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SOUND ABSORPTION IN SOLIDS

Summary

The absorption of plane sound waves in a solid body resulting from the incident wave splitting into two waves of different frequencies is considered. This process is due to the presence of cubic terms in expressions for the density of elastic energy regarded as perturbation. The effect appears to be possible (in the first approximation) only for longitudinal incident waves.

In the work of L. Landau and G. Rumer(1) the absorption of sound in a solid body is regarded as the result of collisions between sound quanta and heat quanta. Here it is found that by virtue of conservation laws, in the first approximation it is only possible to have absorption of transverse sound waves by longitudinal heat waves. The absorption of longitudinal waves is possible, according to this theory, only in the second approximation, taking into account quartic terms in expressions for the density of elastic energy and when at least four waves are involved.

Meanwhile, as we will show, if one considers that the sound quantum splits into two quanta, which is possible under nonlinear conditions, then, contrary to the opinion expressed above, by virtue of the conservation laws it is possible for the longitudinal sound quantum to split and impossible for the transverse sound quantum.

In fact the laws of the conservation of energy and momentum must hold, i.e.,

$$\underset{\sim}{k}_0 = \underset{\sim}{k}_1 + \underset{\sim}{k}_2, \quad \omega_0 = \omega_1 + \omega_2 .$$

Moreover, the theory of elasticity requires that the propagation speed of longitudinal waves c_ℓ should be greater than the propagation speed of transverse waves c_t, i.e., $c_\ell > c_t$.

Thus we have $k_o \leq k_1 + k_2$, where $k_o = |\bar{k}_o|$, $k_1 = |\bar{k}_1|$, $k_2 = |\bar{k}_2|$, and also $k_o = \omega_o/c_o$, $k_1 = \omega_1/c_1$, $k_2 = \omega_2/c_2$, hence

$$\omega_o/c_o \leq \omega_1/c_1 + \omega_2/c_2.$$

Consider the possible cases.

1. $c_o = c_\ell$, $c_1 = c_\ell$, $c_2 = c_t$. Then

$$\omega_o/c_\ell \leq \omega_1/c_\ell + \omega_2/c_t.$$

This inequality holds only when $c_\ell > c_t$.

2. $c_o = c_\ell$, $c_1 = c_t$, $c_2 = c_t$. Then

$$\omega_o/c_\ell \leq \omega_1/c_t + \omega_2/c_t.$$

This also is possible only when $c_\ell > c_t$.

Consequently the absorption of longitudinal waves occurs in the first approximation because of splitting and only in the second approximation because of absorption by heat quanta.

We will note that the transverse waves, as it is easy to see from the conservation laws, do not split in the first approximation.

In fact we will consider the possible cases of the splitting of lateral quanta:

1. $c_o = c_t$, $c_1 = c_t$, $c_2 = c_\ell$. Then

$$\omega_o/c_t \leq \omega_1/c_t + \omega_2/c_\ell.$$

2. $c_o = c_t$, $c_1 = c_\ell$, $c_2 = c_\ell$. Then

$$\omega_o/c_t \leq \omega_1/c_\ell + \omega_2/c_\ell.$$

In both cases the inequalities obtained hold only when $c_t > c_\ell$ which contradicts the elasticity theory.

Note that in this work we are completely neglecting sound dispersion and therefore we are not considering the splitting of the transverse sound quantum into two transverse quanta and the longitudinal quantum into two longitudinal quanta. These effects take place only when dispersion is taken into account, since then the quanta obtained after splitting will have a velocity somewhat different from the velocity of the initial quantum.

Strictly speaking one should take into account not only the splitting of the sound quantum but also the reverse process of reformation, i.e., the formation of one quantum from two. We neglect the second process since, at low temperatures, heat vibrations are practically not excited in a solid body.

Like the work by Landau and Rumer the present work refers to short waves and therefore it cannot be experimentally proven at the present time.

The calculation proceeded as follows:

A plane sound wave with a wave vector $\underset{\sim}{k}_0$ and a frequency ω_0 propagates in an isotropic medium. When the classical theory of elasticity is used it is impossible to have splitting. If, in the expressions for energy density, cubic terms are considered we have the terminal probability of splitting. These cubic terms in expressions for energy density we regard as perturbation causing splitting of the sound wave (the wave vector $\underset{\sim}{k}_0$ and frequency ω_0) into two sound waves (wave vectors $\underset{\sim}{k}_1$ and $\underset{\sim}{k}_2$ and frequencies ω_1 and ω_2).

We restrict ourselves to the first approximation. This means that only three waves will be considered. As shown above, in the presence of three waves it is possible to have splitting only of the longitudinal wave.

Perturbation Energy

If we denote the components of the deformation tensor by $w_{\alpha\beta}$, the density of the elastic energy accurate to terms of the fourth order is expressed by:

$$W = Aw^2_{\alpha\alpha} + Bw^2_{\alpha\beta} + P'w^3_{\alpha\alpha} + Q'w_{\alpha\alpha}w^2_{\alpha\beta} + R'w^3_{\alpha\beta}$$

where $w_{\alpha\alpha} = \mathrm{Spur}\ w_{\alpha\beta}$, $w^2_{\alpha\beta} = \mathrm{Spur}\ w_{\alpha\gamma}w_{\gamma\beta}$, $w^3_{\alpha\beta} = \mathrm{Spur}\ w_{\alpha\gamma}w_{\gamma\delta}w_{\delta\beta}$.

If we discard the cubic terms we get the usual expression for the density of elastic energy.

Coefficients A and B are connected with the velocity c_ℓ and c_t by the following ratio: $c_t = \sqrt{\frac{B}{\rho}}$, $c_\ell = \sqrt{\frac{2(A + B)}{\rho}}$ where ρ is the density. Coefficients P', Q' and R' characterize the deviation from Hooke's law.

The deformation tensor in this case has the form:

$$w_{\alpha\beta} = \frac{1}{2}\left\{\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha} + \delta_{mn}\frac{\partial u_m}{\partial x_\alpha}\cdot\frac{\partial u_n}{\partial x_\beta}\right\},$$

where u_i is the displacement of the point.

We introduce two more tensors

$$u_{\alpha\beta} = \frac{1}{2}\left\{\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha}\right\}$$

and

$$v_{\alpha\beta} = \frac{1}{2}\left\{\frac{\partial u_\alpha}{\partial x_\beta} - \frac{\partial u_\beta}{\partial x_\alpha}\right\}.$$

Expressing $w_{\alpha\beta}$ by $u_{\alpha\beta}$ and $v_{\alpha\beta}$ and substituting in the expression for energy density we get (neglecting terms of the fourth order with respect to deformation)

$$W = \{Au^2_{\alpha\alpha} + Bu^2_{\alpha\beta}\} + \{Pu^3_{\alpha\alpha} + Qu_{\alpha\alpha}u^2_{\alpha\beta} + Ru^3_{\alpha\beta} - Au_{\alpha\alpha}v^2_{\alpha\beta} - Bu_{\alpha\beta}v_{\beta\gamma}v_{\gamma\alpha}\}.$$

Terms in the second set of brackets we regard as perturbation resulting in splitting of the propagating wave.

Transition Probability

We represent sound waves in the form of the sum of harmonic waves:

$$\underset{\sim}{u}\{\underset{\sim}{r}\} = \sum \underset{\sim}{u}\{\underset{\sim}{k}_\alpha, \underset{\sim}{r}\} ,$$

$$\underset{\sim}{u}\{\underset{\sim}{k}_\alpha, \underset{\sim}{r}\} = \underset{\sim}{e}_\alpha(a_\alpha e^{i(\underset{\sim}{k}_\alpha \underset{\sim}{r})} + a_\alpha{}^* e^{-i(\underset{\sim}{k}_\alpha \underset{\sim}{r})}) ,$$

where $\underset{\sim}{e}_\alpha$ is the unit vector of the direction of polarization, a_α is the amplitude, $\underset{\sim}{k}_\alpha$ is the wave vector. Let the number of sound quanta be N_α. We want to find the probability of transition from the state with the numbers of sound quanta N_o, N_1, N_2 into states with numbers of sound quanta of $N_o - 1$, $N_1 + 1$, $N_2 + 1$.

Substituting in the expression for the density of perturbation energy

$$u_{\alpha\beta} = u^{(o)}_{\alpha\beta} + u^{(1)}_{\alpha\beta} + u^{(2)}_{\alpha\beta} , \quad v_{\alpha\beta} = v^{(o)}_{\alpha\beta} + v^{(1)}_{\alpha\beta} + v^{(2)}_{\alpha\beta} ,$$

where the indices 1 and 2 refer to waves obtained after splitting, we find

$$\begin{aligned} W_{\text{perturb.}} = {} & 6Pu^{(o)}_{\alpha\alpha} u^{(1)}_{\alpha\alpha} u^{(2)}_{\alpha\alpha} + 2Q(u^{(o)}_{\alpha\alpha} u^{(1)}_{\alpha\beta} u^{(2)}_{\alpha\beta} + u^{(1)}_{\alpha\alpha} u^{(2)}_{\alpha\beta} u^{(o)}_{\alpha\beta} + \\ & + u^{(2)}_{\alpha\alpha} u^{(o)}_{\alpha\beta} u^{(1)}_{\alpha\beta}) + 6Ru^{(1)}_{\alpha\beta} u^{(2)}_{\beta\gamma} u^{(o)}_{\gamma\alpha} + 2A(u^{(o)}_{\alpha\alpha} v^{(1)}_{\alpha\beta} v^{(2)}_{\alpha\beta} + \\ & + u^{(1)}_{\alpha\alpha} v^{(2)}_{\alpha\beta} v^{(o)}_{\alpha\beta} + u^{(2)}_{\alpha\alpha} v^{(o)}_{\alpha\beta} v^{(1)}_{\alpha\beta}) - 2B(u^{(o)}_{\alpha\beta} v^{(1)}_{\beta\gamma} v^{(2)}_{\gamma\alpha} + \\ & + u^{(1)}_{\alpha\beta} v^{(2)}_{\beta\gamma} v^{(o)}_{\gamma\alpha} + u^{(2)}_{\alpha\beta} v^{(o)}_{\beta\gamma} v^{(1)}_{\gamma\alpha}) . \end{aligned}$$

In this expression all terms are discarded which contain functions with one sign two or three times since they are of no significance in our calculation.

From the theory of harmonic oscillators we know the matrix elements of the displacement. Only the following elements are non-zero:

$$(N_\alpha - 1|a_\alpha|N_\alpha) = \sqrt{\frac{\hbar N_\alpha}{2m\omega_\alpha}}\, e^{-i\omega_\alpha t}; \quad (N_\alpha|a_\alpha^*|N_\alpha - 1) = \sqrt{\frac{\hbar N_\alpha}{2m\omega_\alpha}}\, e^{i\omega\alpha t},$$

where m is the mass of the volume V.

The matrix elements of the components $u_{\alpha\beta}$ and $v_{\alpha\beta}$ we find by differentiating the displacement component:

$$(N|u_{\alpha\beta}|N - 1) = -\frac{i}{2}\, e^{-i(\underset{\sim}{k}\,\underset{\sim}{r})} \cdot (e^\alpha k^\beta + e^\beta k^\alpha)\,(N|a^*|N - 1),$$

$$(N|u_{\alpha\beta}|N + 1) = \frac{i}{2}\, e^{i(\underset{\sim}{k}\,\underset{\sim}{r})} \cdot (e^\alpha k^\beta + e^\beta k^\alpha)\,(N|a|N + 1),$$

$$(N|v_{\alpha\beta}|N - 1) = -\frac{i}{2}\, e^{-i(\underset{\sim}{k}\,\underset{\sim}{r})} \cdot (e^\alpha k^\beta - e^\beta k^\alpha)\,(N|a^*|N - 1),$$

$$(N|v_{\alpha\beta}|N + 1) = \frac{i}{2}\, e^{i(\underset{\sim}{k}\,\underset{\sim}{r})} \cdot (e^\alpha k^\beta - e^\beta k^\alpha)\,(N|a|N + 1),$$

where e^α, k^α denote x, y, z - the components of the vectors $\underset{\sim}{e}$ and $\underset{\sim}{k}$.

For longitudinal waves $[\underset{\sim}{e}\underset{\sim}{k}] = 0$, consequently

$$(N|v_{\alpha\beta}|N - 1) = (N|v_{\alpha\beta}|N + 1) = 0.$$

For transverse waves $(\underset{\sim}{e}\underset{\sim}{k}) = 0$ consequently

$$(N|u_{\alpha\alpha}|N - 1) = (N|u_{\alpha\alpha}|N + 1) = 0.$$

Therefore in case No. 1 ($[\underset{\sim}{e}_0\underset{\sim}{k}_0] = 0$, $[\underset{\sim}{e}_1\underset{\sim}{k}_1] = 0$, $(\underset{\sim}{e}_2\underset{\sim}{k}_2) = 0$) the expression for the density of perturbation energy can be presented in the form:

$$W^1_{\text{perturb.}} = 2Q(u^{(1)}_{\alpha\alpha}\, u^{(2)}_{\alpha\beta}\, u^{(0)}_{\alpha\beta} + u^{(0)}_{\alpha\alpha}\, u^{(1)}_{\alpha\beta}\, u^{(2)}_{\alpha\beta} + 6Ru^{(1)}_{\alpha\beta}\, u^{(2)}_{\beta\gamma}\, u^{(0)}_{\gamma\alpha}.$$

In case No. 2 ($[\underset{\sim}{e}_o\underset{\sim}{k}_o] = 0$, $(\underset{\sim}{e}_1\underset{\sim}{k}_1) = 0$, $(\underset{\sim}{e}_2\underset{\sim}{k}_2) = 0$) the density of perturbation energy has the form

$$W_{\text{perturb.}} = 2Qu^{(0)}_{\alpha\alpha}\, u^{(1)}_{\alpha\beta}\, u^{(2)}_{\alpha\beta} + 6Ru^{(1)}_{\alpha\beta}\, u^{(2)}_{\beta\gamma}\, u^{(0)}_{\gamma\alpha} +$$
$$+ 2Au^{(0)}_{\alpha\alpha}\, v^{(1)}_{\alpha\beta}\, v^{(2)}_{\alpha\beta} - 2Bu^{(0)}_{\alpha\beta}\, v^{(1)}_{\beta\gamma}\, v^{(2)}_{\gamma\alpha}.$$

In further calculations we will compute the following expression:

$$q_1 = \frac{1}{8}(e_1^\alpha k_1^\alpha + e_1^\alpha k_1^\alpha)(e_2^\alpha k_2^\beta + e_2^\beta k_2^\alpha)(e_o^\alpha k_o^\beta + e_o^\beta k_o^\alpha),$$

$$q_3 = \frac{1}{8}(e_o^\alpha k_o^\alpha + e_o^\alpha k_o^\alpha)(e_1^\alpha k_1^\beta + e_1^\beta k_1^\alpha)(e_2^\alpha k_2^\beta + e_2^\beta k_2^\alpha),$$

$$r = \frac{1}{8}(e_o^\gamma k_o^\alpha + e_o^\alpha k_o^\gamma)(e_1^\alpha k_1^\beta + e_1^\beta k_1^\alpha)(e_2^\beta k_2^\gamma + e_2^\gamma k_2^\beta),$$

$$a = \frac{1}{8}(e_o^\alpha k_o^\alpha + e_o^\alpha k_o^\alpha)(e_1^\alpha k_1^\beta - e_1^\beta k_1^\alpha)(e_2^\alpha k_2^\beta - e_2^\beta k_2^\alpha),$$

$$b_1 = \frac{1}{8}(e_o^\alpha k_o^\beta + e_o^\beta k_o^\alpha)(e_1^\beta k_1^\gamma - e_1^\gamma k_1^\beta)(e_2^\gamma k_2^\alpha - e_2^\alpha k_2^\gamma).$$

For case No. 1 we get: $q_1 = k_1(\underset{\sim}{e}_2\underset{\sim}{e}_o)(\underset{\sim}{k}_2\underset{\sim}{k}_o)$, $q_3 = k_o(\underset{\sim}{k}_1\underset{\sim}{k}_2)(\underset{\sim}{e}_1\underset{\sim}{e}_2)$,

$$r = \frac{1}{2}k_o k_1\{(\underset{\sim}{e}_o\underset{\sim}{e}_1)(\underset{\sim}{e}_1\underset{\sim}{e}_2)(\underset{\sim}{k}_2\underset{\sim}{e}_o) + (\underset{\sim}{e}_o\underset{\sim}{e}_2)(\underset{\sim}{e}_o\underset{\sim}{e}_1)(\underset{\sim}{e}_1\underset{\sim}{k}_2)\}.$$

For case No. 2 we get:

$$q_3 = \frac{1}{2}k_o\{(\underset{\sim}{e}_1\underset{\sim}{e}_2)(\underset{\sim}{k}_1\underset{\sim}{k}_2) + (\underset{\sim}{e}_1\underset{\sim}{k}_2)(\underset{\sim}{k}_1\underset{\sim}{e}_2)\},$$

$$r = \frac{1}{4}k_o\{(\underset{\sim}{e}_o\underset{\sim}{e}_1)(\underset{\sim}{k}_1\underset{\sim}{e}_2)(\underset{\sim}{k}_2\underset{\sim}{e}_o) + (\underset{\sim}{e}_o\underset{\sim}{k}_2)(\underset{\sim}{e}_o\underset{\sim}{k}_1)(\underset{\sim}{e}_1\underset{\sim}{e}_2) +$$
$$+ (\underset{\sim}{e}_o\underset{\sim}{e}_2)(\underset{\sim}{e}_o\underset{\sim}{e}_1)(\underset{\sim}{k}_1\underset{\sim}{k}_2) + (\underset{\sim}{e}_o\underset{\sim}{e}_2)(\underset{\sim}{e}_o\underset{\sim}{k}_1)(\underset{\sim}{e}_1\underset{\sim}{k}_2)\}.$$

$$a = \frac{1}{2}k_o([\underset{\sim}{e}_1\underset{\sim}{k}_1]\cdot[\underset{\sim}{e}_2\underset{\sim}{k}_2]) = \frac{1}{2}k_o\{(\underset{\sim}{e}_1\underset{\sim}{e}_2)(\underset{\sim}{k}_1\underset{\sim}{k}_2) - (\underset{\sim}{e}_1\underset{\sim}{k}_2)(\underset{\sim}{k}_1\underset{\sim}{e}_2)\},$$

$$b_1 = \frac{1}{4}\{(\underset{\sim}{e}_0\underset{\sim}{e}_1)([\underset{\sim}{k}_2\underset{\sim}{e}_2]\cdot[\underset{\sim}{k}_0\underset{\sim}{k}_1]) + (\underset{\sim}{k}_0\underset{\sim}{k}_1)([\underset{\sim}{k}_2\underset{\sim}{e}_2]\cdot[\underset{\sim}{e}_1\underset{\sim}{e}_0])\}.$$

Calculating all $u_{\alpha\beta}$ and $v_{\alpha\beta}$ and substituting them in the expression for the density of perturbation energy we find:

$$W^1_{\text{perturb.}} = i[2Q(q_1 + q_3) + 6Rr]\,u^{(0)}\{\underset{\sim}{k}_0,\underset{\sim}{r}\}\,u^{(1)}\{\underset{\sim}{k}_1,\underset{\sim}{r}\}\,u^{(2)}\{\underset{\sim}{k}_2,\underset{\sim}{r}\},$$

$$W^2_{\text{perturb.}} = i[2Qq_3 + 6Rr + 2Aa - 2Bb_1]\,u^{(0)}\{\underset{\sim}{k}_0,\underset{\sim}{r}\}\,u^{(1)}\{\underset{\sim}{k}_1,\underset{\sim}{r}\}\cdot u^{(2)}\{\underset{\sim}{k}_2,\underset{\sim}{r}\}.$$

Hence we find that the matrix element we are interested in has the form

$$(N_0, N_1, N_2|W^1_{\text{perturb.}}|N_0 - 1, N_1 + 1, N_2 + 1) = i[2Q(q_1 + q_3) + 6Rr]\cdot$$
$$\cdot\sqrt{\frac{\hbar N_0}{2m\omega_0}}\cdot\sqrt{\frac{\hbar(N_1 + 1)}{2m\omega_1}}\cdot\sqrt{\frac{\hbar(N_2 + 1)}{2m\omega_2}}\cdot e^{-i[(\underset{\sim}{K}\underset{\sim}{r}) - \Omega t]},$$

$$(N_0, N_1, N_2|W^2_{\text{perturb.}}|N_0 - 1, N_1 + 1, N_2 + 1) = i[2Qq_3 + 6Rr + 2Aa -$$
$$- 2Bb_1]\cdot\sqrt{\frac{\hbar N_0}{2m\omega_0}}\cdot\sqrt{\frac{\hbar(N_1 + 1)}{2m\omega_1}}\cdot\sqrt{\frac{\hbar(N_2 + 1)}{2m\omega_2}}\cdot e^{-i[(\underset{\sim}{K}\underset{\sim}{r}) - \Omega t]},$$

where $\underset{\sim}{K} = \underset{\sim}{k}_0 - \underset{\sim}{k}_1 - \underset{\sim}{k}_2$, $\Omega = \omega_0 - \omega_1 - \omega_2$.

Integrating over the volume V we find the matrix of the perturbation energy $H_{\text{perturb.}}$.

Note that $\int e^{-i(\underset{\sim}{K}\underset{\sim}{r})}d\tau$ is non-zero only under the condition $K = 0$, i.e., only when the law of the conservation of momentum holds.

In this case we have

$$\int_V e^{-i(\underset{\sim}{K}\underset{\sim}{r})}d\tau = \int_V d\tau = V.$$

As a result of integration we find

$$(N_0, N_1, N_2|H^1_{perturb.}|N_0 - 1, N_1 + 1, N_2 + 1) = i[2Q(q_1 + q_3) +$$
$$+ 6Rr]V\sqrt{\frac{\hbar N_0}{2m\omega_0}}\sqrt{\frac{\hbar(N_1 + 1)}{2m\omega_1}}\sqrt{\frac{\hbar(N_2 + 1)}{2m\omega_2}} \cdot e^{i\Omega t},$$

$$(N_0, N_1, N_2|H^2_{perturb.}|N_0 - 1, N_1 + 1, N_2 + 1) = i[2Qq_3 + 6Rr + 2Aa -$$
$$- 2Bb_1]V\sqrt{\frac{\hbar N_0}{2m\omega_0}}\sqrt{\frac{\hbar(N_1 + 1)}{2m\omega_1}}\sqrt{\frac{\hbar(N_2 + 1)}{2m\omega_2}} \cdot e^{i\Omega t}.$$

For calculating the probability of excitation of a given state in a unit of time we have the known formula of the perturbation theory

$$\frac{d}{dt}|a_m|^2 = \frac{2\pi}{\hbar^2} \cdot \sum |a_n(0)|^2 \cdot |H_{mn}|^2\delta(\omega_{mn}),$$

where a_m is the probability amplitude.

For our case ($a_{n_0 n_1 n_2}(0) = 0$, $n_0 n_1 n_2 \neq N_0 N_1 N_2$; $a_{N_0 N_1 N_2}(0) = 1$) this formula takes on the following simpler form:

$$W_0 = \frac{d}{dt}|a_{N_0 - 1,\ N_1 + 1,\ N_2 + 1}|^2 = \frac{2\pi}{\hbar^2}|(N_0, N_1, N_2|H_{perturb.}|N_0 -$$
$$- 1, N_1 + 1, N_2 + 1)|^2\delta(\Omega).$$

W_0 gives the probability of transition in a unit of time of the longitudinal sound quantum of the frequency ω_0 into two sound quanta of the frequencies ω_1 and ω_2 of a given polarization.

To get the transition probability of a sound quantum into two quanta of given frequencies but with arbitrary polarization it is necessary to integrate the expression for W_0 over angles characterizing polarization.

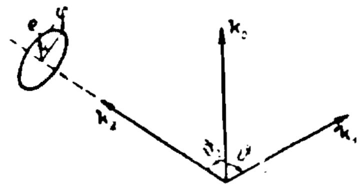

Fig. 1
Case No. 1.

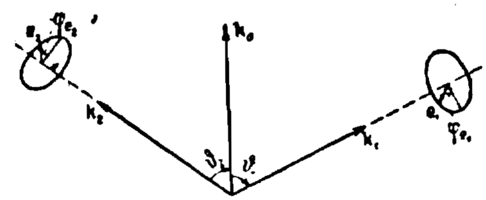

Fig. 2
Case No. 2.

We have (see Figs. 1 and 2)

$$W^{(1)} = \frac{2\pi}{\hbar^2}\,\delta(\Omega)\,\frac{\hbar^3}{(2m)^3}\cdot\frac{N_0(N_1+1)(N_2+1)}{\omega_0\omega_1\omega_2}\,V^2\int_0^{2\pi}[2Q(q_1+q_3)+6Rr]^2\,d\phi,$$

$$W^{(2)} = \frac{2\pi}{\hbar^2}\,\delta(\Omega)\,\frac{\hbar^3}{(2m)^3}\cdot\frac{N_0(N_1+1)(N_2+1)}{\omega_0\omega_1\omega_2}\,V^2\int_0^{2\pi}\int_0^{2\pi}(2Qq_3+6Rr+$$

$$2Aa-2Bb_1)^2\,d\phi_{e_1}d\phi_{e_2}\,.$$

Carrying out the integration, assuming that $N_1 = N_2 = 0$ and dividing both sides of the equation by N_0 we find, for the probability of the decomposition of a sound quantum into two quanta, the expression,

$$W^1 = \left(\frac{\pi}{2}\right)^2\frac{\hbar V^2}{m^3}\cdot\frac{k_0k_1k_2}{c_\ell^{\,2}c_t}\,\delta(\Omega)\cdot I_1(\theta_1,\theta_2),$$

$$W^2 = \left(\frac{\pi}{2}\right)^2\frac{2\pi\hbar V^2}{(2m)^3}\cdot\frac{k_0k_1k_2}{c_\ell c_t^{\,2}}\,\delta(\Omega)\cdot I_2(\theta_1,\theta_2),$$

where

$$I_1(\theta_1,\theta_2) = \{Q[\sin 2\theta_2 + \sin 2(\theta_1+\theta_2)] + 3R\cos\theta_1\sin(\theta_1+2\theta_2)\}^2,$$

$$I_2(\theta_1,\theta_2) = [(2Q+3R)\cos 2(\theta_1+\theta_2) + B + 2A]^2 +$$

$$+\,[2(Q+A)\cos(\theta_1+\theta_2) + (3R+B)\cos\theta_1\cos\theta_2]^2.$$

The number of states at which $\underset{\sim}{k}_1$ is directed towards the element of the solid angle $d\Omega_1$ and the absolute magnitude of k_1 lies in the range between k_1 and $k_1 + dk_1$, is given by the known formula

$$\rho k_1 dk_1 d\Omega_1 = \frac{V}{(2\pi)^3} k_1^2 dk_1 d\Omega_1.$$

The full required probability of splitting in a unit of time is given by the integral,

$$\bar{W} = \int W^i \cdot \rho_{ki} dk_1 d\Omega_1.$$

Carrying out this integration (taking into account the fact that k_1, k_2, θ_1, θ_2 are connected among each other by conservation laws) gives

$$\bar{W}^{(1)} = \frac{1}{256} \cdot \frac{\hbar}{c_\ell^2 c_t^2} \cdot \frac{(2Q + 3R)^2}{\rho^3} \cdot k_o^5 P\left(\frac{c_\ell}{c_t}\right),$$

$$\bar{W}^{(2)} = \frac{\pi}{64} \cdot \frac{\hbar}{c_\ell c_t^3} \cdot \frac{1}{\rho^3} \cdot k_o^5 D\left(Q, R, A, B \frac{c_\ell}{c_t}\right),$$

where

$$P\left(\frac{c_\ell}{c_t}\right) = \frac{c^2 - 1}{c^2} \{(c^2 - 1)^3(1 - 1/u) + 2(c^2 - 1)^2(3c^2 + 1)\ln 1/u -$$

$$- (c^2 - 1)(13c^4 + 6c^2 + 1)(1 - u) + 4c^4(c^2 + 1)(1 - u^2) +$$

$$+ 2/3(c^2 - 1)(7c^4 + 2c^2 + 1)(1 - u^3) +$$

$$+(-7c^6 + c^4 - c^2 - 1)(1 - u^4) + 2/5(c^2 - 1)(7c^4 + 2c^2 + 1)(1 - u^5) +$$

$$+ 4/3\, c^4(c^2 + 1)(1 - u^6) - 1/7\,(c^2 - 1)(13c^4 + 6c^2 + 1)(1 - u^7) +$$

$$+ 1/4(c^2 - 1)^2(1 + 3c^2)(1 - u^8) - 1/9(c^2 - 1)^3(1 - u^9)\},$$

$$D\left(Q,R,A,B,\frac{c_\ell}{c_t}\right) = \left[\frac{1}{240}(15c^4 - 10c^2 + 3) + \frac{(c^2 - 1)^3}{c^3}(1 + 3c^2)\ln\frac{c-1}{c+1} + \right.$$

$$\left. + 1/3 \cdot (18c^4 - 44c^2 + 32) - 2/c^2\right](2Q + 3R)^2 +$$

$$+ 1/240 \cdot (3c^4 - 10c^2 + 15)(3R + B)^2 + 1/60 \cdot (15c^4 - 50c^2 + 43)(Q + A)^2 +$$

$$+ 1/240 \cdot (15c^4 - 10c^2 + 3)(B + 2A)^2 +$$

$$+ 1/60 \cdot (5c^4 - 22c^2 + 25)(Q + A)(3R + B) +$$

$$+ 1/120 \cdot (15c^4 - 90c^2 + 83)(B + 2A)(2Q + 3R),$$

where $c = c_\ell/c_t$ and $u = (c - 1)/(c + 1)$. For $c = \sqrt{3}$ we have:

$$P(\sqrt{3}) \approx \frac{1}{2}, \quad D(Q,R,A,B,\sqrt{3}) = \frac{17}{100}(2Q + 3R)^2 + \frac{1}{20}(3R + B)^2 +$$

$$+ \frac{7}{15}(Q + A)^2 + \frac{9}{20}(B + 2A)^2 + \frac{1}{15}(Q + A)(3R + B) - \frac{13}{30}(B + 2A)(2Q + 3R).$$

Thus longitudinal sound quanta are absorbed because of splitting into longitudinal and transverse quanta and because of splitting into two lateral sound quanta. Thus the probability of the absorption of a longitudinal sound quantum owing to any splitting can be found by combining the probabilities of splitting found for cases 1 and 2.

For the value of $c = \sqrt{3}$ we find that only splitting into two transverse quanta is significant.

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Reference

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