substituting from Eq. 3, we can write Eq. 4 in the form

$$\varphi_{ww}(\mathbf{r};\omega)=\sum_{m,n}\sum_{r,s}\frac{\psi_{mn}(\mathbf{r})\psi_{rs}(\mathbf{r})}{Z_{mn}(\omega)Z_{rs}^*(\omega)}\times\int_A\int_A \psi_{mn}(\mathbf{r}_1)\psi_{mn}(\mathbf{r}_2)\phi_{pp}(\mathbf{r}_1,\mathbf{r}_2;\omega)d\mathbf{r}_1d\mathbf{r}_2, \quad (5)$$

where, if we let $t-t'=\tau$,

$$\phi_{pp}(\mathbf{r}_1,\mathbf{r}_2;\omega)=\int_{-\infty}^{+\infty}\varphi_{pp}(\mathbf{r}_1,\mathbf{r}_2,\tau)\exp(j\omega\tau)d\tau \quad (6)$$

is the cross-power density spectrum and

$$\varphi_{pp}(\mathbf{r}_1,\mathbf{r}_2,t)=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{+T}p(\mathbf{r}_1,t)p(\mathbf{r}_2,t-t')dt' \quad (7)$$

is the cross-correlation function of the pressure at $\mathbf{r}_1$ and $\mathbf{r}_2$. The integrals, with respect to the surface A in Eq. 3, represent the cross-spectral density of the generalized forces in the modes m, n and r, s and will be abbreviated to $I_{mnrs}(\omega)$. Equation 5 is essentially Powell's central result,[7] but Powell wrote the forenamed integrals in a different way, normalizing them to nondimensional factors. The function $I_{mnmn}(\omega)$ gives the effectiveness of a random forcing field in exciting the m, nth mode, and the function $I_{mnrs}(\omega)$ represents the contribution from the coupling effect between the two different modes m, n and r, s.

Equation 5 can be split into two series if the response spectral density is averaged over the surface of the plate. It is assumed that the cross-terms $m\neq r$, $n\neq s$ contribute nothing to this average because of the orthogonality condition, but that the second series with terms involving each mode represents exactly the average spectral density over the surface. In order to obtain the average mean-square velocity, the series may be multiplied by $\Delta\omega\cdot\omega^2$. Thus, if all the results are taken together,

$$\langle v^2(\omega)\rangle=\sum_{m,n}\frac{1/4\omega^2 I_{mnmn}(\omega)}{|Z_{mn}|^2}\Delta\omega. \quad (8)$$

It is possible to rewrite Eq. 8 as

$$\langle v^2(\omega)\rangle=\sum_{m,n}\langle v_{mn}^2(\omega)\rangle, \quad (8a)$$

where $\langle v_{mn}^2(\omega)\rangle$ represents the average mean-square velocity of the panel vibrating in the m, nth mode. Thus,

$$\langle v_{mn}^2(\omega)\rangle=\frac{\omega^2}{4}I_{mnmn}(\omega)\Delta\omega/|Z_{mn}|^2.$$

B. Transmission Loss of the Plate

Equation 8 provides a general formula for the average mean-square velocity of the plate when excited by a random incident sound field characterized by a correlation function $\varphi_{pp}(\mathbf{r}_1,\mathbf{r}_2;\omega)$. In order to predict the transmission loss, it is essential to determine the sound power reradiated from the panel into the medium beyond. Every normal mode produces its own radiated sound power, and the total radiation is given by a summation over all normal modes. It is implied that the cross-terms involving different modes do not contribute to the radiated sound power because of the orthogonality property of the normal modes.

The radiated sound power is formally given:

$$P_{mn}(\omega)=\frac{2\rho_a c_a}{(\pi)^2}\langle v^2_{mn}\rangle\,\mathrm{Re}\int_{\mathbf{k}_r}\frac{|S_{mn}(\mathbf{k}_r)|^2 d\mathbf{k}_r}{(k_a^2-|\mathbf{k}_r|^2)^{\frac{1}{2}}}, \quad (9)$$

when $S_{mn}(\mathbf{k}_r)$ represents the Fourier transform of the m, nth mode ψ_{mn}, which has a velocity amplitude v_{mn}. The definition of the radiation resistance is now given by

$$2R_{mn}=P_{mn}(\omega)/\langle v_{mn}^2\rangle. \quad (10)$$

The transmission loss can be considered to be the quotient of the incident power, given by

$$\frac{\frac{1}{4}A\,\varphi_{p0}(\omega)}{\rho_a c_a}\Delta\omega$$

[$\varphi_{p0}(\omega)$ is the spectral density of the forcing field at a reference point 0] and the power radiated from the panel, viz.,

$$TL=\sum_{m,n}\frac{\frac{1}{2}\omega^2 I_{mn}(\omega)R_{mn}(\omega)}{|Z_{mn}(\omega)|^2\frac{1}{4}A\,\varphi_{p0}(\omega)/\rho_a c_a}, \quad (11)$$

where, for convenience, $I_{mnmn}(\omega)$ is abbreviated to $I_{mn}(\omega)$.

Equation 11 seems to be relatively simple, but the quantities $I_{mn}(\omega)$ and $R_{mn}(\omega)$ are intricate formulae. One simplification can be made by observing that $I_{mn}(\omega)$ and $R_{mn}(\omega)$ have similar forms, apart from some constants, if $I_{mn}(\omega)$ is the quantity found for a completely diffuse sound field.[8] Complete diffusion is characterized by the correlation function,

$$\varphi_{pp}(\mathbf{r}_1,\mathbf{r}_2;\omega)=\sin(k_a|\mathbf{r}_1-\mathbf{r}_2|)/k_a|\mathbf{r}_1-\mathbf{r}_2|\cdot\varphi_{p0}(\omega). \quad (12)$$

With the aid of the Fourier transform of $\varphi_{pp}(\mathbf{r}_1,\mathbf{r}_2;\omega)$: $\Phi_{pp}(\mathbf{k}_r;\omega)$ and of the normal mode $\psi_{mn}(\mathbf{r})$: $S_{mn}(\mathbf{k}_r)$, the expression for $I_{mn}(\omega)$ is converted into its wave-number representation[9]:

$$I_{mn}(\omega)=\int_{\mathbf{k}_r}\Phi_{pp}(\mathbf{k}_r;\omega)|S_{mn}(\mathbf{k}_r;\omega)|^2 d\mathbf{k}_r. \quad (13)$$

The performance of the Fourier transform of $\varphi_{pp}(\mathbf{r}_1,\mathbf{r}_2;\omega)$ turns out to be elementary; φ_{pp} is cylindrically symmetric—i.e., the function depends only upon the distance of two points under consideration. In this case, the Fourier transform reduces to a Hankel transform[10]:

$$\Phi_{pp}(\mathbf{k}_r;\omega)=2\pi\int_0^\infty\varphi_{pp}(r;\omega)J_0(|\mathbf{k}_r|r)rdr$$
$$=2\pi\varphi_{p0}(\omega)k_a^{-1}\,\mathrm{Re}(k_a^2-|\mathbf{k}_r|^2)^{-\frac{1}{2}}. \quad (14)$$

The final result for $I_{mn}(\omega)$ is:

$$I_{mn}(\omega)=2\pi\frac{\varphi_{p0}(\omega)}{k_a}\operatorname{Re}\int_{\mathbf{k}_r}\frac{|S_{mn}(\mathbf{k}_r;\omega)|^2 d\mathbf{k}_r}{(k_a{}^2-|\mathbf{k}_r|^2)^{\frac{1}{2}}} \tag{15}$$

Comparison of Eqs. 9, 10, and 15 exhibits the close relation between $R_{mn}(\omega)$ and $I_{mn}(\omega)$. From a reciprocity principle, one may expect such results. The quantity $I_{mn}(\omega)$ indicates how well the incident sound field couples to a particular normal mode. The reciprocity principle requires that sound radiation must be governed in the same way by the variables involved in $I_{mn}(\omega)$.[11]

C. Transmission Loss (TL) of a Large Panel

From physical considerations, one can expect that transmission loss for a very large panel will approach the TL of an infinite panel. This expectation is now proved.

Assume a rectangular panel, supported at the edges. The normal modes are represented by[12]

$$\psi_{mn}(x,y)=\sin\left(\frac{\pi mx}{a}\right)\sin\left(\frac{\pi ny}{b}\right). \tag{16}$$

The square Fourier transform is given by

$$|S_{mn}(k_x,k_y)|^2=\frac{(2\pi ma)^2(2\pi nb)^2\sin^2(k_xa-m\pi/2)\sin^2(k_yb-n\pi/2)}{(k_x{}^2a^2-m^2\pi^2)^2(k_y{}^2b^2-n^2\pi^2)^2}. \tag{17}$$

On substituting this in Eq. 15, an integration with respect to k_x and k_y is carried out. It may be observed that, for large numbers of $m\pi/a$ and $n\pi/b$, $|S_{mn}(k_x,k_y)|^2$ peaks sharply around $k_x=m\pi/a$ and $k_y=n\pi/b$. If this is the case, then an application of the delta-function approximation is valid because only contributions around the critical points are significant. In this way,

$$I_{mn}(\omega)=A[\varphi_{p0}(\omega)/k_a](\pi/2)\times\operatorname{Re}(k_a{}^2-m^2\pi^2/a^2-n^2\pi^2/b^2)^{-\frac{1}{2}}, \tag{18a}$$

$$R_{mn}(\omega)=A\rho_a c_a k_a\operatorname{Re}(k_a{}^2-m^2\pi^2/a^2-n^2\pi^2/b^2)^{-\frac{1}{2}}. \tag{18b}$$

Equation 18b is exactly the representation for the radiation resistance of an infinite panel in which a flexural wave is travelling with wavenumber $[(m\pi/a)^2+(n\pi/b)^2]^{\frac{1}{2}}$.[8] The wavenumber $[(m\pi/a)^2+(n\pi/b)^2]^{\frac{1}{2}}=k_a$ marks the coincidence phenomenon.

The frequency influence function $Z_{mn}(\omega)$ needs more attention because it contains a convenient property for simplifying results. The damping factor δ_{mn} contains the radiation load and might contain the internal damping of the plate. The latter quantity will be removed to a complex Young's modulus, which implies a natural frequency ω_{mn} with an imaginary part. It develops that δ_{mn} and R_{mn} are related by the following equation:

$$\delta_{mn}=R_{mn}/\omega_{mn}\rho_p A. \tag{19}$$

This makes

$$|Z_{mn}(\omega)|^2=(A\rho_p)^2(\omega^2-\omega_{mn}{}^2)^2+4R_{mn}{}^2\omega^2. \tag{20}$$

For a very large plate, i.e., when a and b are large, one may presume that the normal modes are so close to each other that an integration instead of a summation with respect to the mode numbers is permitted. The fact that both $|Z_{mn}(\omega)|^2$ and R_{mn} contain the wavenumber of the plate in quadratic form, viz.,

$$\omega_{mn}=(B/\rho_p)^{\frac{1}{2}}[m^2\pi^2/a^2+n^2\pi^2/b^2],$$

immediately suggests an application of polar coordinates. The following step is the determination of the number of modes having resonance frequencies lying in the small frequency band between ω_{mn} and $\omega_{mn}+\Delta\omega$. The result is

$$\Delta N=(A\,\Delta\omega/4\pi)(\rho_p/B)^{\frac{1}{2}}, \tag{21}$$

which is independent of frequency.[13]

The double summation reduces itself to an integration with respect to a coordinate k_r, which is defined by

$$k_r{}^2=m^2\pi^2/a^2+n^2\pi^2/b^2. \tag{22}$$

With the aid of this coordinate transformation, we find

$$R_{mn}=\rho_a c_a A\operatorname{Re}(1-k_r{}^2/k_a{}^2)^{-\frac{1}{2}}, \tag{23a}$$

$$\omega_{mn}{}^2=Bk_r{}^4/\rho_p, \tag{23b}$$

and

$$\Delta N=(A/2\pi)k_r\Delta k_r. \tag{23c}$$

For the sake of convenience,

$$k_B{}^4=\rho_p\omega^2/B. \tag{24}$$

The quantity k_B represents the wavenumber of a free-traveling wave in an infinite plate at frequency ω.[14]

Combining these results, one obtains an integral formulation,

$$\mathrm{TL}=\int_0^{k_a}\frac{k_a{}^{-2}\frac{1}{2}\omega^2\rho_a c_a A(1-k_r{}^2/k_a{}^2)^{-\frac{1}{2}}\frac{1}{2}\pi A(1-k_r{}^2/k_a{}^2)^{-\frac{1}{2}}(A/2\pi)\varphi_{p0}(\omega)k_r dk_r}{\frac{1}{4}(\varphi_{p0}(\omega)A/\rho_a c_a)[A^2B^2(k_r{}^4-k_B{}^4)^2+4A^2\rho_a{}^2c_a{}^2(1-k_r{}^2/k_a{}^2)^{-1}\omega^2]}. \tag{25}$$

Removing all common factors in the numerator and denominator and rearranging the equation, one obtains

$$\mathrm{TL}=2\int_0^{k_a}\frac{k_a^{-2}k_r dk_r}{\left(\frac{B}{\omega}\right)^2\frac{(k_r^4-k_B^4)^2}{(2\rho_a c_a)^2}\left[1-\left(\frac{k_r^2}{k_a^2}\right)\right]+1},\qquad(26)$$

which is the same equation as has been found by Cremer[15] for free waves in an infinite panel. This indicates that Cremer's method is not very much of an oversimplification, since the random vibration method is well based on physical principles. Below-coincidence sound transmission is completely governed by the nonresonant modes. This proves that airborne sound insulation according to the "mass-law" is in fact nonresonant transmission.

Powell[16] has shown that, with certain restrictions, the results on infinite-structure and normal-mode methods must be equivalent. He pointed out the importance of damping: even a medium-sized panel can act as an infinite panel if the damping factor is large enough. A difficulty with finite panels arises from the definition of radiation resistance. The delta-function approximation, which was used to obtain the simple form for R_{mn}, holds only for a very large plate. Finite plates show radiation resistances that have a slightly different form from those represented in Eq. 23. For values of frequency where $k_a<k_r$, or below coincidence, radiation can be small and often negligible, as has been assumed so far. Useful investigations concerning the sound radiation below coincidence have been carried out by various authors, including Gösele,[17] Maidanik,[8] and Nikiforov.[18]

Sound radiation in the vicinity of $k_a\approx k_r$ provides major difficulties. Further investigation concerning the impact of the finiteness of the panel upon transmission loss would be interesting. The approach employing statistical methods and normal-mode techniques is extremely useful because it begins with a finite panel. Cremer's method does not fit into this context; it assumes an infinite panel. In the present study, the influence of the condition at the edges is negligible. Although it was assumed initially that the edges were supported, the present analysis holds also for clamped edges, since the delta-function approximation for the Fourier transform of the normal mode is valid in this case.

II. INFLUENCE OF DIFFUSIVITY ON TRANSMISSION LOSS

The foregoing analysis can be generalized by considering different field representations for the incident sound pressure. In the previous sections, ideal diffusivity represented by the correlation function (Eq. 12) was presumed. This equation can be replaced by a different formula affecting only the quantity $I_{mn}(\omega)$. The relation between R_{mn} and $I_{mn}(\omega)$ is broken up, but this is not serious, because it destroys only Cremer's simple representation of the TL. In the field, ideal diffusivity is rarely met. In the middle of the reverberation room, the ideal situation is usually approximated, but in the vicinity of the walls one expects a lack of diffusivity. In field measurements, deviations from ideal diffusivity are almost certain and must be taken into account. One of the possibilities is to convert Eq. 12 to a more general function that includes parameters for obtaining a wide variety of correlation functions. The

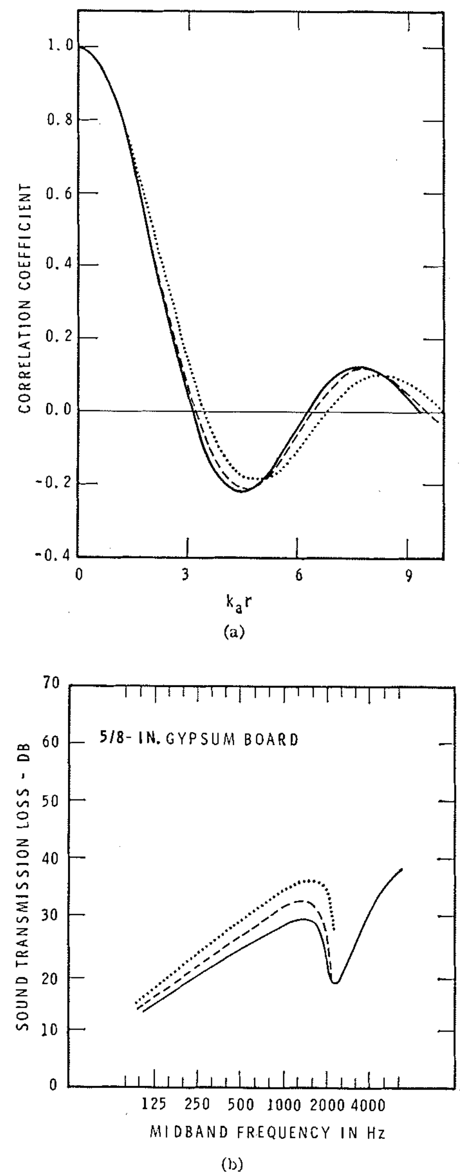

FIG. 1. (a) Correlation coefficient for semidiffuse sound fields in which the grazing incident waves have been neglected; perfectly diffuse $\theta_0=90°$: ——; $\theta_0=85°$: - - - - - $\theta_0=75°$: · · · · · ;(b) sound transmission loss for a $\frac{5}{8}$-in. gypsum-board layer for different incident sound fields of (a).

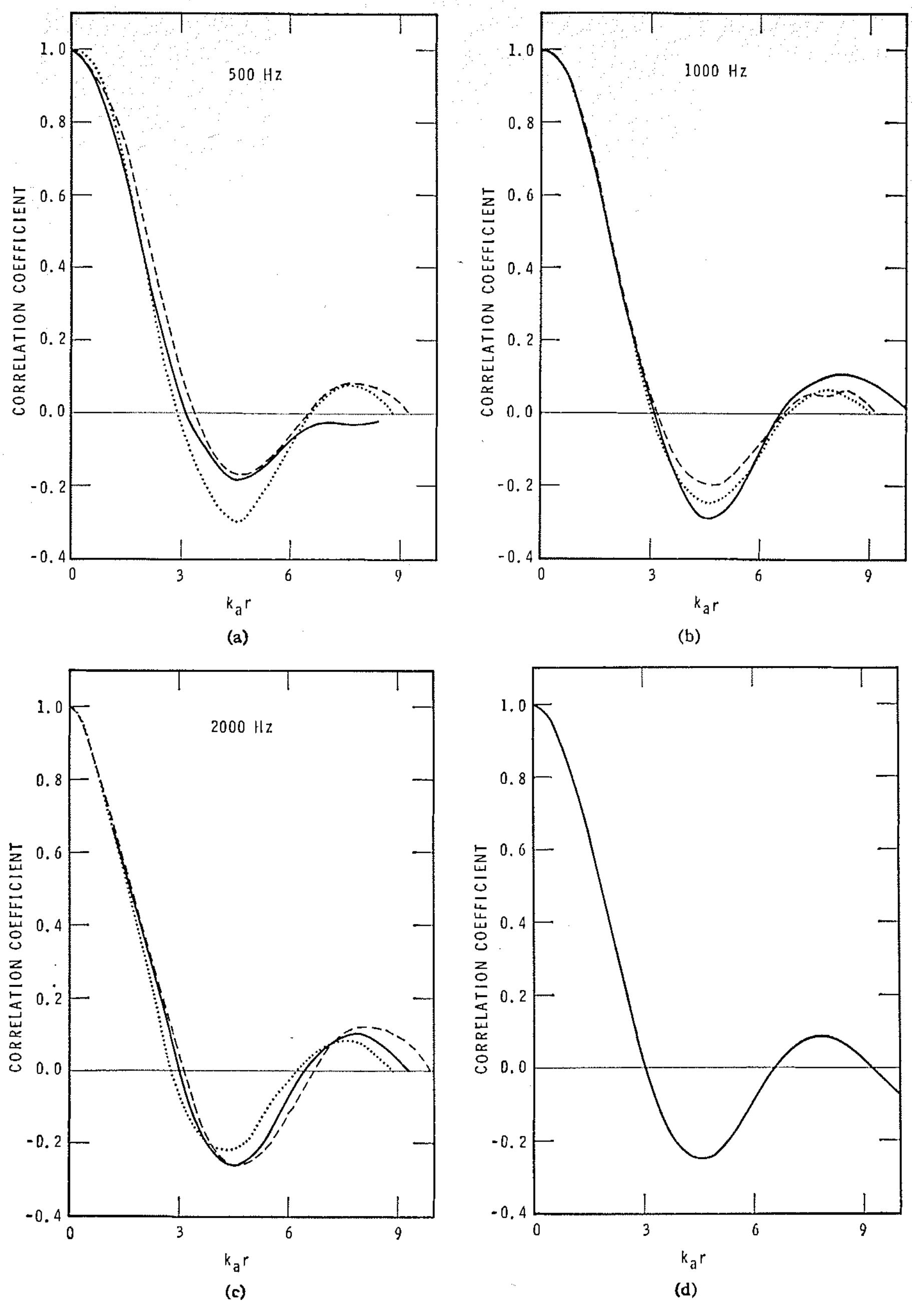

FIG. 2. (a) Three measured correlation coefficients; (b) three measured correlation coefficients; (c) three measured correlation coefficients; (d) correlation coefficient assumed to be the best match for the experimentally found coefficients. $D=0.475$, $E=0.51$, and $F=0.01$.

proposed formula is given by

$$\varphi_{pp}(\mathbf{r}_1,\mathbf{r}_2;\omega)=\varphi_{p0}(\omega)\frac{\sin(Dk_a|\mathbf{r}_1-\mathbf{r}_2|)}{Dk_a|\mathbf{r}_1-\mathbf{r}_2|}\cos(Ek_a|\mathbf{r}_1-\mathbf{r}_2|)\times\exp(-Fk_a|\mathbf{r}_1-\mathbf{r}_2|). \quad (27)$$

Equation 27 reduces to Eq. 12 if $D=1$ and $E=F=0$.

A further advantage of this proposal is its simple Hankel transform, which is essential in making up a simple equation for $I_{mn}(\omega)$. The Hankel transform of Eq. 27 is given by

$$\Phi_{pp}(\mathbf{k}_r,\omega)=\frac{\frac{1}{2}\pi}{2Dk_a}\times\varphi_{p0}(\omega)(\mathrm{Im}\{|\mathbf{k}_r|^2+k_a^2[F+j(D+E)^2]\}^{-\frac{1}{2}}\times\mathrm{Im}\{|\mathbf{k}_r|^2+k_a^2[F+j(D-E)^2]\}^{-\frac{1}{2}}). \quad (28)$$

For a very large plate, again using delta-function approximations, one obtains an analogy of Eq. 18:

$$I_{mn}(\omega)=\tfrac{1}{2}\pi A\Phi_{pp}(k_{r0};\omega), \quad (29)$$

where

$$k_{r0}=[(m\pi/a)^2+(n\pi/b)^2]^{\frac{1}{2}}.$$

Employing the method used previously to switch from a summation to an integration, one obtains an equation analogous to Eq. 26

$$\mathrm{TL}=2\int_0^{k_a}\frac{k_a^{-2}[\Phi_{pp}(k_r;\omega)/\varphi_{p0}(\omega)](1-k_r^2/k_a^2)^{\frac{1}{2}}k_r dk_r}{(B/\omega)^2[(k_B^4-k_r^4)^2/(2\rho_a c_a)^2](1-k_r^2/k_a^2)+1}. \tag{30}$$

III. DISCUSSION OF RESULTS

In order to study the effect of degree of diffusivity on transmission loss, a wall material clearly defined in its physical properties was considered. Plasterboard meets this requirement. In addition, experimental investigations have been carried out in various laboratories and plasterboard constructions represent a common component of modern buildings.

Physical properties such as elasticity and internal damping were measured with the aid of small strips set up to vibrate in flexure as cantilever bars. The resonance frequency provided the elasticity, and the vibrational amplitude at resonance provided the damping factor. The easiest way to include damping in Eq. 30 is to join the elasticity and the damping factors together to form a complex Young's modulus $B'=B(1+j\eta)$. A value for B of 3.10^9 N/m^2 was found, and for the damping factor η a value of 0.012 of critical damping was used for all frequencies. A wall 10 by 8 ft, placed between two reverberation rooms, was employed to determine transmission loss. Use of a reasonably large wall for experiment guaranteed that the nonresonance transmission of sound was predominant. The diffusivity of the sound field in the source room was investigated at a number of locations close to the wall to discover how diffusivity varies with location and eventually to obtain an average value of the cross-correlation coefficient.

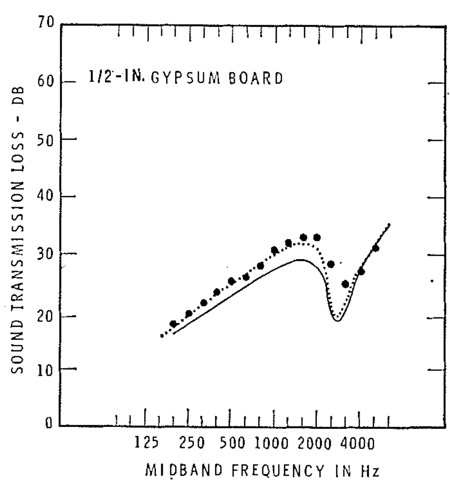

FIG. 3. Transmission loss of a $\frac{1}{2}$-in. gypsum-board layer; calculated with the aid of the measured correlation coefficient: $\cdots\cdots$; transmission loss for a perfectly diffuse sound field: ——. Points indicate measured values.

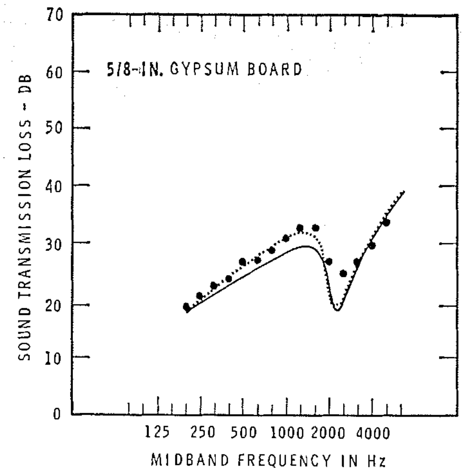

FIG. 4. Transmission loss of a $\frac{5}{8}$-in. gypsum-board layer; calculated with the aid of the measured correlation coefficients: $\cdots\cdots$; transmission loss for a perfectly diffuse sound field: ——. Points indicate measured values.

The source room was excited by an octave of pink noise, and the sound field was picked up by two $\frac{1}{4}$-in. microphones and filtered by two $\frac{1}{3}$-oct filters; the two signals from the filters were correlated by a PAR correlator. From these experimental data the parameters D, E, and F in Eq. 27 were determined and used in Eq. 30 to carry out numerical integration. Numerical evaluation of Eq. 30 was carried out by an adaptive integration method employing Simpson's rule. The transmission loss was averaged with respect to 10 frequencies in the $\frac{1}{3}$-oct frequency band under consideration.

It has been recognized that prediction of transmission loss with Cremer's original formula, averaged with respect to all angles of incidence from 0° to 90°, yields quite disappointing results in comparison with those of experiment. The device usually adopted to fit theory and experiment takes an average with respect to angles of incidence from 0° to, say, 85°, thus neglecting grazing incident waves. To illustrate this, three transmission-loss curves are shown in Fig. 1(b), in which the limiting upper angle of incidence is chosen to be 90°, 85°, and 75°, respectively. The shift of the curve to higher TL data for frequencies below coincidence is much more than would be expected from the three corresponding correlation coefficients [Fig. 1(a)]. These coefficients have been obtained by averaging the correlation coefficient for a plane wave with respect to angles of incidence from 0° to 90°, 0° to 85° and 0° to 75°, respectively. The evaluation of the integral

$$\int_0^{2\pi}d\varphi\int_0^{\theta_0}\cos(k_a r\sin\theta\cos\varphi)\sin\theta d\theta\Big/\int_0^{2\pi}d\varphi\int_0^{\theta_0}\sin\theta d\theta$$
$$=\int_0^{\theta_0}J_0(k_a r\sin\theta)\sin\theta d\theta/1-\cos\theta_0$$

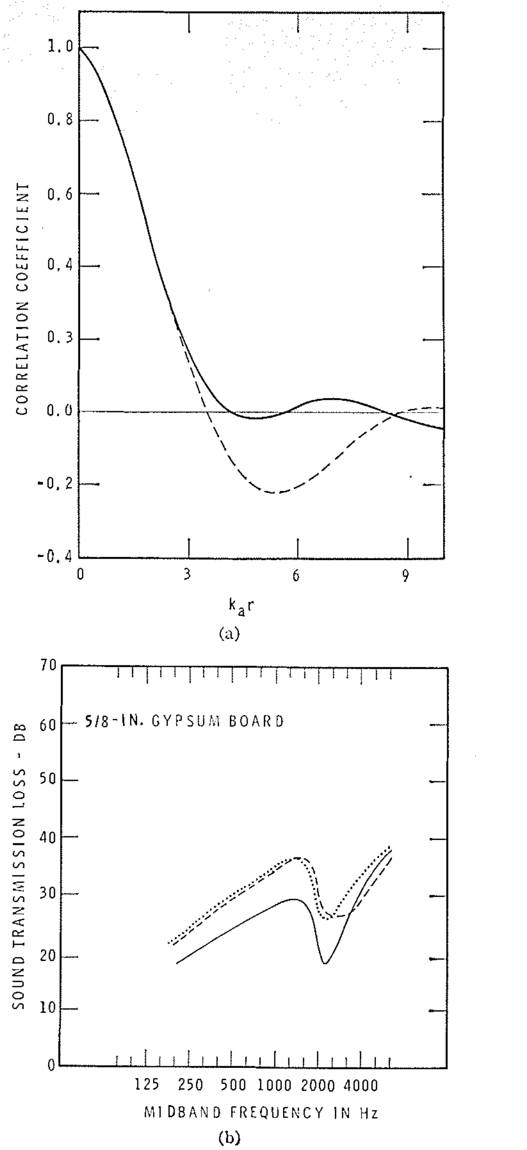

FIG. 5. (a) Correlation coefficients for two fictitious sound fields $D=0.8$; $E=0.3$; $F=0.1$: ——. $D=0.35$; $E=0.45$; $F=0.1$: - - - - - (b) Sound transmission loss of a $\frac{5}{8}$-in. gypsum-board layer, corresponding to the two sound fields, depicted in (a). Perfectly diffuse sound field: ———. $D=0.35$; $E=0.45$; $F=0.1$: - - - - -. $D=0.8$; $E=0.3$; $F=0.1$: · · · · ·.

is then required, θ_0 being 90°, 85°, and 75°, respectively.

The three curves exhibit few departures from each other; in particular, differences between $\theta_0=90°$ and $\theta_0=85°$ are hardly discernible. This confirms Dämmig's observation[19] that for high diffusivity situations the sensitivity of the correlation method in finding deficiencies in diffusivity is restricted. Only the intercepts along the $k_a r$ axis provide a fairly good indication in this matter. This indicates that the experimental investigation must be carried out very carefully because of the small departures one can expect.

Measured correlation coefficients are depicted in Figs. 2(a)–2(c), which show the results for three locations in front of the wall and three different bands of noise, with midband frequencies of 500, 1000, and 2000 Hz, respectively. The deviations from complete diffusivity are small but obvious, as one might expect, in the vicinity of the walls of a reverberation room. The shifts of the intercepts around $k_a r=\pi$ and $k_a r=2\pi$ are especially striking. The minimum around $k_a r=1.5\pi$ is deeper, and the maximum around $k_a r=2.5\pi$ is somewhat lower than it should be. This general trend holds for all curves.

It is possible to match these curves with the trial function of Eq. 27 if $D=0.475$, $E=0.51$, and $F=0.01$ are chosen. The solid line in Fig. 2(d) depicts this

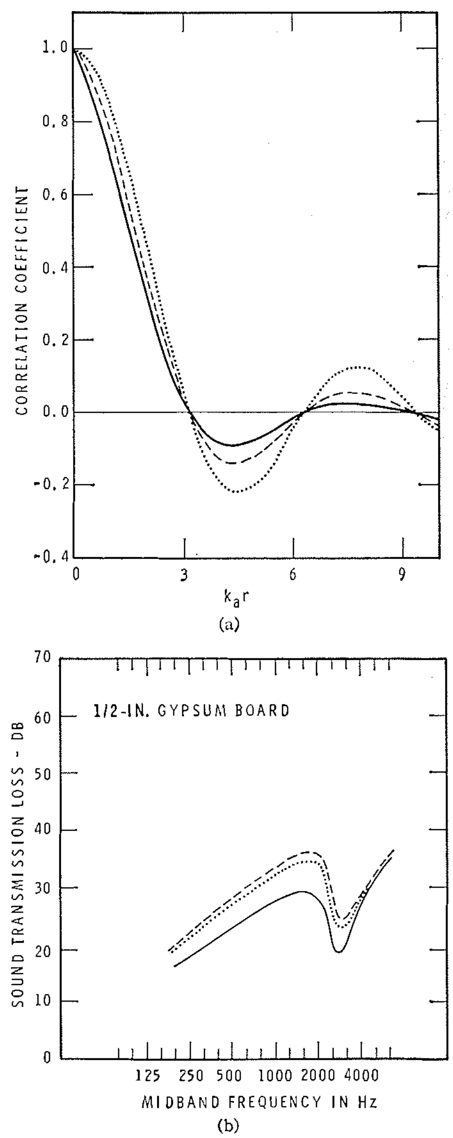

FIG. 6. (a) Correlation coefficients for three fictitious sound fields. $D=1.0$; $E=0.0$; $F=0.0$ (perfectly diffuse): · · · · ·. $D=1.0$; $E=0.0$; $F=0.1$: - - - - -. $D=1.0$; $E=0.0$; $F=0.2$: ——. (b) Sound transmission loss of a $\frac{1}{2}$-in. gypsum-board layer corresponding to the three sound fields, depicted in (a). Perfectly diffuse sound field: ——. $D=1.0$; $E=0.0$; $F=0.1$: · · ·. $D=1.0$; $E=0.0$; $F=0.2$: - - - - -.

function. The calculated transmission-loss curves employing this correlation function confirm very well the experimental data. This may be seen in Figs. 3 and 4 for plasterboard walls $\frac{1}{2}$ in. and $\frac{5}{8}$ in. thick. Below coincidence, the results agree exactly. The slight spread of the experimental data around the theoretical line is believed to be due to factors such as material impurities.

The correlation method thus opens a new way of dealing with the influence of non-ideal sound fields occurring in field measurements upon the transmission loss of actual walls in buildings. For this reason, a few fictitious sound fields, which illustrate deviation that might occur in practice, were investigated (Figs. 5 and 6). Diffusivity is quite important below coincidence, and shifts of about 5 dB can be expected in the curve. The depth of the coincidence remains about 10 dB, and its location in the frequency scale is approximately the same for the range of diffusivities considered. Above coincidence, the situation appears to be more complicated as damping is also important. Semidiffuse sound fields [Figs. 5(a) and 6(a)] have some effect upon the transmission loss [Figs. 5(b) and 6(b)]; it is difficult, however, to indicate any trend in this case. If diffusivity is fairly high, the transmission-loss data are very much the same as those found for ideal diffusivity.

In conclusion, for predominant nonresonant transmission, diffusivity is an important factor to be considered, as is the case with large plates below coincidence. For resonant transmission, diffusivity is not critical.

ACKNOWLEDGMENT

This paper is a contribution from the Division of Building Research, National Research Council of Canada, and is published with the approval of the Director of the Division.

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