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DYNAMIC CHARACTERISTICS OF THE DEGAUSSING FIELD OF HOLLOW FERROMAGNETIC CYLINDERS AND SPHERES

O. PETERSONS

OTTAWA

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ABSTRACT

Dynamic characteristics of the degaussing field of hollow ferromagnetic cylinders and spheres with thin walls have been investigated. Dynamic effects in tilt correction of the degaussing field of large objects, such as hulls of steel ships, may be important, while in small objects they can be neglected.

In thin-walled bodies where the induced flux is small as compared with the exciting flux, the degaussing requirements can be determined by neglecting the effect of iron on the degaussing field. Thick-walled bodies behave as solid bodies with respect to the dynamic effects in the degaussing field.

The results given in this report are also applicable to structures made of non-magnetic conductive materials in evaluation of eddy-current fields.

CONTENTS

FIGURES

LIST OF SYMBOLS

a	Outside radius of the cylinder or sphere (meters)
A	Cross-sectional area of the cylinder or sphere (meters^2)
k	Relative permeability of the material
k_e	Effective permeability of the body
z	$= a\left(\frac{k\mu_o\omega}{\rho}\right)^{\frac{1}{2}}$
DGR	Degaussing Ratio (complex ratio of the degaussing current required for the tilt correction under dynamic conditions to the same current required under static conditions)
δ	Wall thickness of the cylinder or sphere expressed as a fraction of the radius (a)
μ_o	Permeability of free space in mks units = $4\pi \times 10^{-7}$ henry/meter
ρ	Resistivity of the material, (ohm-meter)
ϕ_e	Exciting flux (webers)
ϕ_i	Induced flux (webers)
ϕ_t	Total flux (webers)
ω	Angular frequency of the applied field (radians/second)

The symbols listed here do not include those symbols used in the intermediate steps of derivations given in the Appendix.

SUMMARY OF FORMULAE

Degaussing Ratio (DGR)

Hollow cylinders in transverse exciting field:

$$\mathrm{DGR} = \frac{1 + k\delta}{k\delta} \cdot \frac{\left(\frac{k}{z} - j\frac{z}{k}\right) j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})}{1 + \frac{k}{z} j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})} \; .$$

Hollow cylinders in longitudinal exciting field, and hollow spheres:

$$\mathrm{DGR} = \frac{1 + 2k\delta}{2k\delta} \cdot \frac{\left(\frac{2k}{z} - j\frac{z}{2k}\right) j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})}{1 + \frac{2k}{z} j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})} \; .$$

Total Flux

$$\phi_t = k_e \mu_o H_o A \; .$$

Hollow cylinders in transverse exciting field:

$$k_e = \frac{1 + \frac{k}{z} j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})}{1 + \frac{1}{2}\left(\frac{k}{z} + j\frac{z}{k}\right) j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})} \; ,$$

at $\omega = 0$

$$k_e = \frac{1 + k\delta}{1 + \frac{1}{2}k\delta} \; .$$

Hollow cylinders in longitudinal exciting field:

$$k_e = \frac{1 + \frac{2k}{z} j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})}{1 + j\frac{z}{2k} j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})} \; ,$$

at $\omega = 0$

$$k_e = 1 + 2k\delta \; .$$

Hollow spheres:

$$k_e = \frac{1 + \dfrac{2k}{z} j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})}{1 + \frac{1}{3}\left(\dfrac{2k}{z} + j\dfrac{z}{k}\right) j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})} ,$$

at $\omega = 0$

$$k_e = \frac{1 + 2k\delta}{1 + \frac{2}{3}k\delta} .$$

DYNAMIC CHARACTERISTICS OF THE DEGAUSSING FIELD OF HOLLOW FERROMAGNETIC CYLINDERS AND SPHERES

- O. Petersons -

INTRODUCTION

Most degaussing systems on ships are designed to produce the required degaussing effect when the ship is in an even-keel position. During rolling and pitching conditions mismatch occurs between the earth's field and the applied degaussing field. Some high-quality degaussing systems on minesweepers are equipped with tilt correction, which automatically corrects degaussing currents according to the changing field conditions. The degaussing currents required in the tilt correction are measured or calculated assuming static listing of the ship. Under such conditions eddy-current fields, which originate from dynamic effects, are neglected.

If the ferromagnetic bodies under consideration are of relatively small physical size, the eddy-current fields are small and may be neglected. This, however, is not true of larger bodies, such as hulls of steel ships.

In a previous report [1] the dynamic effects were studied in solid ferromagnetic cylinders. In the present report these investigations are extended to hollow cylinders and spheres with thin walls. Ships with steel hulls would approximate such bodies.

THEORETICAL CONSIDERATIONS

a) General

The problem of tilt correction is formulated and definitions are stated in Reference 1. The basic concepts are reviewed briefly here.

It is proved that conditions applying to a body rolling sinusoidally with frequency ω in a static magnetic field can be closely approximated by those resulting from the application of a sinusoidal field to a stationary body. In the analysis presented in this report, stationary bodies with varying fields are considered.

The "degaussing ratio" (DGR) is defined to be the complex ratio of the degaussing current required for tilt correction under dynamic (rolling, pitching) conditions to the same current required under static conditions. The degaussing ratio is related to various components of flux in the body as follows:

$$\mathrm{DGR} = \frac{\left(\frac{\phi_i}{\phi_t}\right)_\omega}{\left(\frac{\phi_i}{\phi_t}\right)_{\omega = 0}} = \frac{1 - \left(\frac{\phi_e}{\phi_t}\right)_\omega}{1 - \left(\frac{\phi_e}{\phi_t}\right)_{\omega = 0}} , \qquad (1)$$

where ϕ_t is the total flux,

ϕ_e is the exciting flux or the flux which would exist if the body were not present,

ϕ_i is the induced flux.

The above components of flux are phasors being related to each other thus:

$$\phi_t = \phi_e + \phi_i \quad . \qquad (2)$$

The DGR of the body depends on its electric and magnetic properties, on its physical dimensions, and on the frequency of the applied field.

b) Degaussing Ratio of Hollow Cylinders and Spheres

The degaussing ratio of thin-walled cylinders with their axes perpendicular to the applied field is given by

$$\mathrm{DGR} = \frac{1 + k\delta}{k\delta} \cdot \frac{\left(\frac{k}{z} - j\frac{z}{k}\right) j^{-\frac{1}{2}} \tanh\left(\delta z j^{\frac{1}{2}}\right)}{1 + \frac{k}{z} j^{-\frac{1}{2}} \tanh\left(\delta z j^{\frac{1}{2}}\right)} , \qquad (3)$$

$$z = a\left(\frac{k\mu_o\omega}{\rho}\right)^{\frac{1}{2}} ,$$

where μ_o is the permeability of free space (rationalized mks units),

k is the relative permeability of the material,

ρ is the resistivity of the material,

a is the radius of the cylinder,

δ is the wall thickness of the cylinder expressed as a fraction of a,

ω is the angular frequency of the applied field.

The derivation of equation (3) is given in the Appendix.

The degaussing ratios calculated by using equation (3) for a cylinder with $k = 200$ and $\delta = 0.1$, 0.01, and 0.001 are plotted on Fig. 1. On the same figure some points are also plotted for a solid cylinder ($\delta = 1$). These have been computed by using formula 23 of Reference 1. It is apparent that the characteristics of the solid cylinder and those of the cylinder with $\delta = 0.1$ are almost identical. This becomes evident by observing that induced fluxes in the two cases at $\omega = 0$

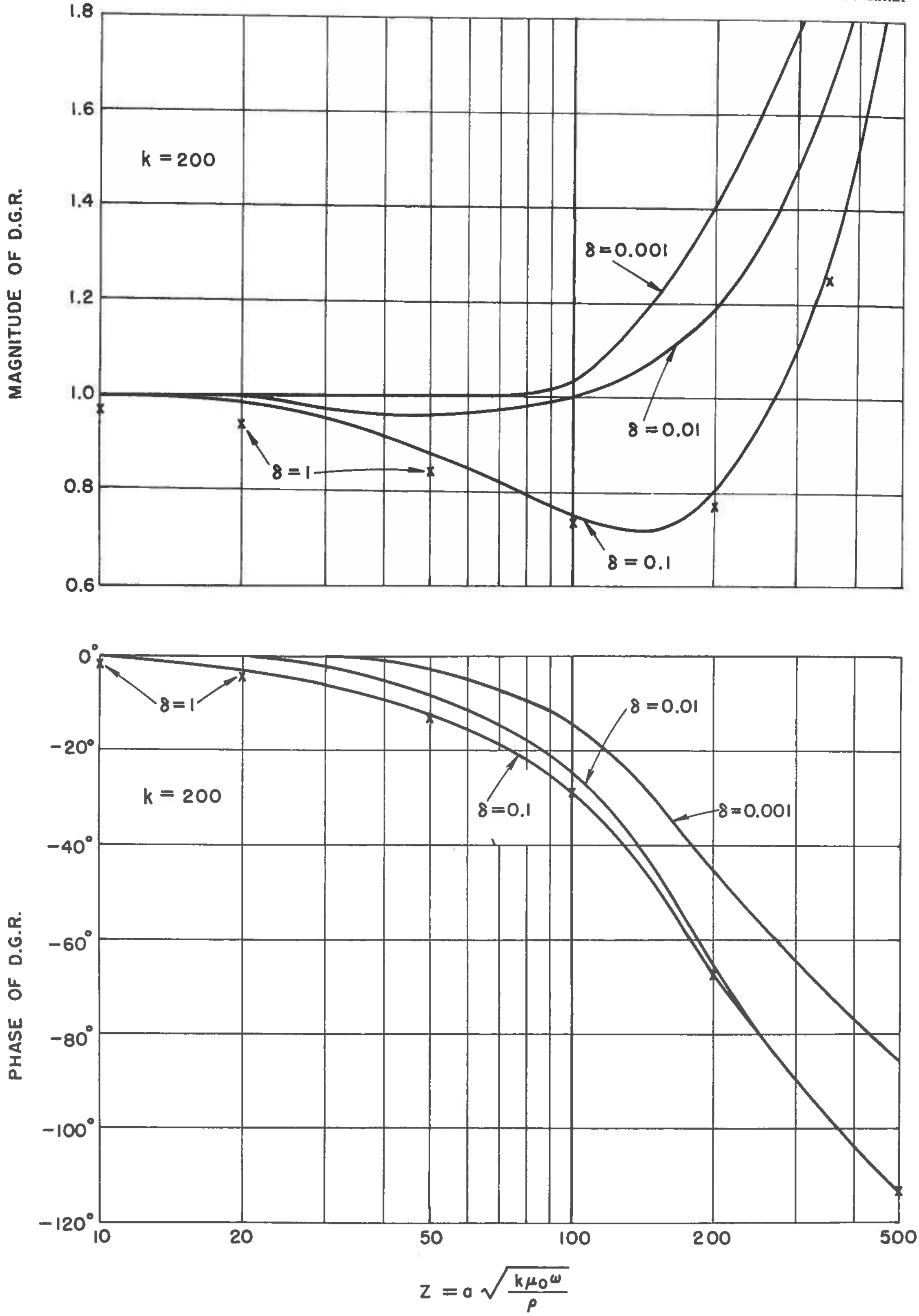

FIG. 1 COMPUTED DEGAUSSING FIELD CHARACTERISTICS FOR FERROMAGNETIC CYLINDERS IN A TRANSVERSE EXCITING FIELD

are almost equal (see formulae A-7, A-9, and Reference 2). At alternating fields when z is large enough for the degaussing ratio to differ significantly from unity, most of the flux is concentrated in the outer shell, and the two cases are again identical.

In this report rationalized mks units are used, while in Reference 1 emu units are employed. The graphical representations of DGR in both cases include the dimensionless, independent variable z. Thus, they are directly comparable.

The degaussing ratio of infinitely long thin-walled cylinders with their axes parallel to the applied magnetic field is given by

$$\mathrm{DGR} = \frac{1 + 2k\delta}{2k\delta} \cdot \frac{\left(\frac{2k}{z} - j\frac{z}{2k}\right) j^{-\frac{1}{2}} \tanh\left(\delta z j^{\frac{1}{2}}\right)}{1 + \frac{2k}{z} j^{-\frac{1}{2}} \tanh\left(\delta z j^{\frac{1}{2}}\right)} . \qquad (4)$$

The derivation of the above equation is given in the Appendix. The formula (4) is also the expression of the degaussing ratio for hollow ferromagnetic spheres.

Calculated degaussing ratios for a cylinder or sphere with $k = 200$ and $\delta = 0.01$ and 0.001 are given in Fig. 2.

c) Matching of Degaussing Coils

Degaussing of a body is perfect when at every point in the body the induced flux is reduced to zero. The degaussing coil system is then perfectly matched to the body to be degaussed.

In practice, perfect degaussing is seldom possible. Degaussing is accomplished by reducing the induced flux to zero over a certain area with concentrated degaussing coils. As an example, coils are placed around whole ships. Sometimes perfect degaussing is theoretically possible, as in the case of an infinitely long solid cylinder with field parallel to its axis, which can be degaussed at static fields with a solenoidal degaussing coil. However, the degaussing will cease to be perfect when the applied field is varying, since the field inside the cylinder then varies with the radius. Degaussing can be applied so as to reduce the total induced flux over the whole cross section of the cylinder to zero. Hollow cylinders in the same field conditions can only be perfectly degaussed with two degaussing coils, one outside and one inside the cylinder. In practice, such cylinders are usually degaussed with coils on the outside only. Under these circumstances perfect degaussing is not achieved either in static or dynamic conditions. However, the induced flux over the whole cross section of the cylinder can be reduced to zero in either case. The expression of the degaussing ratio will be derived for such considerations.

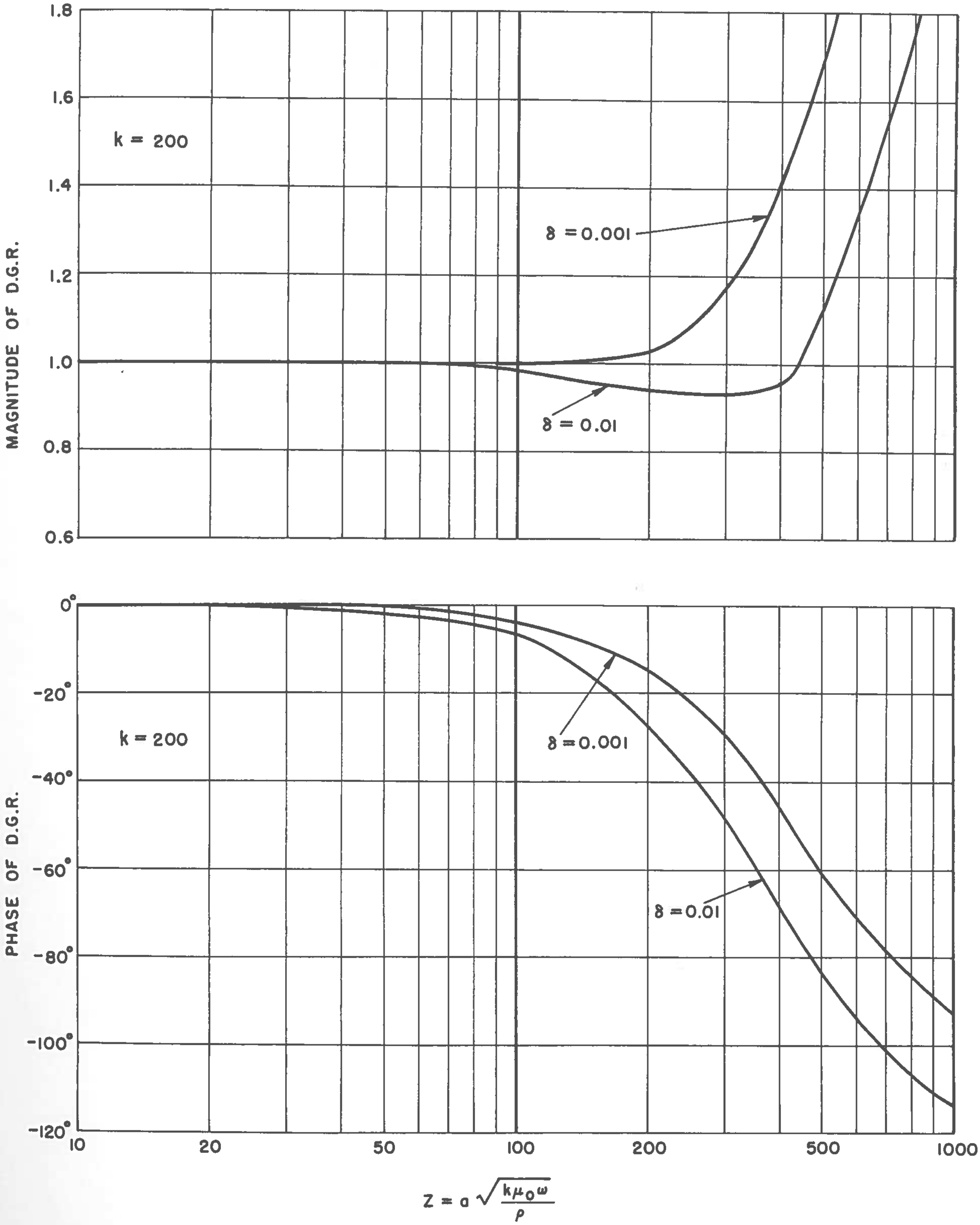

FIG. 2 COMPUTED DEGAUSSING FIELD CHARACTERISTICS FOR FERROMAGNETIC CYLINDERS IN A LONGITUDINAL EXCITING FIELD

Similarly, perfect degaussing of cylinders in a transverse exciting field cannot be achieved with concentrated degaussing coils. The degaussing requirements are calculated to reduce the induced flux to zero in the axial plane of the cylinder normal to the exciting field. In the case of spheres, the induced field is reduced to zero in the diametrical plane normal to the exciting field.

d) Degaussing of Thin-walled Bodies

In thin-walled bodies with finite permeabilities the induced flux may be much smaller as compared with the exciting flux. For such structures the degaussing requirements may be calculated by neglecting the presence of iron on the degaussing field. This principle is applicable to static as well as dynamic exciting fields.

The total flux in the body which is placed in a uniform exciting field is given by

$$\phi_t = k_e \mu_o H_o A \quad , \tag{5}$$

where H_o is the strength of the exciting field,
A is the area of the plane in the body in which the flux is considered.

The term $\mu_o H_o A$ is the exciting flux. Since the presence of the body increases the flux by the factor k_e, it can be considered as an effective relative permeability of that body under particular conditions. Further discussion on effective permeabilities is given in the Appendix.

From the above considerations, $\phi_t = k_e \phi_e$.

The induced flux becomes $\phi_i = \phi_t - \phi_e = \phi_e (k_e - 1)$.

To degauss the body the induced flux must be counteracted by the degaussing flux (ϕ_d) .

$$\therefore \phi_d = -\phi_i \quad .$$

Provided the degaussing field (H_d) can be matched to the exciting field its value is given by

$$H_d = \frac{\phi_d}{k_e \mu_o A}$$

$$= -H_o \left(1 - \frac{1}{k_e}\right) \quad . \tag{6}$$

To calculate H_d by neglecting the presence of iron ($k_e = 1$)

$$H_d = \frac{\phi_d}{\mu_o A} = -H_o(k_e - 1) \quad . \tag{7}$$

As an example, it is required to calculate the degaussing field for a cylinder with $\delta = 0.001$ and $k = 100$ in a transverse exciting field. For this cylinder, under static field conditions, $k_e = 1.048$. The degaussing fields calculated by using equations (6) and (7) differ by approximately 5%.

Since the degaussing requirements can be designed by neglecting the presence of iron, the location of coils can be such that the induced flux caused by the degaussing field is opposite to the exciting flux, as would be obtained by placing the degaussing coils inside the cylinder to be degaussed.

The principle discussed previously is also applicable to the tilt effect correction. The degaussing ratio is defined as

$$DGR = \frac{\left(\frac{\phi_i}{\phi_t}\right)_\omega}{\left(\frac{\phi_i}{\phi_t}\right)_{\omega=0}} \quad .$$

The changes due to dynamic effects, although being significant in ϕ_i, may be neglected in ϕ_t. Thus

$$(\phi_t)_\omega = (\phi_t)_{\omega=0} \quad ,$$

and

$$DGR = \frac{(\phi_i)_\omega}{(\phi_i)_{\omega=0}} \quad . \tag{8}$$

This simplified definition of degaussing ratio does not lead to simpler mathematical expressions. However, equation (8) shows that the frequency characteristics of the degaussing field are the same as those of the induced field, and therefore can be measured directly using the established techniques (References 3 and 4).

e) Example

To illustrate the importance of dynamic effects in tilt correction, consider the following example. Given a cylinder with

$$\text{radius } (a) = 20' \quad ,$$

$$\text{wall thickness} = 0.25'' \quad .$$

It is subjected to an oscillating magnetic field having a frequency of 0.1 cps. It is required to find the degaussing ratio for this cylinder in transverse and longitudinal exciting fields. Assume that the material has the following properties:

$$k = 200 \ ,$$

$$\rho = 0.10 \ \mu\text{ohm-meter} \ .$$

Then,

$$\left(\frac{k\mu_o\omega}{\rho}\right)^{\frac{1}{2}} = 39.7 \ \frac{1}{m} = 1.01 \ \frac{1}{in} \ ,$$

$$z = 242 \ ,$$

$$\delta z = 0.252 \ ,$$

$$j^{-\frac{1}{2}}\tanh\left(\delta z j^{\frac{1}{2}}\right) \cong 0.252 \ ,$$

$$k\delta = 0.208 \ .$$

For the transverse direction

$$DGR = 1.78\underline{/-55.6°} \ .$$

For the longitudinal direction

$$DGR = 1.06\underline{/-20.2°} \ .$$

Next, consider a cylinder made of similar material, but with a radius of 2'.

$$z = 24.2 \ ,$$

$$\delta z = 0.252 \ ,$$

$$k\delta = 2.08 \ .$$

For an exciting field in the transverse direction

$$DGR = 1\underline{/-0.8°} \ ,$$

and in the longitudinal direction

$$DGR = 1\underline{/-0.2°} \ .$$

The above results indicate the significance of the physical size of a body on the degaussing characteristics. Dynamic effects are negligible in small bodies, but become quite important in relatively large structures.

f) Applications to Non-magnetic Bodies

The results presented in this report are also applicable to non-magnetic bodies. In this case the induced flux in the structure is caused entirely by dynamic effects. Since degaussing is not needed at static conditions, the concept of degaussing ratio cannot be used. However, the formulae governing the fluxes in the cylinders derived in the Appendix are equally well applicable to non-magnetic structures.

EXPERIMENTAL DATA

The degaussing ratio of two hollow cylinders was determined experimentally. The dimensions of the cylinders and properties of the materials are as follows:

Cylinder No. 1

length = 24" ,

radius (a) = 2" ,

wall thickness = 0.125" ,

δ = 0.0625 ,

k = 107 ,

ρ = 0.180 μohm-meter .

Cylinder No. 2

length = 24" ,

radius (a) = 2" ,

wall thickness = 0.025" ,

δ = 0.0125 ,

k = 182 ,

ρ = 0.142 μohm-meter .

The measurement technique was the same as that described in Reference 1. Uniform field was provided by a large Helmholtz coil, in which the cylinders were placed.

The degaussing ratio is given by

$$\mathrm{DGR} = \frac{1 - \left(\dfrac{\phi_e}{\phi_t}\right)_{\omega}}{1 - \left(\dfrac{\phi_e}{\phi_t}\right)_{\omega = 0}} .$$

The total flux in the cylinder (ϕ_t) is determined by measuring the induced voltage in a pickup coil placed around the cylinder. The corresponding exciting flux (ϕ_e) can be measured by removing the cylinder but leaving the coil intact. The ratio of the two voltages is equal to the corresponding flux ratio. In order to avoid moving the cylinder at each measurement point, a second reference coil was used for detecting the exciting flux. Initially, with the cylinder removed, the reference coil was adjusted to detect the same voltage as the pick-up coil.

The eddy-current effects in the cylinders became negligible at low frequencies, and therefore the measurement at 10 cps was considered to be equivalent to the d-c measurement.

a) Cylinders in a Transverse Exciting Field

The degaussing ratio of both cylinders in a transverse exciting field was measured. The theoretical analysis presented in this report is applicable to infinitely long cylinders, while the samples on which measurements were done could not be considered as such, especially with respect to eddy currents in them. A closer equivalent to infinite cylinders can be obtained by short-circuiting the ends of the cylinders. In some tests this was done by soldering copper end plates to the cylinders. To illustrate the effect of discontinuities in eddy-current paths, cylinder No. 2 was cut in the middle perpendicular to its axis, and the degaussing ratio was measured.

The experimental results are plotted in Figs. 3 and 4. In order to make these results readily applicable to other cylinders, the dimensionless quantity $z = a\left(\frac{k\mu_0\omega}{\rho}\right)^{\frac{1}{2}}$ is used as the independent variable in these plots, k and δ being parameters. The curves computed by using equation (3) are included in Figs. 3 and 4.

From the above curves it is observed that experimental results on cylinders with end plates follow the computed values closely. In cylinders with open ends, eddy-current effects are less pronounced; they become significant at higher frequencies. Discontinuities in eddy-current paths further minimize their effects, as seen from Fig. 4.

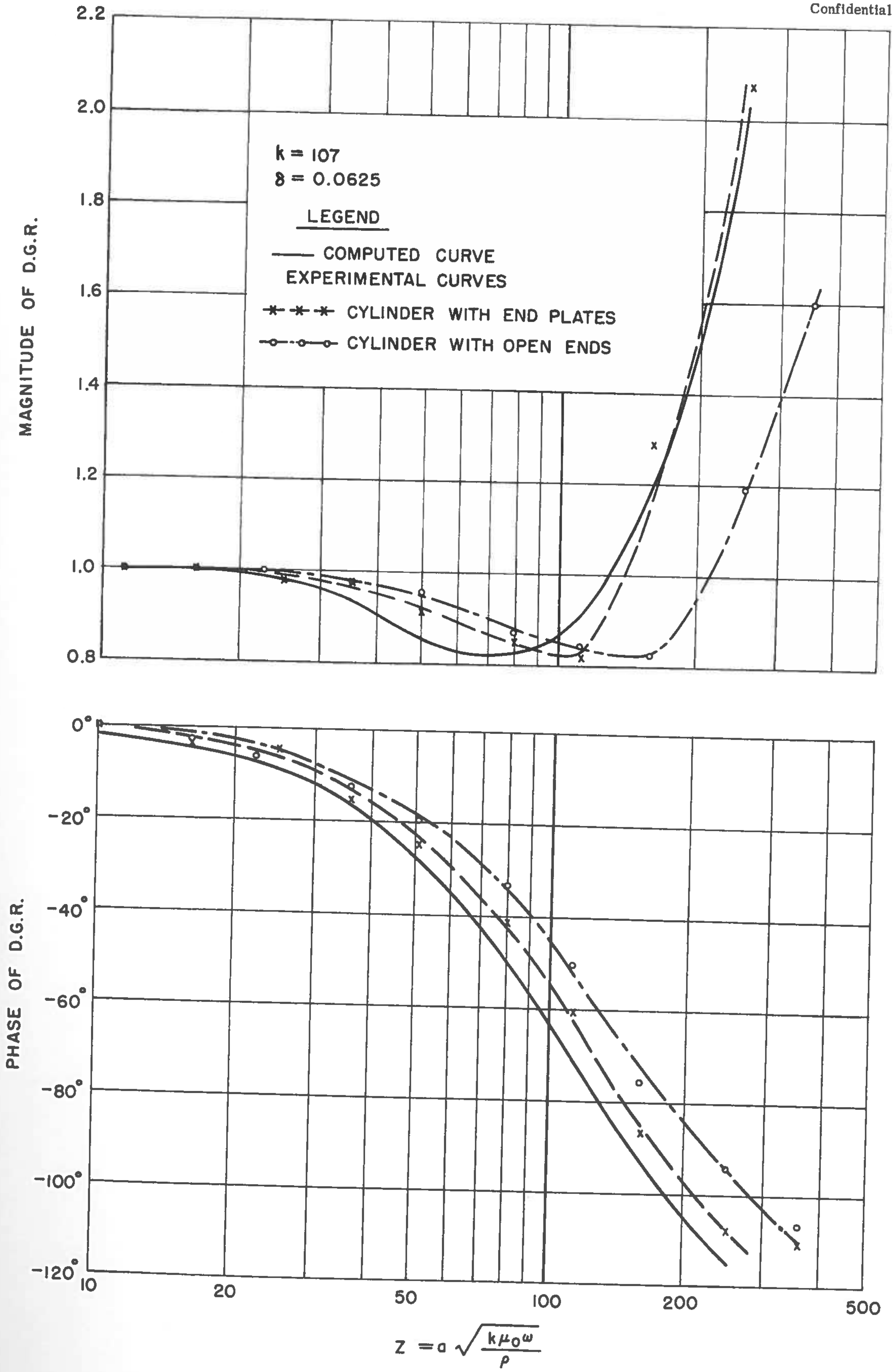

FIG. 3 DEGAUSSING FIELD CHARACTERISTICS FOR A HOLLOW FERROMAGNETIC CYLINDER (k = 107, δ = 0.0625), TRANSVERSE EXCITING FIELD

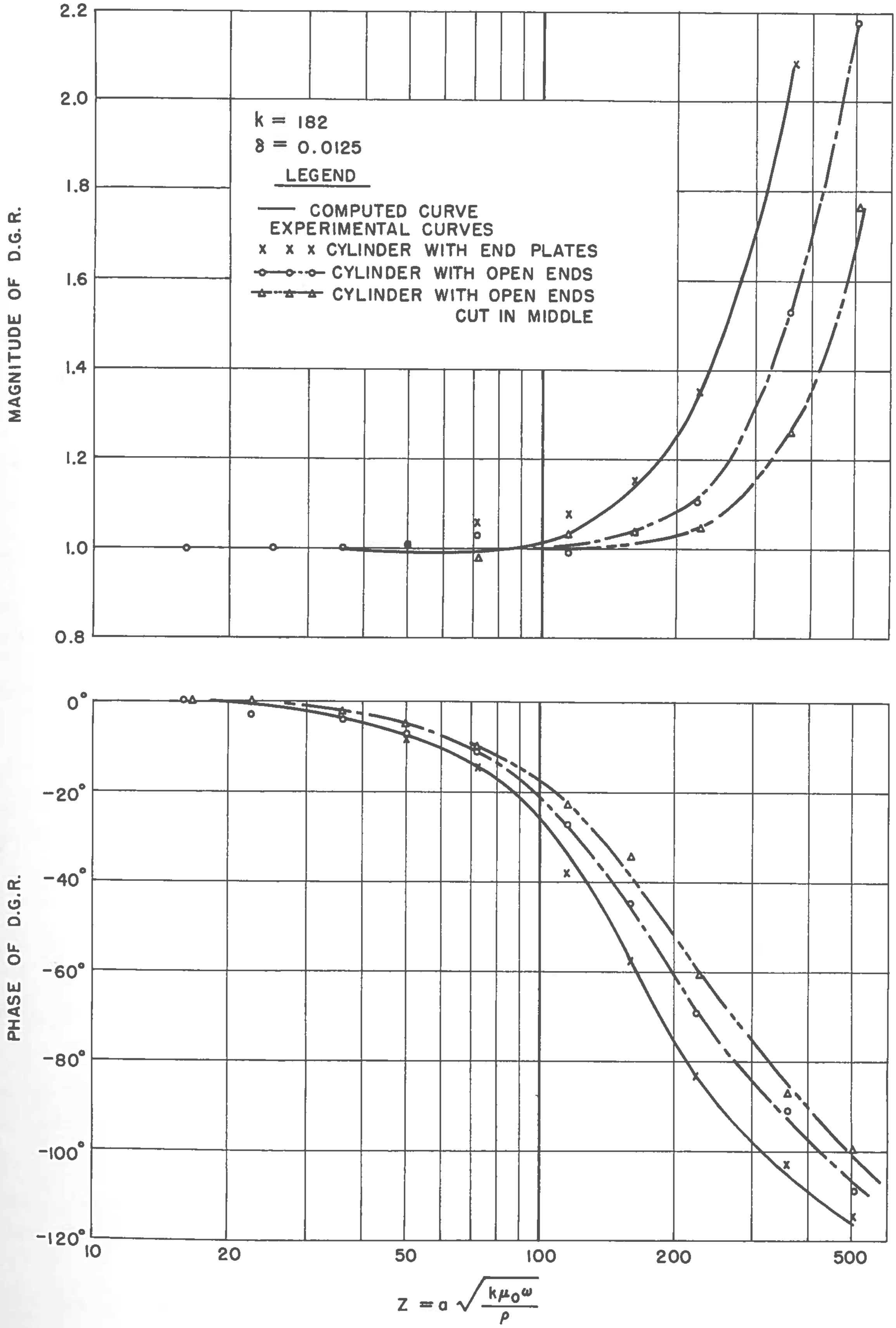

FIG. 4 DEGAUSSING FIELD CHARACTERISTICS FOR A HOLLOW FERROMAGNETIC CYLINDER (k = 182, δ = 0.0125), TRANSVERSE EXCITING FIELD

b) Cylinders in a Longitudinal Exciting Field

The degaussing ratio of cylinder No. 1, having its original length, and of a sample of the same cylinder having a length of 4" was measured in a longitudinal exciting field. The results of these measurements, and the results computed using equation (4), are plotted in Fig. 5.

The eddy currents in this case are along the circumference of the cylinder and end plates are not required. Since the expression for the degaussing ratio is the same for cylinders and for spheres, it is to be expected that measurement results should be relatively independent of the length of the cylinder.

c) Permeability Measurements

The permeability of the materials tabulated above was obtained from the measurements of $k_e = \left(\frac{\phi_t}{\phi_e}\right)$ at 10 cycles per second using a transverse exciting field. Permeability is calculated by using formula (A-9) given in the Appendix. As a check of the above measurement, the permeability was also determined from the attenuation properties of the cylinders. For static magnetic fields (transverse) the ratio of the undisturbed field outside the cylinder to the field inside is given in Reference 5 by

$$1 + \tfrac{1}{2}k\delta \quad .$$

As before, a magnetic field alternating at 10 cycles per second is considered to be equivalent to a static field. The permeabilities as determined by this method are:

Cylinder No. 1 — k = 103 ,

Cylinder No. 2 — k = 163 .

CONCLUSIONS

The foregoing investigation indicates that if tilt correction is introduced in degaussing systems of large ferromagnetic bodies, such as hulls of steel ships, consideration must be given to the dynamic (eddy current) effects. In the example given in this report it is shown that in a ferromagnetic body whose dimensions approach those of an actual ship, the degaussing ratio differs significantly from unity, both in magnitude and phase. In smaller ferromagnetic bodies, the dynamic effects are negligible.

Many ferromagnetic bodies can be approximated by cylinders or spheres, and an estimate of the dynamic effects obtained.

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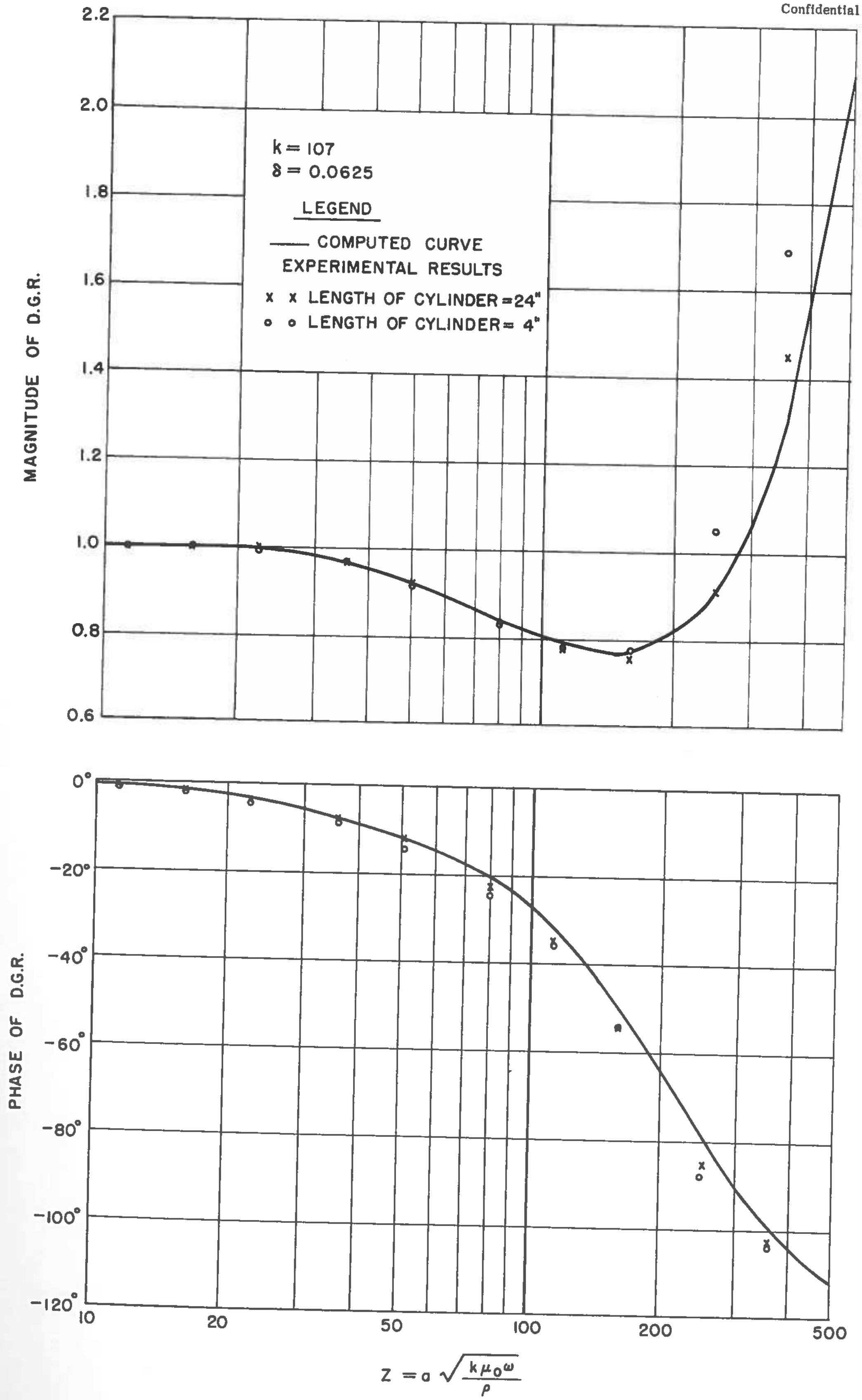

FIG. 5 DEGAUSSING FIELD CHARACTERISTICS FOR A HOLLOW FERROMAGNETIC CYLINDER (k = 107, δ = 0.0625), LONGITUDINAL EXCITING FIELD

In cylindrical structures, the transverse field is the most important one for eddy-current effects.

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APPENDIX

DERIVATION OF THE DEGAUSSING RATIO FOR HOLLOW FERROMAGNETIC CYLINDERS

The general equations governing electric and magnetic fields in conductive media are derived in Reference 5 (p. 6). Sinusoidally varying fields can be represented by the following Poisson's equations:

$$jp\vec{E} = \nabla^2 \vec{E} \quad , \qquad \text{(A-1)}$$

$$jp\vec{H} = \nabla^2 \vec{H} \quad , \qquad \text{(A-2)}$$

where $p = \dfrac{k\mu_o \omega}{\rho}$,

k is the relative permeability of the medium,

μ_o is the permeability of free space,

ρ is the resistivity of medium,

$\vec{E}$ is the electric field strength (vector),

$\vec{H}$ is the magnetic field strength (vector),

∇^2 is the Laplacian Operator.

a) EXCITING FIELD PERPENDICULAR TO THE AXIS OF THE CYLINDER

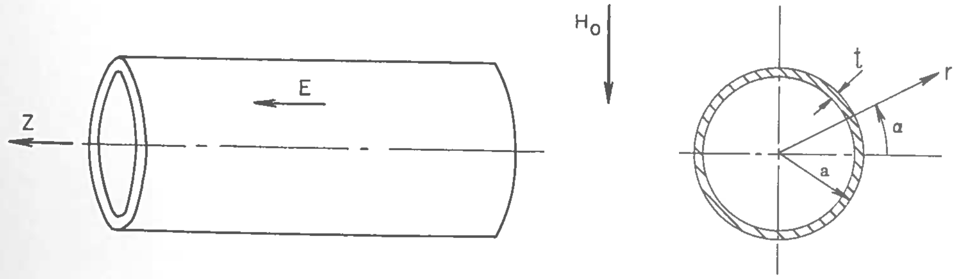

FIG. 6 CYLINDER IN A TRANSVERSE EXCITING FIELD

Consider a hollow cylinder, as shown in Fig. 6, and assume that this cylinder is very long. The equations of the electric and magnetic fields for this case are derived in Reference 5, pp. 78-80. The following is the derivation of the degauss -ing ratio.

The electric field inside the cylinder walls is parallel to the axis of the cylinder and is given by

$$E = (Ae^{\sqrt{jp}\, r} + Ce^{-\sqrt{jp}\, r})\cos\alpha \ , \qquad (A\text{-}3)$$

from which the component of magnetic field in the α direction is found to be

$$H_\alpha = \frac{\sqrt{jp}}{j\omega k\mu_o}(Ae^{\sqrt{jp}\, r} - Ce^{-\sqrt{jp}\, r})\cos\alpha \ . \qquad (A\text{-}4)$$

A and C are constants.

Inside the cylinder $(r \leqslant a - t)$

$$H_\alpha = QH_o \cos\alpha \ ,$$

where H_o is the undisturbed magnetic field,

$$Q = \frac{1}{\cosh(\delta z j^{\frac{1}{2}}) + \frac{1}{2}\left(\frac{k}{z} + j\frac{z}{k}\right) j^{-\frac{1}{2}} \sinh(\delta z j^{\frac{1}{2}})} \ ,$$

$$z = ap^{\frac{1}{2}} \ ,$$

$$\delta = \frac{t}{a} \ .$$

The constants in (A-3) and (A-4) have values:

$$A = \tfrac{1}{2}QH_o \frac{j\omega\mu_o a}{e^{\sqrt{jp}(a-t)}}\left(1 + \frac{k}{z\sqrt{j}}\right) \ ,$$

$$C = \tfrac{1}{2}QH_o \frac{j\omega\mu_o a}{e^{-\sqrt{jp}(a-t)}}\left(1 - \frac{k}{z\sqrt{j}}\right) \ .$$

For calculating the degaussing ratio we are interested in the total flux crossing the axial plane perpendicular to the applied field. In this plane the magnetic field can be described by H_α at $\alpha = 0$.

The flux density B inside the iron becomes

$$B = \frac{\sqrt{jp}}{j\omega}(Ae^{\sqrt{jp}\, r} - Ce^{-\sqrt{jp}\, r}) \ ,$$

and flux (ϕ) in the iron per unit length of the cylinder is

$$\phi = 2\int_{a-t}^{a} B dr$$

$$= 2\mu_0 H_0 a \frac{\cosh(\delta z j^{\frac{1}{2}}) + \frac{k}{z j^{\frac{1}{2}}} \sinh(\delta z j^{\frac{1}{2}}) - 1}{\cosh(\delta z j^{\frac{1}{2}}) + \frac{1}{2}\left(\frac{k}{z} + j\frac{z}{k}\right) j^{-\frac{1}{2}} \sinh(\delta z j^{\frac{1}{2}})} \quad . \qquad (A-5)$$

Inside the cylinder,

$$B = \mu_0 Q H_0 \ ,$$

$$\phi = 2\int_{0}^{a} B dr \ ,$$

$$= 2\mu_0 H_0 a \frac{1}{\cosh(\delta z j^{\frac{1}{2}}) + \frac{1}{2}\left(\frac{k}{z} + j\frac{z}{k}\right) j^{-\frac{1}{2}} \sinh(\delta z j^{\frac{1}{2}})} \ , \qquad (A-6)$$

Since $t \ll a$, the limit of integration in deriving (A-6) is taken as a, instead of (a - t).

The total flux per unit length of the cylinder (ϕ_t) is the sum of the fluxes in the iron and inside the cylinder.

$$\therefore \ \phi_t = 2\mu_0 H_0 a \ . \ \frac{1 + \frac{k}{z} j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})}{1 + \frac{1}{2}\left(\frac{k}{z} + j\frac{z}{k}\right) j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})} \ ,$$

$$= 2 k_e \mu_0 H_0 a \ , \qquad (A-7)$$

where k_e can be considered as the effective permeability of the body.

$$k_e = \frac{1 + \frac{k}{z} j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})}{1 + \frac{1}{2}\left(\frac{k}{z} + j\frac{z}{k}\right) j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})} \ ; \qquad (A-8)$$

at $\omega = 0$

$$k_e = \frac{1 + k\delta}{1 + \frac{1}{2}k\delta} \quad . \tag{A-9}$$

The exciting flux per unit length of the cylinder is

$$\phi_e = \mu_o H_o a \quad ,$$

$$\therefore \quad 1 - \left(\frac{\phi_e}{\phi_t}\right)_\omega = \frac{\frac{1}{2}\left(\frac{k}{z} - j\frac{z}{k}\right) j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})}{1 + j\frac{k}{z} j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})} \quad .$$

$$1 - \left(\frac{\phi_e}{\phi_t}\right)_{\omega = 0} = \frac{\frac{1}{2}k\delta}{1 + k\delta} \quad .$$

$$\therefore \; DGR = \frac{1 - \left(\frac{\phi_e}{\phi_t}\right)_\omega}{1 - \left(\frac{\phi_e}{\phi_t}\right)_{\omega = 0}} = \frac{1 + k\delta}{k\delta} \cdot \frac{\left(\frac{k}{z} - j\frac{z}{k}\right) j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})}{1 + \frac{k}{z} j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})} \quad . \tag{A-10}$$

a) EXCITING FIELD PARALLEL WITH THE AXIS OF THE CYLINDER

Consider the cylinder shown in Fig. 7. The general equations for the electric and magnetic fields are given in Reference 5, pp. 77-78.

Inside the iron the magnetic field is parallel to the axis of the cylinder and is given by

$$H = Ae^{\sqrt{jp}\, r} + Ce^{-\sqrt{jp}\, r} \quad , \tag{A-11}$$

where A and C are constants.

Inside the cylinder,

$$H = QH_o \quad , \tag{A-12}$$

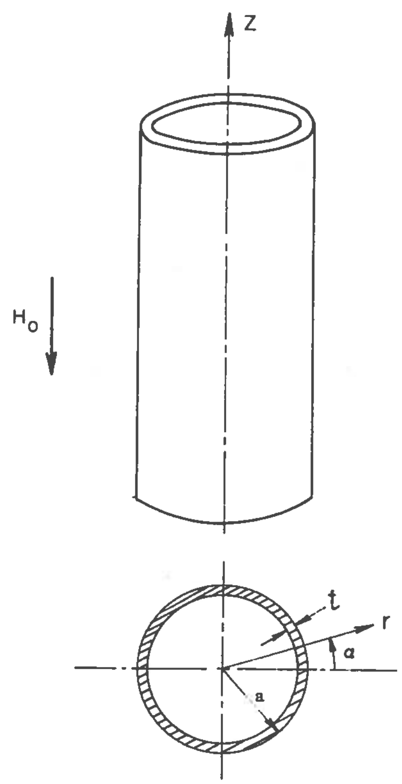

FIG. 7 CYLINDER IN A LONGITUDINAL EXCITING FIELD

where H_o is the undisturbed magnetic field and

$$Q = \frac{1}{\cosh(\delta z j^{\frac{1}{2}}) + \frac{z j^{\frac{1}{2}}}{2k} \sinh(\delta z j^{\frac{1}{2}})} .$$

The values of the constants in (A-11) are:

$$A = \tfrac{1}{2} Q H_o \left(\frac{1 + \frac{z j^{\frac{1}{2}}}{2k}}{e^{\sqrt{jp}(a - t)}} \right) ,$$

$$C = \tfrac{1}{2}QH_o \left(\frac{1 - \dfrac{zj^{\frac{1}{2}}}{2k}}{e^{-\sqrt{jp}\,(a - t)}} \right) .$$

The flux in the iron is

$$\phi = 2\pi a \int_{a-t}^{a} B dr$$

$$= \mu_o H_o \pi a^2 \left(\frac{\cosh(\delta z j^{\frac{1}{2}}) + \dfrac{2k}{zj^{\frac{1}{2}}} \sinh(\delta z j^{\frac{1}{2}}) - 1}{\cosh(\delta z j^{\frac{1}{2}}) + \dfrac{zj^{\frac{1}{2}}}{2k} \sinh(\delta z j^{\frac{1}{2}})} \right) \qquad \text{(A-13)}$$

and the flux inside the cylinder

$$\phi = B\pi a^2$$

$$= \frac{\mu_o H_o \pi a^2}{\cosh(\delta z j^{\frac{1}{2}}) + \dfrac{zj^{\frac{1}{2}}}{2k} \sinh(\delta z j^{\frac{1}{2}})} . \qquad \text{(A-14)}$$

It is to be noted that $t \ll a$, and therefore for calculating the flux in the hollow cylinder the radius is assumed to be a, instead of $(a - t)$.

The total flux becomes

$$\phi_t = \mu_o H_o \pi a^2 \quad \frac{1 + \dfrac{2k}{z} j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})}{1 + j\dfrac{z}{2k} j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})} \qquad \text{(A-15)}$$

$$= k_e \mu_o H_o \pi a^2 \ ,$$

where $$k_e = \frac{1 + \dfrac{2k}{z} j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})}{1 + j\dfrac{z}{2k} j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})} , \qquad \text{(A-16)}$$

k_e can be considered as an effective permeability for the structure.

At $\omega = 0$, $k_e = 1 + 2k\delta$. (A-17)

The exciting flux is $\phi_e = \mu_o H_o \pi a^2$.

$$\therefore \quad 1 - \left(\frac{\phi_e}{\phi_t}\right)_\omega = \frac{\left(\frac{2k}{z} - j\frac{z}{2k}\right) j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})}{1 + \frac{2k}{z} j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})} \; .$$

$$1 - \left(\frac{\phi_e}{\phi_t}\right)_{\omega = 0} = \frac{2k\delta}{1 + 2k\delta} \; .$$

$$\therefore \text{DGR} = \frac{1 - \left(\frac{\phi_e}{\phi_t}\right)_\omega}{1 - \left(\frac{\phi_e}{\phi_t}\right)_{\omega = 0}} = \frac{1 + 2k\delta}{2k\delta} \cdot \frac{\left(\frac{2k}{z} - j\frac{z}{2k}\right) j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})}{1 + \frac{2k}{z} j^{-\frac{1}{2}} \tanh(\delta z j^{\frac{1}{2}})} \; . \quad \text{(A-18)}$$

The degaussing ratio for a hollow ferromagnetic sphere can be derived by an analogous method. See Reference 5, p. 85.