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SOME PROPERTIES OF FLUTTAR WAVEFORMS WITH APPLICATIONS TO DETECTION SYSTEMS

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ABSTRACT

Theoretical extensions of some results previously dealt with in two reports (ERA-284, -285) on Fluttar systems are given.

SOME PROPERTIES OF FLUTTAR WAVEFORMS WITH APPLICATIONS TO DETECTION SYSTEMS

- E. L. R. Webb -

The wave scattered by the "target" is delayed relative to the direct wave, and this phase delay is a well behaved and most significant property of the signals generated by fluttar systems.

In Appendix 1 we show a typical arrangement (Fig. 1) of transmitter T and receiver R and state some definitions. In Section 1 we derive the equation

$$g = \frac{V^2 \sin^2\theta}{\lambda DL(1 - L)}$$

giving the glide "g", or rate of change of fluttar frequency, in terms of the various parameters. This relation is valid for crossings in the middle portion of the link and has been verified experimentally in considerable detail. The expected frequencies tend to be low; in the range zero to several tens of cycles per second, and the duration of the waveform is normally tens of seconds, dependent on antenna patterns, etc.

In Section 2 of Appendix 1 we deal with crossings over a terminal and Fig. 2 is a "universal" curve of the variation of frequency during a crossing. We also derive an expression for the maximum value of glide

$$g_o = V^2/\lambda h.$$

Most of the action occurs in the short time it takes the target to go a distance comparable to twice its altitude. For low fliers this may be a few seconds and the value of g may be as high as several hundred cycles per second per second.

In two previous reports [1,2] the mid-link relationship has been used to determine location of crossing in a special arrangment of one transmitter and two closely spaced receivers.

In Appendix 2 we extend this idea to some more general cases but still retain close lateral spacing between overlapping links. The expressions for location may become multivalued and some care in siting or in interpretation might be necessary to avoid ambiguity. Once the location is determined it is easy to determine V sin θ — the crossing

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[1] ERA-284, Catalog of Fluttar Waveforms
[2] ERA-285, Location and Velocity of Crossing in Twin Fluttar Systems

component of velocity.

In Appendix 3 we indicate how the information obtained from a widely spaced pair of receivers and a single transmitter could at least in principle yield complete track data.

APPENDIX 1

Definitions:

λ = wave length
ϕ = phase lag of scattered wave (in cycles)
t = time (seconds) from cross over
D = length of link
L = location (normalized)
V = ground speed
θ = angle of crossing
h = altitude
f = fluttar frequency ($d\phi/dt$)
g = glide (df/dt)

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Section 1 Derivation of: $g = \dfrac{V^2 \sin^2\theta}{\lambda DL(1-L)}$, valid for mid-link crossings.

From Fig. 1 we can write by inspection the equation,

$$\lambda\,\phi(t) = [(DL + Vt\cos\theta)^2 + (Vt\sin\theta)^2 + h^2]^{\frac{1}{2}} + [(D - DL - Vt\cos\theta)^2 + (Vt\sin\theta)^2 + h^2]^{\frac{1}{2}},$$

which on differentiating gives

$$\frac{\lambda\,f(t)}{V} = \frac{DL\cos\theta + Vt}{[(DL + Vt\cos\theta)^2 + (Vt\sin\theta)^2 + h^2]^{\frac{1}{2}}} - \frac{D\,(1 - L)\cos\theta - Vt}{[(D - DL - Vt\cos\theta)^2 + (Vt\sin\theta)^2 + h^2]^{\frac{1}{2}}}.$$

We now make the assumption that Vt is an order of magnitude smaller than either of DL or D(1 - L) so that the square of Vt sin θ is negligible compared with the square of DL or D(1 - L). We can dispose, at least temporarily of h in the same way. Thus we get --

$$\frac{\lambda\,f(t)}{V} \doteq \frac{DL\cos\theta + Vt}{DL + Vt\cos\theta} - \frac{D\,(1 - L)\cos\theta - Vt}{D\,(1 - L) - Vt\cos\theta}$$

which simplifies to

$$\frac{\lambda\,f(t)}{V} \doteq \frac{Vt\sin^2\theta}{DL\,(1 - L)},$$

if we discard terms containing higher orders of $\dfrac{Vt}{D}$.

Finally we differentiate once more to arrive at

$$\frac{\lambda\, g(t)}{V} = \frac{V \sin^2\theta}{DL\ (1 - L)}.$$

Thus we see that g (t) is approximately constant for a significant portion of mid-link crossings, and equal to

$$g = \frac{V^2 \sin^2\theta}{\lambda DL\ (1 - L)}.$$

The special case of L = 0.5 has been previously derived in the form

$$g = \frac{4\, V^2 \sin^2\theta}{\lambda D}.$$

<u>Section 2</u> Derivation of $g_o = \frac{V^2}{\lambda h}$ for crossings over a terminal.

For crossings in the vicinity of one of the terminals it is necessary to make somewhat different simplifying assumptions. At the receiver, L = 1, and we have

$$\frac{\lambda f(t)}{V} = \frac{D \cos\theta + Vt}{[(D + Vt \cos\theta)^2 + (Vt \sin\theta)^2 + h^2]^{\frac{1}{2}}} + \frac{Vt}{[V^2t^2 + h^2]^{\frac{1}{2}}}.$$

A similar result is obtained for crossings over the transmitter, i.e., L = 0.

We now assume $(Vt)^2$ and h^2 small compared with D^2 and get as a first step

$$\frac{\lambda f(t)}{V} \doteq \frac{D \cos\theta + Vt}{D + Vt \cos\theta} + \frac{Vt}{[V^2t^2 + h^2]^{\frac{1}{2}}}.$$

This result was derived in a slightly different manner in January, 1953, and communicated privately. We pointed out at that time that a single curve $\frac{1}{\sqrt{1 + (h/Vt)^2}}$ could be drawn to indicate, in normalized form, the variations of frequency with time for different headings and altitudes. Such a graph is shown in Fig. 2.

When proper care is taken to preserve the correct plus and minus signs for the square root quantities, the expression for frequency agrees with the physically obvious. In particular, an aircraft flying into the system from either end (θ = 0 or 180°) generates a decreasing frequency, which is initially asymptotic to the radar doppler frequency and finally to zero. This is in contrast with the case of an aircraft flying across one end (θ = ±90°) which generates a waveform whose frequency is initially and finally asymptotic to half the radar frequency. Of course, in any physical arrangement, other factors, such as antenna directivity will limit the range over which a recognizable waveform may be recorded.

We now proceed to differentiate the expression for frequency and obtain

$$\frac{\lambda g(t)}{V} = \frac{h^2/V^2t^3}{(1 + h^2/V^2t^2)^{3/2}} .$$

It will be noted that g (t) goes to zero for large (absolute) values of $\underline{t}$, and has a maximum at t = 0. This maximum, $g_o = V^2/\lambda h$, could be an important design factor.

The foregoing is critically dependent on L being exactly 0 or 1; however, on looking back we see in a general way that failure to cross directly over a terminal has the same effect as increasing the altitude. The significant quantity is the minimum radial distance from the terminal.

<u>Numerical Examples</u>: $\lambda = 0.5$ m; $V = 340$ m/s (Mach 1)

$D = 100$ km; $h = 340$ m (1150 ft)

i) Mid link:

$L \simeq 0.5$

$$g = \frac{4 \times 340 \times 340 \sin^2\theta}{0.5 \times 100,000}$$

$$= 9 \sin^2\theta \qquad c/s/s$$

ii) Terminal:

$L = 0$ or 1

$$f = \frac{340}{0.5}\left(\pm \cos\theta \pm \frac{1}{\sqrt{1 + h^2/V^2t^2}} \right)$$

$$= 680 \left(\pm \cos\theta \pm \frac{1}{\sqrt{1 + 1/t^2}} \right) \; c/s$$

$$g_o = \frac{340 \times 340}{0.5 \times 340} = 680 \; c/s/s.$$

<u>APPENDIX 2</u> Application of $g = \dfrac{V^2 \sin^2\theta}{\lambda DL(1 - L)}$ to the determination of location of crossing.

It has been previously shown [1,2] that a pair of closely spaced receivers a distance S apart yield more than sufficient data to determine location of crossing by means of the relation

$$L = \frac{1}{1 + S^2/\lambda D g t^2} .$$

We now extend our analysis to the more general system indicated in Fig. 3. The distances S_1 and S_2 are not necessarily equal or unequal but are an order of magnitude smaller than D_1 or D_2. The link lengths D_1 and D_2 may or may not be equal but should overlap an appreciable amount M. Because of the small angle between links there will be little error if we refer distances across and along the system to the two directions making equal angles with both links.

An aircraft crossing both links will do so at instants a time $\underline{t}$ apart. If the links are parallel, $(S_1 = S_2 = S)$, $\underline{t}$ will be given by

$$t = \frac{S}{V \sin \theta}$$

independent of location. If they are not parallel we can write an equation for $\underline{t}$ in terms of either L_1 or L_2,

$$t = \frac{1}{V \sin \theta}\left[S_1 + \frac{L_2 D_2}{M}(S_2 - S_1)\right]$$

$$\text{or } t = \frac{1}{V \sin \theta}\left[S_2 + \frac{(1 - L_1)}{M} D_1 (S_1 - S_2)\right].$$

In any case we can solve either of these equations simultaneously with an equation of the type

$$g_i = \frac{V^2 \sin^2\theta}{\lambda D_i L_i (1 - L_i)}.$$

For example let $S_1 = S_2 = S$; then

$$L_i (1 - L_i) = \frac{S^2}{\lambda D g_i t^2}.$$

This expression is multivalued, the two values of L_i being symmetrical about the value $L = 0.5$. If the system were staggered with M equal to or less than half the smaller of D_1 or D_2, no ambiguity could result.

For the non-parallel case the situation is a little more complicated; however it can be seen in a general way that there will be a pair of simultaneous solutions each in itself multivalued. With this redundancy it should always be possible to sort out the actual location of crossing. The two equations for L_i are given in unreduced form

$$g_1 t^2 \lambda_1 D_1 L_1 (1 - L_1) = \left[S_2 + (1 - L_1)\frac{D_1}{M}(S_1 - S_2)\right]^2.$$

and

$$g_2 t^2 \lambda_2 D_2 L_2 (1 - L_2) = \left[S_1 + L_2 \frac{D_2}{M}(S_2 - S_1)\right]^2.$$

It will be noted that for $S_1 = S_2$ both reduce to the equation given above for the parallel case. For routine solution of these equations the best method would be to tabulate or plot the functions $L_i(g_i t^2)$ for each fixed combination of parameters.

APPENDIX 3

The redundancy of information in the previous analyses suggest that it might be possible to determine more completely the straight line path followed by an aircraft. It is felt intuitively that rather wide spacings may be necessary in order to yield answers of acceptable accuracy. In the following "for instance" system we indicate how this might be done.

Referring to Fig. 4, and noting that ϕ now represents the angle between links, we may write the following equations:

$$g_2 = \frac{V^2 \sin^2(\theta_1 - \phi)}{\lambda_2 D_2 L_2 (1 - L_2)} \quad (1)$$

$$g_1 = \frac{V^2 \sin^2(\theta_1)}{\lambda_1 D_1 L_1 (1 - L_1)} \quad (2)$$

$$L_2 D_2 \cos(\theta_1 - \phi) - L_1 D_1 \cos\theta_1 = Vt \quad (3)$$

$$L_2 D_2 \sin(\theta_1 - \phi) = L_1 D_1 \sin\theta_1 . \quad (4)$$

Here we have four equations in the four unknowns L_1, L_2, θ_1 and V, which ought to be enough.

Equations (3) and (4) may be replaced by their equivalents

$$\frac{Vt}{\sin\phi} = \frac{L_1 D_1}{\sin(\theta_1 - \phi)} \quad (3a)$$

and

$$\frac{Vt}{\sin\phi} = \frac{L_2 D_2}{\sin\theta_1} \quad (4a)$$

which could have equally well been deduced from first principles.

Substituting (3a) in (1) and (4a) in (2) we get

$$g_2 t^2 = \frac{L_1^2 D_1^2 \sin^2\phi}{\lambda_2 D_2 L_2 (1 - L_2)} ,$$

and

$$g_1 t^2 = \frac{L_2^2 D_2^2 \sin^2\phi}{\lambda_1 D_1 L_1 (1 - L_1)} .$$

These equations are of the form

$$L_1^2 = A_2 L_2 (1 - L_2) \quad \text{and} \quad L_2^2 = A_1 L_1 (1 - L_1)$$

where

$$A_2 = \frac{g_2 t^2 \lambda_2 D_2}{D_1^2 \sin^2\phi} \quad \text{and} \quad A_1 = \frac{g_1 t^2 \lambda_1 D_1}{D_2^2 \sin^2\phi} ,$$

and from these we get

$$L_1^2 = A_2 \sqrt{A_1 L_1 (1 - L_1)} \left(1 - \sqrt{A_1 L_1 (1 - L_1)} \right) \quad (5)$$

and

$$L_2^2 = A_1 \sqrt{A_2 L_2 (1 - L_2)} \left(1 - \sqrt{A_2 L_2 (1 - L_2)} \right) , \quad (6)$$

which are essentially cubic equations and we will not attempt a solution here. Even the parametric type of solution, $L_i \, (g_i t^2)$, previously used, has its usefullness restricted because we now have two parameters, i.e., $L = L_i\ (A_1, A_2)$. Assuming, however, that we have obtained the roots of Equations (5) and (6) and that we have resolved any ambiguities arising from the multivalued solutions or have agreed to live with them, we can easily return to Equation (4) for the course angle θ, and then to Equation (3) to get ground speed V.

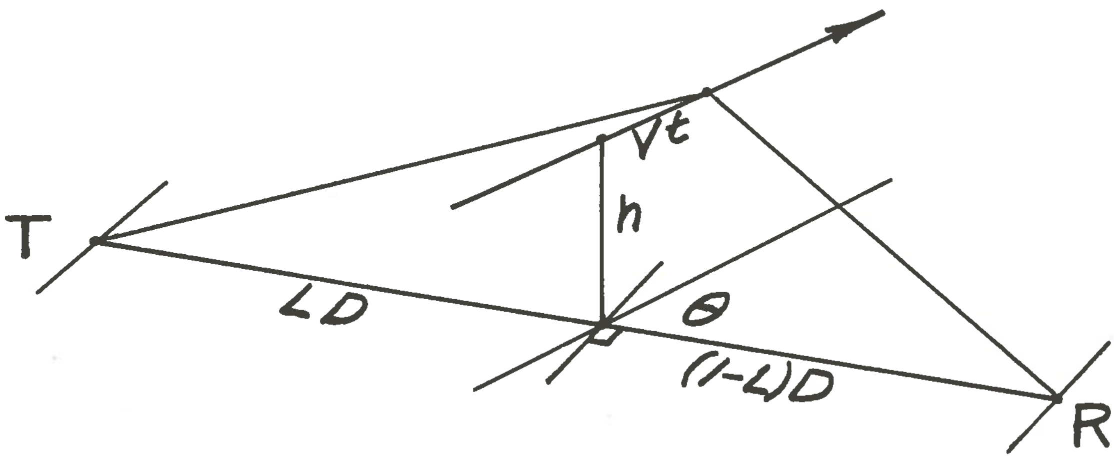

FIG. 1

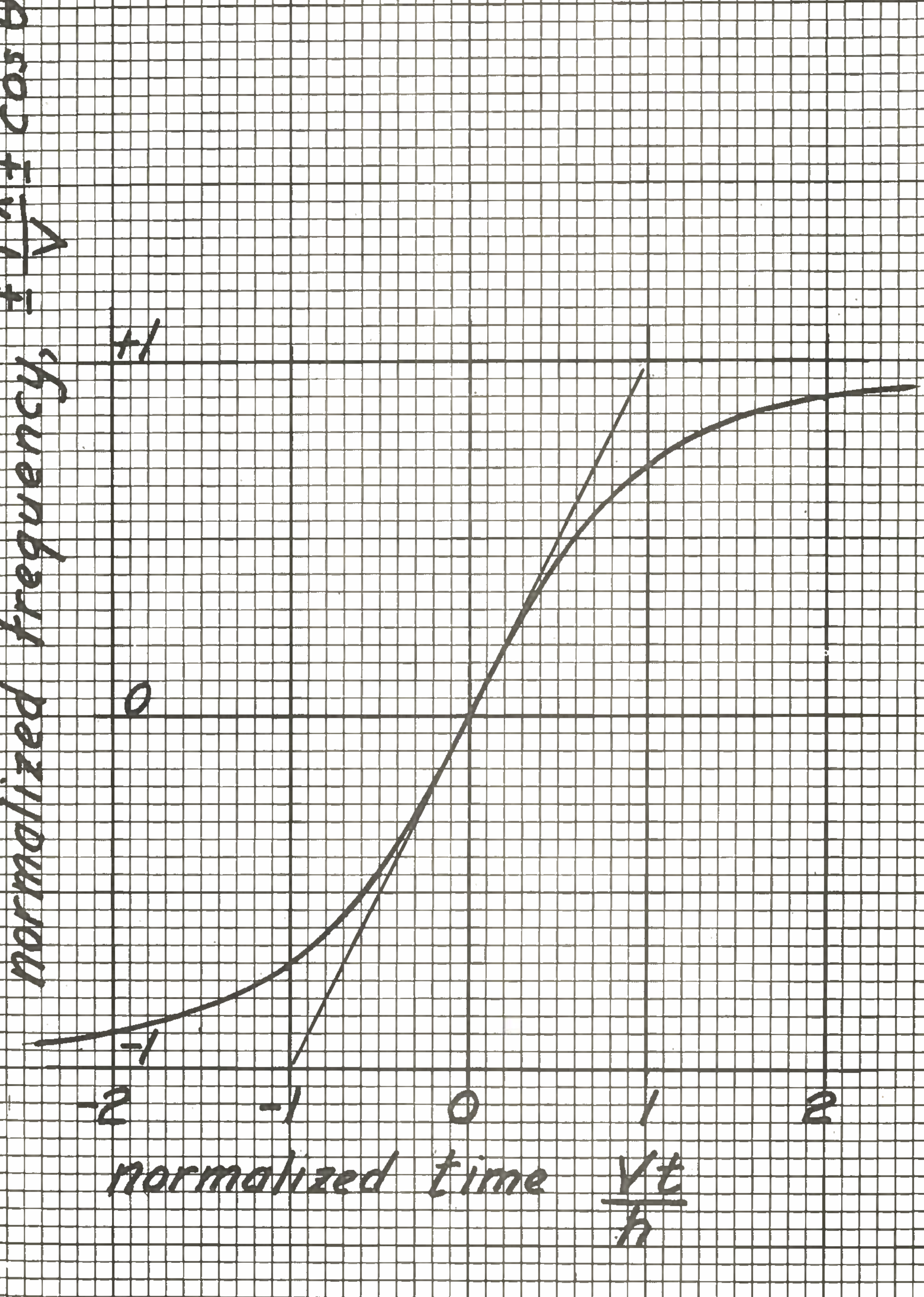
normalized frequency, $\pm \frac{f\lambda}{V} \pm \cos\theta$
+1
0
-1
-2
-1
0
1
2
normalized time $\frac{Vt}{h}$

FIG. 2

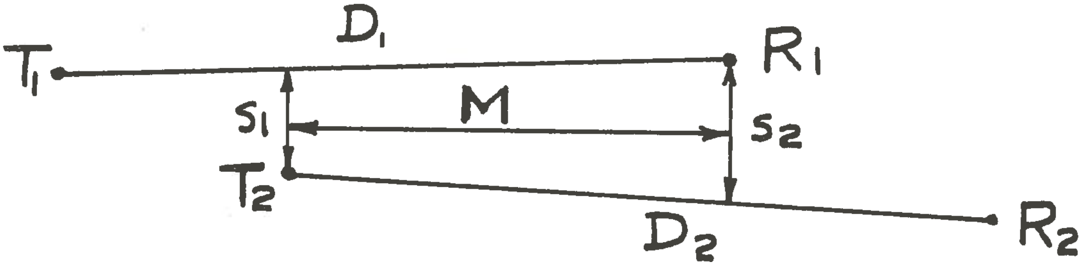

FIG. 3

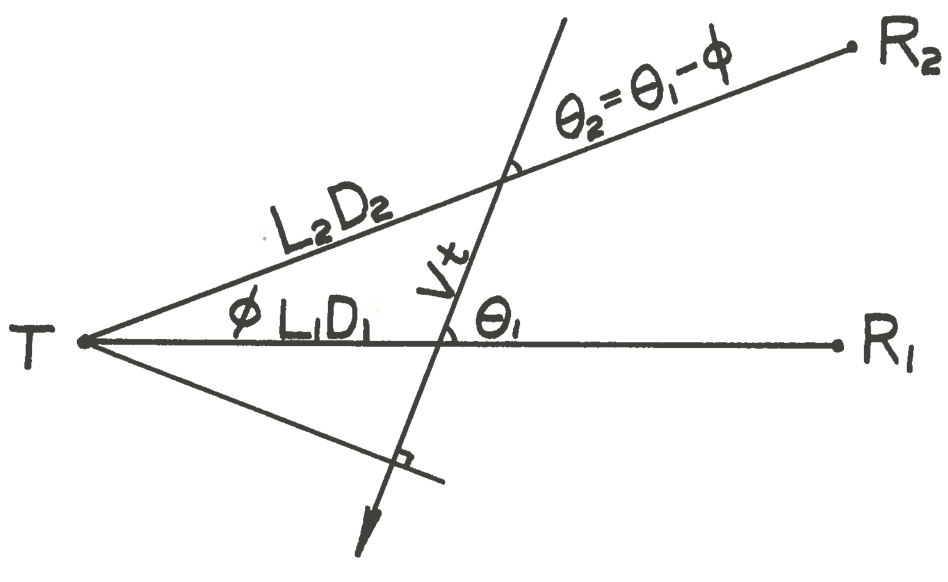

FIG. 4