Electrical pattern calculator for linear arrays

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ELECTRICAL PATTERN CALCULATOR FOR LINEAR ARRAYS

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ELECTRICAL PATTERN CALCULATOR FOR LINEAR ARRAYS

I. SUMMARY

A calculator for the electrical addition of twelve vectors of arbitrary phase and magnitude is described. Examples are given of its use in calculating the patterns of linear arrays of antennas with arbitrary spacing, phase and magnitude. This may be done directly for asymmetrical arrays of 12 elements, or symmetrical arrays of 23 elements; and indirectly in steps for arrays of n elements.

II. GENERAL

A linear antenna array may be defined as a collinear spacial distribution of similarly oriented radiating sources with identical radiation patterns. Although in general the relative spacings of the elements are not identical, the most common type of array is equispaced, and for that reason will be the only case dealt with in detail.

Consider a linear array of equispaced elements (see Fig.1). The complex field strength at an angle θ from the perpendicular to the array may be written:

$$Ee^{i\alpha} = A_o + \sum_{1}^{n-1} A_m e^{i\alpha_m} \qquad (1)$$

$$\alpha_m = m\psi + \phi_m$$

$$\psi = \frac{2\pi}{\lambda}\ell \sin\theta + \phi$$

where A_o, A_1, ... A_m, ... A_{n-1} are the relative amplitudes of the antenna elements.

ϕ is the progressive phase angle between elements

ϕ_1, ϕ_2, ... ϕ_m, ... ϕ_{n-1} are the phase deviations from the above progressive phase angle.

λ is the wavelength

ℓ is the spacing between elements.

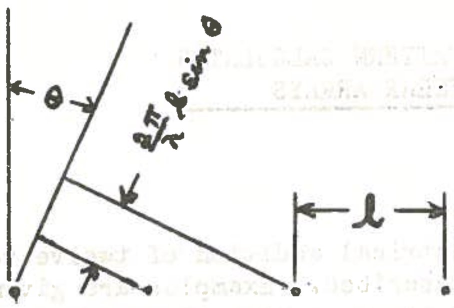

FIG. I

The calculation of an array pattern then consists of the addition of n co-planer vectors for each angle θ considered. The magnitude of the vectors are directly proportional to the amplitudes of the elements, and the phases are functions of the azimuth angle, the spacing, the wavelength and the relative antenna phases. This is shown for a four element array in Fig. 2.

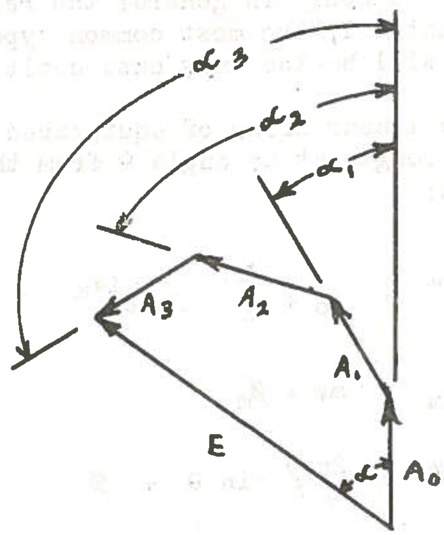

FIG. 2

Resultant field strength for any angle θ

A number of circuits for making this calculation electrically were suggested, nearly all of them being more interesting than the one chosen.

(a) The first method involved a set of Selsyns, one for each vector. They were to have three phase windings on both stator and rotor. The Selsyns were tc be cascaded by connecting the stator of one to the rotor of the preceding. The rotors were to be ganged on a single shaft with provision for adjusting the initial phase deviations ϕ_1, ϕ_2, ... ϕ_{n-1} independently. Fractions of voltage from each stator corresponding to the amplitudes A_o, A_1, ... A_{n-1} were then to be added to give the resultant voltage E. In this way the phase angles α_1, α_2, ... α_{n-1} could have been controlled by a single knob since the phase displacement of the m^{th} vector is the sum of m displacements, $\Delta\psi$. A simple slide rule style of chart could be used to relate ψ and Θ, with ℓ/λ and ϕ as parameters.

(b) A second method consisted of cascaded phase shift stages of the resistance-condenser type. This too would use a single knob to change the phase of all vectors simultaneously.

(c) Another suggestion involved the use of ironless goniometers driven from a common voltage source. A mechanical step-up connection between the rotors would give the m^{th} vector the required $m(\Delta\psi)$ phase displacement.

(d) The most interesting method proposed was the use of a real or artificial line to obtain the phase shifts, the vector voltages being taken off at suitable points along its length. The phase angle ψ would depend directly on the frequency, and the frequency would be variable.

(e) The method adopted is a simplification of (b). It uses a master polyphase source of the resistance-condenser type to supply all the vector voltages. Since these voltages are quite independent of one another, each phase angle α_1, α_2, ... α_{n-1} must be set individually for every angle Θ considered. This is the principal defect of the system. A special chart to be described later (see Fig.10) is used to relate α_1, α_2 ..... α_{n-1} to Θ.

Method (a) was eliminated because of the cost of the Selsyns, method (c) because of the machine shop time required, and methods (b) and (d) because of the time required for development.

III. DESIGN

The polyphase source is star connected, and operates at the line frequency of 60 cycles per second. Each point in the star (V_1 ---- V_q) is fed from a phase shifter of the familiar resistance-condenser type (see Fig.3). Adjacent points are 20° apart in phase, but only half of the 18 phase source is actually provided, an optional 180° phase shift in the output circuit allowing all angles to be covered (see Fig.4B). For each vector there is a selector switch and interpolator I_m (see Fig.4A) to interpolate to degree accuracy between any two points of the star. The amplitude of each vector is set by an amplitude control potentiometer, B_m.

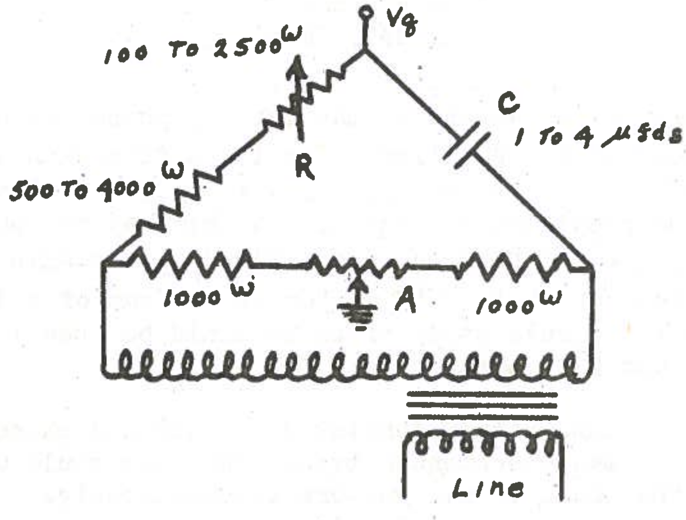

FIG. 3

The values of R and C (Fig.3) lie within the ranges shown on the diagram. They are chosen to give approximately the correct phase, the exact values being obtained by a simple method described later. A is a centre adjustment.

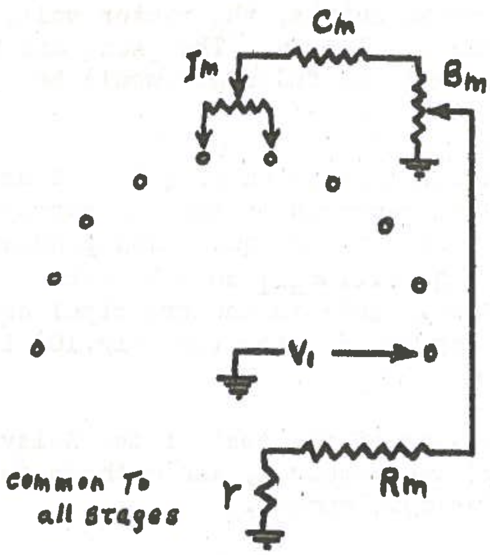

FIG. 4A

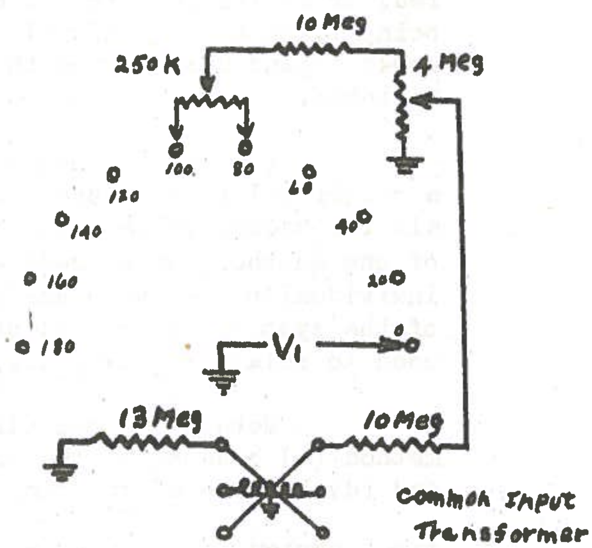

FIG. 4B

The vector relations in the interpolation potentiometer are shown in Fig. 5. It will be seen that the voltage V_m fed to the m^{th} vector changes gradually in phase as the movable arm travels across the

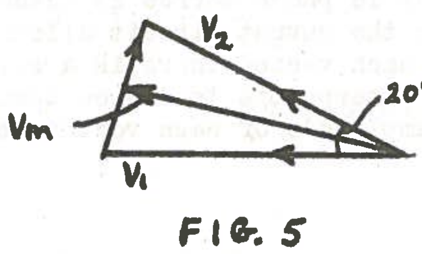

FIG. 5

potentiometer I_m, and remains approximately constant in amplitude. R_m is a high "isolating" resistance. The low resistance r is common to all stages.

Suppose the potentiometer B_m has been adjusted so that the voltage developed across the common resistance r is proportional to the amplitude of the m^{th} vector. Then the total voltage E_R developed across r is the vector sum of the voltage drops due to the currents from all stages. This resultant voltage E_R is read on a voltmeter provided for the purpose. An extra stage whose phase angle and amplitude may be adjusted to cancel out the resultant may be used to measure the angle of the resultant. This is required if in equation (1), $n > 11$, and the sum must be calculated in steps of 12.

Since the line voltage is used as a source of supply, harmonics of the supply frequency are present. It is easier to remove these at the voltmeter than before the phase source. A tuned transformer was found adequate when the instrument is operated from the line, but is not good enough to permit the use of a magnetic stabilizer.

To avoid having the resultant E_R proportional to the line voltage, an inverse feed back amplifier with incandescent lamps in the inverse feed back circuit is used as the voltmeter. The lamps are heated by a direct current equal to the average value of line voltage derived from a rectifier. The circuit is arranged to have a gain proportional to $\frac{1}{\text{line voltage}}$ over a range of $\pm$ 10%. However, due to the instability of the lamp characteristic, the circuit is not entirely satisfactory. Other methods of eliminating changes in output due to line voltage are being tried.

A "calibration" circuit is included to permit checking and correcting the gain at any time without disturbing the calculation.

IV. ADJUSTMENTS

(1) The balance of the centre tapped source required for the generation of the phase voltages may be checked on a bridge which has the other two arms obviously balanced to ground. The centre tapped output transformer is checked in the same way.

(2) When this is done, the two phase voltages V_1 and V_{10} are known to be equal and accurately 180° apart. Adjacent pairs are then made equal in magnitude and subtracted, starting with a pair containing either V_1 or V_{10}. The resultant should be $2\ V_1 \sin \frac{\theta}{2}$. The phase of the unknown voltage in the pair is adjusted until the resultant is correct. A check is obtained when the last pair is reached.

(3) In Fig. 3, the condenser is shown as a pure reactance. If the power factor is greater than zero, the result of greatest importance is a decrease in phase voltage depending upon the phase. The error has a maximum value numerically equal to the power factor, when the phase angle is 90°. A simple adjustment to the input voltage to this phase may be made if the error is large enough to be serious.

(4) Both voltmeter and amplifier must, of course, be accurately linear. The most difficult element was the input transformer, which had to be tuned rather carefully to minimize the bad effects of change of permeability with flux density.

V. OPERATION

The phase chart shown in Fig. 10 consists of twelve (the twelfth is used only when calculating an array in steps) concentric paper discs of radius r, 2r, 3r .... 12r, mounted on a replaceable thirteenth sheet. The discs are free to rotate with respect to one another and the back sheet, but may be clamped securely in any position. A radial cursor is provided. Plotted uniformly around the concentric discs are the phase angles α_1, α_2 .... α_{12} in 1° intervals; and sinusoidally around the back sheet, the azimuth angle θ from 0° to 90° in 1/2° intervals. Separate back sheets are used for each element spacing ℓ/λ considered. An equal angular rotation of all the discs relative to the back sheet will set the progressive phase angle $\emptyset$, while individual rotations of the discs will set the phase deviations $\emptyset_1$, $\emptyset_2$ .... $\emptyset_{12}$. If the cursor is set on any azimuth angle, the corresponding phase angles α_1, α_2 .... α_{n-1} are read directly. In order to cover a range of θ from 0° to 90°, for any element spacing up to 1.0 λ, and to confine the chart to a reasonable size, it is necessary to allow for six revolutions of the cursor. Each 10° point of α_n has six angles marked on rather than one, and the back sheets have up to six circles for the azimuth angle θ, depending on the spacing. Two-colour lettering helps avoid confusion. The chart is of course only useful for equispaced arrays. Patterns of arrays with arbitrary element spacing can be handled by the calculator, but it is then necessary to work out the phase angles α_1, α_2, ... α_{n-1} for each azimuth angle θ, a somewhat laborious task.

From the viewpoint of pattern calculations, linear arrays with equispaced elements can be grouped in the following manner:

(1) Up to 12 elements - arbitrary spacing, amplitude and phase:- The parameters ℓ , λ, $\emptyset$ and $\emptyset_1$, $\emptyset_2$, ... $\emptyset_{n-1}$ are set on the phase chart. The amplitudes A_0, A_1, A_2, ... A_{n-1} are set on the vector panel. For each azimuth angle θ the phase angles α_1, α_2, ... α_{n-1}, read from the chart, are set on the vector panel, and the field strength read from the meter. If many calculations are to be made of arrays with the elements varying im amplitude but all in phase, it is convenient to tabulate the phase angle α_1 .... α_{n-1} for each ℓ/λ required and use this table rather than the chart.

(2) Odd numbers of elements up to 23 - arbitrary spacing, amplitude and phase, but symmetrical about the centre element:-

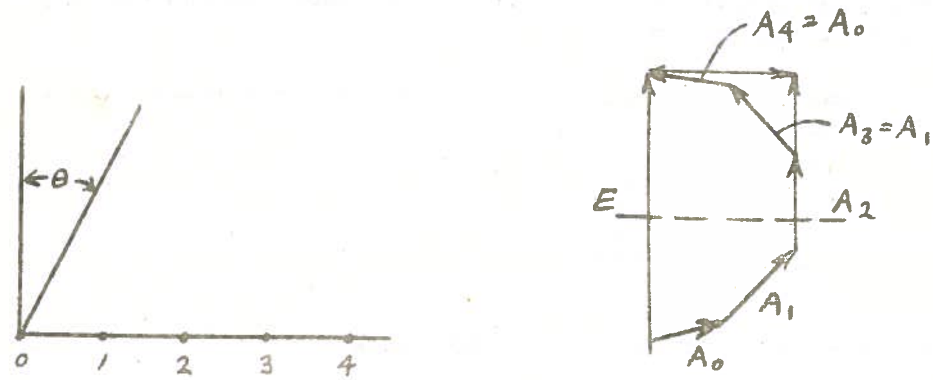

FIG. 6

On the vector panel set up the amplitudes of the centre to end elements, with the centre element at one-half value. It can be seen from Fig. 6 that if an additional vector, with its phase in quadrature with the centre element, is added in so as to minimize the resultant, that resultant will be one-half the required value.

(3) More than 12 elements - arbitrary spacing, amplitude and phase:- Set up the array approximately twelve elements at a time, and for each group determine the magnitude and phase angle of the resultant. To the phase angles of each group after the first will have to be added α_{12}, α_{24} ($\alpha_{24} = 2\alpha_{12}$ if all elements are in phase), etc. The addition of these group resultants in another step will give the required value.

Representative patterns worked out on the calculator illustrating these three cases are appended. (See Figs. 11, 12, 13.)

VI. ERRORS

The errors in the instrument are the result of a compromise between accuracy and economy. They may be reduced to any desired value by proper choice of circuit constants. In many cases our choice had to be made on the basis of components available. In this section we shall give values for the various errors in the instrument built.

(1) From the vector diagram of Fig. 5, it may be seen that V_m, the length of the vector representing the voltage on the movable arm of the interpolation potentiometer I_m, is not constant, but follows the straight

line joining the ends of the vectors V_1 and V_2. The relative error is greatest at the mid-point and is equal to $(1 - \cos \phi/2)$ where ϕ is the angle between phase steps, or approximately $\phi^2/8$ (ϕ in radians). 20° was chosen as a suitable angular step so that the maximum error is 1.5%. For 10° it would be 1/4 of this value or 0.4%. There is also an angular error, but this is negligible.

(2) A similar error arises from the internal resistance of the interpolation potentiometer. The maximum value is

$$\frac{\frac{I_m}{4}}{C_m + B_m}$$

approximately (Fig. 4). It is 0.5% in this case.

(3) The loading of the phase source by the interpolation potentiometer causes (a) a phase error, (b) a voltage error.

The circuit of Fig. 7 represents one of the phase sources.

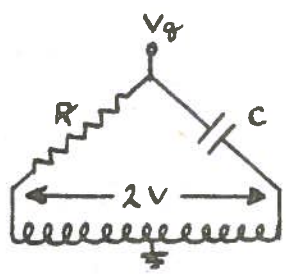

FIG. 7

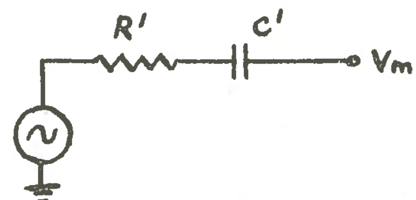

FIG. 8

Fig. 7 is equivalent to Fig. 8 when

$$R' = \frac{R}{1 + R^2\omega^2C^2}$$

$$X_{C'} = -\frac{R^2\omega C}{1 + R^2\omega^2C^2}$$

If all the interpolation potentiometers were placed in the first phase step, the load would be

$$\frac{250\text{ K}}{12} \cong 20\text{ K ohms}$$

$$R' = 107.2 \text{ ohms}$$

$$X_{C'} = 647 \text{ ohms}$$

The phase error, $\frac{R'}{20\ K}$ is thus 0.5%.

The voltage drop across $X_{C'}$ is $\frac{0.347\ V}{20\ K} \times X_C = 1.1\%$

This voltage is essentially 180° out of phase with the phase voltage V, and is therefore the voltage error at this end of the potentiometer.

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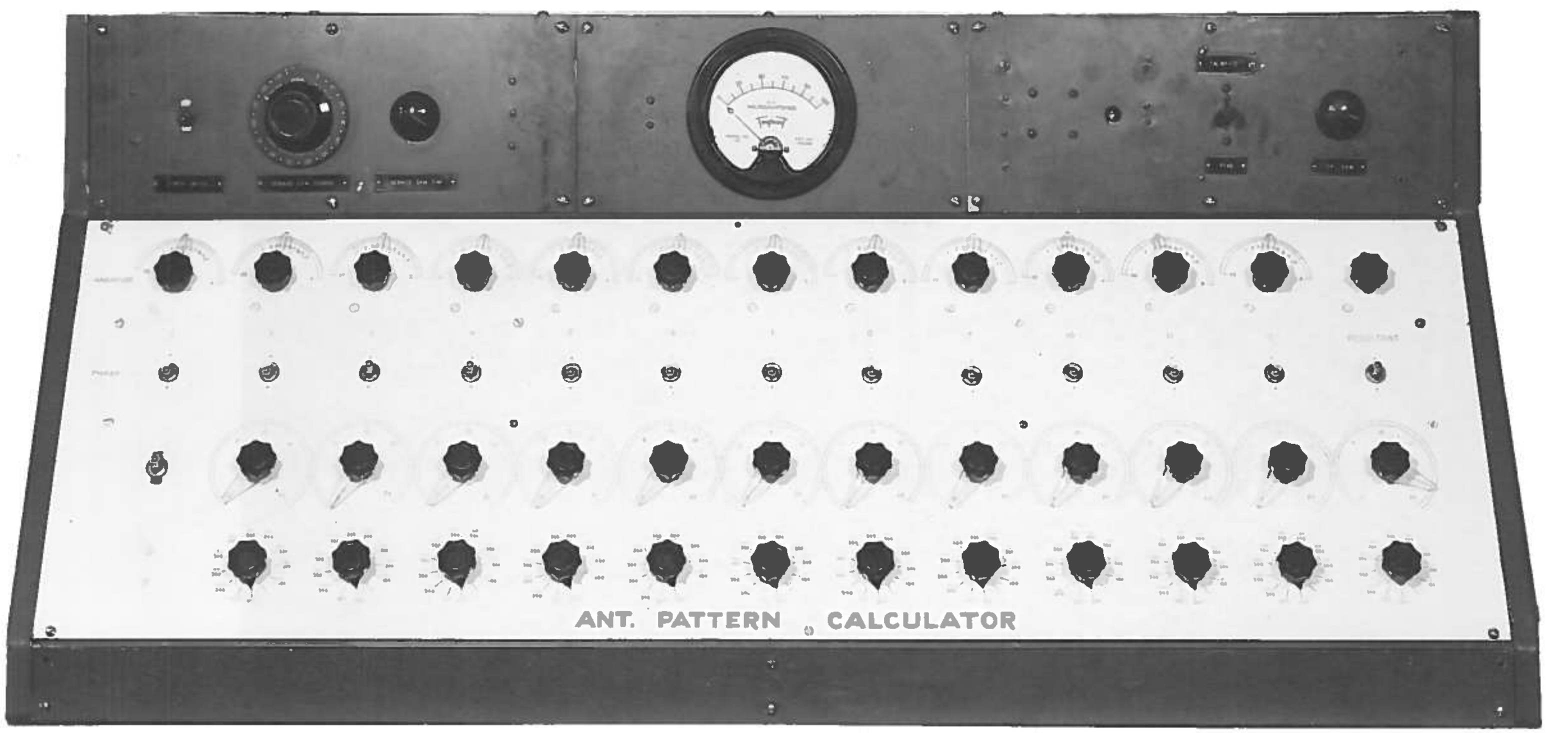

N.R.C. PHOTO
FIG. 9

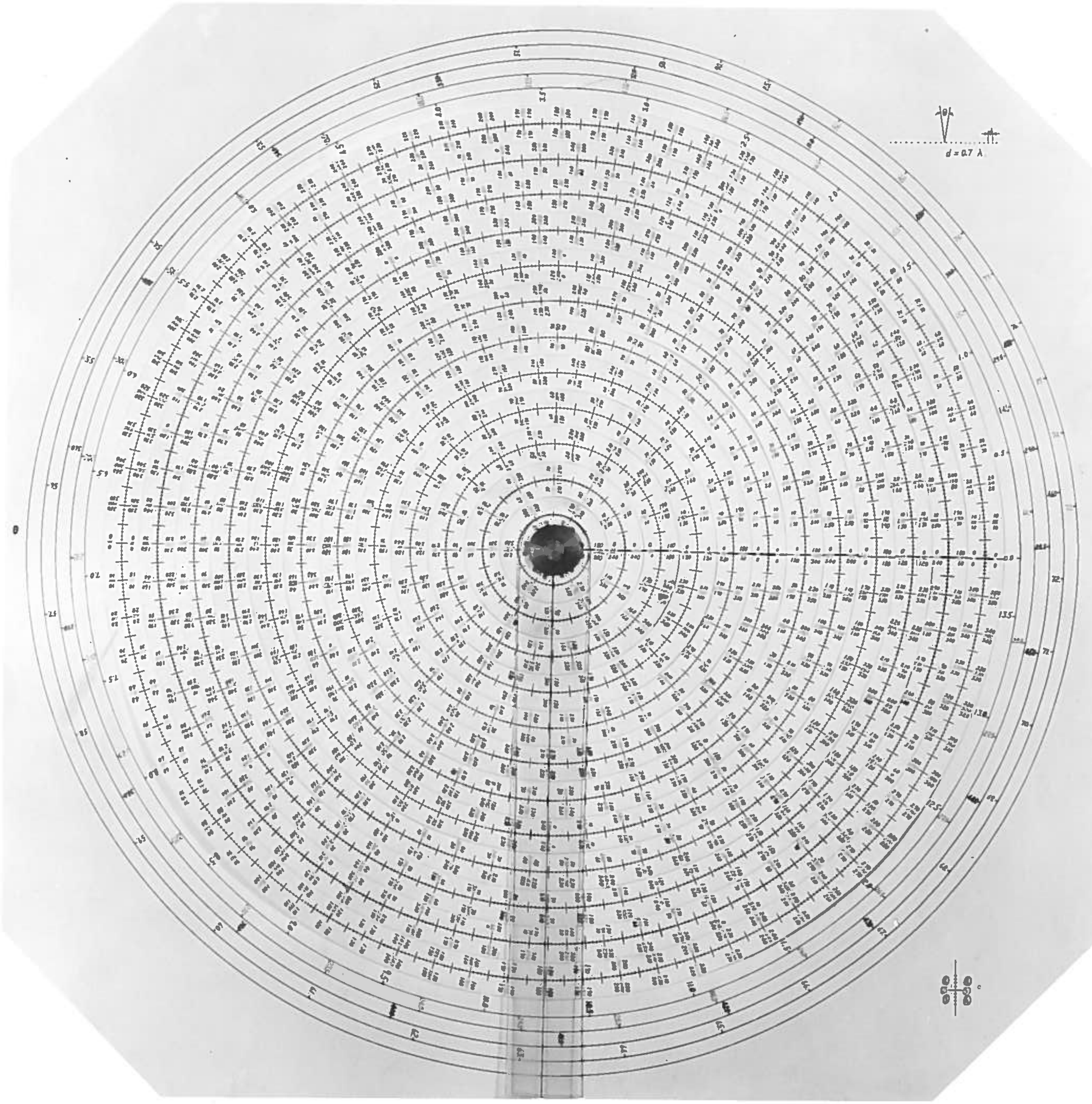

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FIG.10

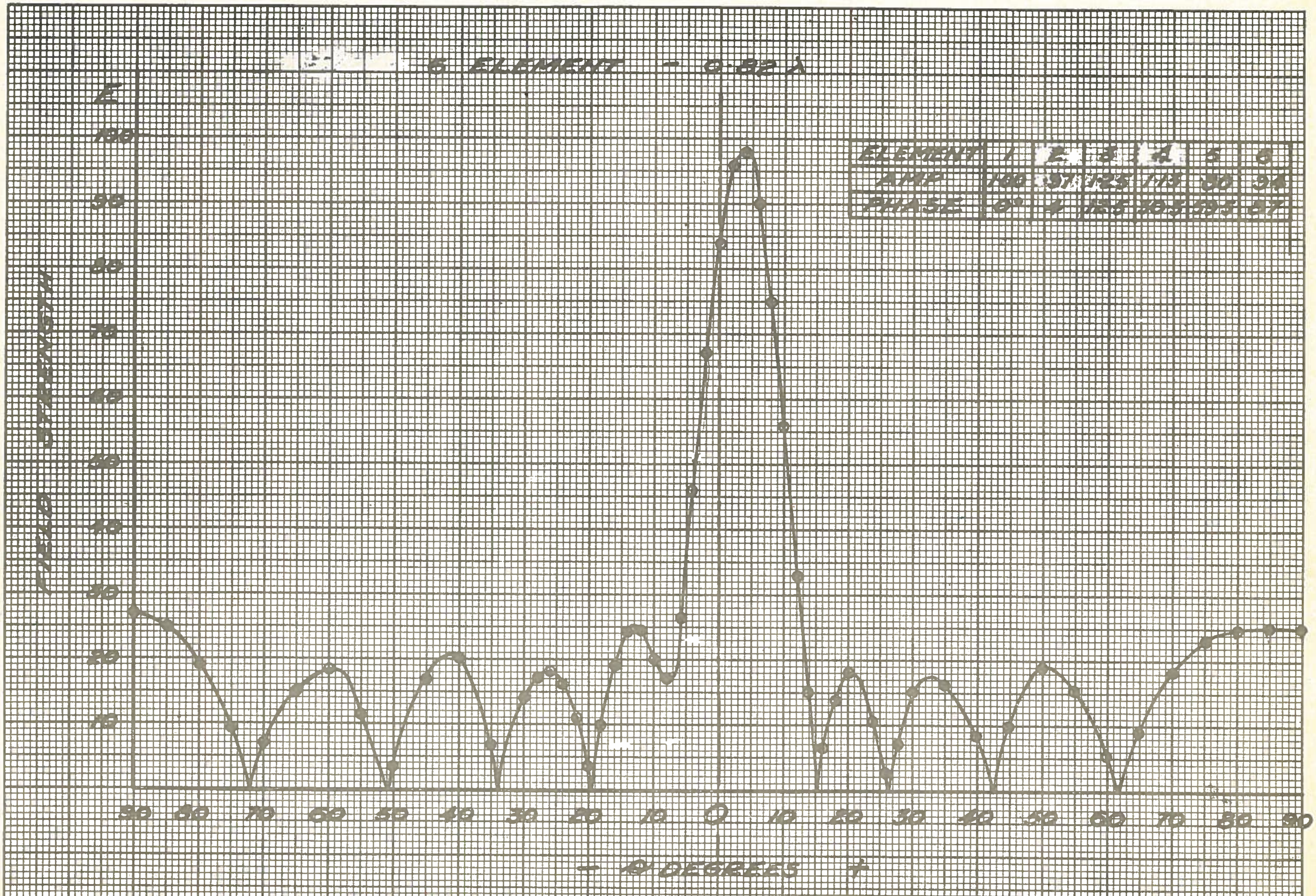

FIGURE 11

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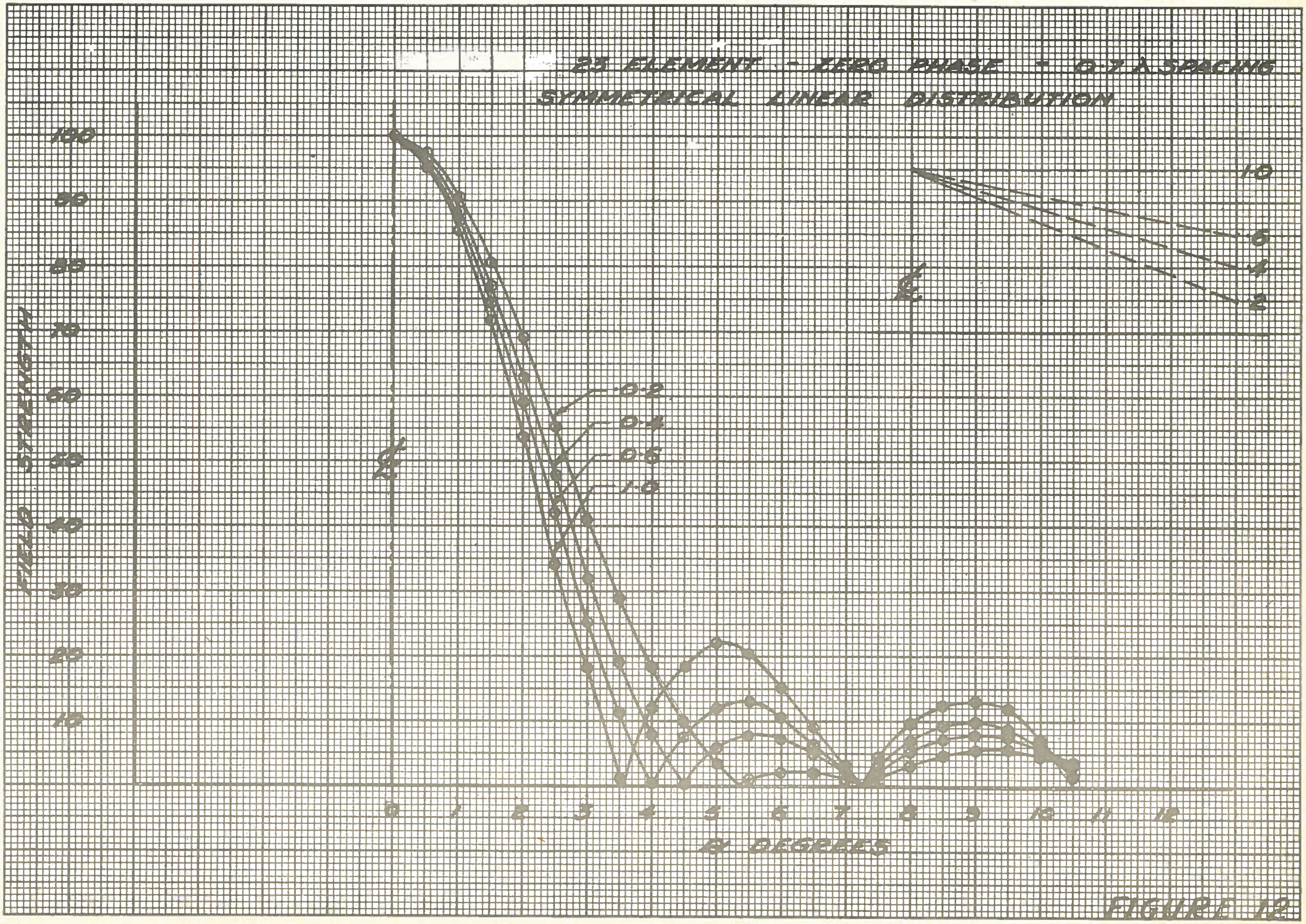

FIGURE 12

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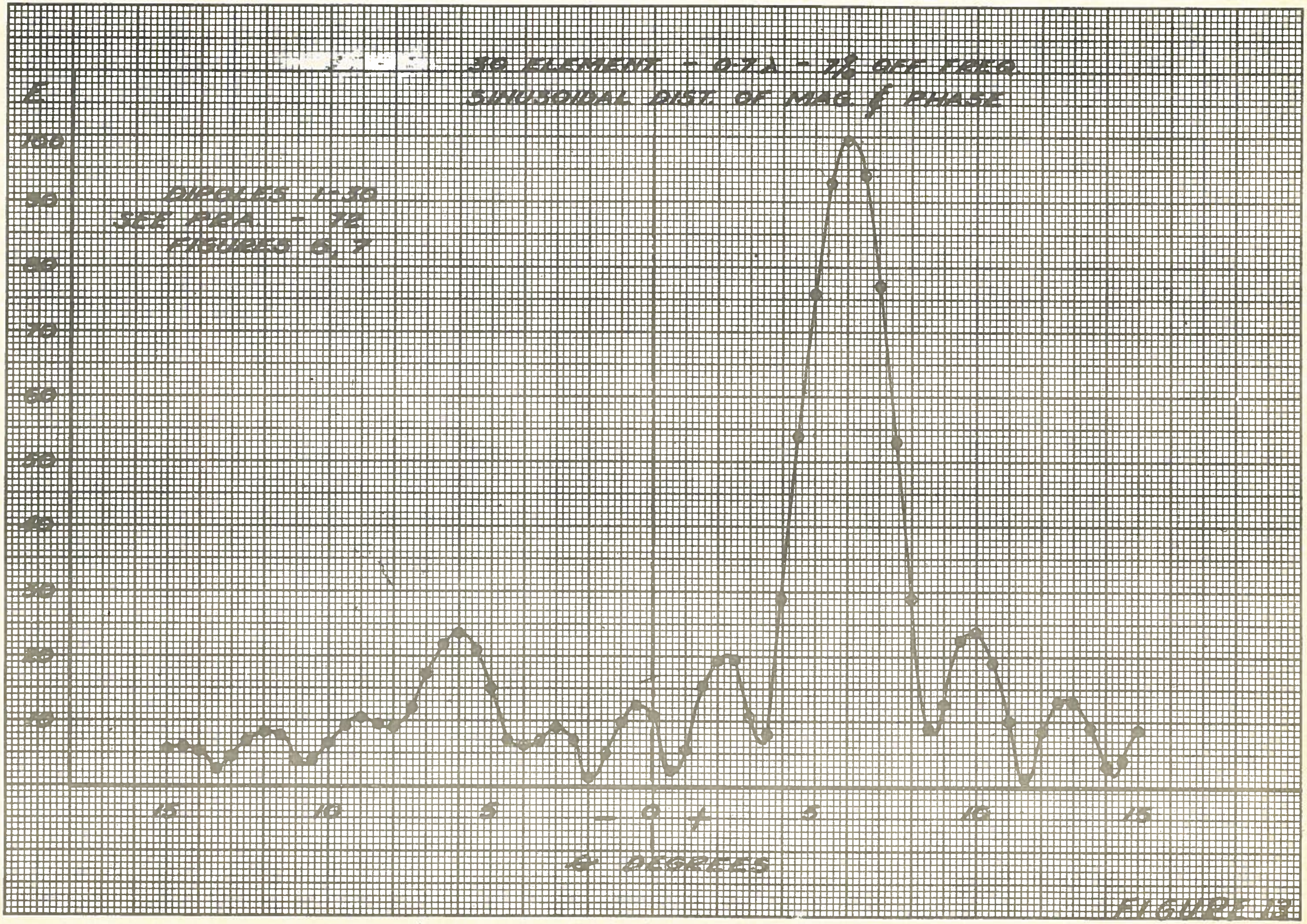

FIGURE 13

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