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RADIO BRANCH

NON-RESONANT ARRAYS

author's approval — R.E. Bell

Final approval — June 15/45

OTTAWA

MAY, 1945

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NATIONAL RESEARCH COUNCIL OF CANADA
RADIO BRANCH

NON-RESONANT ARRAYS

1. INTRODUCTION

A non-resonant antenna array is defined as an array whose radiating elements are fed as shunt loads distributed along a transmission line or waveguide, the spacing of the radiators being different from an integral number of half wavelengths. Such an array is fed from one end, and is terminated at the other end by a matched load. The main beam of radiation is produced at some angle other than 90° to the array.

We may picture the array as a "leaky pipe", in which each radiator extracts a certain portion of the power incident upon it, and allows the remainder to pass on. In one respect this picture is slightly incorrect, because each radiator causes a certain amount of reflection of the incident power. However, since the radiators are at non-half-wavelength spacing, these small reflections do not add up, and so cause no trouble. In other words, since the array is terminated in a matched load, the transmission line remains very nearly matched along the whole length of the array. Under such conditions the fraction of the incident power which a single radiator extracts from the transmission line is proportional to its conductance measured as a fraction of the characteristic admittance of the transmission line (its "normalized conductance").

It will be seen that as we progress along the array from the input end, the amount of power remaining in the transmission line is steadily decreasing. Hence, in order to maintain the amount of radiation constant along the array, the more remote radiators must have progressively larger and larger conductances. As a slight further complication, arrays are usually arranged to be fed more strongly in the centre, so as to reduce side lobes, and this must be taken into account in calculating the conductances to be used.

For purposes of calculation, we will treat the array as a long transmission line continuously loaded with conductance, and instead of calculating the conductance of individual radiators, will merely find the conductance per unit length as a function of the position along the array. This procedure has the advantage of applying to all arrays at any non-resonant spacing. The conductance of individual radiators can be found from the conductance per unit length, as shown later.

The distribution of power over the array has been chosen as a 4:1 linear power taper from centre to end of the array. This is a taper of 2:1 in amplitude, and serves to reduce the side lobes below 10% voltage (theoretically) while widening the main beam only about 10%.

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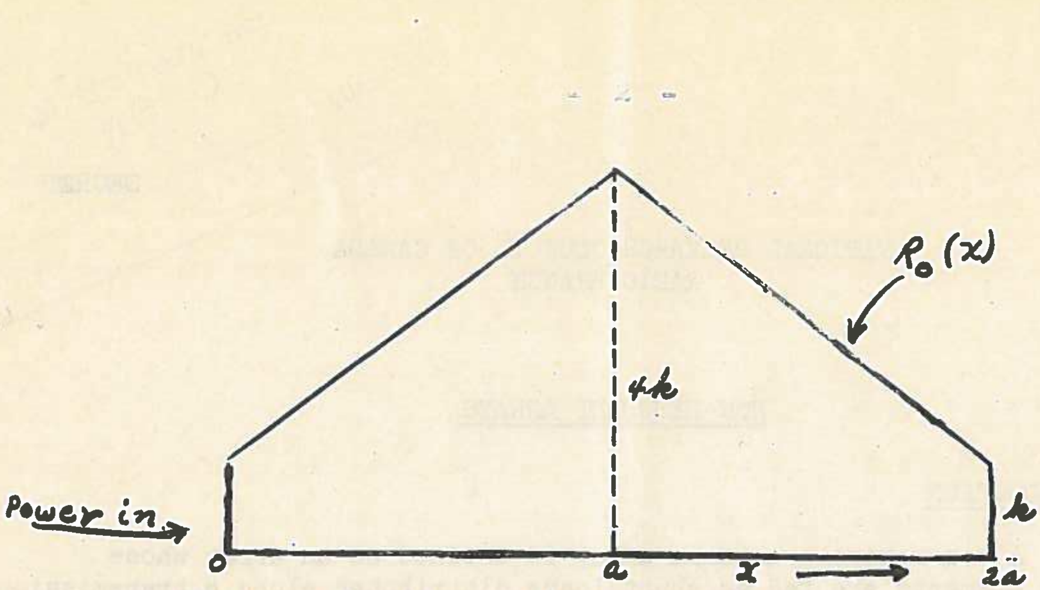


The power function can be expressed as:

$$R_o = k(1 + 3\tfrac{x}{a}) \quad (o < x < a)$$
$$R_o = k(7 - 3\tfrac{x}{a}) \quad (a < x < 2a) \qquad \ldots\ldots\ldots\ldots(1)$$

A reasonable value for power lost in the matched load is 5% of the input power: to obtain a lower figure, the ratio between the radiator conductance at the input end and that at the remote end would have to be higher, which might be hard to obtain with practical radiators.

II. CALCULATION OF CONDUCTANCES

All quantities with the subscript $_o$ (such as R_o above) refer to the centre design wavelength, λ_o. At other wavelengths the same quantities are written without subscripts.

Let the power radiated per unit length be $R_o(x)$, as defined in (1) above.

Let the normalized conductance per unit length be $G_o(x)$. This is the quantity we wish to calculate in order to design an array.

Let the power remaining in the transmission line at position x be $P_o(x)$. The incident power, P_{oo}, is taken as unity. Obviously, if we are to believe in conservation of energy,

$$P_o = P_{oo} - \int_o^x R_o\,dx$$

Or, since $P_{oo} = 1$,

$$P_o = 1 - \int_o^x R_o\,dx \qquad \ldots\ldots\ldots(2)$$

As stated in the explanation above, if a radiator of conductance G_o finds itself in parallel with a matched line containing a power P_o, the amount of power extracted by this load will be:

$$R_o = P_o G_o$$

$$= (1 - \int_0^x R_o dx) G_o \qquad \text{..........(3)}$$

If the flat load absorbs power in the amount .05, then the portion of the array from 0 to a absorbs power equal to .95/2 = .475, and we have .525 left in the transmission line at x = a. Hence, from (2)

$$\left[P_o\right]_{x=a} = 1 - \int_0^a R_o dx \quad \text{or} \quad .525 = 1 - \int_0^a k(1 + \frac{3x}{a})dx = 1 - \frac{5}{2} ka$$

$$\therefore \quad k = .19/a \qquad \text{..........(4)}$$

Hence, in the region $o < x < a$, (3) becomes:

$$\frac{.19}{a}(1 + 3\frac{x}{a}) = \left[1 - \int_0^x \frac{.19}{a}(1 + \frac{3x}{a})\,dx\right] G_o$$

$$= \left[1 - .19\frac{x}{a} - .285\frac{x^2}{a^2}\right] G_o$$

$$\therefore \quad G_o = \frac{.19}{a} \frac{(1 + 3\,x/a)}{(1 - .19\frac{x}{a} - .285\,(\frac{x}{a})^2\,)} \qquad \text{..........(5)}$$

$$(o < x < a)$$

So far as the region $a < x < 2a$ is concerned, the input power is .525, and (2) becomes:

$$P_o = .525 - \int_a^x R_o dx$$

Thus (3) becomes:

$$\frac{.19}{a}(7 - \frac{3x}{a}) = \left[.525 - \int_a^x \frac{.19}{a}(7 - 3\frac{x}{a})\,dx\right] G_o$$

$$= \left[1.57 - 1.33\frac{x}{a} + .285\,(\frac{x}{a})^2\right] G_o$$

$$\therefore \quad G_o = \frac{.19}{a} \quad \frac{(7 - 3\,x/a)}{(1.57 - 1.33\,\frac{x}{a} + .285(\frac{x}{a})^2} \qquad \text{..........(6)}$$

$$(a < x < 2a)$$

The quantity in brackets in the denominator of (5) and (6) is P_o, the power remaining in the guide, and hence,

$$P_o = 1 - .19\frac{x}{a} - .285\,(\frac{x}{a})^2 \qquad (o \leq x < a) \qquad \text{..........(7)}$$

$$= 1.57 - 1.33\,\frac{x}{a} + .285\,(\frac{x}{a})^2 \quad (a < x < 2a)$$

To use the foregoing equations in the design of an array of N discrete radiators, we must:

(a) replace 2a, the total length of the array, by N, the total number of radiators;

(b) replace x, the distance along the array, by n, the number of the radiator under consideration, counting from the input end;

(c) replace $G_o(x)$ by G_n, the conductance of the n^{th} radiator.

We will then find that G_nN is a constant, no matter what the number of radiators used in an array, because of the factor 1/a which appears in (5) and (6).

n/N	x/a	$\frac{1}{k} R_o(x) = (1+3\,x/a)$	$P_o(x)$	$G_o(x)$	G_nN
0	0	1.0	1.0	.19a	.380
.1	.2	1.6	.951	.319/a	.638
.2	.4	2.2	.878	.475/a	.950
.3	.6	2.8	.781	.681/a	1.362
.4	.8	3.4	.665	.972/a	1.924
.5	1.0	4.0	.525	1.445/a	2.890
n/N	x/a	=(7-3 x/a)			
.5	1.0	4.0	.525	1.445/a	2.89
.6	1.2	3.4	.385	1.680/a	3.36
.7	1.4	2.8	.267	1.99/a	3.98
.8	1.6	2.2	.172	2.43/a	4.86
.9	1.8	1.6	.099	3.0[illegible]/a	6.14
1.0	2.0	1.0	.050	3.8/a	7.60

The curve $G_n N$ versus n/N can be used to design any non-resonant array, provided that there is available an elementary radiator whose conductance can be adjusted to the curve as we progress along the array. Curves of $G_n N$, R_o and P_o are plotted in Figures 1 and 2.

III. OFF-FREQUENCY PERFORMANCE

As the wavelength is changed, the following effects take place in a non-resonant array:

(1) the spacing in electrical degrees of the radiators changes;

(2) the admittance of each radiator changes, having, in most cases, greatest conductance at the centre wavelength. In all that follows we will assume that the admittance of each radiator changes in the same way, and that different radiators in the array differ only in the absolute magnitude of their admittances.

Radiation Pattern:

Effect (1) above tends to change the phasing between adjacent dipoles, and hence to swing the beam. It is not hard to show that if the separation between elements is S, and the wavelength in the transmission line is λ_g, that the "squint angle" by which the main beam deviates from the normal to the array is:

$$\alpha = \sin^{-1}\left[\frac{\lambda}{\lambda_g} - \frac{\lambda}{2S}\right] \qquad \text{..........(8)}$$

Effect (2) alters the distribution of radiated power over the aperture of the array from the designed function. This effect takes place relatively slowly, and very little deterioration of the beam takes place over quite large wavelength limits. If all the conductances of the radiators change simultaneously by a factor K, it can be shown that the distribution of radiated amplitude over the array is:

$$A = \sqrt{k\left(1 + 6\frac{n}{N}\right)\left(1 - .38\frac{n}{N} - 1.14\left(\frac{n}{N}\right)^2\right)^{k-1}} \qquad \text{..........(9)}$$

$$(o < n < n/2)$$

$$A = \sqrt{k\left(7 - 6\frac{n}{N}\right)\left(1.57 - 2.66\frac{n}{N} + 1.14\left(\frac{n}{N}\right)^2\right)^{k-1}}$$

$$\left(\frac{N}{2} < n < N\right)$$

Curves of A for various values of k are plotted in Figure 3.

Power lost in Matched Load:

If all the conductances in the array change by a factor K, it is not hard to show that the power remaining in the guide at any point is:

$$P = (P_o)^k \quad \text{..........(10)}$$

where P_o was the power in the guide at that point before the change in conductance. Hence, if the flat load absorbs 5% of the power at the design wavelength, it will absorb:

$$(.05)^k$$

at the new wavelength.

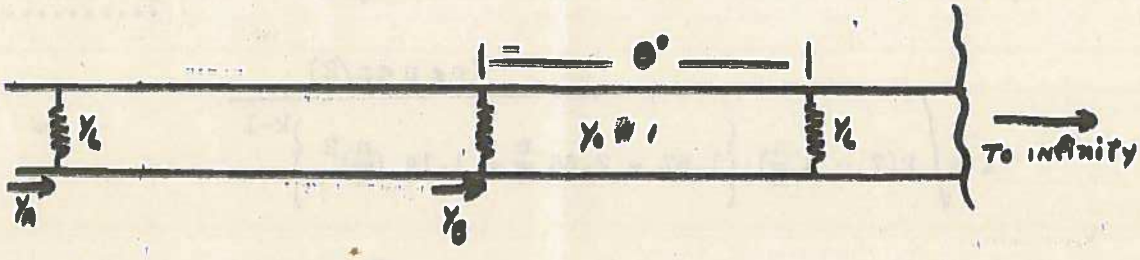
A curve of power absorbed in the flat load, versus k, is given in Figure 4.

Thus we see that the whole off-frequency performance of the array can be predicted if the radiator conductance as a function of frequency is known. For some types of radiators, mutual effects between adjacent array elements may be appreciable, and must be taken into account. If, in a preliminary design, the conductances of the radiators in the array are in doubt, due to unknown mutual effects, or to difficulty in measuring the conductances of the radiators, Figures 3 and 4 can be compared with experimental results of the array to deduce the value of k, and hence to obtain the true conductances from the supposed value, for use in an improved design.

In all the foregoing it has been assumed that half-wavelength spacing (resonant spacing) has been avoided, because almost all the analysis given above breaks down at or very near half-wavelength spacing.

IV. INPUT STANDING WAVE RATIO

A good approximation to the input admittance of a non-resonant array can be obtained by taking the array as infinitely long, and the radiators as all identical with the input radiator of the actual array.



Let the radiators have an admittance Y_L, and let their spacing be θ electrical degrees in the transmission line. Since the line is infinitely long, conditions at every radiator must be identical, so that $Y_B = Y_A$. Remembering this fact, and also that $Y_o = 1$, we may write

$$Y_A = Y_L + \frac{1 - j\,Y_A \cot\theta}{Y_A - j\cot\theta} \quad \text{..........(11)}$$

Simplifying,

$$Y_A{}^2 - Y_A\,Y_L = 1 - j\,Y_L \cot\theta$$

We know Y_A is of the order of $Y_o = 1$, while Y_L is always small. Hence, to a good approximation

$$Y_A{}^2 = 1 - j\,Y_L \cot\theta \quad \text{..........(12)}$$

Now $Y_A = 1$ represents a perfectly matched input, so we see from (12) that Y_L and θ determine only the deviation from a perfect match, and that, so long as $Y_L \cot\theta$ is small, the input admittance will be good. It is easy to use a circle diagram to make up a table of standing wave ratios in terms of $Y_L \cot\theta$. The following table applies to the case where Y_L is a pure conductance: results for cases where Y_L is complex will be similar.

$Y_L \cot\theta$	VSWR
0	1.0
0.1	1.04
0.2	1.09
0.3	1.15
0.4	1.22
0.5	1.30
0.6	1.39
0.7	1.49
0.8	1.60
0.9	1.73
1.0	1.99

Even when the V.S.W.R. is relatively large, it is not a rapid function of wavelength, and a standard matching devide at the input will give a good input match over a large wavelength band.

The dependence of V.S.W.R. on $Y_L \cot\theta$ shows that short arrays (high Y_L) are harder to match than long arrays, and that arrays with small squint angles (θ near 180°) are harder to match than arrays with large squint angles. In practical cases, however, a broadband input match is always obtainable with great ease.

V. DESIGN OF AN ARRAY

The design of an array will normally proceed according to the following steps:

(1) The wavelength and desired beamwidth fixes the total array length.

(2) The tolerable squint angle determines the radiator spacing by equation (8). The squint angle should be large enough at the center wavelength to obtain the necessary bandwidth. A radiator spacing of 200 electrical degrees is a common value, which in S-band waveguide at $\lambda = 10.70$ cm. gives a squint angle of 3.5°.

(3) The total number of radiators, N, can now be computed.

(4) Each radiator is assigned a value of conductance from the curve, Fig. 1. Often the radiators are grouped into about 10 groups, all radiators in a group being given the conductance value for the center radiator of the group.

(5) The parameters which govern the conductance of whatever type of radiator is to be used are adjusted to the values of step (4). The array design is now complete for the center wavelength. Its off-frequency performance may be estimated from equation (8) and the curves of Figs. 3 and 4, provided the frequency characteristics of the elementary radiator are known.

VI. CONCLUSION

The theory and procedure outlined above have been available in memorandum form, and have been used at this Laboratory and at R.A.E. and T.R.E. in England for the past year. During that time a number of successful slotted arrays have been built at each of the wavelength bands, S, X and K. When the properties of the elementary radiators (slots) are well known in advance, it is usually possible to design the array completely on paper, although in some cases it may be necessary to use the experience of a first attempt to build the final array. Radiation patterns and input impedances have been good in all cases.

Ottawa
April 27, 1945.

R. E. Bell

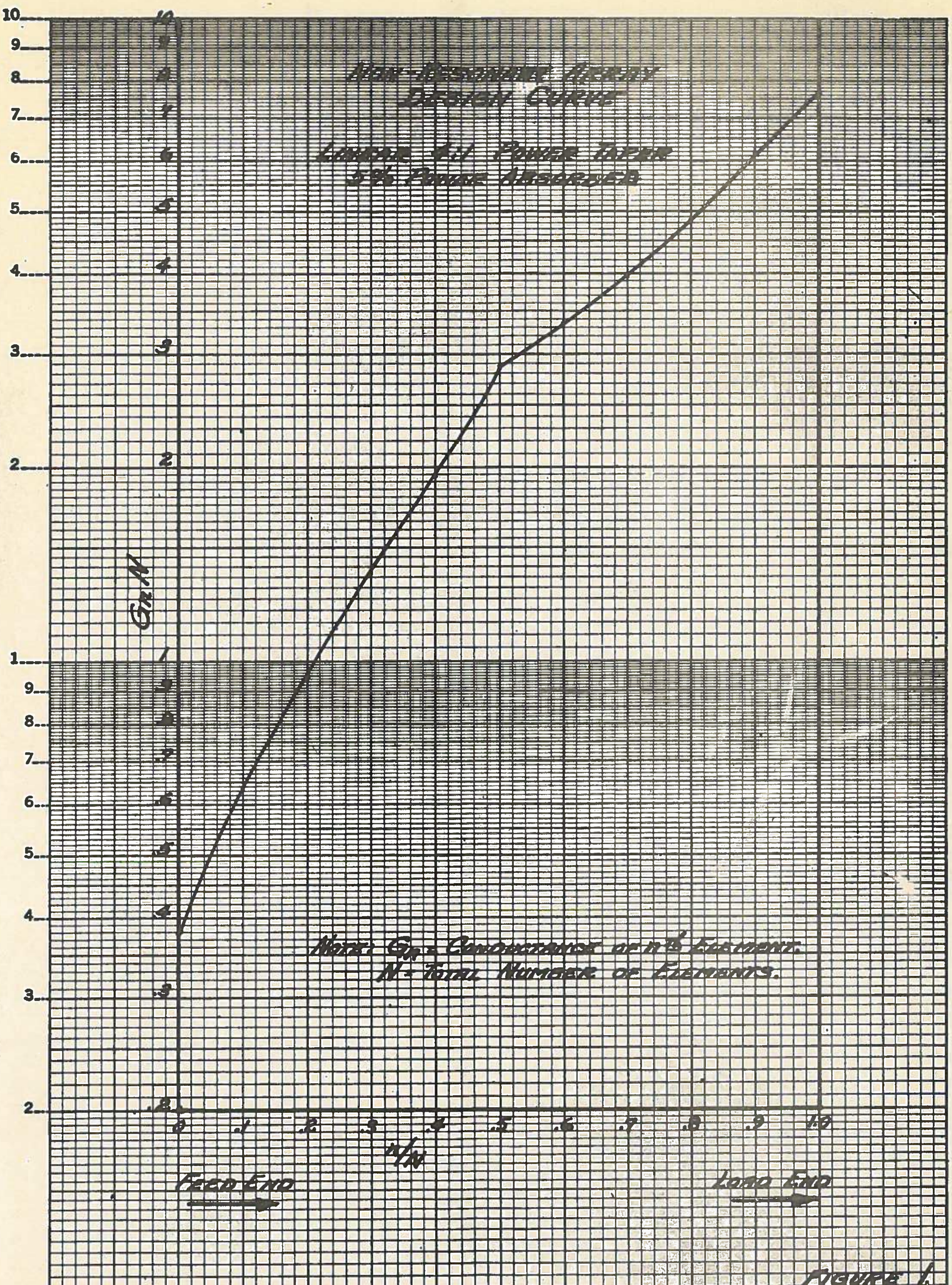
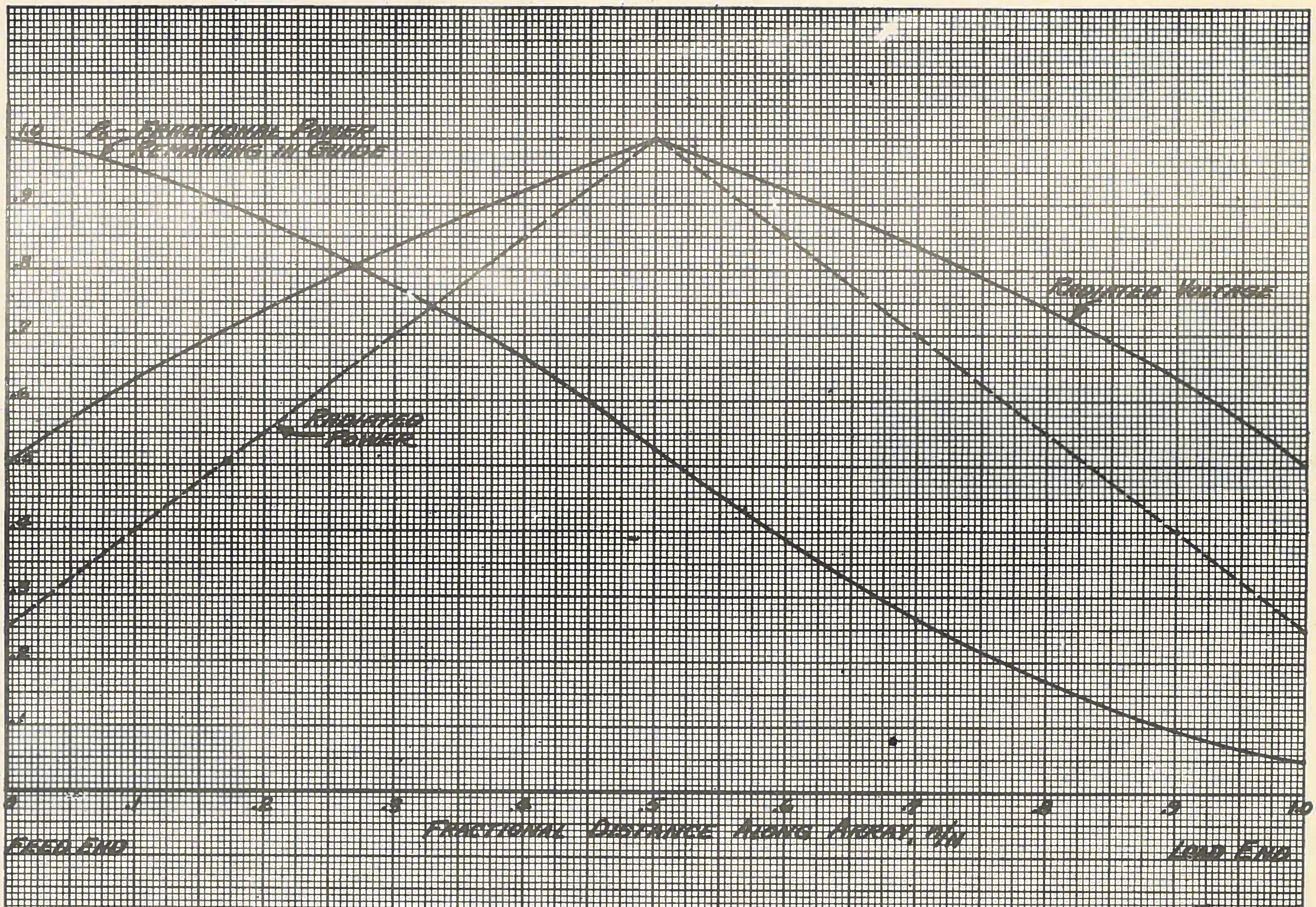
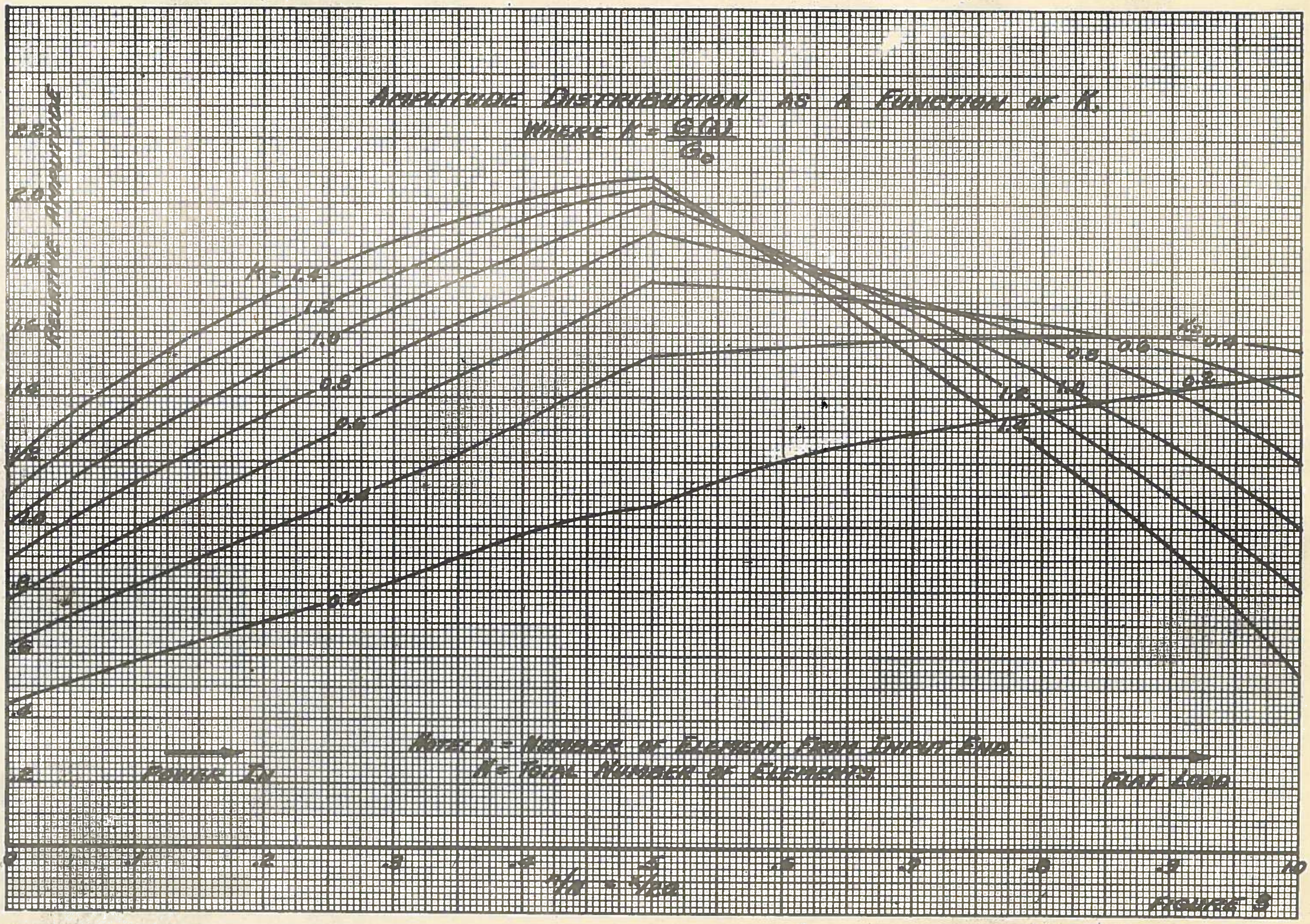

NON-RESONANT ARRAY
DESIGN CURVE
LINEAR 4:1 POWER TAPER
5% POWER ABSORBED
GnN
NOTE: Gn = CONDUCTANCE OF nth ELEMENT.
N = TOTAL NUMBER OF ELEMENTS.
n/N
0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0
FEED END
LOAD END

FIGURE 1.



1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
Fractional Power Remaining in Guide
Radiated Voltage
Radiated Power
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
Fractional Distance Along Array, n/N
Feed End
Load End

FIGURE 2



AMPLITUDE DISTRIBUTION AS A FUNCTION OF K.
WHERE K = G(a) / G₀
RELATIVE AMPLITUDE
K = 1.4
1.2
1.0
0.8
0.6
0.4
0.2
NOTE: n = NUMBER OF ELEMENT FROM INPUT END.
N = TOTAL NUMBER OF ELEMENTS
POWER IN
FLAT LOAD
FIGURE 3

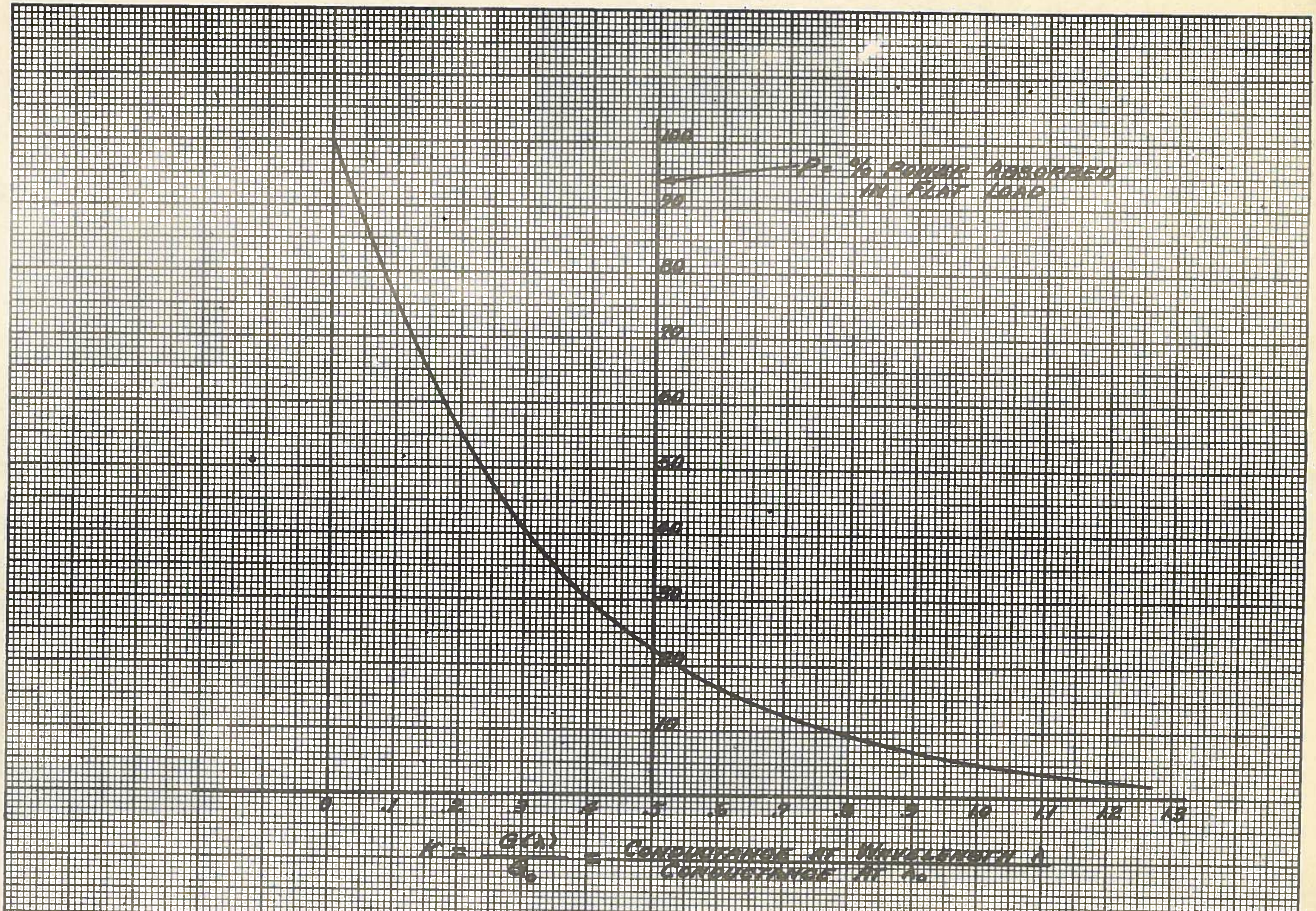

100
90
80
70
60
50
40
30
20
10
P = % POWER ABSORBED IN FLAT LOAD
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
$K = \frac{G(\lambda)}{G_0} = \frac{\text{CONDUCTANCE AT WAVELENGTH } \lambda}{\text{CONDUCTANCE AT } \lambda_0}$

FIGURE 4