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## **A Procedure for Deriving Thermal Transfer Functions for Walls from Hot-Box Test Results**

D.G. Stephenson, K. Ouyang and W.C. Brown

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A Procedure for Deriving Thermal Transfer Functions  
for Walls from Hot-Box Test Results: Part I

by

D.G. Stephenson, K. Ouyang, W.C. Brown

IRC/NRC

1988

Abstract

A guarded hot-box is used to determine the U-value of a wall specimen at two different mean temperatures. With the hot side temperature held constant, the cold side temperature is varied at a constant rate during the transition between the two steady-state conditions. The results of this test are used to determine three time constants of the wall, which are then used to calculate the coefficients of the z-transfer function for that wall.

Introduction

The transfer function approach for calculating heat flow through walls and ceilings has been endorsed by ASHRAE for the past 20 years, and is widely used throughout North America. The procedure requires, as data, the coefficients of the wall's thermal transfer functions, and hourly values of the temperature for the outer face of the wall. Transfer function coefficients for many walls have been published in the ASHRAE Handbook of Fundamentals (1).

Unfortunately, these data are all based on the assumption that the heat flow through these walls is one dimensional. There is no allowance for framing members that act as heat bridges through insulation, nor for such common building materials as hollow concrete blocks. Therefore, the transfer function data in the ASHRAE Handbook of Fundamentals may not accurately represent the thermal performance of real walls. Consequently there is a growing interest in being able to derive these data from the results of tests on full scale wall specimens. This paper presents a procedure for doing this. It involves using a guarded hot-box wall testing facility; the IRC facility is shown schematically in Figure 1. A test consists of three phases:

- I) An initial steady-state U-value test with the room-side environmental temperature at  $T_h$  and the climate side temperature at  $T_{ci}$ .
- II) A transition phase during which the climate chamber temperature is changed from  $T_{ci}$  to  $T_{cf}$ . The temperature is changed at a constant rate during this transition.
- III) A final steady-state U-value test with the room-side temperature at  $T_h$  and the climate side temperature at  $T_{cf}$ .

Figure 2 shows the temperature vs time curve obtained for a typical test, and the corresponding values for the heat flux into the room-side face of the test wall. These results can be used to determine three time-constants for the wall, plus their associated residues. These time-constants and residues plus the U-value are used as the data to calculate the coefficients for a rational z-transfer function that relates the heat flux through the room-side face of the wall to the temperature at the outside surface. The procedure for this latter calculation is basically

the same as is described in Stephenson and Mitalas (2). The difference is that the time-constants and residues are derived from test results rather than from the dimensions and thermal properties of the materials that make up the wall.

### Theory

The Laplace transforms of  $T_h$  and  $T_c$  are respectively  $\theta_1$  and  $\theta_2$ , and the transforms of the heat fluxes  $Q_h$  and  $Q_c$  are  $\phi_1$  and  $\phi_2$  respectively. These are related by

$$\begin{bmatrix} \theta_1 \\ \phi_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} \theta_2 \\ \phi_2 \end{bmatrix} \quad (1)$$

where the elements A, B, C and D of the transmission matrix are functions of the thermal properties and dimensions of the materials in the wall and the heat transfer resistance at the surfaces. This formulation, which is taken from Carslaw and Jaeger (3), assumes that the thermal properties of the wall are constant. Equation 1 can be recast as

$$\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} D/B & -1/B \\ 1/B & -A/B \end{bmatrix} \cdot \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \quad (2)$$

The element C has been eliminated by using the fact that  $A \cdot D - B \cdot C = 1$ . The functions  $D/B$ ,  $1/B$  and  $A/B$  are referred to as the Laplace transfer functions of the wall.

The transfer function  $1/B(s)$  can be represented as

$$\frac{1}{B(s)} = U \sum_{n=1}^{\infty} \frac{\alpha_n}{1 + \tau_n s} \quad (3)$$

where  $\tau_n$  are the time constants of the wall (i.e., the poles of  $1/B(s)$  are at  $s = -1/\tau_n$ )

$\alpha_n$  are the residues at the poles

All the transfer functions of Equation (2) have their poles at  $s = -1/\tau_n$ , since  $B(s)$  is the common denominator.

When calculating the heat flux through a wall or roof it is more convenient to use z-transforms of the temperatures and fluxes rather than the Laplace transforms. (The z-transforms are sometimes referred to as the time-series representations of these quantities). An equation similar to equation 2 relates the z-transforms of heat flux to the z-transforms of temperatures: viz,

$$Z\{Q_h\} = \frac{D(z)}{B(z)} \cdot Z\{T_h\} - \frac{1}{B(z)} \cdot Z\{T_c\} \quad (4)$$

where

$$Z\{Q_h\} = Q_{h,t} + Q_{h,t-\delta} z^{-1} + Q_{h,t-2\delta} z^{-2} + \dots$$

$$Z\{T_h\} = T_{h,t} + T_{h,t-\delta} z^{-1} + T_{h,t-2\delta} z^{-2} + \dots$$

and

$$Z\{T_c\} = T_{c,t} + T_{c,t-\delta} z^{-1} + T_{c,t-2\delta} z^{-2} + \dots$$

The z-transfer functions can be approximated by

$$\frac{D(z)}{B(z)} = \frac{U \sum_{n=0}^N a_n \cdot z^{-n}}{\sum_{n=0}^N d_n \cdot z^{-n}} \quad (5)$$

and

$$\frac{1}{B(z)} = \frac{U \sum_{n=0}^N b_n \cdot z^{-n}}{\sum_{n=0}^N d_n \cdot z^{-n}} \quad (6)$$

where  $U$  = the overall conductance of the wall

$z = e^{s\delta}$ ,

$\delta$  = the time interval between successive terms in the time series for the temperatures and heat fluxes,

$s$  = the parameter in Laplace transforms,

$N$  = the number of terms in the various sums ( $N$  may be different for each summation)

$a_n, b_n, d_n$  =  $z$ -transfer function coefficients.

Equation 4 is usually written in the time domain as

$$d_o \cdot Q_{h,t} = U \sum_{n=0}^N a_n \cdot T_{h,t-n\delta} - U \sum_{n=0}^N b_n \cdot T_{c,t-n\delta} - \sum_{n=1}^N d_n \cdot Q_{h,t-n\delta} \quad (7)$$

where  $T_{h,t}$  is the environmental temperature in the building at time  $t$ ,  
 $T_{c,t}$  is the environmental temperature outside at time  $t$ .

The approach used in this paper is to approximate  $1/B(s)$  by the sum of only three (or in some cases four) terms, and then to use these values of  $a_n$  and  $\tau_n$  to calculate  $b_n$  and  $d_n$ . When values of  $a_n$  are required (i.e., when inside temperature is changing), they are obtained from experimentally determined values of the wall's frequency response and the time constants, as described in Part II of this paper.

#### Step #1 Determination of the $\alpha_n$ and $\tau_n$ of a wall

If the start of phase II (see Figure 2) is taken as  $t = 0$ ,  $Q_1$  as the value of heat flux during the steady-state at the end of phase I, and  $Q_f$  at the steady-state in phase III, then the heat flux,  $Q_t$ , at any time  $t$  during phase II can be represented (3) by:

$$\int \left\{ \left( \frac{Q_t - Q_1}{Q_f - Q_1} \right) t^* \right\} dt = \frac{1}{s^2} \cdot \sum_{n=1}^{\infty} \frac{\alpha_n}{1 + \tau_n s} \quad t \leq t^* \quad (8)$$

where  $t^*$  is the duration of phase II. This can be inverted to give

$$Q_t = Q_1 + \left( \frac{Q_f - Q_1}{t^*} \right) (t - \Gamma) + \epsilon_t \quad 0 \leq t \leq t^* \quad (9)$$

where

$$\Gamma = \sum_{n=1}^{\infty} \alpha_n \tau_n \quad (10)$$

and

$$\epsilon_t = \frac{Q_f - Q_1}{t^*} \sum_{n=1}^{\infty} \alpha_n \tau_n e^{-t/\tau_n} \quad (11)$$

As  $t \rightarrow \infty$ ,  $\epsilon_t \rightarrow 0$

and 
$$Q_t \rightarrow Q_1 + \frac{Q_f - Q_1}{t^*} (t - \Gamma)$$

This asymptote is a straight line with a slope of  $\frac{Q_f - Q_1}{t^*}$ , and it equals  $Q_1$  when  $t = \Gamma$ . Thus the value of  $\Gamma$  can be obtained from the intersection of this asymptote and the value of  $Q_1$ .

In phase III,

$$Q_f - Q_t = \epsilon_t = \frac{Q_f - Q_1}{t^*} \sum_{n=1}^{\infty} \alpha_n \tau_n e^{-t'/\tau_n} \quad t^* \leq t \quad (12)$$

where  $t' = t - t^*$

Since  $\epsilon_t (0 \leq t \leq t^*)$  is equal to  $\epsilon_t (t^* \leq t)$ , Equation 12 can be combined with Equation 9 to give the following expression for  $\Gamma$

$$\Gamma = t - \frac{t^*}{Q_f - Q_1} (Q_t + Q_{t^*+t} - Q_1 - Q_f) \quad 0 \leq t \leq t^* \quad (13)$$

A value of  $\Gamma$  can be obtained in this way from each value of  $Q_t$ , and the corresponding  $Q_{t^*+t}$ , for  $0 \leq t \leq t^*$ .

Equation 12 indicates that as  $t'$  becomes large

$$\frac{Q_f - Q_t}{Q_f - Q_1} \rightarrow \frac{\alpha_1 \tau_1}{t^*} e^{-t'/\tau_1} \quad t^* \leq t \quad (14)$$

where  $\tau_1$  is the largest time-constant.

Thus 
$$\ln \left( \frac{Q_f - Q_t}{Q_f - Q_1} \right) \rightarrow \ln \left( \frac{\alpha_1 \tau_1}{t^*} \right) - \frac{t}{\tau_1} \quad t^* \leq t \quad (15)$$

This asymptote is a straight line with a slope of  $-\frac{1}{\tau_1}$ , and its value at  $t=0$  (i.e.,  $t = t^*$ ) is  $\ln \left( \frac{\alpha_1 \tau_1}{t^*} \right)$ . Thus the asymptote gives values for  $\tau_1$  and  $\alpha_1$ .

If equation 12 is integrated from  $t^*$  to  $\infty$  it gives

$$\int_{t^*}^{\infty} (Q_f - Q_t) dt = \left( \frac{Q_f - Q_1}{t^*} \right) \sum_{n=1}^{\infty} \alpha_n \tau_n^2$$

Thus 
$$\sum_{n=1}^{\infty} \alpha_n \tau_n^2 = \Delta \quad (16)$$

where 
$$\Delta = \frac{t^* \int_{t^*}^{\infty} (Q_f - Q_t) dt}{Q_f - Q_1} \quad (17)$$

One more relationship can be derived from the fact that

$$\left. \frac{d}{dt}(Q_t) \right|_{t=0} = 0$$

When equations 9 and 11 are differentiated they give

$$\frac{d}{dt}(Q_t) = \left( \frac{Q_f - Q_1}{t^*} \right) \left( 1 - \sum_{n=1}^{\infty} \alpha_n e^{-t/\tau_n} \right)$$

Therefore

$$\sum_{n=1}^{\infty} \alpha_n = 1 \quad (18)$$

Equations 10, 16 and 18 are used to determine the time constants and residues. However, if the transfer function  $\frac{1}{B(s)}$  is to be approximated by a finite number of terms rather than an infinite number, the sums of the various finite series must be the same as the sums of the corresponding infinite series, i.e.



$$\sum_{n=1}^N \alpha_n = 1$$

$$\sum_{n=1}^N \alpha_n \tau_n = \Gamma \quad (19)$$

$$\sum_{n=1}^N \alpha_n \tau_n^2 = \Delta$$

Values of  $\alpha_1$  and  $\tau_1$  can be derived directly from the test results using equation 15. Thus, if  $N=2$  there would be only two undetermined constants,  $\alpha_2$  and  $\tau_2$ , so it would not be possible to satisfy all three relations. On the other hand, if  $N=3$  there would be four undetermined constants, and the three relations are not sufficient to determine all four constants. One of the constants must be used as an undetermined constant to be determined by some other constraint. It is convenient to let  $\alpha_3$  be the undetermined constant.

$$\begin{aligned} \text{Let } \alpha_2 + \alpha_3 &= F = 1 - \alpha_1 \\ \alpha_2 \tau_2 + \alpha_3 \tau_3 &= G = \Gamma - \alpha_1 \tau_1 \\ \alpha_2 \tau_2^2 + \alpha_3 \tau_3^2 &= H = \Delta - \alpha_1 \tau_1^2 \end{aligned} \quad (20)$$

where  $\Gamma$  and  $\Delta$  are determined from Equation 13 and 17, respectively. These can be solved to give

$$\begin{aligned} \alpha_2 &= F - \alpha_3 \\ \tau_3 &= \frac{G + [(G^2 - FH)(-\frac{\alpha_2}{\alpha_3})]^{\frac{1}{2}}}{F} \\ \tau_2 &= \frac{G - \alpha_3 \tau_3}{\alpha_2} \end{aligned} \quad (21)$$

There is the additional constraint that  $\tau_2$  and  $\tau_3$  must be real and positive. This limits the possible range for  $\alpha_3$  as follows:

- a) if  $G^2 > FH$   
 $\alpha_3$  must have the opposite sign to  $F$

- b) if  $G^2 = FH$   
 $\alpha_3$  must be zero and  $\tau_2 = \frac{G}{F}$
- c) if  $G^2 < FH$   
 $\alpha_3$  must be between zero and  $F$ .

The best value for  $\alpha_3$  is determined by trying several values within the allowable range, and for each of these determining a consistent set of values for  $\alpha_2$ ,  $\tau_2$  and  $\tau_3$ . Then using these various sets of values a variance  $\eta_t$  is calculated as:

$$\eta_t = \frac{Q_f - Q_i}{t^*} - \sum_{n=1}^3 \{ \alpha_n \tau_n e^{-t/\tau_n} \} - \epsilon_t \quad 0 \leq t \leq t^* \quad (22)$$

and  $\epsilon_t$  is determined from experiment ( $\epsilon_t = Q_f - Q_t$  for  $t^* \leq t$ )

The best value for  $\alpha_3$  is the one that gives the smallest value for  $\int_0^\infty \eta_t^2 \cdot dt$ . This is a "least squares" curve fitting procedure that satisfies the constraints on  $Q_t$  and its first derivative.

#### Example 1

This procedure is illustrated in example 1. In this example, however, the values of  $Q_t$  have been calculated from the exact solution for a homogeneous slab rather than being experimental results. Hence, the values of  $\eta_t$  are due to the approximation of  $\frac{1}{B(s)}$ . In a real test there would also be some experimental error in the measured values that would also contribute to  $\eta_t$ .

For a homogeneous slab of the following properties

$$R = 2.0 \text{ m}^2 \cdot \text{K/W}$$

$$Lpc = 1.728 \times 10^5 \text{ J/(m}^2 \cdot \text{K)}$$

under the following test conditions,

$$T_h = 20.0^\circ\text{C}$$

$$T_{ci} = 10.0^\circ\text{C}$$

$$T_{cf} = 30.0^\circ\text{C}$$

$$t^* = 50.0 \text{ h}$$

the following values are calculated using the exact transfer function:

$$RLpc = 3.456 \times 10^5 \text{ s} = 96 \text{ h}$$

$$\Gamma = \frac{96}{6} = 16 \text{ h}$$

$$\tau_n = \frac{6}{n^2 \pi^2}$$

$$\alpha_n = 2(-1)^{n+1}$$

$$\Delta = 179.19 \text{ h}^2$$

The resulting heat flux,  $Q_t$ , and  $\epsilon_t$  are given in Table I.

Table I

t	$Q_t$	$\epsilon_t$	t	$Q_t$	$\epsilon_t$
0	15.0000	3.2000	13	15.4178	1.0178
1	15.0000	3.0000	14	15.5194	.9194
2	15.0000	2.8000	15	15.6302	.8302
3	15.0002	2.6002	16	15.7496	.7496
4	15.0014	2.4014	17	15.8767	.6767
5	15.0066	2.2066	18	16.0108	.6108
6	15.0184	2.0184	19	16.1513	.5513
7	15.0404	1.8404	20	16.2975	.4975
8	15.0734	1.6734	21	16.4490	.4490
9	15.1184	1.5184	22	16.6052	.4052
10	15.1758	1.3758	23	16.7656	.3656
11	15.2452	1.2452	24	16.9298	.3298
12	15.3260	1.1260			

For the remainder of phase II,  $Q_t$  is given by

$$Q_t = 11.800 + 0.200 \{t + 19.4536 e^{-t/9.7268}\}$$

$$\text{where } \alpha_1 = 2.000 \text{ and } \tau_1 = \frac{96}{\pi^2} = 9.7268 \text{ h}$$

(These are the values that would be indicated by the results of a perfect ramp test).

From equation 20

$$\begin{aligned} F &= 1.000 - 2.000 = -1.000 \\ G &= 16.000 - 19.4536 = -3.4536 \text{ [h]} \\ H &= 179.19 - 189.22 = -10.03 \text{ [h}^2\text{]} \\ G^2 - FH &= 1.8974 \text{ [h}^2\text{]} \end{aligned}$$

Thus  $\alpha_3$  must have the opposite sign of  $F$ , i.e. positive.

Table II gives the values of  $\alpha_2$ ,  $\tau_2$  and  $\tau_3$  that are consistent with various values of  $\alpha_3$ . The lower part of the table gives the values of  $\eta_t$  for three of the cases. These values show that the approximation gets better as  $\alpha_3$  increases, but beyond  $\alpha_3 = 3$  the improvement is very slight.

Table II

Case #	1	2	3	4	5
$\alpha_1$	2.00	2.00	2.00	2.00	2.00
$\alpha_2$	-2.00	-3.00	-4.00	-5.00	-6.00
$\alpha_3$	1.00	2.00	3.00	4.00	5.00
$\tau_1$	9.7268	9.7268	9.7268	9.7268	9.7268
$\tau_2$	2.4799	2.3293	2.2611	2.2220	2.1966
$\tau_3$	1.5062	1.7671	1.8636	1.9141	1.9452

	↓		↓		↓
t	$\eta_t$		$\eta_t$		$\eta_t$
0	.0000		.0000		.0000
	.0029		.0021		.0020
2	.0046		.0030		.0028
	.0032		.0015		.0013
4	.0010		-.0002		-.0003
	-.0009		-.0014		-.0015
6	-.0015		-.0015		-.0015
	.0020		-.0016		-.0016
8	-.0019		-.0013		-.0013
	-.0016		-.0009		-.0008
0	-.0013		-.0006		-.0005
	-.0010		-.0004		-.0003
2	-.0007		-.0002		-.0001
	-.0005		-.0001		-.0001
4	-.0004		.0000		.0000
	-.0002		.0001		.0001
6	-.0002		.0001		.0001
	-.0001		.0001		.0001
8	-.0001		.0001		.0001
	-.0001		-		-
20	-		-		-
	-		-		-
21	-		-		-
	-		-		-
22	-		-		-
	-		-		-
24	-		-		-
	-		-		-
$\Sigma \eta_t^2 = 58 \times 10^{-6}$		$\Sigma \eta_t^2 = 26 \times 10^{-6}$		$\Sigma \eta_t^2 = 23 \times 10^{-6}$	

Step #2 Determining the z-transfer function coefficients

The problem at this step is to determine the coefficients  $b_n$  and  $d_n$  that make

$$\frac{R}{B(z)} = \frac{\sum_{n=0}^N b_n z^{-n}}{\sum_{n=0}^N d_n z^{-n}}$$

equivalent to

$$\frac{R}{B(s)} = \sum_{n=1}^3 \frac{\alpha_n}{1 + \tau_n s}$$

Values of  $\alpha_n$  and  $\tau_n$  were determined in step #1. Equivalence of the two transfer functions requires that the poles of the two correspond, and that they have the same frequency response at frequencies that are of particular importance in the intended application.

The poles of the Laplace transfer function are at  $s = -1/\tau_n$ . Hence the poles of the z-transfer function must be at  $z = e^{-\delta/\tau_n}$ .

$$\text{Therefore } \sum_{n=0}^3 d_n z^{-n} = \prod_{n=1}^3 (1 - e^{-\delta/\tau_n} \cdot z^{-1}) \quad (23)$$

which gives the coefficients  $d_n$  as:

$$\begin{aligned} d_0 &= 1.0000 \\ d_1 &= - \sum_{n=1}^3 e^{-\delta/\tau_n} \\ d_2 &= \left( \prod_{n=1}^3 e^{-\delta/\tau_n} \right) \sum_{n=1}^3 e^{\delta/\tau_n} \\ d_3 &= - \prod_{n=1}^3 e^{-\delta/\tau_n} \end{aligned} \quad (24)$$

The frequency response of  $R/B|_{s=i(2\pi f)}$  can be calculated from

$$\frac{R}{B(s)} \Big|_{s=i(2\pi f)} = \sum_{n=1}^3 \frac{\alpha_n}{1 + i(2\pi f \tau_n)} = X_f + i Y_f \quad (25)$$

where

$$X_f = \sum_{n=1}^3 \frac{\alpha_n}{1+(2\pi f \tau_n)^2} \quad (26)$$

$$Y_f = - \sum_{n=1}^3 \frac{\alpha_n (2\pi f \tau_n)}{1+(2\pi f \tau_n)^2} \quad (27)$$

Let

$$\sum_{n=0}^3 d_n \cdot z^{-n} \Big|_{z=e^{j2\pi f \delta}} = V_f + j W_f \quad (28)$$

where

$$V_f = \sum_{n=0}^3 d_n \cdot \cos(2\pi n \delta f) \quad (29)$$

$$W_f = - \sum_{n=0}^3 d_n \cdot \sin(2\pi n \delta f) \quad (30)$$

Then matching the frequency response of  $R/B(z)$  to  $R/B(s)$  for a harmonic driving function with a frequency of  $f$  requires

$$\sum_{n=0}^N b_n \cos(2\pi n \delta f) = X_f V_f - Y_f W_f \quad (31)$$

and

$$- \sum_{n=0}^N b_n \sin(2\pi n \delta f) = X_f W_f + Y_f V_f \quad (32)$$

At steady-state (i.e.,  $f = 0$ )

$$X_0 = \sum_{n=1}^3 \alpha_n = 1$$

$$Y_0 = 0$$

$$V_0 = \sum_{n=0}^3 d_n$$

$$W_0 = 0$$

Therefore

$$\sum_{n=0}^N b_n = \sum_{n=0}^3 d_n \quad (33)$$

Equations 31, 32 and 33 can be solved to find three  $b_n$  coefficients.  
In matrix form

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & \cos(2\pi\delta f) & \cos(4\pi\delta f) \\ 0 & -\sin(2\pi\delta f) & -\sin(4\pi\delta f) \end{bmatrix} \cdot \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} \sum_{n=0}^3 d_n \\ X_f V_f - Y_f W_f \\ X_f W_f + Y_f V_f \end{bmatrix} \quad (34)$$

If it is desired to match the frequency response at a second frequency, there would have to be five  $b_n$  coefficients. Two more rows and two more columns would be added to the square matrix, and two more terms in the column matrix on the right side of the equation.

If the square matrix in equation 34 is  $M$ , then  $b_n$  are given by

$$\begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = [M]^{-1} \begin{bmatrix} \sum_{n=0}^3 d_n \\ X_f V_f - Y_f W_f \\ X_f W_f + Y_f V_f \end{bmatrix} \quad (35)$$

#### Example II

Taking the output of step 1 in Example I to be

$$\alpha_1 = 2.00 \quad \tau_1 = 9.7268 \text{ h}$$

$$\begin{aligned}\alpha_2 &= -4.00 & \tau_2 &= 2.2611 \text{ h} \\ \alpha_3 &= 3.00 & \tau_3 &= 1.8636 \text{ h}\end{aligned}$$

With  $\delta = 1 \text{ h}$ , and  $f = 1 \text{ cycle/24 hours}$

$$[M] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0.9659 & 0.8660 \\ 0 & -0.2588 & -0.5000 \end{bmatrix}$$

Thus

$$[M]^{-1} = \begin{bmatrix} 14.6739 & -13.6739 & 5.6639 \\ -28.3478 & 28.3478 & -7.5958 \\ 14.6739 & -14.6739 & 1.9319 \end{bmatrix}$$

And from Equation 24

$$\begin{aligned}d_0 &= 1.0000 \\ d_1 &= -2.1296 \\ d_2 &= 1.4381 \\ d_3 &= -0.3390 \\ \sum_{n=0}^3 d_n &= 0.0145\end{aligned}$$

$$\begin{aligned}X_{24} &= -0.27165 \\ Y_{24} &= -0.109306 \\ V_{24} &= -0.0123426 \\ W_{24} &= 0.0493402\end{aligned}$$

$$XV - YW = 0.008746$$

$$XW + YV = -0.012054$$

$$\begin{aligned}b_0 &= 0.02491 \\ b_1 &= -0.07155 \\ b_2 &= \underline{0.06114}\end{aligned}$$



$$\sum_{n=0}^2 b_n = 0.01450$$

Table III compares the frequency responses of  $R/B(s)$  and  $R/B(z)$  with the correct values obtained from an analytical solution for this homogeneous slab. These values show that  $R/B(z)$  matches the correct values well when  $\frac{L^2 \rho c f}{\lambda} < 4$ . But when this number is larger, the difference between  $R/B(z)$  and the correct frequency response is substantial. Whether this difference will lead to a significant error in  $Q_t$  depends on the frequency content of the driving function  $T_c$ .

$\frac{L^2 \rho c f}{\lambda}$  is a dimensionless parameter  
where  $L$  = thickness of slab  
 $\rho$  = density  
 $c$  = specific heat  
 $\lambda$  = thermal conductivity  
 $f$  = frequency

Table III

Case 3  $\frac{L^2_{pc}}{\lambda} = 96$

$\frac{L^2_{pcf}}{\lambda}$	R/B(s) = X + i Y				R/B(z) = X + i Y				"Exact"			
	X	Y	Mod	Arg	X	Y	Mod	Arg	X	Y	Mod	Arg
2	-0.0833	-0.5737	0.5797	-98.3°	-0.07976	-0.5733	0.5789	-97.9°	-0.0832	-0.5733	0.5793	-98.2°
4	-0.2716	-0.1093	0.2928	-158.1°	-0.2716	-0.1093	0.2928	-158.1°	-0.2686	-0.1078	0.2894	-158.1°
8	-0.0544	0.0941	0.1087	-240.0°	-0.0556	0.1024	0.1166	-241.5°	-0.0439	0.0834	0.0942	-242.2°
16	0.0376	0.0215	0.0433	-330.3°	0.0461	0.0357	0.0583	-322.3°	0.0167	-0.0003	0.0167	-361.2°

$$\frac{R}{B(s)} = \frac{2}{1 + 9.7268s} - \frac{4}{1 + 2.2611s} + \frac{3}{1 + 1.8632s}$$

$$\frac{R}{B(z)} = \frac{0.0249}{1.0000} - \frac{0.0716 z^{-1} + 0.0612 z^{-2}}{2.1296 z^{-1} + 1.4831 z^{-2} - 0.3390 z^{-3}}$$

Example III - Results of a Ramp Test on a Heavy Wall with Heat Bridges

A ramp test was carried out on a wall with a 127 mm layer of foam polystyrene sandwiched between two 89 mm wythes of concrete. The sample was brought to a steady-state with  $T_h = 21^\circ\text{C}$  and  $T_{ci} = -7.1^\circ\text{C}$ . Then the temperature on the cold side was reduced to  $T_{cf} = -30.0^\circ\text{C}$  at a constant rate over a period of  $t^* = 60$  hours. Table IV gives the values of temperature and total heat flow at 3-hour intervals over the 7-day test period.

The U-value of the specimen can be determined from the initial and final steady-state results. The variation of U with mean temperature can then be calculated from

$$U = \frac{Q}{\text{Area} (T_h - T_c)} = U_o + \frac{dU}{dT_m} \left( \frac{T_h + T_c}{2} \right)$$

For this test  $Q_i = 86.6$  W,  $Q_f = 156.6$  W, and the test area  $A = 5.946$  m<sup>2</sup>.

These values lead to:

$$U_o = 0.517 \text{ Wm}^{-2}\text{K}^{-1}$$

and

$$\frac{dU}{dT_m} = 1.66 \times 10^{-4} \text{ Wm}^{-2}\text{K}^{-2}$$

As  $\frac{dU}{dT_m}$  is very small it is valid to analyze the results from the transient part of the test as though the U-value were a constant.

Table IV

Results of a Ramp Test

	t	T <sub>c</sub>	Q <sub>t</sub>	Γ
	[h]	[°C]	[W]	[h]
		- 7.1	86.6	
		- 7.1	86.5	
		- 7.1	86.6	
Phase I		- 7.1	86.6	
		- 7.1	86.7	
		- 7.1	86.5	
		- 7.1	86.5	
Ramp Started at 0.0				
	0.3	- 7.3	86.6	13.4
	3.3	- 8.4	86.6	13.4
	6.3	- 9.5	87.0	13.5
	9.3	-10.7	88.2	13.5
	12.3	-11.8	89.9	13.5
	15.3	-13.0	92.1	13.5
	18.3	-14.6	94.7	13.5
	21.3	-15.3	97.6	13.5
Phase II	24.3	-16.5	100.6	13.5
	27.3	-17.6	103.8	13.5
t*=60.0h	30.3	-18.7	107.1	13.5
	33.3	-20.0	110.4	13.5
	36.3	-21.1	113.8	13.5
	39.3	-22.2	117.2	13.5
	42.3	-23.4	120.6	13.5
	45.3	-24.4	124.0	13.5
	48.3	-25.5	127.5	13.5
	51.3	-26.7	131.0	13.4
	54.3	-27.9	134.5	13.4
	57.3	-29.2	138.0	13.4
Ramp Finished at 60.0				

Table IV (cont'd)

	$t$	$T_C$	$Q_t$	$\epsilon_t$	$t' = t - t^*$
	[h]	[°C]	[W]	[W]	[h]
Phase III	60.3	-30.0	141.5	15.3	0.3
	63.3	-30.0	144.9	11.8	3.3
	66.3	-30.0	147.9	8.8	6.3
	69.3	-30.0	150.2	6.5	9.3
	72.3	-30.0	151.9	4.7	12.3
	75.3	-30.0	153.2	3.4	15.3
	78.3	-30.0	154.1	2.5	18.3
	81.3	-30.0	155.7	1.9	21.3
	84.3	-30.0	155.2	1.4	24.3
	87.3	-30.0	155.5	1.1	27.3
	90.3	-30.0	155.7	0.9	30.3
	93.3	-30.0	155.9	0.7	33.3
	96.3	-30.0	156.0	0.6	36.3
		-30.0	156.1	0.5	39.3
	102.3	-30.0	156.2	0.4	42.3
	105.3	-30.0	156.3	0.3	45.3
	108.3	-30.0	156.3	0.3	48.3
	111.3	-30.0	156.4	0.2	51.3
	114.3	-30.0	156.4	0.2	54.3
	117.3	-30.0	156.4	0.2	57.3
	120.3	-30.0	156.4	0.2	60.3
	123.3	-30.0	156.5	0.1	63.3
	126.3	-30.0	156.5	0.1	66.3
	129.3	-30.0	156.5	0.1	69.3
	132.3	-30.0	156.6		
	135.3	-30.0	156.6		
	138.3	-30.0	156.6		
	141.3	-30.0	156.6		

Values of  $\Gamma$  are given in Table IV for each value of  $Q_t$  in Phase II. These were calculated using Equation 13, i.e.

$$\Gamma = t - \frac{t^*}{Q_f - Q_i} \{Q_t + Q_{t^*+t} - Q_i - Q_f\} \quad 0 \leq t \leq t^*$$

The values in table IV show that the best value for  $\Gamma$  is 13.5 hours.

Table IV shows that  $\epsilon_t$ , has not decayed to zero when  $t'$  reaches 60.0 h (a period that equals  $t^*$ ). Thus  $\epsilon_t$  will also not be zero when  $t = t^*$  (i.e. at the end of phase II). This must be taken into account when calculating  $\epsilon_t$ , for the early part of phase III. Subtracting  $\epsilon_{t^*+t}$  from  $Q_t$  (for  $t \geq t^*$ ) corrects for the residual effect of phase II, that is

$$\epsilon_{t'} = Q_f - (Q_t - \epsilon_{t^*+t}) \quad t \geq t^*$$

For example:

$$\epsilon_{0.3} = 156.6 - (141.5 - 0.2) = 15.3$$

$$\epsilon_{3.3} = 156.6 - (144.9 - 0.1) = 11.8$$

etc.

$$\epsilon_{60.3} = 156.6 - 156.4 = 0.2$$

$$\epsilon_{63.3} = 156.6 - 156.5 = 0.1$$

The values of  $\epsilon_t$ , are then used to determine  $\alpha_1$  and  $\tau_1$ , (or as in this example  $\alpha_0$ ,  $\tau_0$ ,  $\alpha_1$  and  $\tau_1$ ). Figure 3 shows values of  $\ln \epsilon_t$ , vs  $t'$ . At large values of  $t'$  the points lie along a curve that is slightly concave upward and this curvature is due to the heat bridges, which conduct some heat through the insulation in parallel with the main heat flow path. To account for this effect, values of  $\epsilon_t$ , for  $t' \geq 12$  hours can be represented by the sum of two exponential terms:

$$\epsilon_{t'} = \frac{Q_f - Q_i}{t^*} \{ \alpha_0 \tau_0 e^{-t'/\tau_0} + \alpha_1 \tau_1 e^{-t'/\tau_1} \}$$

A regression fit of the data to this equation gives:

$$\alpha_0 = 0.056 \quad \tau_0 = 25.0 \text{ h}$$

$$\alpha_1 = 1.670 \quad \tau_1 = 8.33 \text{ h}$$

Integrating  $\epsilon_t$ , from  $t' = 12$  to  $t' = \infty$  yields a value of 57.3 W.h. The integral from  $t' = 0$  to  $t' = 12$  can be evaluated numerically. It equals 114.3 W.h. (This value was obtained using hourly values rather than the 3-hour values in Table IV.)

Thus, from Equation 17

$$\Delta = \frac{60(114.3+57.3)}{70.0} = 147.1 \text{ h}^2$$

Also from Equation 20

$$\begin{aligned} F &= 1 - 0.056 - 1.670 = -0.726 \\ G &= 13.50 - 1.40 - 13.92 = -1.82 \text{ h} \\ H &= 147.1 - 35.0 - 116.0 = -3.90 \text{ h}^2 \end{aligned}$$

$$\text{Therefore } G^2 - FH = 0.48 \text{ h}^2$$

As  $G^2 > FH$  and  $F < 0$ ,  $\alpha_2$  must be less than zero and  $\alpha_3$  must be greater than zero in order for  $\tau_2$  and  $\tau_3$  to be real and positive. Various pairs of values for  $\alpha_2$ ,  $\alpha_3$  were tried and

$$\alpha_2 = -2.00, \alpha_3 = 1.274$$

gave results in good agreement with the experimental values for the early part of the test. From Equation 21 the corresponding values for  $\tau_2$  and  $\tau_3$  are:

$$\tau_2 = 1.745 \text{ h}$$

$$\tau_3 = 1.31 \text{ h}$$

These values can be used to determine the coefficients of  $R/B(z)$  just as was done in Example II.

Table V  
Summary of Results

$n$	$\alpha_n$	$\tau_n$ [h]	$\alpha_n \tau_n$ [h]	$\alpha_n \tau_n^2$ [h <sup>2</sup> ]
0	0.056	25.0	1.40	35.0
1	1.670	8.33	13.92	116.0
2	-2.000	1.74	-3.49	-6.1
3	1.274	1.31	1.67	2.2
$\sum_{n=0}^3$	1.000		13.50	147.1

### Discussion

Figure 3 shows the importance of obtaining values of  $\epsilon_t$  over as long a period as possible. When there are some heat bridges through layers of insulation they give rise to a long time-constant with a small associated residue. This means that the wall may appear to have reached a steady-state, but the heat flux will continue to change very slowly for many hours. Thus it is good practice to maintain the constant temperature conditions for at least a day after it seems that the steady-state has been reached.

The concept of a transfer function that embodies the thermal characteristics of a wall is valid only if the thermal properties of the wall are independent of the temperature and time. The initial and final steady-state results can be used to check whether this assumption is valid. This check should be made before making a detailed analysis of the results from the transient part of the test.

The output from a ramp test should be the values of  $U_0$ ,  $\frac{dU}{dt}_m$ ,  $\tau_n$  and  $\alpha_n$ . These values will enable a user to calculate the values of  $b_n$  and  $d_n$  for any time-step. When values of  $a_n$  are required it is necessary to have values of the A/B and D/B transfer function for the frequencies of particular interest. This procedure is explained in Part II.

### Conclusion

A ramp test starting from a steady-state and ending with another steady-state yields enough information to determine three time constants of the wall, which are then used to calculate the z-transfer junction coefficient  $1/B(z)$  for the wall.

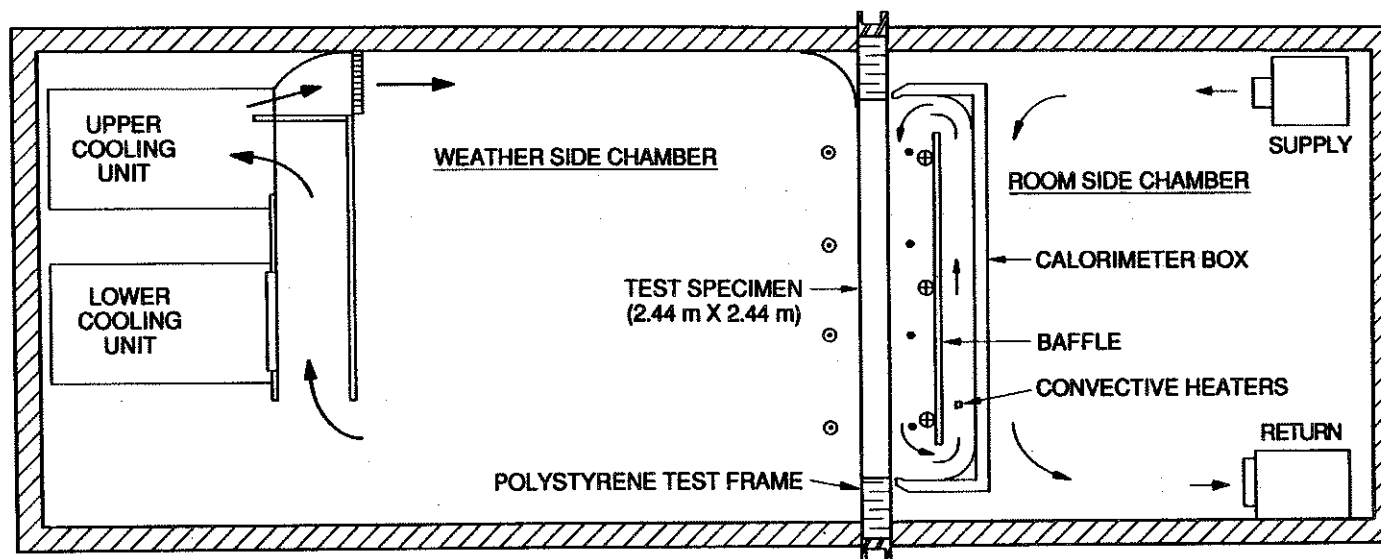
### Acknowledgement

The authors wish to express their appreciation to Mr. J. Richardson and Mr. G. Keatley for their assistance in preparing the wall specimen for test and in carrying out the ramp test. The quality of the results is due to their skill in operating this complex facility.



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- INSIDE AIR TEMPERATURE THERMOCOUPLES ( $T_h$ )
- ⊕ BAFFLE TEMPERATURE THERMOCOUPLES ( $T_b$ )
- ⊙ OUTSIDE AIR TEMPERATURE THERMOCOUPLES ( $T_c$ )

→ INDICATES AIR FLOW

FIGURE 1

NRC/IRC HOT-BOX TEST FACILITY

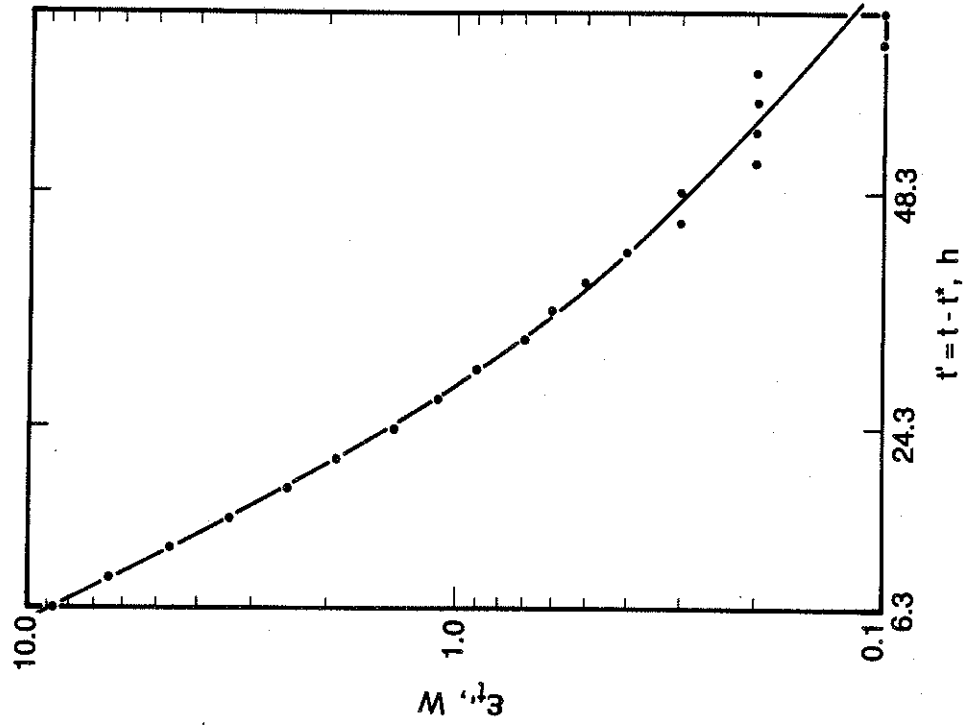


FIGURE 3  
VALUE OF  $\epsilon'$  AGAINST TIME

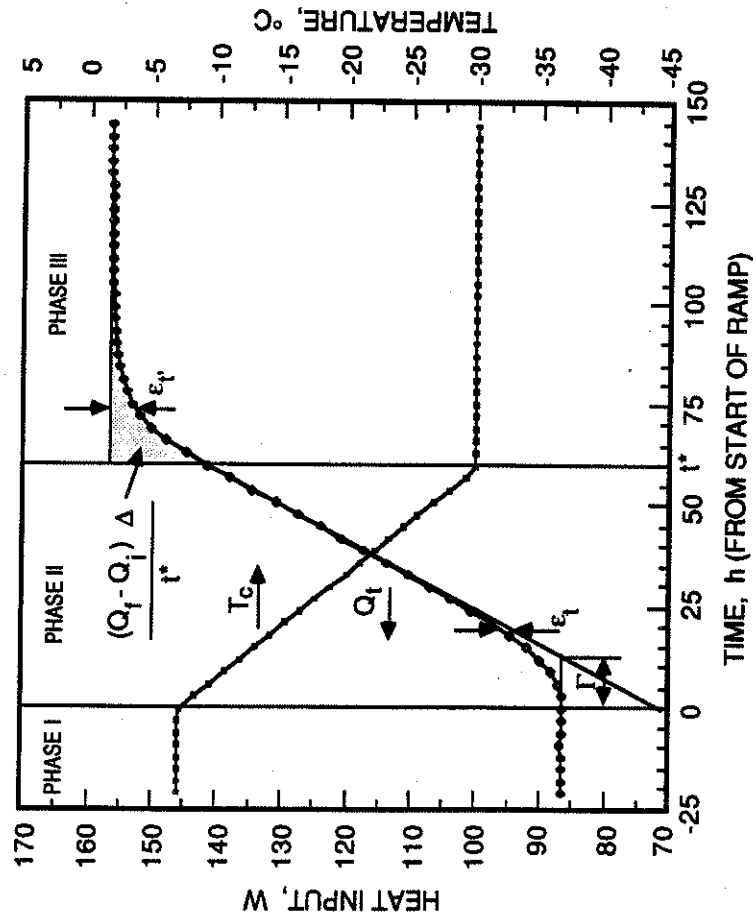


FIGURE 2  
TYPICAL TEST RESULTS

# A Procedure for Deriving Thermal Transfer Functions for Walls from Hot-Box Test Results: Part II

By

D.G. Stephenson, K. Ouyang, W.C. Brown

Part I of this paper presents a procedure for determining the time-constants (and associated residues) for a wall from the results of a hot-box wall test. This second part presents a procedure for obtaining the frequency response of a wall from the results of a second set of hot-box tests. Both the time-constants and the frequency response are required when calculating the coefficients of the z-transfer functions.

This second part of the paper also presents a procedure for the calibration, or obtaining the transfer functions, for the hot-box.

## Determining the Frequency Response of a Wall

A section of a wall is installed in a hot-box wall testing apparatus just as it would be for a U-value test. The arrangement is shown schematically in figure 1. To determine the frequency response of the  $\frac{D}{B}$  transfer function the temperature in the climate chamber is kept constant while the power to the metering box is varied sinusoidally at various frequencies. The resulting variation in the temperature in the metering box is measured at regular intervals, and from these data a value of  $\frac{D}{B} \Big|_{s=i2\pi f}$  can be derived for the particular frequency,  $f$ , used for the test.

The  $\frac{1}{B}$  transfer function can also be obtained from cyclic test results. In this case the climate chamber temperature is varied sinusoidally and the power to the metering box is kept constant.

The metering box is assumed to be a linear system with three transfer functions E, I and J. The E function allows for the heat capacity and transport lag of the heating system, and I and J account for the heat capacity of the air and baffle in the metering box and for the heat that flows into (or out of) the shell of the metering box, that is

$$E \cdot \Phi = A \cdot \Phi_w + I \cdot \Theta_1 - J \cdot \Theta_g \quad (1)$$

$\Phi$  = Laplace transform of the power,  $P$ , supplied to the metering box

$A$  = area of the test wall

$\Phi_w$  = Laplace transform of the heat flux into the test wall from the metering box, represented by:

$$\Phi_w = \left\{ \frac{D}{B} \cdot \Theta_1 - \frac{1}{B} \cdot \Theta_2 \right\} \quad (2)$$

$\Theta_1$  = Laplace transform of  $T_h$ , the temperature in the metering box

$\Theta_2$  = Laplace transform of  $T_c$ , the temperature in the climate chamber

$\Theta_g$  = Laplace transform of  $T_g$ , the temperature in the guard space surrounding the metering chamber

If  $T_g$  is constant it can be subtracted from  $T_h$  and  $T_c$ , and then  $J \cdot \Theta_g$  can be dropped from equation 1 (i.e.,  $T_g$  is the reference or zero level for the temperatures). Thus, from Equations 1 and 2

$$\frac{E}{A} \cdot \Phi = \left( \frac{D}{B} + \frac{I}{A} \right) \cdot \Theta_1 - \frac{1}{B} \cdot \Theta_2 \quad (3)$$

This is the basic relation that can be used to determine the value of  $\frac{D}{B} \Big|_{s=i\omega}$  and  $\frac{1}{B} \Big|_{s=i\omega}$  when  $\frac{E}{A}$  and  $\frac{I}{A}$  are known for  $s=i\omega$  ( $\omega = 2\pi f$  where  $f$  is the frequency). Conversely when a wall is tested that has known values of  $\frac{D}{B}$  and  $\frac{1}{B}$  the results can be used to determine  $\frac{E}{A}$  and  $\frac{I}{A}$ . This is the way the metering box is calibrated.

#### Determination of $\frac{D}{B} \Big|_{s=i\omega}$

The climate chamber temperature is kept at a constant value,  $T_c$ , and the power to the metering-box is varied sinusoidally at an angular velocity  $\omega$ , that is

$$P = \bar{P} + \tilde{P} e^{i(\omega t + \beta)} \quad (4)$$

This cyclical variation of the power produces a corresponding cyclical variation of  $T_h$

$$T_h = \bar{T}_h + \tilde{T}_h e^{i(\omega t + \gamma)} \quad (5)$$

Thus with  $T_c$  constant

$$\left( \frac{D}{B} + \frac{I}{A} \right)_{s=i\omega} = \left( \frac{E}{A} \right)_{s=i\omega} \cdot \left( \frac{\tilde{P}}{\tilde{T}_h} \right) \cdot e^{i(\beta - \gamma)} \quad (6)$$

The values of the real and imaginary parts in this complex expression depend on the value of  $\omega$ . When  $\frac{E}{A} \Big|_{s=i\omega}$  and  $\frac{I}{A} \Big|_{s=i\omega}$  are known from a calibration of the hot box (as explained later), the value of  $\frac{D}{B} \Big|_{s=i\omega}$  can be obtained from the results of a cyclical test using equation 6.

#### Example I

Data for the metering box obtained from calibration for  $f = 1$  cycle/day ( $\omega = 0.2618$  rad/h)

$$A = 5.95 \text{ [m}^2\text{]}$$

$$E = 1.00 \text{ [-0.8}^\circ\text{]}$$

$$I = 24.5 \quad \underline{54.1^\circ} \quad [W/K]$$

Test conditions and measured quantities:

$$T_c = -20.0 \quad [^\circ C]$$

$$P = 150.0 + 90.0 \quad \underline{0.0^\circ} \quad [W]$$

$$T_g = 20.0 \quad [^\circ C]$$

$$\omega = 0.2618 \quad [rad/h]$$

$$T_h = 20.0 + 1.92 \quad \underline{-54.2^\circ} \quad [^\circ C]$$

Derived quantities:

$$U = \frac{\bar{P}}{A(\bar{T}_h - \bar{T}_c)} = \frac{150.0}{5.95 (40.0)} = 0.630 \quad [W/(m^2 \cdot K)]$$

$$R = \frac{1}{U} = 1.586 \quad [m^2 \cdot K/W]$$

$$\begin{aligned} \frac{D}{B} + \frac{I}{A} &= \left( \frac{1.00 \quad \underline{-0.8}}{5.95} \right) \left( \frac{90.0 \quad \underline{0.0}}{1.92 \quad \underline{-54.2}} \right) \\ &= 7.87 \quad \underline{53.4^\circ} \\ &= 4.69 + i \quad 6.32 \quad [W/(m^2 \cdot K)] \end{aligned}$$

But  $\frac{I}{A}$  for the box is represented as

$$\frac{I}{A} = 2.41 + i \quad 3.33 \quad [W/(m^2 \cdot K)]$$

Therefore

$$\begin{aligned}\frac{D}{B} &= 2.28 + i 2.99 \\ &= 3.76 \angle 52.67^\circ \quad [W/(m^2 \cdot K)]\end{aligned}$$

Determination of  $\frac{1}{B}$   $\Big|_{s=i\omega}$

For this test the climate chamber temperature,  $T_c$ , is varied sinusoidally and the power to the metering chamber is kept constant. This causes a sinusoidal variation of  $T_h$ .  
In this case

$$T_c = \bar{T}_c + \tilde{T}_c e^{i(\omega t + \psi)}$$

and from Equation 3:

$$\left(\frac{1}{B}\right)_{s=i\omega} = \left(\frac{D}{B} + \frac{I}{A}\right)_{s=i\omega} \cdot \left(\frac{\tilde{T}_h \angle \gamma}{\tilde{T}_c \angle \psi}\right)_{s=i\omega} \quad (7)$$

### Example II

Test conditions and measured quantities:

$$\begin{aligned}T_c &= -20.0 + 10.0 \angle 0.0^\circ [^\circ C] \\ P &= 150.0 [W] \\ T_H &= 20.0 + 0.22 \angle -211.1^\circ [^\circ C] \\ T_g &= 20.0 [^\circ C] \\ \omega &= 0.2618 [\text{rad/h}]\end{aligned}$$

Derived quantities:

From the previous test

$$\frac{D}{B} + \frac{I}{A} = 7.87 \angle 53.4^\circ$$

$$\begin{aligned} \text{Thus } \frac{1}{B} &= (7.87 \angle 53.4^\circ) \left( \frac{0.22 \angle -211.1^\circ}{10.0 \angle 0.00^\circ} \right) \\ &= 0.173 \angle -157.7^\circ \text{ [W/(m}^2\text{.K)]} \end{aligned}$$

$$\begin{aligned} \text{or } \frac{R}{B} &= 0.275 \angle -157.7^\circ \\ &= -0.254 - i 0.104 \end{aligned}$$

These values for the real and imaginary parts of  $\frac{R}{B} \Big|_{s=i\omega}$  can be compared with the values derived from the time-constants and residues. It should be noted, however, that the values derived from the results of the cyclical tests may have a relatively large uncertainty because the amplitude  $\tilde{T}_h$  can be quite small. The value of  $\frac{D}{B} \Big|_{s=i\omega}$ , on the other hand, will have a much smaller probable error because of a larger magnitude of  $\tilde{T}_h$ .

#### Calibration of the Metering Box

The transfer functions  $\frac{E}{A} \Big|_{s=i\omega}$  and  $\frac{I}{A} \Big|_{s=i\omega}$  for the metering box can be determined for various frequencies in the same way as the frequency response of a wall is determined. But for this calibration the test wall must be one whose transfer functions are known. This is best obtained with a wall that is made of a homogeneous material whose thermal properties are known. In this case the transfer functions  $\frac{D}{B} \Big|_{s=i\omega}$  and  $\frac{1}{B} \Big|_{s=i\omega}$  can be derived from the dimensions and thermal property values. Thus for a calibration test  $\frac{E}{A} \Big|_{s=i\omega}$  and  $\frac{I}{A} \Big|_{s=i\omega}$  are the unknowns.

In this case the first test has P held constant while  $T_c$  is varied sinusoidally, i.e. the same test conditions as for  $\frac{1}{B}$ .

Equation 7 can be rearranged to



$$\left(\frac{D}{B} + \frac{I}{A}\right) = \left(\frac{1}{B}\right) \left( \frac{\tilde{T}_c \mid \psi}{\tilde{T}_h \mid \gamma} \right) \quad (8)$$

As  $\frac{D}{B}$  and  $\frac{1}{B}$  are known for the wall that is used for the calibration, equation 8 gives the value for  $\frac{I}{A}$ .

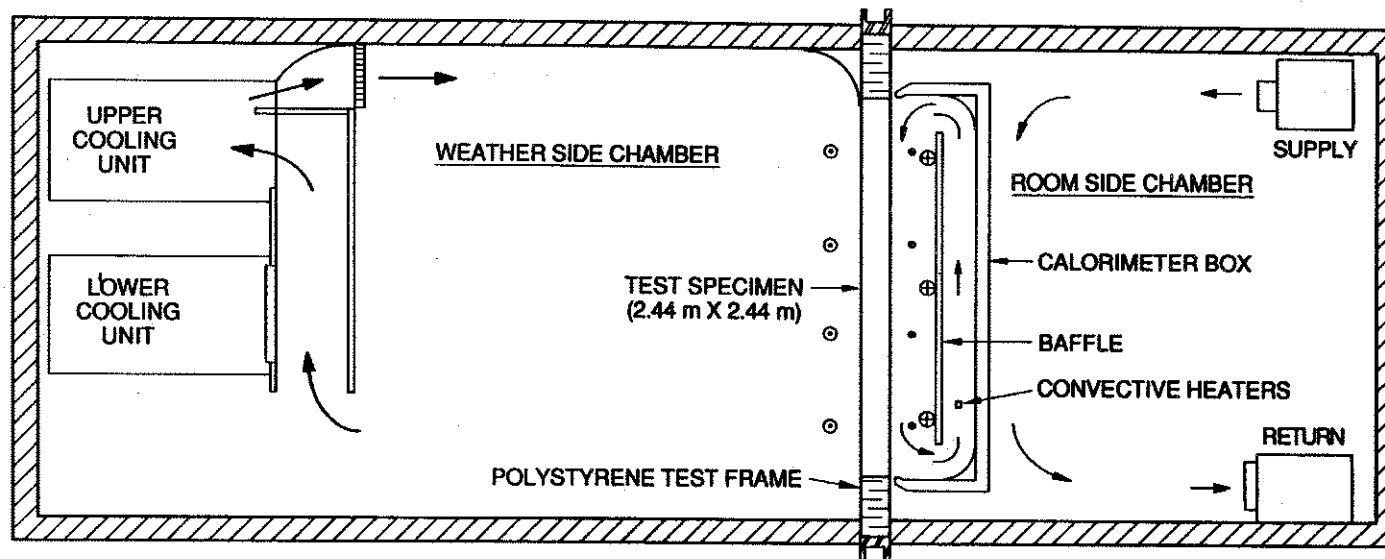
For this case a 100 mm expanded polystyrene wall was used and the values of  $\frac{I}{A}$  and  $\frac{E}{A}$  were found to be

$$\frac{I}{A} = 4.117 \mid \underline{54.1}$$

and  $\frac{E}{A} = 0.168 \mid \underline{-0.8}$

Then having determined the value of  $\frac{D}{B} + \frac{I}{A}$ , it can be used in equation 6 to obtain  $\frac{E}{A}$  as:

$$\frac{E}{A} = \left(\frac{D}{B} + \frac{I}{A}\right) \left( \frac{\tilde{T}_H \mid \gamma}{\tilde{P} \mid \beta} \right)$$



- INSIDE AIR TEMPERATURE THERMOCOUPLES ( $T_h$ )
- ⊕ BAFFLE TEMPERATURE THERMOCOUPLES ( $T_b$ )
- ⊙ OUTSIDE AIR TEMPERATURE THERMOCOUPLES ( $T_c$ )

→ INDICATES AIR FLOW

FIGURE 1

NRC/IRC HOT-BOX TEST FACILITY