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MEASUREMENT OF CONVERTER CRYSTAL PARAMETERS AND OVER-ALL NOISE FIGURES AT 10.7 CENTIMETERS

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ABSTRACT

Apparatus and experimental procedures are described for the determination at 10.7 centimeters of conversion loss, L, and noise temperature, t, of converter crystals, and of the noise figure of the intermediate frequency amplifier. An over-all noise figure is thus obtained from the formula $F_r = L(F_{if} + t - 1)$. This is compared with an independent determination of F_r using a fluorescent lamp mounted across the wave guide as a source of radio-frequency noise power.

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MEASUREMENT OF CONVERTER CRYSTAL PARAMETERS AND OVER-ALL NOISE FIGURES AT 10.7 CENTIMETERS

- I -

CRYSTAL TEST SET

The crystal test set to be described is designed to measure the following parameters of type-1N21 (A, B, or C) converter crystals at 10-centimeter wavelengths:

(a) The Intermediate-Frequency Impedance - It is found that this is independent of the actual intermediate frequency used, and therefore may be measured on an audio bridge or even by d-c methods. In the apparatus to be described the intermediate-frequency impedance is determined at 60 cycles and is an integral part of the measurement of conversion loss.

(b) Noise Temperature - A crystal of specific intermediate-frequency impedance will generate more noise than the Johnson noise power developed within a carbon resistor of equal impedance. This "noisiness" varies from crystal to crystal, and it is therefore necessary to specify it for each particular crystal.

(c) Conversion Loss - The intermediate-frequency power available at the input to the i-f amplifier is less than the radio-frequency power available at the input to the crystal mixer itself. This loss of power also varies from crystal to crystal.

SUMMARY OF BASIC THEORY

(a) Noise Figure

The noise figure (F) of any network, which is usually, but not necessarily, an amplifier, is defined in the following manner:

$$F = \frac{\frac{S_{in}}{N_{in}}}{\frac{S_o}{N_o}} = \frac{S_{in}}{KT_oB} \cdot \frac{N_o}{GS_{in}} = \frac{N_o}{KT_oBG},$$

where S_{in} and N_{in} are, respectively, the available signal and noise powers at the input terminals of the network, N_{in} is, by Nyquist's Theorem, equal to $KT\int df$ or KTB. S_o and N_o are respectively, the available signal and noise powers at the output terminals,

K is Boltzman's constant, 1.37×10^{-23} joules/degree
B is the effective bandwidth, and
G is the gain of the amplifier.

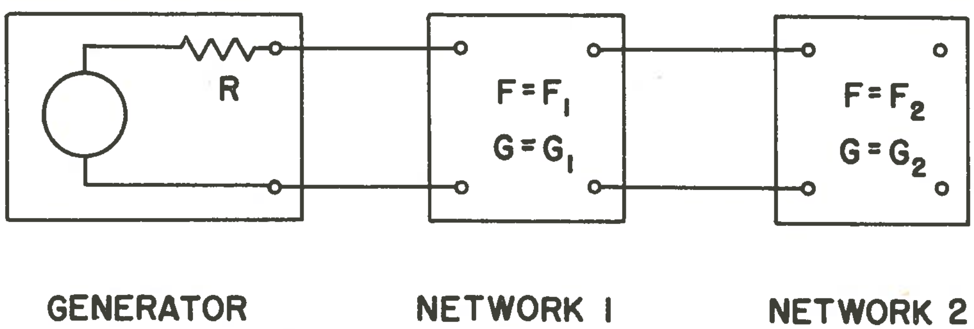
Strictly, the terms should be put in differential form, but in most methods of measurement the total noise powers are considered, and B therefore cancels out. If not, it is taken that

$$B = \frac{1}{G_{max}} \int_0^{\infty} G\,df$$

It is to be noted that F, and also G, are functions of the match employed between the generator and network.

The noise figure of two networks in cascade is given by the following formula:

$$F_{1+2} = F_1 + \frac{F_2 - 1}{G_1}$$



TWO NETWORKS IN CASCADE

The two networks may be, for example, two amplifiers, or two stages of the same amplifier, but in the present application the first network will be the crystal mixer and the second the intermediate-frequency amplifier.

(b) Conversion Loss (Specified as L, or $\frac{1}{G_x}$)

This is a very complicated function of the radio-frequency impedances presented to the crystal mixer at signal, image, and local oscillator frequencies. Values, as ordinarily quoted, are understood to have been measured under completely matched conditions for both channels.

(c) Noise Temperature

The noise temperature (t) of a crystal converter is given by

$$t = \frac{N_o}{KTB},$$

where N_o is the noise power available at the i-f terminals of the crystal (for a specified r-f input), and KTB is the Johnson noise from a resistor.

Considering the crystal as a network, its noise figure

$$F_x = \frac{N_o}{KTB\ G_x} = Lt.$$

(d) As N_o and KTB are of very low absolute value it is customary to employ an intermediate parameter, Y, defined in the same manner as t, except that the noise powers involved are those at the output of the receiver. Thus $Y = \frac{N_{ox}}{N_{or}}$, where N_{ox} is the noise output with the crystal at the input terminals of the amplifier (again with the local oscillator power or rectified crystal current specified), and N_{or} is the noise output with a resistor of impedance equal to the intermediate-frequency crystal impedance at the input. It can be shown that the over-all receiver noise figure of the crystal and amplifier in series is $F_r = L\ F_{if}\ Y$, and from the formula for two networks in cascade:

$$F_r = F_x + \frac{1}{G}(F_{if} - 1) = L(F_{if} + t - 1),$$

which shows the dependence of the over-all noise figure of the receiver upon the crystal parameters.

To relate t and Y we have:

$$F_r = L\ F_{if}\ Y = L(F_{if} + t - 1),$$

from which $$t = F_{if}(Y - 1) + 1.$$

EFFECT OF LOCAL OSCILLATOR POWER LEVEL

L, t, and F_{if} are all functions of the local oscillator power level. t is, in general, a linear function, while L rises rapidly towards the lower values of rectified crystal current and settles down to an almost constant value at high crystal current (see Fig. 4).

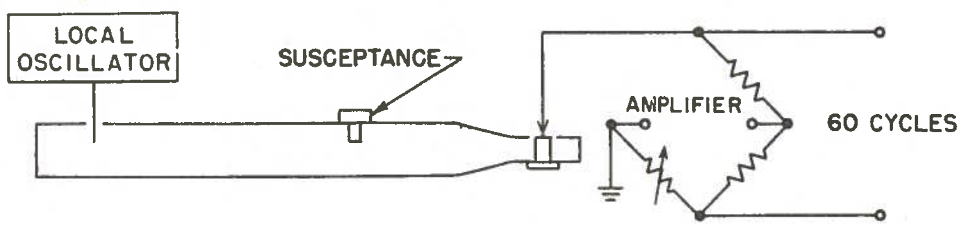
As the intermediate-frequency impedance is a function of local oscillator level (Fig. 4), F_{if} will be, as well. However, in the neighbourhood of 0.6 milliampere rectified crystal current the over-all noise figure of the receiver is usually a minimum, and fortunately, a slowly varying function of the crystal current. This is so because in this neighbourhood t and L are changing in opposite directions.

Unless otherwise specified the intermediate-frequency impedances, L, and t, are measured in the apparatus to be described at 0.6 milliampere crystal current.

MEASUREMENT OF CONVERSION LOSS

Dicke's Impedance Method is used in the present apparatus. This method depends upon a theoretical supposition, namely that the reciprocity condition which holds in the analysis of passive mixer networks (Ref.3) will also hold in the case of the crystal mixer. Although this condition fails partially for germanium crystals, where Dicke's method is thus unsuitable, no silicon crystal has been found for which the reciprocity condition fails.

Diagramatically the equipment is as follows (see also Fig.2):



The formula for conversion loss is

$$L = 2\,\frac{p-1}{p+1}\,\frac{R_o(R_2 - R_1)}{(R_o - R_1)(R_2 - R_o)}$$

The experimental procedure is as follows:

1. The crystal is placed in its holder and accurately matched to the line at 0.6 ma. crystal current, and the i-f impedance R_o is determined by adjusting one arm of the bridge. The 60-cycle voltage used is effectively the i-f voltage, and the impedance so obtained is the same as at 30 or 60 Mc. for completely matched conditions.

2. The standard susceptance is placed in the wave guide and its voltage standing wave ratio (p) determined. It is not necessary, of course, to repeat this step for subsequent readings.

3. As the standard susceptance is varied in position along the line the i-f impedance, R, goes through a cyclic variation and the position is selected at which R is a maximum. The crystal current must be readjusted. This value, determined from the bridge as before, is R_2.

4. Step 3 is repeated to obtain the minimum value of R, which is R_1.

Thus R_o, p, R_2 and R_1 have been determined and L may be calculated from the formula. A check on the internal consistency of the measurements is afforded by the relationship $R_o = \sqrt{R_1 R_2}$.

A typical set of values is as follows:

$$\begin{aligned} R_o &= 558 \text{ ohms,} \\ R_2 &= 837 \quad " \\ R_1 &= 375 \quad " \\ \sqrt{R_1 R_2} &= 560 \quad " \\ p &= 2.45 \end{aligned}$$

In actual work the formula has been recast in the form

$L = K \dfrac{\sqrt{x} + 1}{\sqrt{x} - 1}$, where $x = R_2/R_1$, $K = 2 \dfrac{p - 1}{p + 1}$, and the values of L are read from a graph.

The results of measurements made on a number of crystals are shown in Fig. 6. The scatter falls fairly well within the limits of the specifications for the various types, but it must be noted that all the used (and possibly abused) crystals were not eliminated from the group.

NOISE TEMPERATURE MEASUREMENTS

(a) Accuracy of Method Employed

The accurate determination of t is complicated by the fact that it is for practical reasons first necessary to measure Y, and Y itself is a function of the circuit parameters. In other words, the simple formula $Y = \dfrac{1}{F_{if}}(t - 1) + 1$ is true only in the event that the i-f impedance of the crystal is equal to the resistance replacing the crystal. In general, it is found (Ref. 3) that

$$Y = \frac{1}{F_{if}}\left[\frac{(tp + m)\ (1 + m)^2}{(p + m)^2} - m - 1\right] + 1,$$

where $p = \frac{g_1}{g_s} = \frac{r_s}{r_1}$, $m = \frac{g_2}{g_s} = \frac{r_s}{r_2}$, and

where g_1 = admittance of the crystal,
g_2 = input admittance of i-f amplifier, and
g_s = admittance of resistor replacing the crystal.

If p=1, this reduces to the simpler equation above.

An examination of this general formula reveals that Y is a critical function of p, but varies almost negligibly for changes in m. The procedure that has been adopted, therefore, is to hold p close to unity, with a resultant error in t only in the second decimal place.* For example, say the input impedance r_2, of the i-f amplifier is 400 ohms, and the i-f impedance, r_1, of the crystal is 310 ohms. N_{ox} is determined with this crystal in place (the actual absolute value of power is not measured in the experimental procedure, but this is irrelevant to the present argument). Now, to determine N_{or}, the crystal is replaced with a resistance of, say, 300 ohms.

$$\text{Thus } r_s = 300 \quad p = \frac{r_s}{r_1} = 0.968, \text{ and } m = \frac{r_s}{r_2} = 0.75.$$

Suppose $F_{if} = 4$, and assume that the correct value of t for this crystal is 2.00,

$$\text{Then } Y = \frac{1}{4}\left[\frac{(1.936 + 0.75)\,(1.75)^2}{(1.718)^2} - 1.75\right] + 1$$

$$= 1.258.$$

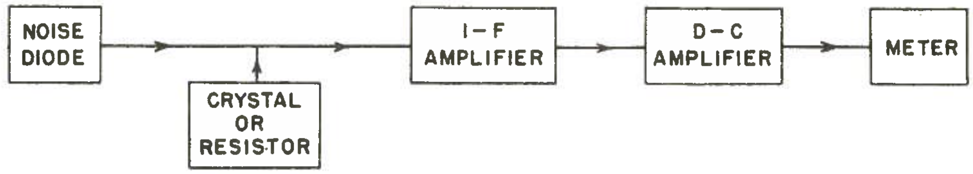
Then, from $t = F(Y-1) + 1$,
$t = (4 \times 0.258) + 1 = 2.03.$

We have assumed that the correct value of t is 2.00, and have shown that experimentally we would obtain a value of 2.03, which is not a significant difference. In general, p is held to values of about $1 \pm 4\%$, or less.

In the derivation of the general formula above it was assumed that F_{if} was measured with the standard resistance, r_s, across the input. As r_1 is very nearly equal to r_s, F_{if} will not appreciably change, when this substitution is made. Since r_1 and r_s may together vary from about 200 to 800 ohms it would seem necessary to know F_{if} over this range. However, this is obviated in the procedure employed, by which F_{if} if effectively, although not explicitly, measured for each determination of t.

*The more usual method, particularly where testing is done on a production basis, is to use an input coupling circuit devised by Roberts which makes Y very nearly independent of m and p over the range of crystal admittances usually encountered.

Experimental Procedure



BLOCK DIAGRAM

(SEE ALSO FIG. 3)

With the crystal in place and the rectified crystal current set at 0.6 ma, the output meter is read. Now, the crystal is replaced by a resistance whose impedance is approximately equal to the i-f impedance of the crystal. The resistor is mounted in the same manner as the crystal. The output reading will be somewhat less than previously. The noise diode is turned on and adjusted so that the reading of the output meter is the same as with the crystal input.

From diode theory $i^2 = 2eI\Delta f$,

where i is the alternating or noise current,
I is the direct current through the diode,
e is the charge on an electron, and
Δf is the element of bandwidth.

$$\text{Available power} = \frac{i^2}{4g} = \frac{2eI\Delta fR}{4}$$

Total available power, including Johnson noise from the resistor,

$$= \tfrac{1}{2}eI\Delta fR + KT_o f$$

Thus we may define a noise temperature of the resistance as

$$t_r = \frac{KT_o\Delta f + \frac{1}{2}eI\Delta fR^{*}}{KT_o\Delta f}$$

$$= 1 + \frac{eIR}{2KT_o} = 1 + 20IR, \text{ for } T_o = 292°K.$$

If I is adjusted to obtain the same reading in the two cases, then $t_r = t_x = 1 + 20IR$. t then is determined from the value of R and the reading of the diode direct current, I. It will be noted that with this procedure it is unnecessary to know the law of the second detector.

*(See following page)

* This expression must be the same as $t_r = F(Y-1) + 1$.

Proof: $t_r = F(Y-1) + 1$

$$= \frac{N_{or}}{N_{in}G} \frac{(N_{ox}-N_{or})}{N_{or}} + 1, \text{ where } N_{in} = KT_oB.$$

Let $N_eG = (N_{ox} - N_{or}) = (\frac{1}{2}eI\Delta fR)G$.

$$\text{Then } t_r = \frac{N_eG}{N_{in}G} + 1$$

$$= \frac{\frac{1}{2}eI\Delta fR + KT\Delta f}{KT\Delta f}.$$

- II -

EXPERIMENTAL CONFIRMATION OF THE EQUATION FOR OVER-ALL NOISE FIGURE

The over-all noise figure, F_r, of a receiver with crystal input has been determined by two distinct methods of measurement:

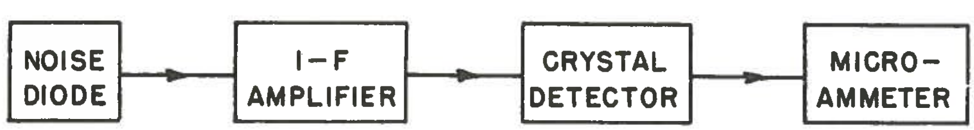
(a) L, t, and F_{if} were measured separately and F_r calculated from the formula $F_r = L(F_{if} + t - 1)$, and

(b) F_r was measured directly using a fluorescent lamp across the waveguide as a source of signal noise power.

(a) Measurement of L, t, and F_{if}

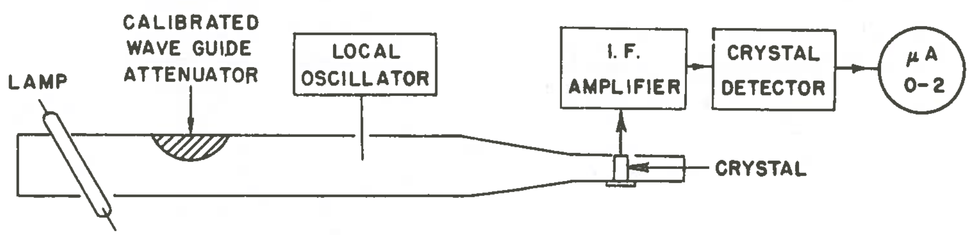
L and t were determined as outlined in Part I of this report.

F_{if} was determined with an experimental set-up, as indicated in the following block diagram:



It is customary to make the signal and noise output powers equal, so that the formula reduces to $F_{if} = \frac{S_{in}}{KT_oB} = 20IR_g$ when a noise diode is used as the source of signal power. The use of a crystal detector at the output obviates the need for a 3-decibel attenuator and post-amplifier, but this advantage is offset by the fact that the crystal itself must be calibrated. In this experiment the crystal was used at low level, with current of the order of one microampere through it. Several crystals were calibrated against a piston attenuator as a primary standard, and showed a response which was closely, although not exactly, linear with power input. As the resultant discrepancy in results was comparatively small, a linear response for the crystal detector was assumed in these experiments.

(b) Direct Measurement of F_r



BLOCK DIAGRAM OF APPARATUS

The application of a fluorescent lamp mounted across a wave guide as a convenient source of r-f noise power which is at once definitely specified, of a comparatively high level, and broad band, was first described by Mumford, (Ref. 5).

The noise power available from this lamp was determined* by comparison with a heat load source upon apparatus originally designed for measurement of solar radio noise. Thus an effective temperature of the lamp, T_h, was determined as 11,400°K.

The determination of F_r is then carried out in two steps. First, a reading, R_1, proportional to output noise power is taken from the microammeter with the crystal matched to the wave guide and with no signal imposed. Second, the fluorescent lamp is switched on and the wave-guide attenuator adjusted to such a setting, a, that the output reading, R_2, is equal to twice R_1. In the particular case we have here, where $R_2 = 2R_1$ and there is no suppression of the image frequency, it is shown in the Appendix that $F = 2\frac{(T_e - T_o)}{T_o}$ where T_e is the effective temperature at the input to the receiver. As the

* (in collaboration with A.C. Hudson)

effective temperature of the lamp is T_h, we may put

$$\frac{T_e - T_o}{T_o} = \frac{T_h - T_o}{T_o} \cdot \frac{1}{a},$$

where a is the attenuation in the wave guide expressed as a ratio.

For the expression $10 \log \frac{T_h - T_o}{T_o}$, Mumford has obtained experimental values of 15.86 and 15.80 db at 3930 megacycles/second. Our calibration at 2800 megacycles/second gives $10 \log \frac{(11{,}400 - 292)}{292}$, or 15.8 db.

Thus we may put $10 \log \frac{(T_e - T_o)}{T_o} = (15.8 - A)$ db, where A is the attenuation in the waveguide expressed in decibels. The overall noise figure of the receiver is then given in decibels by the expression $F = 15.8 - A + 3$.

(c) Results

1. Crystal No. 26 (NRC serial number):

I-F impedance	(R_3) =	445 ohms
Conversion loss	(L) =	3.88
Noise temperature	(t) =	1.41
Noise figure of I-F amplifier	(F_{if}) =	5.5 (average of five readings)

$\therefore F = L(t + F_{if} - 1) = 23.$

Using the fluorescent lamp method:

(First Measurement)

A	=	5.0 db
F_r (db)	=	$15.8 - 5.0 + 3 = 13.8$
Fr (ratio)	=	24

(Second Measurement)

A	=	5.2 db
F_r	=	23

2. Crystal B-35:

R_3	=	450 ohms
L	=	4.68
t	=	1.44
F_{if}	=	5.5
F_r	=	27.8

By direct measurement,

F_r = 31.6, 29.6 and 27.6 for three measurements.

Average F_r = 29.6

3. <u>Crystal C-11</u>:

R_3 = 441
L = 3.46
t = 1.14
F_{if} = 5.5
F_r = 19.5.

By direct measurement, F_r = 21, 19, and 20.4 for three measurements.

Average F_r = 20.1

As the probability of error of individual measurements is of the order of 5 to 10 per cent, the results obtained may be taken as a confirmation within these limits of the formula $F_r = L(F_{if} + t - 1)$.

APPENDIX

NOISE FIGURE OF A SUPERHETERODYNE RECEIVER

The equations for noise figure are often expressed in the form

$$F = \frac{t-1}{r-1}$$

for a single channel receiver, and

$$F = \frac{n(t-1)}{r-1}$$

for a receiver in which the image frequencies are partially or wholly accepted.

Here,

$$n = \frac{\int_0^\infty G\,df}{\int_B G\,df}$$

where the integral in the denominator is taken over the useful signal channel only. It is understood that an initial reading of output noise power, R_o, is taken with the generator resistor at $T_o = 292°K$, and a second reading, rR_o, taken at some higher temperature $T = tT_o$. A source of sufficient band width to cover all necessary channels may be used which generates noise power equivalent to KT_eB, where $T_e = tT_o$ (the effective noise temperature), is then calculated or determined experimentally.

The above formulae are obtained as follows:

A. - <u>For the single channel case</u>:

(1) $R_o = FN$, where $N = KT_oBG$,

(a) $rR_o = FN + (t-1)N$,

whence $F = \dfrac{t-1}{r-1}$. For $r = 2$, $F = t - 1 = \dfrac{T - T_o}{T_o}$.

B. - <u>For the case where the image channel is wholly or partially accepted</u>:

(1) $R_o = N + (F-1)N = FN$,

(2) $rR_o = FN + n(t-1)N$,

whence $F = \dfrac{n(t-1)}{r-1}$.

If $r = 2$ and $n = 2$, then $F = 2t - 2 = \dfrac{2(T-T_o)}{T_o}$.

Appendix

It will be noted that the Johnson noise from the image channel has been included with the receiver noise — i.e., in the expression $(F - 1)N$. Therefore, in the experiment described in Part II of this report, where F_r has been determined by two different methods, each method should be essentially the same insofar as image channel reception is concerned. It would seem that this condition has been fulfilled, as there was no image suppression in the determination of F_r, L, or t, and each measurement was taken with the crystal mixer matched to both channels.

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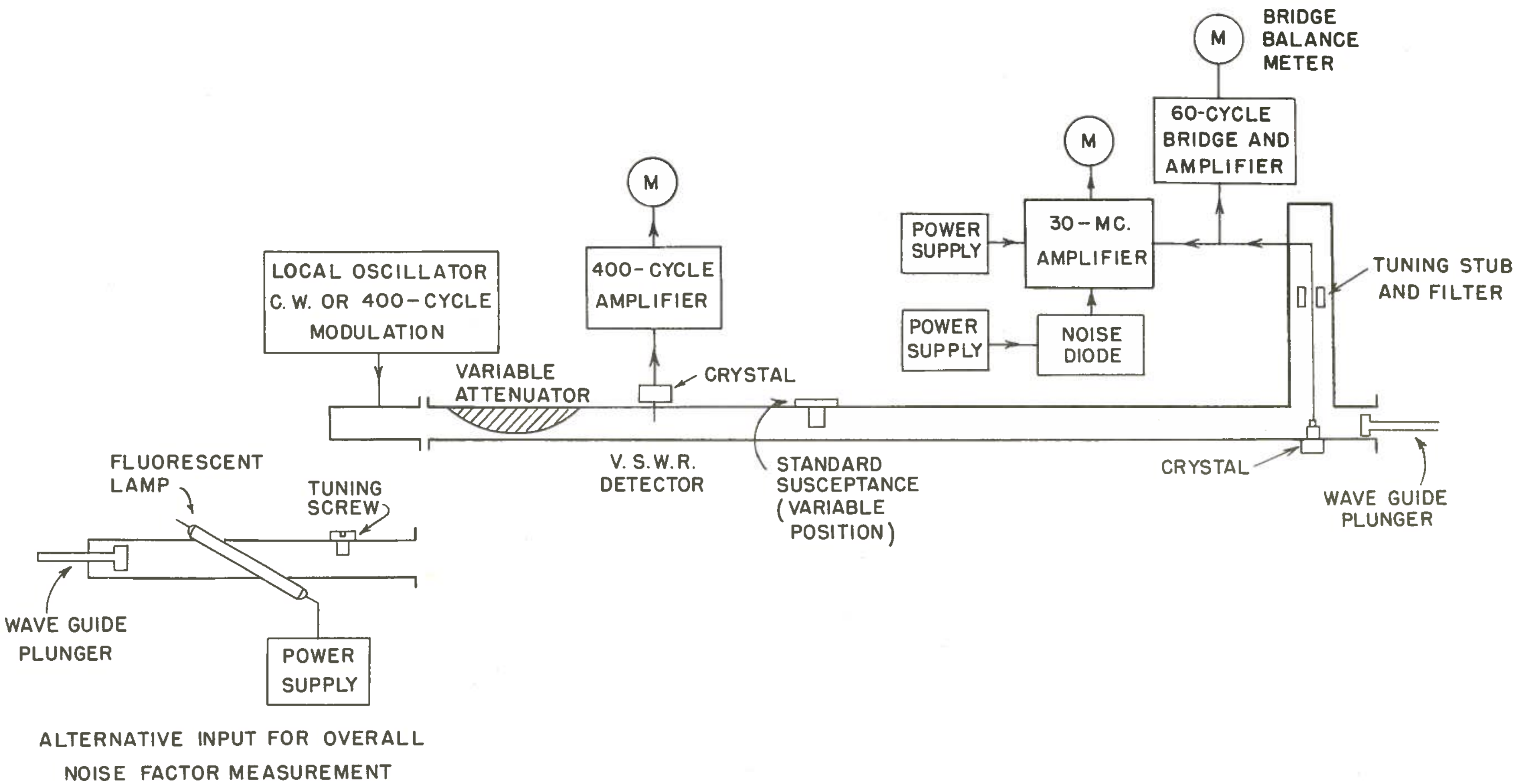

FIG. I

FUNCTIONAL DIAGRAM OF CRYSTAL MEASUREMENTS SET

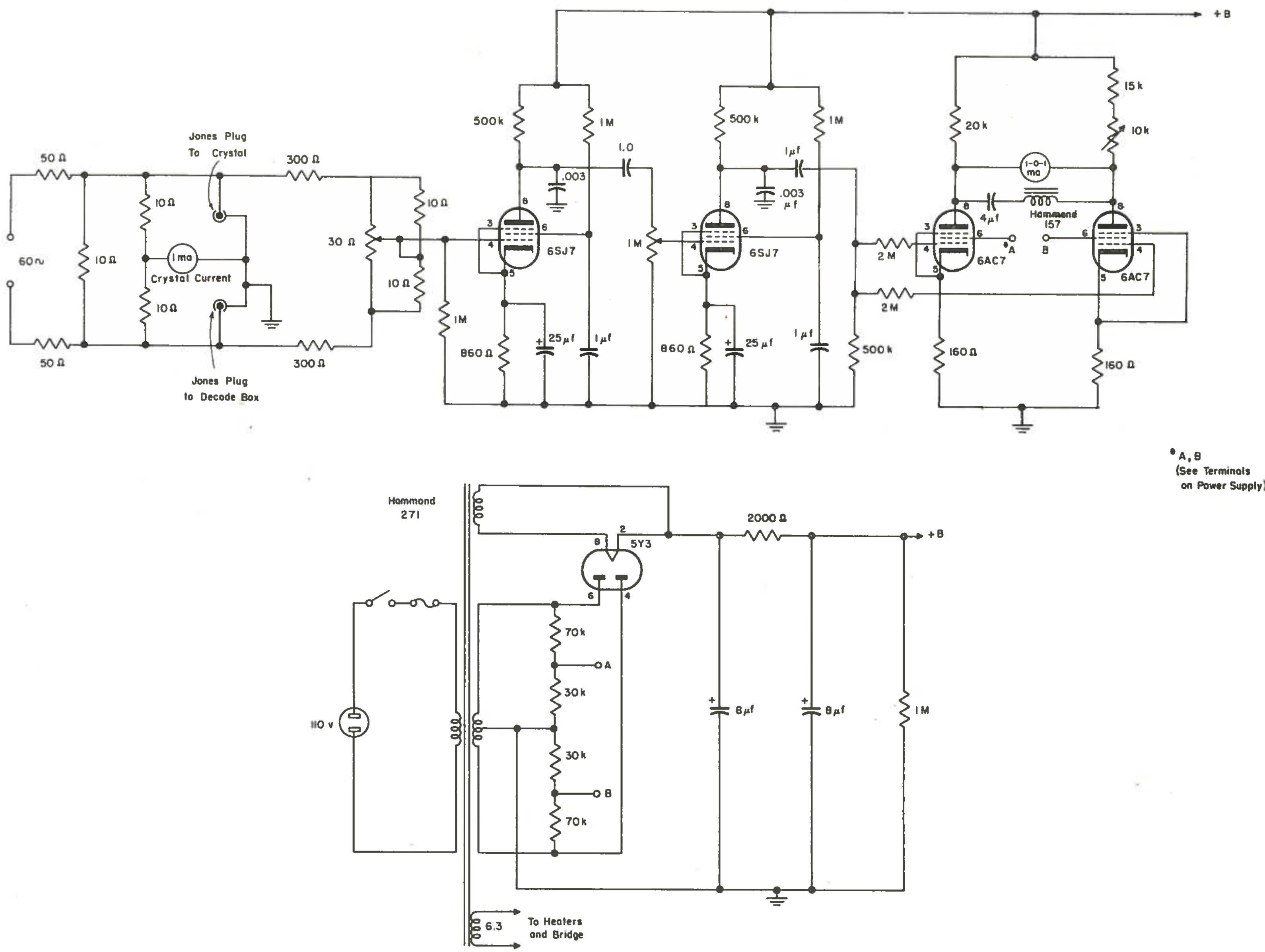

FIG. 2

60-CYCLE BRIDGE AND AMPLIFIER

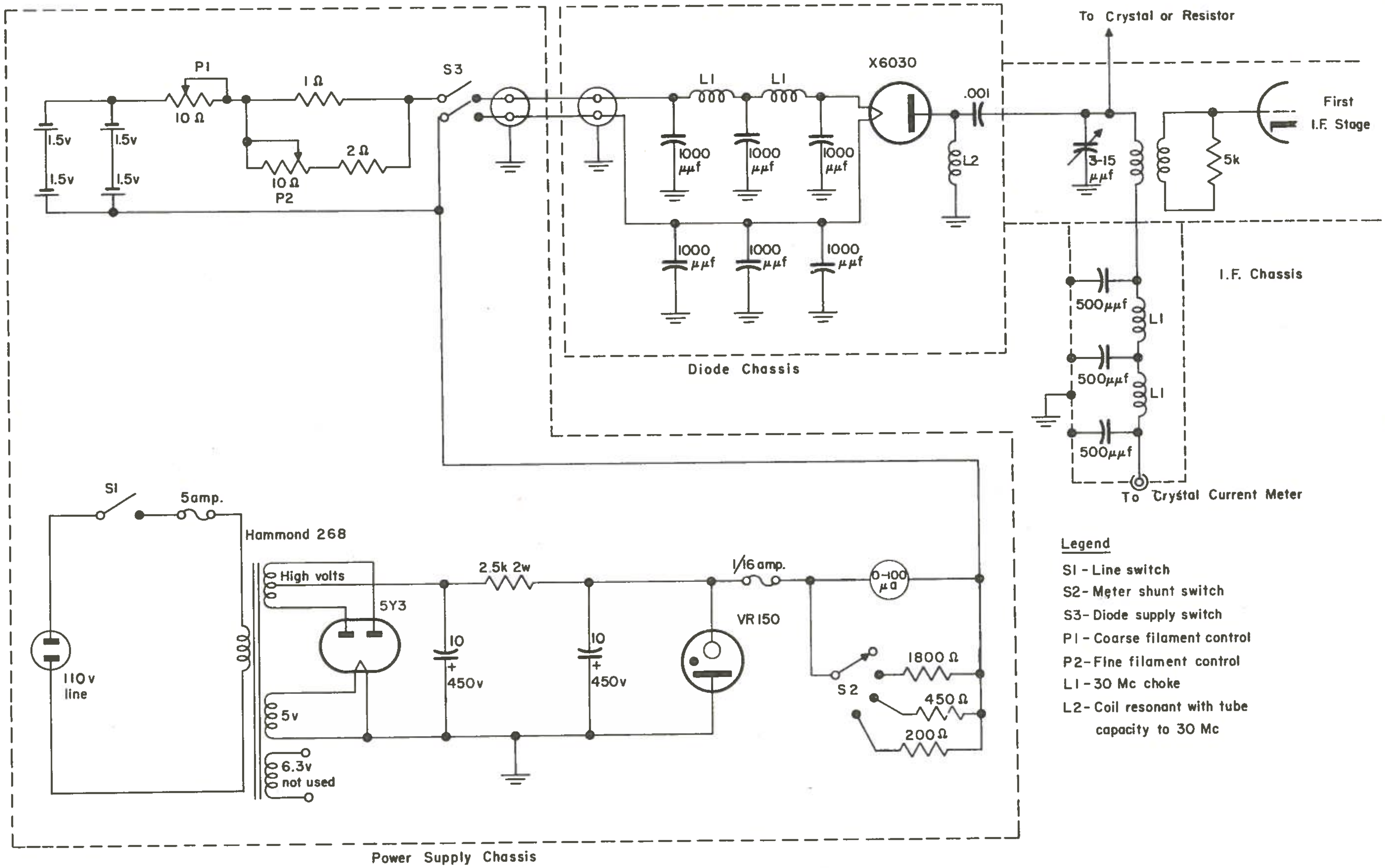

FIG. 3

NOISE DIODE CIRCUIT WITH ASSOCIATED POWER SUPPLIES

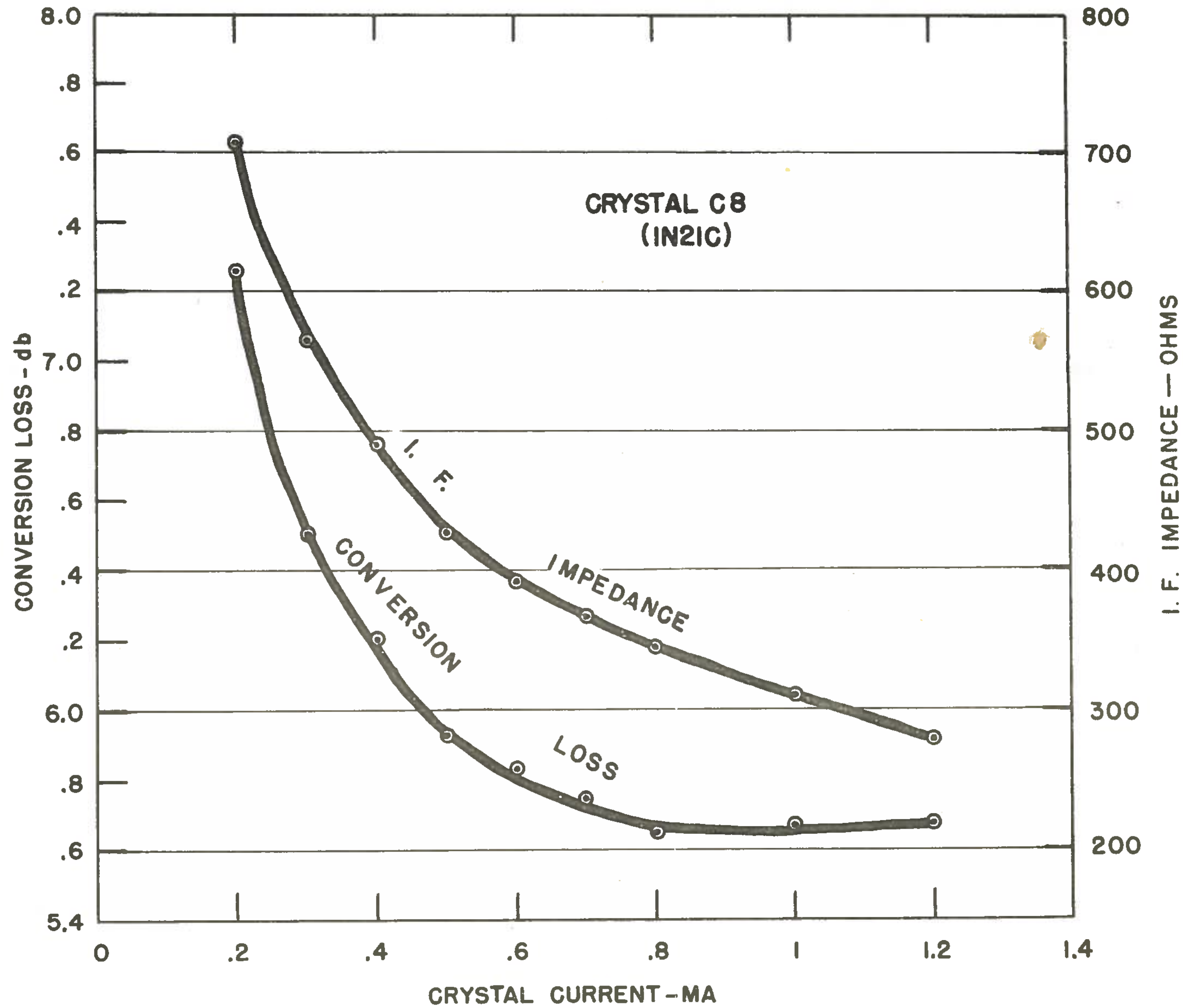

FIG. 4
CONVERSION LOSS AND I.F. IMPEDANCE
AS A FUNCTION OF RECTIFIED CRYSTAL CURRENT

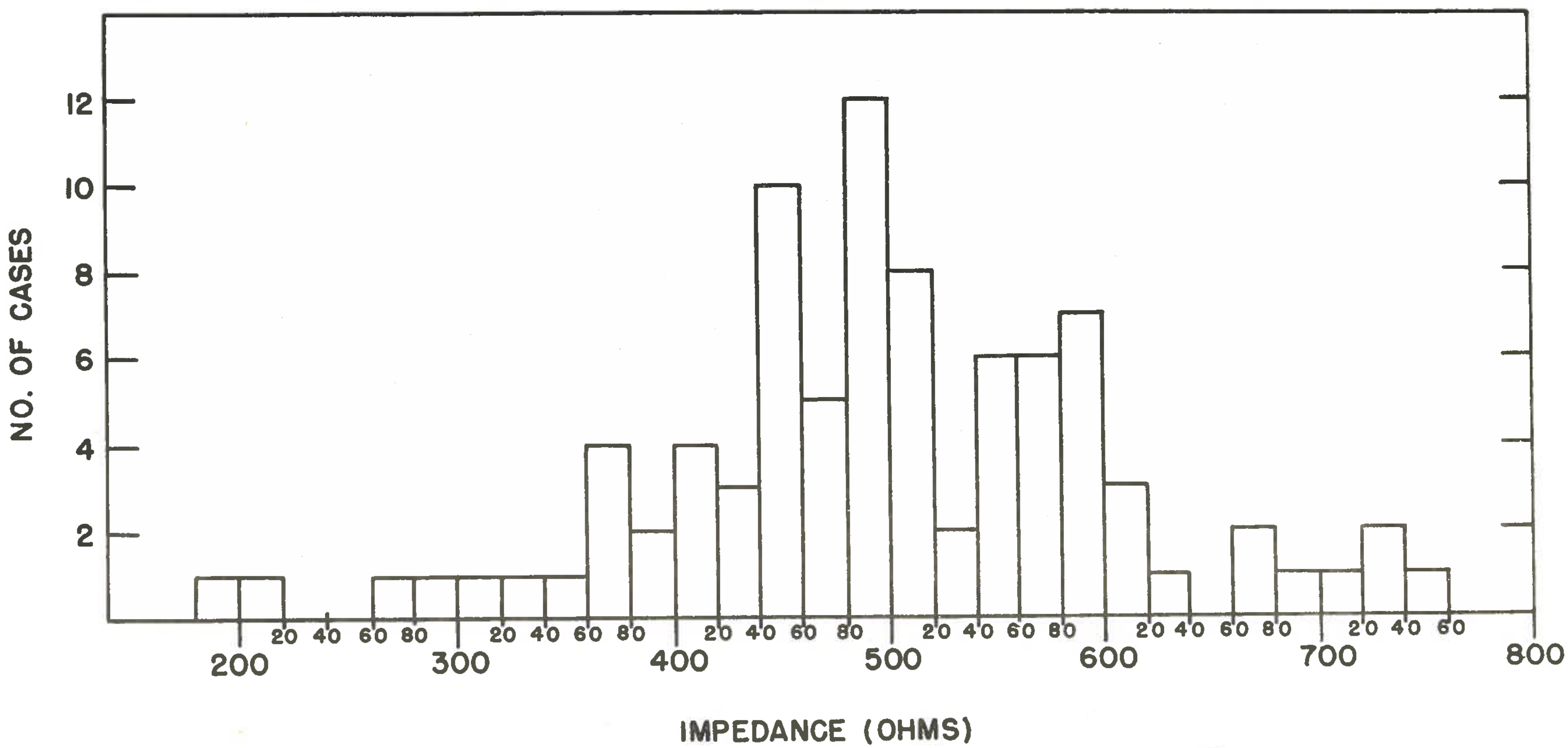

FIG. 5

RANGE OF I.F. IMPEDANCES OF 87 CRYSTALS MEASURED AT 10.7 CENTIMETERS (INCLUDING TYPES IN21, IN21B, IN23B AND IN21C)

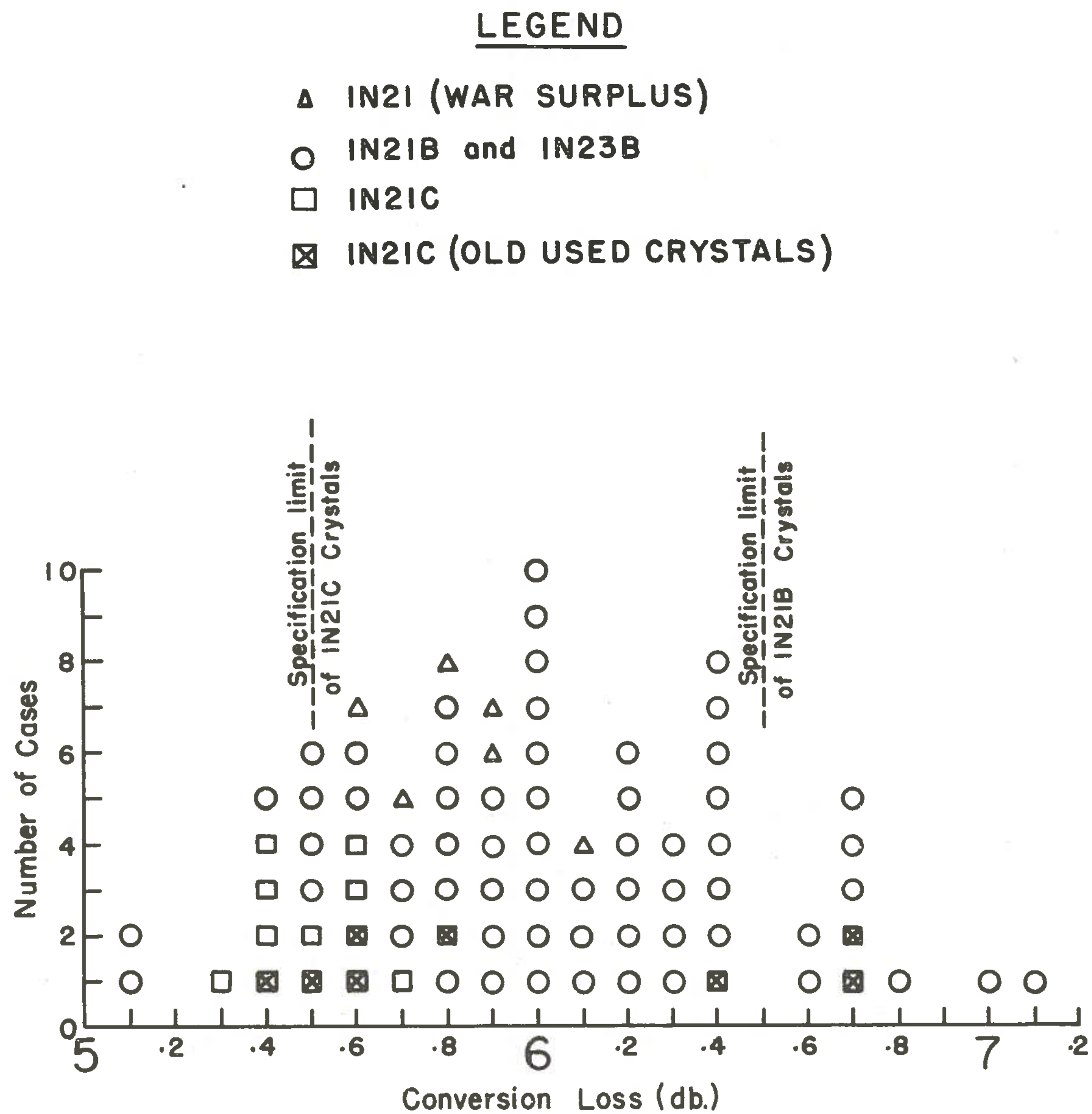

FIG. 6
CONVERSION LOSS OF CRYSTALS MEASURED AT 10.7 CENTIMETERS