Input and output impedance concepts in nonlinear am detectors

Whitford, B.G.

Publisher's version / Version de l'éditeur:
https://doi.org/10.4224/21274364
Report (National Research Council of Canada. Radio and Electrical Engineering Division : ERB), 1962-08

ERB-617

UNCLASSIFIED

NATIONAL RESEARCH COUNCIL OF CANADA

RADIO AND ELECTRICAL ENGINEERING DIVISION

INPUT AND OUTPUT IMPEDANCE CONCEPTS IN NONLINEAR AM DETECTORS

B. G. WHITFORD

OTTAWA

AUGUST 1962

ABSTRACT

Theory and experiment are combined in a study of basic principles of RF and video impedance in nonlinear detectors under small-signal and low-frequency conditions; large-signal behaviour is considered where the extension is easy. Expressions for these impedances in three elementary detector circuits containing a general nonlinear resistor are derived, with effects of load resistance, nonlinear resistor bias, and input voltage being considered. Properties of microwave crystal diodes as nonlinear resistors at low frequency are discussed. The report aims to present a more rigorous definition of RF and video impedance and of the terminology connected therewith, and generally to point out some less well-known aspects of nonlinear detectors.

<u>CONTENTS</u>

INPUT AND OUTPUT IMPEDANCE CONCEPTS IN NONLINEAR AM DETECTORS

- B.G. Whitford -

I. INTRODUCTION

Video detection with nonlinear resistance is of importance in low radio-frequency and microwave instrumentation. The literature on the subject is extensive. This includes both the writings concerned with low-frequency circuit behaviour of nonlinear resistances and those concerned directly with microwave detection with crystal diodes. From the practical standpoint, the application of nonlinear resistance to detection is of greater importance at microwave frequencies. Unfortunately the nonlinear element used, the crystal diode, is not a pure resistance at microwave frequencies and its behaviour at these frequencies cannot be readily explained in terms of its low-frequency circuit behaviour. For example, attempts have been made, but so far only qualitative agreements have been obtained to correlate the shape of the voltage-current characteristic of a crystal diode with its observed performance in microwave detectors. Even so, a considerable amount of information regarding some concepts of microwave detection may be obtained by studying the behaviour of low-frequency equivalents of microwave detectors; that is, the input frequency is assumed to be low enough so that the crystal diode may be represented by a pure nonlinear resistor devoid of any reactive components. In this manner, the complexity of the problem is reduced. Basic principles of nonlinear detector performance which apply equally well to the microwave case and the low-frequency case may thus be more easily studied.

This approach will be used here in a discussion of the concepts of the input and output impedance of a nonlinear detector circuit, commonly referred to as the RF and video impedance,respectively. Definitions of these parameters appear in the open literature, but it is felt that some of these definitions, in particular that of video impedance, have been too general and sometimes misleading. As far as is known to the writer, no papers have appeared in the open literature pointing out the restrictions governing use of these concepts, or giving a detailed account of their evolution out of the realm of linear circuit theory. It is hoped that this can be done here, and in essence this paper will be a review, but the main objective is to give a more rigorous definition to both RF impedance and video impedance.

By far the larger part of the report will be devoted to video impedance, since it is the belief of the writer that this is the more complex and the more often misunderstood of the two. Effects of input power level, nonlinear resistor bias, and detector load will be considered. Simplicity will be strived for in the exposition, and therefore the simplest means available will be used to explain a point. The analytic approach is preferred if this results in relatively simple final expressions; otherwise, experimental results are used. The rectifying element will be considered as a nonlinear resistor with no reactive components and having a voltage-current characteristic continuous throughout the operating range. The impedances will therefore always be resistances. In all analyses, the characteristic will be represented by a Taylor series in the region of interest. The waveform of the detector input signal will be restricted to a c-w sinusoid originating from a zero impedance source, and consequently the output will be a DC current or voltage. Use of AM input signals emanating from sources with internal impedances would make the treatment more realistic, but would also add to the complexity without contributing significantly to the presentation of the basic ideas.

As the nonlinear resistor is the heart of the matter, the paper will begin with a review of its properties.

II. RF RESISTANCE

1. RESISTANCE CONCEPTS APPLIED TO A CONTINUOUS NONLINEAR V-I CHARACTERISTIC

Consider the simple circuit of Fig. 1 comprised of a nonlinear resistance element R_N and a zero internal resistance DC voltage source V. The applied voltage V and resulting current I will be related by some function

$$I = f(V). \tag{1}$$

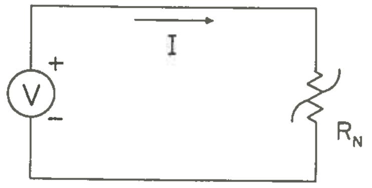

Fig. 1 Simple circuit with nonlinear resistance

A plot of this function may result in a curve such as that shown in Fig. 2. Equation (1) and also the curve in Fig. 2 are frequently referred to as the voltage-current (V-I) characteristic of the nonlinear resistor R_N. It is obvious that no simple proportionality constant relating the voltage across, and current through R_N can be defined since the proportionality is not independent of V. However, for a fixed value of applied voltage $V = V_o$, corresponding to point P in Fig. 2, three proportionality constants may be defined which constitute the possible definitions of resistance for a nonlinear resistor [1]. These are DC resistance, differential resistance, and average resistance. Together with their inverses, the corresponding conductances, they constitute six parameters which are useful in computations involving nonlinear resistors.

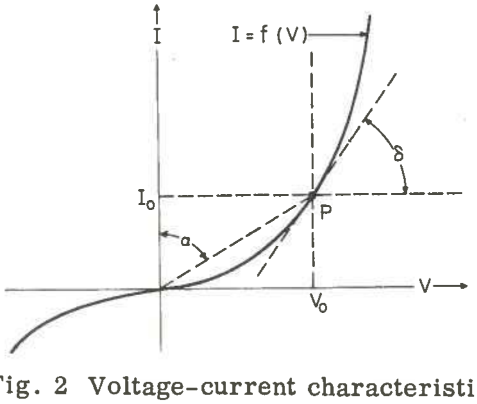

Fig. 2 Voltage-current characteristic of a hypothetical nonlinear resistor

(i) DC Resistance

This relates DC voltage applied and the resulting DC current in the resistor. It can be used only with the value of voltage for which it was defined.

$$R_{DC} = \frac{V_o}{f(V_o)} = \frac{V_o}{I_o} = \tan\alpha \ . \tag{2}$$

$$\text{DC Power Dissipated} = \frac{V_o^2}{R_{DC}} \ . \tag{3}$$

(ii) Differential Resistance

A small change in the value of applied voltage from $V = V_o$ to $V = V_o + \Delta V$ results in a resistor current change from $I = I_o = f(V_o)$ to $I = I_o + \Delta I = f(V_o + \Delta V)$. In the limit, as ΔV is made infinitesimally small, the relationship between voltage increment and current increment defines the differential resistance.

$$R_{DI} = \lim_{\Delta V \to 0} \frac{\Delta V}{\Delta I} = \frac{1}{f'(V_o)} = \cot\ \delta \ . \tag{4}$$

Since in practice voltage and current changes cannot be infinitesimally small, the proportionality constant is only approximate when dealing with finite increments. For example:

$$R_{DI} \approx \frac{\Delta V}{\Delta I} \ . \tag{5}$$

In practice, this constant is useful in relating a small AC voltage applied to the resistor and the resulting AC current when the former is superimposed on a DC voltage V_o (which may be zero). For example:

$$I_{rms} \approx \frac{E_{rms}}{R_{DI}} \ . \tag{6}$$

$$\text{AC Power Dissipated} = P_{AC} \approx \frac{E_{rms}^2}{R_{DI}} \ . \tag{7}$$

(iii) Average Resistance

If the AC voltage superimposed on a DC voltage V_o and applied to the nonlinear resistor is large, the approximations (6) and (7) become very poor and are unsuitable even for general engineering accuracy requirements. A definition for a resistance suitable for dealing with large sinusoidal AC voltages may be obtained by considering the average power dissipation in the nonlinear resistor.

The application of a sinusoidal voltage $e(t) = E_m \sin \omega t$ to the resistor will result in a periodic current $i(t)$. The average power absorbed in the nonlinear resistor is given by

$$P_{AV} = \frac{1}{T}\int_o^T e(t).i(t)\,dt, \tag{8}$$

where T is the period of the voltage waveform and is equal to $\frac{2\pi}{\omega}$. The DC power dissipation due to any bias is neglected.

If the equation (8) is evaluated by expanding i (t) into a Fourier series of harmonics, the integrals of the products of terms of unlike frequency are zero, and we are left with

$$P_{AV} = \frac{1}{T}\int_o^T (E_m \sin\omega t \,.\, I_m \sin\omega t)\,dt, \tag{9}$$

where I_m is the maximum value of the fundamental component of resistor current.

$$\text{Hence,}\quad P_{AV} = \frac{E_m I_m}{2}\,. \tag{10}$$

We wish to define a resistance R_{AV} that, in conjunction with E_m, will also enable average power to be calculated in the manner:

$$P_{AV} = \frac{E_m^2}{2R_{AV}} \tag{11}$$

If equations (10) and (11) are equated :

$$R_{AV} = \frac{E_m}{I_m}\,. \tag{12}$$

Equation (12) defines an average resistance at the operating point determined by V_o, relating the applied voltage and the fundamental component of diode current for any value of E_m, large or small. The value so defined depends on V_o and E_m. It can be easily shown that if V_o is kept constant and E_m becomes very small,

$$R_{AV} = \lim_{E_m \to 0} \frac{E_m}{I_m} = R_{DI} = \frac{1}{f'(V_o)}\,. \tag{13}$$

The constants R_{DI} and R_{AV}, as defined in (ii) and (iii), are AC resistances, generally applicable only when AC voltages are applied to the resistor. A plot of the AC resistance of a 1N23B diode as a function of RMS diode voltage is shown in Fig. 3 for 2 operating points. The curves were determined experimentally using equation (12), the fundamental component of diode current, I_m, being measured with a calibrated superheterodyne receiver.

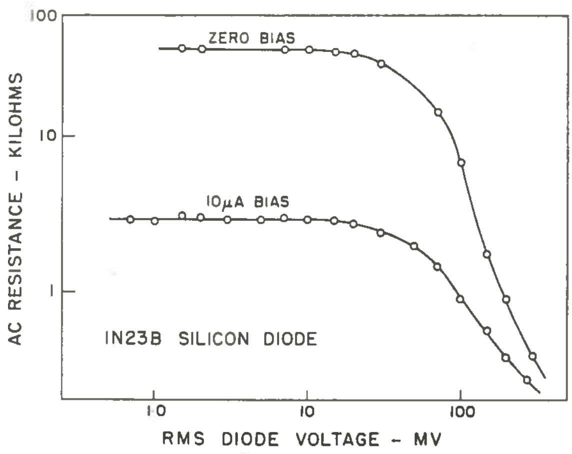

Fig. 3 AC resistance of a crystal diode as a function of applied AC voltage and bias, experimentally determined using equation (12) and an input frequency of 40 kc/s

2. RF RESISTANCE IN NONLINEAR DETECTOR CIRCUITS

RF resistance can be defined as the input resistance to a detector circuit. As such, it is an inherent property of the circuit and not of the nonlinear resistor (i.e., crystal diode), although the latter may be the major component of the circuit. The definition of RF resistance is illustrated in Fig. 4(a). The RF resistance is the resistance looking to the right of the detector input terminals AA' in Fig.4(a). In most practical forms of detector circuits where the major element is the nonlinear resistor, the circuit of Fig.4(a) may be replaced by that of Fig.4(b), as far as the AC input signal is concerned. The detector is replaced by a nonlinear resistor R_D whose V-I characteristic may be

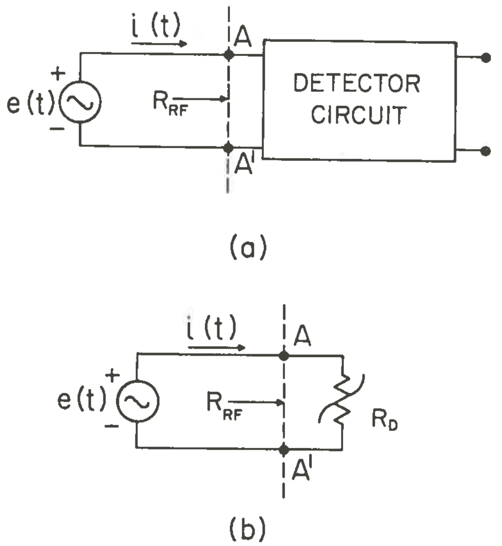

Fig. 4 Definition of RF resistance, R_{RF}

the same as, or different from that of the rectifying element in the circuit, R_N. A detector input signal $e(t) = E_m \sin \omega t$ now sees, as the detector input resistance (RF resistance) at the terminals AA', a nonlinear resistor R_D. The RF resistance can now be defined as the AC resistance of the nonlinear resistor R_D in the manner discussed in the previous section. In general, for a sinusoidal input of any amplitude

$$R_{RF} = \frac{E_m}{I_m} , \tag{14}$$

where E_m is the maximum value of the sinusoidal input voltage at AA' and I_m is the maximum value of the fundamental component of current at AA' .

If the input signal is very small,

$$R_{RF} \approx \frac{1}{F'(V_o)} , \tag{15}$$

where I = F' (V) is the derivative of the V-I characteristic of the nonlinear resistor R_D and V_o is the DC voltage determining the operating point. I = F(V) cannot always be directly determined by the usual procedure of applying a variable DC voltage and measuring the resulting current at the terminals AA' . In some practical detectors, the input resistance would be close to an open circuit for DC voltages (approximately equal to the load resistance when this is high). However, it can always be determined by AC methods or by altering the detector circuit for the measurement so that the R_D characteristic can be measured by the usual point by point method. However, R_{RF} can be determined most rapidly for large or small RF inputs by making use of equation (14). The average RF power absorbed by the detector with a c-w input is

$$P_{AV} = \frac{E_m^2}{2R_{RF}} . \tag{16}$$

(i) Simple Detector with Short-circuited Output

A very elementary form of detector using a nonlinear resistance element is shown in Fig.5. The detector input terminals are AA' and the output terminals, BB' .

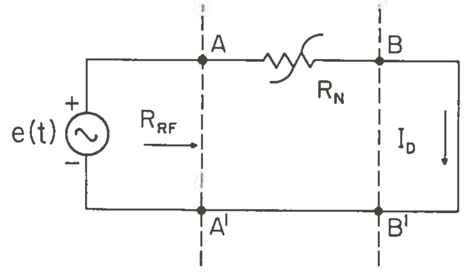

Fig. 5 Simple detector using a nonlinear resistor and zero load resistance

The load is a short circuit. For simplicity, the input is a c-w RF signal and the output is a DC current. The RF resistance of this simple detector is equal to the AC resistance of the nonlinear element at the operating point V_o (bias is not shown in Fig.5). For small RF signals,

$$R_{RF} \approx \frac{1}{f'(V_o)} . \tag{17}$$

(ii) Simple Loaded Detector

The form of this elementary detector is shown in Fig.6. It is similar to the previous case except for the load resistance R_L connected to the output terminals.

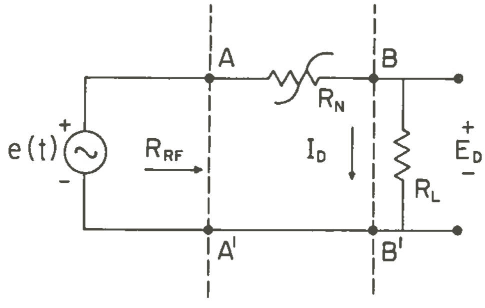

Fig. 6 Simple loaded detector

This alters the behaviour somewhat. The output is now a DC voltage E_D (excluding bias) developed across R_L. The AC voltage appearing across the nonlinear element R_N is no longer sinusoidal and is superimposed on a DC component equal to $-E_D$. In general, the RF input resistance of the detector is given by equation (14). However, with small signals and a bias V_o,

$$R_{RF} \approx R_L + \frac{1}{f'(V_o - E_D)} , \qquad (18)$$

and since E_D is small if the input is small

$$R_{RF} \approx R_L + \frac{1}{f'(V_o)} , \qquad (19)$$

and with small input and zero bias,

$$R_{RF} \approx R_L + \frac{1}{f'(0)} . \qquad (20)$$

(iii) Detector with RF Bypass Capacity

This type of detector, shown in Fig. 7, resembles very closely what is actually used in practice when the load resistance is very high. We will assume that the reactance of C at the RF frequency used is negligible relative to the AC resistance of R_N. Therefore, the voltage across R_N is the input voltage e(t), superimposed on a DC voltage, $-E_D$ (where $-E_D$ is the DC voltage due to signal only). The RF resistance is the AC resistance of the nonlinear element R_N at the operating point $V_o - E_D$. If the input is small,

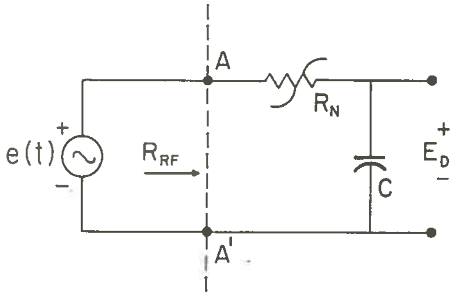

Fig. 7 Detector with capacity output (RF bypass) and infinite load resistance

$$R_{RF} \approx \frac{1}{f'(V_o - E_D)} \quad . \tag{21}$$

A further, coarser, approximation may be obtained by assuming E_D to be very small. Then

$$R_{RF} \approx \frac{1}{f'(V_o)} \quad . \tag{22}$$

Because of the practical importance of this latter case, it is worth while to discuss it further with reference to the characteristics of a nonlinear element actually used in practice, the microwave crystal diode. A survey of the literature on this subject shows that some inconsistencies exist in the explanations offered for the observed behaviour. On a crystal diode V-I characteristic, operation with a capacity output is frequently shown as in Fig. 8(a) and explained as in the following quotation [2]:

> "Using the opposite extreme, an open-circuited load resistance, the video capacitance becomes charged to the peak R-F voltage, after many cycles, and the crystal's conduction becomes zero provided the crystal back resistance is infinite."

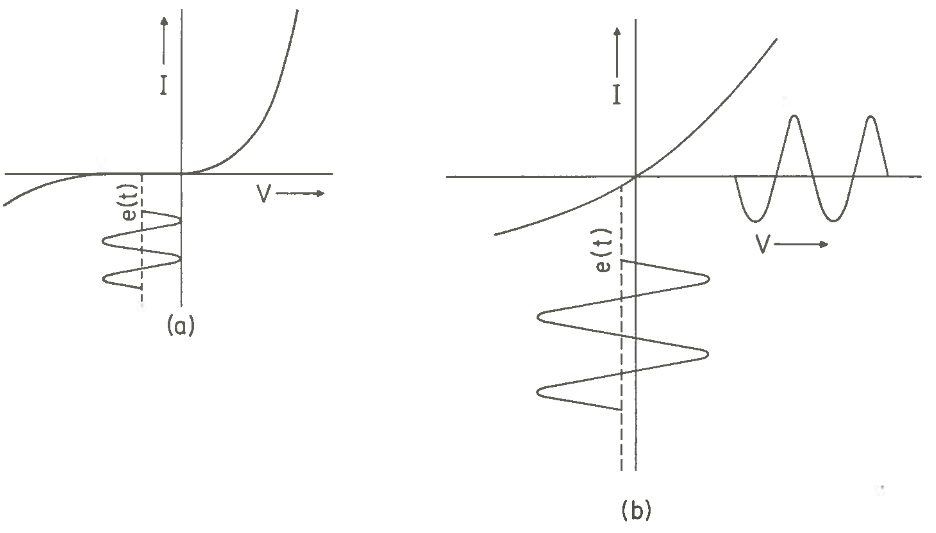

Fig. 8 Operation of detector with capacity output: (a) very large signal, (b) small signal. The bias $V_o = 0$.

The statement is true, but is helpful only in understanding very large signal behaviour. It sheds no light on the practical small-signal case (i.e., in detector applications) where the back resistance is almost equal to the forward resistance, the crystal conducts throughout the RF cycle, the RF input resistance is not infinite, and the crystal absorbs RF power. The practical low-level case is more closely approximated by Fig. 8(b). Forward and reverse conduction are almost equal and operation is almost identical to the short-circuit load case (i), except that the crystal now operates with a small, negative, self-generated, DC voltage bias. This moves the operation to a point of slightly higher value of average resistance, thus decreasing the fundamental frequency component of current. The voltage across the diode is almost sinusoidal. Hence, the effect of replacing the short circuit (as in Fig. 5) by a capacity (as in Fig. 7) is a small increase in RF resistance. For small inputs there is negligible change in RF resistance as the load is changed from short circuit to open circuit, if the output capacity (often called RF bypass capacity) is large enough to satisfy $\left|\frac{1}{j\omega C}\right| << R_{AV}$. With large amplitude inputs, the difference in RF resistance between the open and short circuit cases for a particular diode depends on E_D. The manner in which E_D varies and its dependence on the various circuit parameters will be discussed in detail in a later

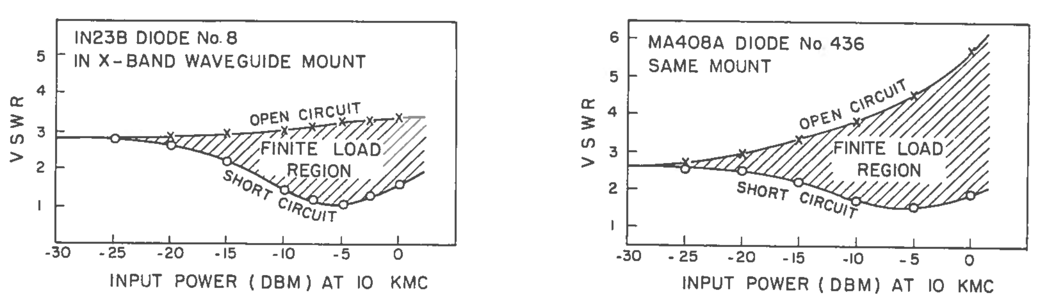

Fig. 9 Effect of self-bias on the input impedance of one X-band microwave detector

section. The effect of E_D on the input impedance of one particular microwave detector is illustrated in Fig. 9. It is seen that the effect is dependent on the type of diode used in the detector. On open circuit a bias voltage $-E_D$ exists across the diode, while on short circuit the diode bias is zero. As one would expect, the effect of E_D on RF resistance becomes less and less significant as the input power decreases, since E_D decreases with decreasing input. With a load R_L connected to the output terminals, the input impedance would lie somewhere in between the open and short circuit curves, and it appears that a load can be chosen so that the input impedance would remain practically constant over a wide range of input power variation.

III. VIDEO RESISTANCE

1. INTRODUCTION

The terms "DC impedance" and "video resistance" appear to have originated in the MIT Radiation Laboratory, and are first mentioned by Beringer in 1943 [3] and 1944 [4], respectively. In his earlier report only the term "DC impedance" is used and is defined as "the dynamic resistance for the crystal" obtained by connecting the crystal into a DC circuit (consisting of a resistance and current meter in series) and, with RF power applied, determining the ratio of the differential change in crystal voltage to the differential change in circuit current, due to a differential change in DC circuit resistance. In the later report Beringer introduced the term "crystal's video resistance" and uses this synonomously with "crystal's DC resistance". A number of points are not made clear in the 1943 report (originally defining video resistance) which are essential to a proper understanding of the video resistance concept. Firstly, neither the method of connecting the crystal in the detector circuit nor the manner in which RF power is applied are mentioned. Secondly, although it is now generally agreed that the crystal voltage and circuit current, referred to above, are the DC components of the total voltage and current, this is not stated explicitly. Thirdly, Beringer states that empirical results show the DC impedance as defined above to be equal to the slope of the crystal's volt-ampere characteristic at the operating point; however, the presence of an RF bypass capacity in the detector circuit which, it will be shown later, is necessary for the above to be true, is not referred to. Lastly, it is not made clear whether there is some significance to calling it an "impedance" in one case and a "resistance" in another; nor is there any justification shown for use of the adjective "DC", for, if anything, the inverse slope of a nonlinear characteristic at the operating point is more of the nature of, and is more generally known, as an AC resistance [1].

The main objective of the introduction of this concept of video resistance appears to have been to define a resistance suitable for use in the representation of the detector, as seen by the load, as a voltage generator in series with an internal resistance or, as a current generator in shunt with this resistance. From this point of view, the video resistance assumes the role of a "source resistance" or "output resistance" of the detector. It is believed that this is the point of view Beringer had in mind. It is unfortunate then, that the video resistance was defined by him as a "dynamic resistance for the crystal" and again in slightly different form by Torrey and Whitmer [5] as a "dynamic output resistance for the crystal", rather than as a dynamic output resistance of the detector in which the crystal is only one component. As will be shown later, the video resistance as the effective output resistance of the detector is dependent on the detector circuit configuration. In general, it is not a property of the crystal alone.

Recent applications of detectors in nanosecond microwave pulse work have increased the importance of video resistance. The problem here is to minimize output pulse rise time. It is believed that rise time is dependent on video resistance [2, 6], generally decreasing as the resistance decreases owing to increasing power level and bias [2]. However, the exact relationship between video resistance and rise time is not entirely clear, particularly at the higher RF power levels (greater than -30 dbm) usually used in laboratory work with short pulses. In fact, the meaning of video resistance under conditions of high power input (if it has any meaning here at all) is itself vague. Falconer [2] measures a video resistance at power levels as high as +3 dbm by noting the value of load resistance required to decrease the detector output by one-half from its near open-circuit value; but he does not justify the technique as resulting in a value for video resistance which is more meaningful than that obtained by, for example, measuring the ratio of open-circuit voltage to short-circuit current. These two different techniques, though giving identical results in linear circuit source resistance measurements, and similar results in certain types of nonlinear detectors at low RF input levels, would yield different results here. In other words, at high-level inputs the superposition principle does not apply.

In attempting to correlate detector output pulse rise time and video resistance, particularly under varying conditions of crystal bias and input power, it became apparent that the foundation for a successful theory, as exemplified by the preceding remarks, was insufficient. As a result, the entire concept of video resistance in nonlinear detectors was reviewed and in the process certain limitations of the concept became apparent which, it is believed, are not generally known. This section of the report attempts to relate these findings and to give a more rigourous definition of video resistance.

2. THE SOURCE RESISTANCE CONCEPT IN LINEAR CIRCUITS

In linear systems, most practical voltage sources have an internal resistance R_g which makes the output voltage E_o less than the internal generated voltage E_g by an amount depending on the current I delivered to a load resistance R_L. These relationships are shown in Fig. 10. Knowledge of the internal resistance, or as it is often called, "source resistance" or "output resistance" is important as it enables the determination of:

a) load resistance $R_L = R'_L$ required for maximum power transfer;

b) regulation, or change in E_o due to a change in R_L;

c) the change in I due to a change in E_g;

d) the form of the transient in E_o due to a transient in E_g;

e) the thermal noise voltage appearing across the load;

f) an equivalent circuit for the source if its internal structure is not exactly known, or simplification is desired.

Many relationships exist which can be considered as defining R_g. If E_R is the voltage drop across R_g, some of these are:

a) $$R_g = \frac{E_R}{I} \, , \tag{23}$$

b) $$R_g = \frac{dE_R}{dI} \, , \qquad R_g = \frac{\Delta E_R}{\Delta I} \, . \tag{24}$$

The quantities dE_R and ΔE_R are differential and measurable increments, respectively, in E_R, and are due to a change in R_L with E_g constant or a change in E_g with R_L constant.

c) $$R_g = -\frac{dE_o}{dI} \, , \quad R_g = -\frac{\Delta E_o}{\Delta I} \, . \tag{25}$$

The change in E_o and I is again the result of a change in R_L with E_g constant or a change in E_g with R_L constant.

d) $$R_g = \frac{E_o \text{ (open circuit)}}{I \text{ (short circuit)}} \, . \tag{26}$$

E_g is kept constant and R_L changed from an open to a short circuit.

e) $$R_g = R'_L \, . \tag{27}$$

R'_L is the value of load giving maximum power transfer.

f) $$R_g = \frac{E_g}{I} \, . \tag{28}$$

This is the result of setting $R_L = 0$.

g) $$R_g = \frac{R_{L_2}(1 - E_{01}/E_{02})}{E_{02}/E_{01} - R_{L_2}/R_{L_1}} \tag{29}$$

This is based on a change in E_o from E_{01} to E_{02} when the load is changed from R_{L_1} to R_{L_2}.

Other definitions are possible. In an existing linear circuit, R_g may be measured by making use of any one of these relations. The value obtained would be independent of the method used. The one actually used in practice is a matter of convenience, or perhaps the use of one definition in preference to another is indicated by the form of the circuit. For example, the node A in Fig. 10 is often inaccessible and E_R cannot be measured; hence, the definitions given by equations (23) and (24) cannot be used.

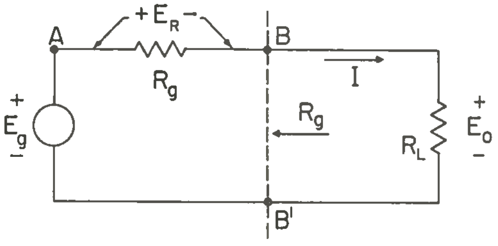

Fig. 10 Definition of symbols used in discussion of source resistance in linear circuits

3. ANALYSIS OF NONLINEAR DETECTOR CIRCUITS

To enable one to discuss video resisistance, it is necessary to have some background knowledge of the mathematical relationships of the detection phenomenon. These depend greatly on the form of the detector circuit. In this section expressions for the DC output currents and voltages for the detector circuits discussed in the section on RF resistance will be derived. The same circuit diagrams will be used.

(i) Simple Detector with Short-circuited Output

The circuit diagram is shown in Fig. 5. In this simple case, the instantaneous detector output current and the input voltage are related by a V-I characteristic which is the same as the V-I characteristic of the nonlinear element R_N. The relationship can be expressed in the form of equation (1):

$$I = f(V) \ .$$

This equation cannot always be expressed exactly and analytically in closed form, but may always be represented graphically using data obtained experimentally. In general, the voltage V can be expressed as the sum of two components (yet to be defined) so that

$$V = V_o + v \ . \tag{30}$$

The current becomes a function of these two components:

$$I = f(V_o + v) \ . \tag{31}$$

If now V_o is assumed to remain constant and v is allowed to vary over a limited range, either side of zero, the function $f(V_o + v)$ may be expressed in the form of a Taylor series about V_o:

$$I = f(V_o + v) = f(V_o) + \frac{f'(V_o)}{1!} v + \frac{f''(V_o)}{2!} v^2 + \frac{f'''(V_o)}{3!} v^3 + \; . \; . \qquad (32)$$

$f'(V_o)$ represents the first derivative of $f(V)$, evaluated at $V = V_o$. Equation (32) gives the circuit current as a function of the new variable v. The series converges rapidly and the first few terms give a good approximation to $f(V_o + v)$, if $|v|$ is kept small. The coefficients $f'(V_o)$, $f''(V_o)/2!$, etc., may be determined from the analytic expression for the characteristic, if this is available, or from the graphical respresentation of the characteristic as determined by experiment.

In the operation of this circuit as a detector, any DC voltage appearing across the input terminals is identified as V_o, and is termed the "bias" or "operating point". The instantaneous value of the input signal voltage is identified as v. The term $f(V_o)$ in the series denotes a bias current due to the bias voltage V_o and remains constant if V_o is constant. Other than serving to fix the operating point, it plays no part in the detection process and will be ignored. The circuit current due to v then becomes

$$i(v) = f'(V_o)v + \frac{f''(V_o)v^2}{2} + \frac{f'''(V_o)v^3}{6} + \; . \; . \; . \; . \qquad (33)$$

To simplify the notation this will be written as

$$i(v) = G_1 v + G_2 v^2 + G_3 v^3 + \; . \; . \; . \; . \qquad (34)$$

The coefficients G_1, G_2, G_3, etc., are functions of the bias voltage, but are constant once V_o has been fixed. The voltage v, as the instantaneous value of the input signal, is a function of time. If the input is a c-w signal,

$$v = E_m \sin \omega t ,$$

the circuit current is

$$i(t) = G_1 E_m \sin \omega t + G_2 E_m^2 \sin^2 \omega t + G_3 E_m^3 \sin^3 \omega t + \; . \; . \; . \qquad (35)$$

After expansion and application of the proper trigonometric identities, it can be seen that the expression for i(t) contains constant terms denoting DC currents, terms denoting currents of frequency ω, and terms denoting currents of frequencies which are harmonics of ω. In the detector application one is interested only in the DC components of i(t). The total DC current in the circuit, generated by the rectifying action of R_N and excluding the bias current due to V_o, is the sum of these components:

$$I_D = \tfrac{1}{2} G_2 E_m^2 + \tfrac{3}{8} G_4 E_m^4 + \tfrac{10}{32} G_6 E_m^6 + \ldots \qquad (36)$$

At low input levels E_m is very small and to a good approximation

$$I_D \approx \frac{G_2 E_m^2}{2} \quad . \qquad (37)$$

Since $G_2 = f''(V_o)/2$, the output (short-circuit DC current) of the detector of Fig. 5 at low level inputs is directly proportional to the "rate of change of conductance", $f''(V_o)$, and the "square of the maximum value of the input voltage", E_m^2. At higher input levels, the output increases more rapidly than E_m^2. These considerations hold true for any V-I characteristic which can be expressed in the form of a Taylor series in the region of interest; any device having such a characteristic and used in a detector circuit will result in so called "square law" operation when the input is small.

The discussion so far has been limited to a general nonlinear resistor and any operating point. It is of practical importance to examine the behaviour of a nonlinear resistor actually used in practice, such as a 1N23B silicon diode (microwave mixer). The "rate of change of diode conductance", as determined by

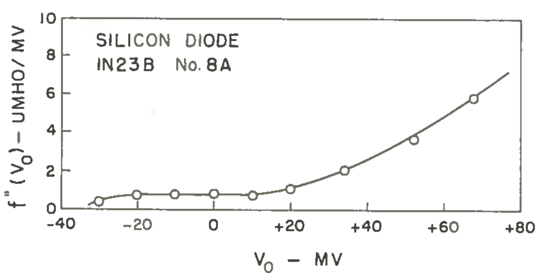

Fig. 11 Rate of change of conductance as a function of bias in the region of zero bias

measurement, is plotted in Fig. 11 against diode bias V_o for one particular 1N23B silicon diode. For this diode, $f''(V_o)$ is constant at 0.8 μmhos/mv from V_o = -20 mv to V_o = + 10 mv. Hence, if the bias and input are such that operation is confined to this region, the V-I characteristic can be represented by the first three terms of equation (32); the coefficients of succeeding terms are zero. The approximate equality of equation (37) then becomes exact:

$$I_D = \frac{G_2 E_m^2}{2} \quad . \qquad (38)$$

In the example shown, equation (38) holds for the maximum possible value of E_m if the bias is set to about -5mv. Since the curve shown is representative of this type of diode, one may conclude that operation of 1N23B or MA408A silicon diodes at low levels (a few millivolts) and near zero bias gives perfect square law detection. The value of $f''(0)$, however, varies widely from one diode to another of the same type. This is exemplified by the data of Table I, as obtained by measurement. Therefore, the magnitude of the rectified current for a given value of E_m varies from one diode to another.

TABLE I

RATE OF CHANGE OF CONDUCTANCE OF SEVERAL MICROWAVE DIODES AT ZERO BIAS

Diode Type	Laboratory Designation	$f''(0)$ (μmhos/mv)
1N23B	555	1.2
1N23B	8A	.80
MA408A	441	.35
MA408A	436	1.1

It is of interest to note that if the operating point is moved away from the origin by the application of forward bias ($V_0 > 0$) so that it falls on the curved portion of the graph in Fig. 11, the generated DC component of current is no longer given by equation (38), but by the more general approximate relation, equation (37). Detection becomes only approximately "square law", even for small inputs. Despite this deficiency, it would sometimes be advantageous to operate with a forward bias because a greater output can be obtained owing to $f''(V_0)$ increasing with V_0, as shown in Fig. 11.

(ii) Simple Loaded Detector

The circuit diagram is shown in Fig. 6. In this case the V-I characteristic relating input voltage and circuit current is not the V-I characteristic of the nonlinear resistor. However, the circuit characteristic may be found if the nonlinear resistor element characteristic and the load are known. The output of the detector is a DC voltage equal to the generated DC component of circuit current multiplied by the load resistance. The circuit characteristic, often called the "dynamic characteristic", may be found graphically by drawing a load line on the resistor characteristic curve, as shown in Fig. 12. The instantaneous circuit current flowing when the applied voltage is E and the load is R_L. is determined by the intersection of the load line drawn through E and the static curve at an angle $\tan^{-1} R_L$ to the vertical, and the V-I characteristic of the nonlinear resistor. Fig. 12 may be used as an aid

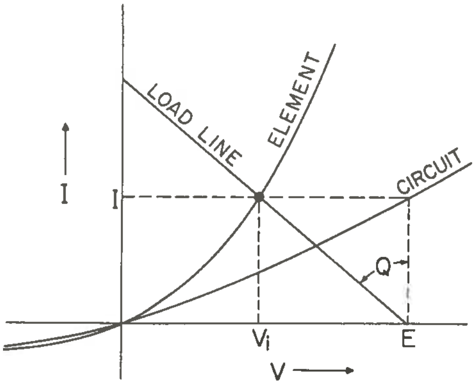

Fig. 12 Determination of current I in a series circuit of a nonlinear and linear resistor with applied voltage E

to the derivation of an analytic expression for the circuit current I as a function of the input voltage E. For simplicity, it will be assumed that the R_N characteristic may be represented by the first three terms of the Maclaurin form of a Taylor series. This implies that the voltage variation across R_N either is restricted to a very small range, or that the R_N characteristic actually has no derivatives higher than the second. The V-I characteristic of the nonlinear element is

$$I = G_1'V + G_2'V^2 , \tag{39}$$

where $G_1' = f'(0)$ and $G_2' = f''(0)/2$. The equation of the load line is

$$I = G_L(E - V) , \tag{40}$$

where G_L is the load conductance. The circuit current with an applied voltage E is given by the current at the point of intersection of (39) and (40). At the point of intersection

$$G_1'V + G_2'V^2 = G_L (E - V) . \tag{41}$$

This can be written

$$G_2'V^2 + (G_1' + G_L)V - G_L E = 0 . \tag{42}$$

The solution of equation (42) gives the voltage at the point of intersection:

$$V_i = -\frac{(G_1' + G_L)}{2G_2'} + \frac{1}{2G_2'}\left[(G_1' + G_L)^2 + 4G_2' G_L E\right]^{\frac{1}{2}} . \tag{43}$$

This actually represents the nonlinear resistor voltage when the applied voltage is E. The circuit current is found by substituting (43) into (39) or (40):

$$I = G_L \left\{ E + \frac{(G_1{}' + G_L)}{2G_2{}'} - \frac{1}{2G_2{}'} \left[(G_1{}' + G_L)^2 + 4G_2{}' \, G_L \right]^{\frac{1}{2}} \right\} \qquad (44)$$

If one now allows E to become the independent variable, equation (44) gives the circuit current I as a function of the applied voltage E and is, in fact, the dynamic characteristic of R_N and R_L in series. It is applicable over a restricted range of input voltage and is represented by the lower curve of Fig. 12. It may also be represented by a Taylor series in the region of interest. If equation (44) is written as $I = F(E)$ and v is used as a new independent variable, the Taylor series expansion about an operating point E_o (DC bias voltage across input terminals) is

$$I = F(E_o + v) = F(E_o) + \frac{F'(E_o)v}{1!} + \frac{F''(E_o)v^2}{2!} + \frac{F'''(E_o)v^3}{3!} + \dots \qquad (45)$$

As was done in the case of the previous detector, the term $F(E_o)$ will be ignored and it will be assumed that v is small so that the current due to v is, to a good approximation, given by the first two of the remaining terms:

$$i(v) = F'(E_o)v + \frac{F''(E_o)\, v^2}{2} \ . \qquad (46)$$

The coefficients $F'(E_o)$ and $F''(E_o)/2$ may be found by repeated differentiation of equation (44) and subsequent substitution of E_o for E. Equation (46) then becomes

$$i(v) = \left\{ G_L - G_L{}^2 \left[(G_1{}' + G_L)^2 + 4G_2{}' \, G_L \, E_o \right]^{-\frac{1}{2}} \right\} v + \left\{ G_2{}' \, G_L{}^3 \left[(G_1{}' + G_L)^2 + 4G_2 \, G_L \, E_o \right]^{\frac{3}{2}} \right\} v^2 \ . \qquad (47)$$

Substitution of $E_m \sin \omega t$ for v enables the generated DC circuit current to be determined for any bias E_o satisfying the conditions under which equation (39) remains a good approximation to the voltage-current characteristic of R_N.

It is of interest to compare the generated DC current in the previous short-circuited detector with that in the loaded detector. To be more meaningful, the comparison should be made with the same value of bias in both cases. Zero bias will be chosen. Thus the generated DC current in the short-circuit case with $V_o = 0$ is

$$I_D \approx \frac{G_2{}' E_m}{2} \ . \qquad (48)$$

After substituting $E_o = 0$ and $v = E_m \sin \omega t$ into equation (47) and simplifying, one obtains for the DC component of current due to the signal in the loaded detector:

$$I_D \approx \frac{G_2'}{2}\left(\frac{G_L}{G_1' + G_L}\right)^3 E_m^2 \quad . \tag{49}$$

Hence, if the output short circuit is replaced by a load of conductance G_L, the DC current decreases by the factor

$$\left(\frac{G_L}{G_1' + G_L}\right)^3 \quad ,$$

but remains proportional to E_m^2 . The detector output is actually the voltage

$$E_D \approx \frac{G_2'}{2G_L}\left(\frac{G_L}{G_1' + G_L}\right)^3 E_m^2 \quad . \tag{50}$$

(iii) Detector with RF Bypass Capacity

The circuit is shown in Fig. 7. The output is a DC voltage across the capacitor which also appears across R_N and forms a bias voltage on R_N . This is the generated component of bias due to the input signal and is zero when the input is zero. In addition, any externally applied diode bias voltage also appears across the diode and across the capacitor. The DC component of circuit current is zero. These facts provide sufficient data to enable one to calculate the output as a function of the input and the circuit parameters.

All signal voltages will be measured relative to the external bias voltage V_o which will be used as a reference. In this manner the analysis is made quite general and applicable to the detector with any value of externally applied bias. If the output capacitor is short-circuited (the circuit then becomes equivalent to that of Fig. 5) the circuit characteristic in the region of the operating point determined by the external bias can be expressed as :

$$i = G_1 V + G_2 V^2 + G_3 V^3 + \ldots\ldots \tag{51}$$

$$= P(V) \, ,$$

where V is the applied voltage measured relative to the external bias voltage V_o and G_1 , G_2 , etc., are coefficients which are functions of V_o . The circuit current measured relative to the bias current is defined by i.

Then

$$P'(V) = G_1 + 2G_2 V + 3G_3 V^2 + \ldots\ldots + \tag{52}$$

$$P''(V) = 2G_2 + 6G_3V^2 + \ldots + \quad (53)$$

$$P'''(V) = 6G_3 + \ldots + \quad (54)$$

etc.

It will now be assumed that the input to this short-circuited detector is the sum of a DC voltage E_d and the signal $v = E_m \sin \omega t$, both measured relative to the bias reference V_o. The circuit current may therefore be written as a function of these two voltages in the form of a Taylor series:

$$i = P(E_d + v) = P(E_d) + \frac{P'(E_d)v}{1!} + \frac{P''(E_d)v^2}{2!} + \frac{P'''(E_d)v^3}{3!} + \ldots + \quad (55)$$

After substitution of (52), (53) and (54) this becomes

$$\begin{aligned} i = {} & (G_1 E_d + G_2 E_d^2 + G_3 E_d^3 + \ldots +) \\ & + (G_1 + 2G_2 E_d + 3G_3 E_d^2 + \ldots +)v \\ & + (2G_2 + 6G_3 E_d + 12G_4 E_d^2 + \ldots +)\frac{v^2}{2!} \\ & + (6G_3 + 24G_4 E_d + \ldots +)\frac{v^3}{3!} + \ldots + \end{aligned} \quad (56)$$

After substituting $E_m \sin \omega t$, for v, expanding and simplifying, one obtains for the generated DC component of circuit current

$$\begin{aligned} I_D = {} & (G_1 E_d + G_2 E_d^2 + G_3 E_d^3 + \ldots +) \\ & + \tfrac{1}{2}(2G_2 + 6G_3 E_d + 12G_4 E_d^2 + \ldots +)\frac{E_m^2}{2!} \\ & + \tfrac{3}{8}(24G_4 + \ldots +)\frac{E_m^4}{4!} + \ldots + \end{aligned} \quad (57)$$

A value for the voltage E_d may be chosen so that

$$I_D = 0. \quad (58)$$

The value of E_d required to satisfy equation (58) may be determined by setting equation (57) equal to zero and solving for E_d. This would be a formidable, if not an impossible task. However, if one assumes E_m to be very small or if operation is confined to the "square law" region of a crystal diode, equation (58) may be written as

$$I_D = G_2 E_d^2 + G_1 E_d + \frac{G_2 E_m^2}{2} = 0. \qquad (59)$$

A further simplification may be made as a result of E_d being small when the input is small. If this is assumed:

$$G_2 E_d^2 \ll G_1 E_d \ , \qquad (60)$$

and the solution for E_d is

$$E_d \approx \frac{-G_2}{G_1} \ \frac{E_m^2}{2} \ . \qquad (61)$$

The conditions in the short-circuited detector existing when the input signal contains a DC component given by (61) are identical to those existing when the short circuit is removed and replaced by the capacitor C, and the DC component of the input made equal to zero. The voltage E_d still appears across the diode as a result of the rectifying action of the circuit. The output voltage across the capacitor is $E_D = -E_d$. Thus, except for sign, equation (61) gives the output voltage of the detector as a function of the input. The output voltage of the detector of Fig. 7 is:

$$E_D \approx \frac{G_2}{G_1} \ \frac{E_m^2}{2} \ . \qquad (62)$$

TABLE II

RATIO G_2/G_1 FOR SEVERAL MICROWAVE DIODES AT ZERO BIAS

Diode Type	Laboratory Designation	G_2/G_1 Volts^{-1}
1N23B	555	5.1
1N23B	8A	4.0
MA408A	441	2.5
MA408A	436	15.2

For a given E_m, the output depends on the ratio G_2/G_1 of the nonlinear resistor used in the circuit. This is quite different from the behaviour of the circuit of Fig. 5 where the output varied as G_2, the parameter G_1 having no effect. Values of G_2/G_1

for 4 silicon microwave diodes, as determined experimentally, are shown in Table II, and a plot of G_2/G_1 against bias for one of these diodes is shown in Fig. 13. These are the diodes previously considered in Table I. It is of importance to note that of the 4 diodes considered, the one (No. 555) which results in the largest output in the circuit of Fig. 5 is not the one (No. 436) which results in the largest output when in the circuit of Fig. 7.

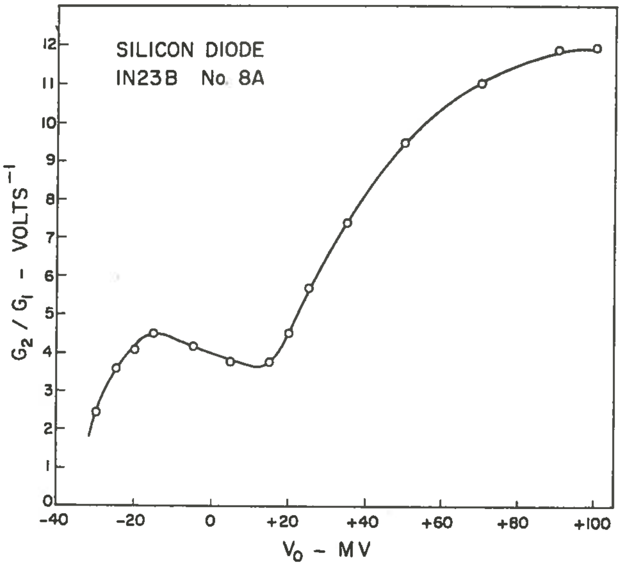

Fig. 13 G_2/G_1 as a function of bias in a typical 1N23B silicon diode

4. VIDEO RESISTANCE IN NONLINEAR DETECTOR CIRCUITS

From the point of view of a load, the detector may be considered as a DC voltage generator E_g in series with a resistance R_V which will be known as the output resistance of the detector and will be called the detector's "video resistance"; or, it may be considered as a current generator I_g in shunt with a "video conductance" $G_V = 1/R_V$. Both types of representation are shown in Fig. 14. The two circuits may be called the equivalent circuits of the detector. However, because of the presence

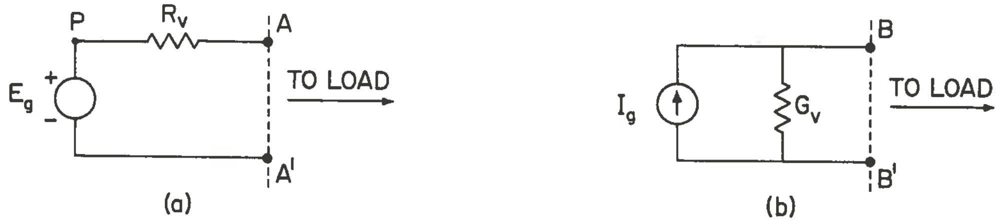

Fig. 14 Detector output equivalent circuits: (a) voltage generator type, (b) current generator type

of the nonlinearity, it must be anticipated that superposition will not always apply; hence, in general, these are not equivalents in the Helmholtz-Thévenin and Helmholtz-Norton sense. Nevertheless, the representation is always useful for purposes of discussion. The details of the equivalent circuit parameters of each of the detectors being studied will be introduced as the discussion proceeds.

(i) Simple Detector with Short-circuited Output

The circuit is shown in Fig. 5. The detector load is a short circuit and, in terms of the voltage equivalent circuit of Fig. 14(a), the only definition which can be given to the video resistance R_V is perhaps that of the "effective resistance" limiting the flow of short-circuit current. However, because the node P (Fig. 14) is inaccessible, the voltage E_g cannot be measured. In a linear circuit one could determine E_g by removing the short circuit from the terminals AA'. E_g then appears at AA' as the open-circuit voltage and the source resistance is simply the ratio of open-circuit voltage to short-circuit current. In the case being considered, this cannot be done. An open circuit at the output is transformed into an open circuit at the RF end. The nonlinear resistor therefore absorbs no RF power; there is no conversion of AC to DC and hence $E_g = 0$. This is true regardless of the level of input and is one case where the circuit of Fig. 14(a) does not behave as a Helmholtz-Thévenin equivalent. Another assumption might be that the resistance limiting DC current flow in the short circuit is the low level AC resistance of the nonlinear resistor, $1/f'(V_o)$. This appears to be doubtful, since the expressions for short-circuit DC current (equations 37 and 38) indicate that this current is entirely independent of $f'(V_o)$. It will be shown in the following section that video resistance for this short-circuited detector can be defined only by considering it as a special case of the detector of Fig. 6 or of the detector of Fig. 7, obtained by letting $R_L \rightarrow 0$. This will result in two different values of R_V being obtained for the short-circuited detector.

(ii) Simple Loaded Detector

The circuit is shown in Fig. 6. As in the previous case, the generated DC voltage corresponding to E_g in the equivalent circuit cannot be measured. However, there now exist a DC output voltage and current; and a video resistance may be defined for this detector using, for example, the first of equations (25). After a change of symbols to make it conform to detector circuit notation this is

$$R_V = -\frac{dE_D}{dI_D} \Big\|_{R_L \text{ varied}} \quad . \tag{63}$$

This will now be regarded as defining R_V, regardless of whether the parameter we have labelled as E_g (in the equivalent circuit) does, or does not remain constant when the load is changed. Expressions for I_D and E_D have been derived in the form of equations (49) and (50).

$$I_D = \frac{G_2'}{2}\left(\frac{G_L}{G_1 + G_L}\right)^3 E_m^2 \;, \quad E_D = \frac{G_2'}{2G_L}\left(\frac{G_L}{G_1 + G_L}\right)^3 E_m^2 \;.$$

To study the behaviour of R_V, defined by equation (63), $G_L = 1/R_L$ will be considered as the independent variable and the above equations then constitute a parametric representation of the E_D vs I_D relationship, with parameter G_L. It is assumed that G'_1 and G'_2 are constant and $G'_2 \neq 0$. The differentials, dE_D and dI_D may be readily obtained from the above equations and after simplification

$$R_V = -\frac{dE_D}{dI_D} = \frac{G_L - 2G'_1}{3G_L G'_1} \;. \qquad (64)$$

The video resistance of the detector of Fig. 6, as defined in this manner, is a function of the load conductance G_L, but is independent of the input signal amplitude at low levels. A plot of R_V as a function of load resistance $R_L = 1/G_L$ is shown in Fig. 15 for a value of $G'_1 = 1 \times 10^{-4}$ mhos, which is typical for a 1N23B diode.

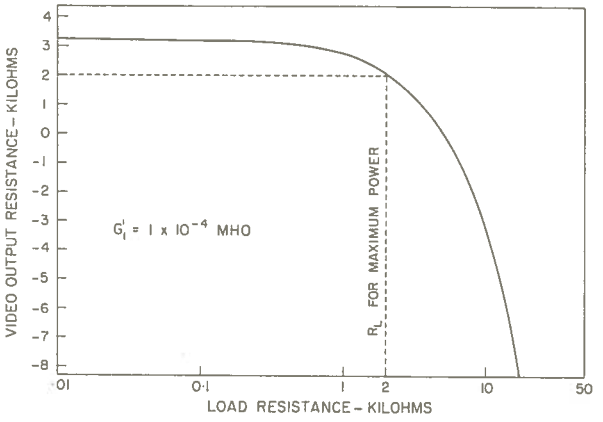

Fig. 15 Calculated video resistance as a function of load for the simple loaded detector of Fig. 6

Some peculiar features (when compared with the behaviour of source resistance in a linear circuit, where R_g is constant) to be observed are:

a) R_V varies with load
b) $R_V \rightarrow \frac{1}{3}G'_1$ as $G_L \rightarrow \infty$ (i.e., a short circuit)
c) $R_V = 0$ at $G_L = 2G'_1$
d) $R_V \rightarrow -\infty$ as $G_L \rightarrow 0$ (i.e., an open circuit)
e) $R_V = R_L$ ($= 1/G_L$) at $G_L = 5G'_1$.

One value for the video resistance for the previous short-circuit case (detector of Fig. 5) is defined by the relationship given by (b).

A second definition of video resistance, applicable in the linear case, is R_V = open-circuit voltage/short-circuit current. If this is applied to this detector, the result is $R_V = 0$, since the open-circuit DC voltage is zero.

A third definition of video resistance for the loaded detector may be obtained by determining the value of load for maximum power transfer. The DC power delivered to the load in the detector circuit of Fig. 6 is

$$P_D = I_D E_D \quad \frac{G'^2_2 G_L^5 E_m^4}{4(G'_1 + G_L)^6} \ . \tag{65}$$

Differentiation of equation (65) with respect to G_L and setting the result equal to zero, enables one to determine the value of load for maximum power. This is

$$G_L' = 5G'_1 , \tag{66}$$

or, in terms of resistance

$$R'_L = \frac{1}{5G'_1} \ . \tag{67}$$

The primes indicate the value of load for maximum power. Plots of power, voltage, and current as a function of load resistance showing the relative positions of maximum power and maximum voltage appear in Fig. 16. Typical values of

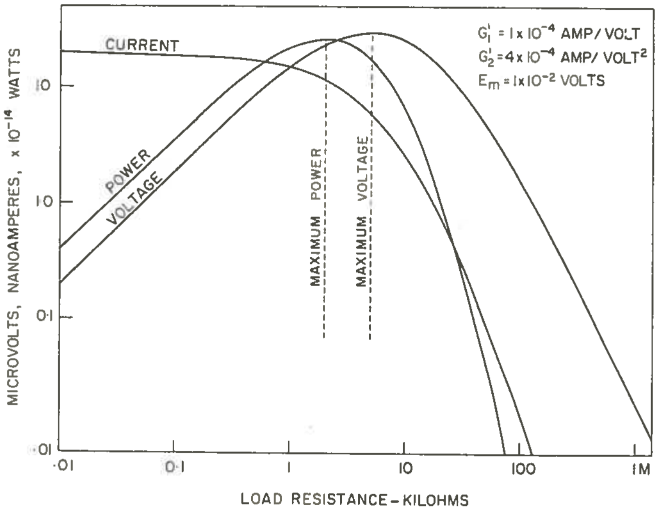

Fig. 16 Calculated values of DC voltage, DC current, and DC power output as a function of load for the detector of Fig. 6

G'_1 and G'_2 obtained by measurements on a 1N23B diode are used. If Figs. 15 and 16 are compared, it can be seen that although $-dE_D/dI_D = R_V$ varies with load resistance, it equals R_L at the point of maximum power output. In terms of maximum power considerations video resistance of the detector of Fig. 6 may also be defined as:

$$R_V = R_L' = \frac{1}{5G'_1} \; . \tag{68}$$

This defines a video resistance which is independent of load.

Still other values for video resistance for this circuit may be obtained by use of the other definitions for source resistance in a linear circuit. For example, $R_V = -\frac{\Delta E_D}{\Delta I_D}$ would give a value which is dependent on the magnitude of the voltage and current increments. However, the examples given are sufficient to show that the value obtained for the video resistance of the detector of Fig. 6 depends on the manner in which this is defined and measured, even though the various definitions used would result in one constant value being obtained for the source resistance in a linear system. The examples also show that video resistance is a function of certain parameters of the nonlinear resistor in the circuit, but is not necessarily equal to any one of these parameters. This is in contrast to the often expressed statement, sometimes made with no qualifications or restrictions mentioned, that the video resistance is equal to the inverse slope of the V-I characteristic of the nonlinear resistor at the operating point.

(iii) Detector with RF Bypass Capacity — Low-level Input

The circuit is shown in Fig. 7. Of the three detectors considered, this one behaves most ideally in the sense that at low-level inputs it may be represented by an equivalent circuit which has most of the properties of a Helmholtz-Thévenin or Helmholtz-Norton equivalent. One experiment may be performed which confirms the previous statement and generally provides much information about the behaviour of this detector. The results of such an experiment on one particular detector are shown in Fig. 17.

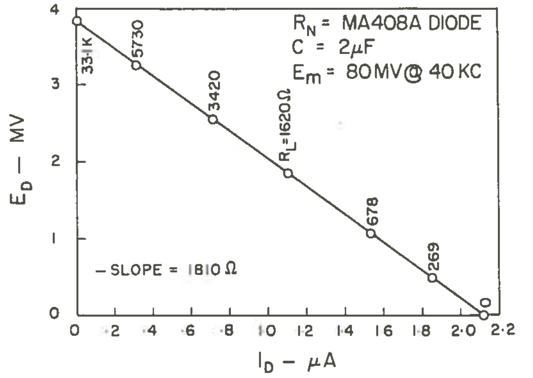

Fig. 17 DC output voltage — DC load current relationship in the detector of Fig. 7 as a function of load with low-level input

The relationship between output voltage and output current, with load as the independently variable parameter, is linear. This being the case, the same value for video resistance is obtained by using either one of

$$R_V = -\frac{dE_D}{dI_D} \; ; \quad R_V = -\frac{\Delta E_D}{\Delta I_D} \; ; \quad R_V = \frac{E_D\,(\text{open})}{I_D\,(\text{short})} \; .$$

The video resistance so defined is thus independent of load for small inputs. Further, since the DC voltage and current behave exactly as in a linear circuit, it should be expected that maximum output power would occur at the point where the load is equal to the value of R_V, defined by the three previous equations. That this

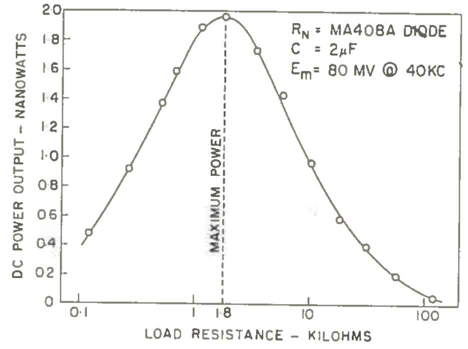

Fig. 18 Power output as a function of load for the detector of Fig. 7 with low-level input

is true, is shown in Fig. 18 where the DC power output as a function of load is plotted using the same data as in Fig. 17. The peak power output occurs when the load resistance equals the negative of the slope of the curve of Fig. 17. This fact may also be used to determine R_V, and gives the same result.

The video resistance may also be determined in terms of the nonlinear resistor parameter G_1 by making use of the definition $R_V = E_D(\text{open})/I_D(\text{short})$. The open-circuit voltage is given by equation (62).

$$E_D\,(\text{open}) \approx \frac{G_2}{G_1} \cdot \frac{E_m^2}{2} \; .$$

The short-circuit current is the same as in the detector of Fig. 5 and is given by equation (37).

$$I_D\,(\text{short}) \approx G_2 \cdot \frac{E_m^2}{2} \; .$$

Hence, for low-level inputs:

$$R_V = \frac{E_D\,(\text{open})}{I_D\,(\text{short})} \approx \frac{1}{G_1} \quad . \tag{69}$$

The video resistance of the detector of Fig. 7 at low-level inputs is equal to the inverse slope of the V-I characteristic at the operating point of the nonlinear resistor used. The detector has most of the properties of a DC generator with internal resistance in linear circuits, and any of the methods used to determine R_g in the linear circuit may also be used for R_V. Care must be exercised, however, in methods such as the "ohmmeter method" where the DC voltage applied from the load side must be kept very small (in the order of 1 millivolt, in the case of silicon microwave diodes) in order to give a resistance equal to the value obtained by the other methods.

It has been shown that the above definition is true for low-level inputs and any value of load resistance. Therefore, it must be true for $R_L = 0$. With $R_L = 0$, the detector of Fig. 7 is identical to that of Fig. 5, and it must be concluded that the video resistance of the short-circuited detector is $1/G_1$ at low-level inputs. However, we had previously found it to be $1/3G_1$, by letting $R_L = 0$ in the detector of Fig. 6. Thus, the video resistance of the simple short-circuited detector depends on the detector configuration prior to letting R_L approach zero. In other words, it depends on the manner in which the short-circuited condition is approached.

(iv) Detector with RF Bypass Capacity — High-level Input

As the input voltage E_m is increased to higher values, the video resistance becomes a function of input and of load. As a consequence, none of the definitions for R_V given in the previous section result in the value $1/G_1$ being obtained as the video resistance. Furthermore, each of the definitions results in a different value. To illustrate, the detector output voltage and current relationship for one particular detector as a function of changing load is shown in Fig. 19, where the input E_m is

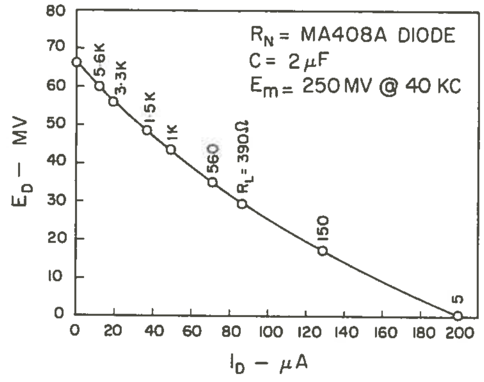

Fig. 19 DC output voltage — DC load current relationship in the detector of Fig. 7 as a function of load with high-level input

250 mv. Except for E_m, all other parameters are the same as in the case of Fig. 17. It is obvious that now the relationship between voltage and current is no longer linear. Thus, if the video resistance is defined by

$$R_V = -\frac{dE_D}{dI_D} \Bigg\| \; E_m \text{ constant}$$

it becomes a function of load. In this example, R_V varies from about 800 ohms (minus slope at $I_D = 0$) with an open-circuited output to about 200 ohms (minus slope at $E_D = 0$) with short-circuited output. If one chooses to define video resistance in terms of the ratio of open-circuit voltage to short-circuit current, the value 320 ohms is obtained. A plot of DC power output against load for the

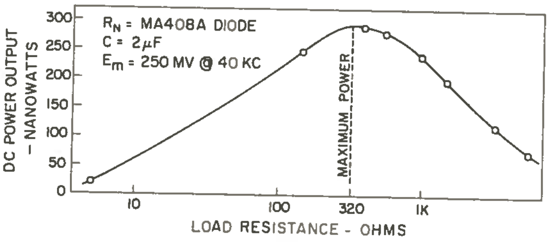

Fig. 20 Power output as a function of load for the detector of Fig. 7 with high-level input

same value of E_m (250 mv) is shown in Fig. 20. The value of load for maximum power is about 320 ohms, which can also be defined as the video resistance. That the same value of 320 ohms is obtained by the two methods above is only peculiar to this case; it is not believed to be generally true. It should be noted here that, as in the case of the detector of Fig. 6, maximum power output occurs at the point where $R_L = -dE_D/dI_D$. Only at this point do the "maximum power" definition and the "minus slope" definition give the same value for R_V. It is obvious from the preceding that still other values of video resistance may be found by using the definition $R_V = -\Delta E_D/\Delta I_D$. If this is used the value obtained would, in general, depend on the magnitude of the increments. However, if the increments are kept small, it provides a good approximation to $R_V = -dE_D/dI_D$ and is useful in experimental work.

In addition to the variation in values of R_V obtained as a result of using different methods of measurement, the values obtained in each case vary with the input E_m, as well as with load resistance. This is shown graphically in Fig. 21. The curves show the variation of video resistance with power level and load for an X-band microwave detector.

To sum up, the concept of video resistance as applied to linear circuits and to the capacity-output detector with low-level inputs becomes meaningless when applied to the detector under the influence of a large-amplitude input signal. During detection of high-level AM signals, the video resistance, as given by any of the previous

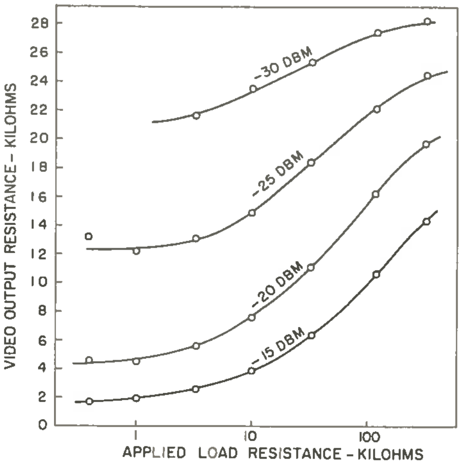

Fig. 21 Video resistance of microwave detector determined experimentally using the definition $R_V = \Delta E_D / \Delta I_D$, as a function of load resistance and input power. Nonlinear detector element was a 1N23B diode mounted in a standard waveguide mount. Input signal frequency = 9.4 kmc/s

definitions, is constantly changing as the instantaneous input power varies. As the subject is too complex, there will be no further mention of it here.

IV. CONCLUSIONS

An attempt has been made to show the relationships between the resistance properties of a device with a continuous nonlinear voltage-current characteristic and the concepts of RF resistance and video resistance as applied to detectors employing such a device, and to point out the restrictions to be observed in using these concepts. The detector models studied have been kept as simple as the requirements of the subject would allow. This was done with the belief that it in no way hindered the attainment of the goals set forth at the beginning of this report. Although low radio frequencies, c-w inputs, and zero internal impedance RF sources have been used — to name a few of the tools by which simplicity was achieved — the main points discussed, but not always the quantitative results obtained, apply equally well to microwave detectors subjected to pulsed RF input signals.

The main conclusions to be reached on the basis of the material presented are the following:

a) RF impedance and video impedance are properties of a detector and not of the nonlinear element in the circuit.

b) A detector may be represented by a Helmholtz-Thévenin or a Helmholtz-Norton equivalent circuit, if the circuit configuration is of the type with an RF bypass capacity, and only if the input level is low. In general, detectors without this capacity cannot be so represented.

c) The detector with RF bypass capacity and with low-level input may be represented by a Helmholtz equivalent circuit. It has a video resistance which is independent of signal level and load, and is equal to the inverse slope at the operating point of the V-I characteristic of the nonlinear resistor used in the circuit. In other words, it is equal to the differential AC resistance of the resistor at this point.

d) Unless the detector can be represented by one of the Helmholtz equivalent circuits, the value obtained for video resistance depends entirely on the technique used for its measurement.

e) The output of the detector with RF bypass capacity and with low-level input may be written as:

$$E_D = \frac{R_L}{R_V + R_L} \cdot \frac{G_2 \, E_m^2}{G_1 \, 2} = \frac{G_2}{G_1 + G_L} \, \frac{E_m^2}{2} \, .$$

Therefore, the output is very closely proportional to the nonlinear resistor parameter $G_2 = f''(V_o)/2$, if $G_L \gg G_1$ (i.e., for very low load resistance) but becomes approximately proportional to the ratio $G_2/G_1 = f''(V_o)/2f'(V_o)$ when $G_L \ll G_1$ (i.e., for very high load resistance).

f) Both RF resistance and video resistance, as defined in this report, vary conjointly with the instantaneous amplitude of a high-level AM input signal as the amplitude varies owing to modulation.

References

1. Kharkevich, A.A., "Nonlinear and Parametric Phenomena in Radio Engineering", (translated from the Russian by J.G. Adashko) Electronic Design, 6:64, 1958
2. Falconer, R., "The Effects of RF Signal Level and Bias on the Characteristics of Three Crystal Detectors", Stanford Electronics Lab., Tech. Rept. No. 150-1, October 19, 1955
3. Beringer, R., "Low Level Crystal Detectors", MIT Radiation Lab., Rept. No. 61-15, March 16, 1943
4. Beringer, R., "Crystal Detectors and the Crystal Video Receiver", MIT Radiation Lab., Rept. No. 638, November 16, 1944
5. Torrey, H.C. and Whitmer, C.A., "Crystal Rectifiers", MIT Radiation Lab. Series, vol. 15, McGraw-Hill Book Co. (New York and London, 1948)
6. Whitford, B.G., "Frequency Response of Nanosecond RF Pulse Detectors", J. Sci. Instr., 39:303, 1962

Bibliography

Barrow, W.L., "Contribution to the Theory of Non-linear Circuits with Large Applied Voltages", Proc. IRE, 22: 964, 1934

Bridges, J.E., "Pseudo-rectification and Detection by Simple Bilateral Non-linear Resistors", Proc. IRE, 49: 469, 1961

Cooper, W.H.B., "A Method of Solving Certain Non-linear Circuit Problems", Wireless Engr., 21: 323, 1944

de Broekert, J.D., "Two Wideband Video Techniques for Microwave Detectors", Stanford Electronics Labs., Tech. Rept. No. 152-2, November 27, 1957

Everitt, W.L., "Communication Engineering", 2nd Ed., McGraw-Hill Book Co. Inc., (New York and London, 1937)

Huss, P.O., "An Analysis of Copper-oxide Rectifier Circuits", Trans. AIEE, 56: 354, 1937

Kilgour, C.E. and Glessner, J.M., "Diode Detection Analysis", Proc. IRE, 21: 930, 1933

Marks, R.C., "Low Level Detector Crystals and the Detector Video Receiver", Ministry of Supply, Telecomm. Res. Estab., Gt. Malvern, Worcs. Tech. Note No. 102, November 30, 1950

Moullin, E.B. and Turner, L.B., "The Thermionic Triode as a Rectifier", JIEE, 60: 706, 1922

Pantell, R.H., "General Power Relations for Positive and Negative Non-linear Resistive Elements", Proc. IRE, 46: 1910, 1958

Peterson, E., "Impedance of a Non-linear Circuit Element", AIEE Trans., 46: 528, 1927

Shelton, E.E., "Non-linear Current Potential Characteristics", Electronic Eng., 15: 339, 1943

Staniforth, A. and Craven, J.H., "Improvement in the Square Law Operation of 1N23B Crystals from 2 to 11 kmc", IRE Trans., MTT-8: 111, 1960

Tucker, D.G., "Rectifier Resistance Laws", Wireless Engr., 25: 117, 1948