Preliminary Analysis of Structure-Ground Interaction for Seismic Design of a Nuclear Reactor Structure

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TECHNICAL NOTE

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SUBJECT PRELIMINARY ANALYSIS OF STRUCTURE-GROUND INTERACTION FOR SEISMIC DESIGN OF A NUCLEAR REACTOR STRUCTURE

The following is a preliminary analysis of the effects of interaction of the structure with the ground under seismic loading. Various approximations and assumptions had to be made either because more definitive information was not available or there was insufficient time for a more exact analysis.

This approximate treatment indicated the important parameters involved and what further steps are required for a more rigorous treatment. The methods contained herein should not be used for final design without a critical evaluation of their accuracies and applicability to the structure at hand.

It should be noted that this analysis does not consider the amplification of ground motion due to deep soft soil deposits.

DATA

Structure Dimensions

Diameter of circular rigid base = 156 ft (47.5 m)
Height = 175 ft (53.3 m)
C.G. at approximately mid-height = 26.7 m
Weight of structure = 50,000 tons approx. (50,000,000 kg)
Estimated natural period of rigid base structure,

$$T_o = 0.2 \text{ sec.}$$

Soil Constants

Winkler foundation, $k_v = 0.8 \text{ kg/cm}^3$

$k_H = 0.6 \text{ kg/cm}^3$

Boussinesq foundation should also be used

STRUCTURAL AND FOUNDATION PROPERTIES

1. Determination of Natural Frequencies

Vertical stiffness

$$K_v = \text{Area} \cdot k_v$$

$$\text{Area} = \frac{\pi d^2}{4} = \frac{\pi(47.5)^2}{4} = 1772 \text{ m}^2$$

$$K_V = 1772 \times (0.8) \times 10{,}000$$

$$= 14{,}200{,}000 \text{ kg/cm} = 13{,}900{,}000{,}000 \text{ N/m}$$

Horizontal stiffness

$$K_H = 1772 \times 0.6 \times 10{,}000$$

$$K_H = 10{,}630{,}000 \text{ kg/cm} = 10{,}430{,}000{,}000 \text{ N/m}$$

Rocking stiffness

$$K_\theta = k_v . I = k_v . \frac{\pi r^4}{4} = 0.8 \times \frac{\pi(4750)^4}{64}$$

$$K_\theta = 20 \times 10^{12} \text{ kg cm/rad} = 1.96 \times 10^{12} \text{ Nm/rad}$$

Translational frequency

$$\omega_H = \sqrt{\frac{K}{M}} = \sqrt{\frac{10.63 \times 10^6 \times 982}{50{,}000{,}000}} = \sqrt{209} = 14.5 \text{ rad/sec}$$

or $f_H = 2.31$ Hz

$T_H = 0.433$ sec.

Rocking Frequency

Mass moment of inertia of structure:

assume uniform mass distribution

$$w = \frac{50{,}000{,}000}{47.5 \times 53.3} = 19{,}700 \text{ kg/m}^2$$

$$I = \frac{bd^3}{3} \cdot w = \frac{47.5 \times (53.3)^3}{3} \cdot 19,700$$

$$I = 4.73 \times 10^{10} \text{ kg m}^2$$

$$\omega_\theta = \sqrt{\frac{K_\theta}{I}} = \sqrt{\frac{196 \times 10^{10} \text{ Nm}}{4.73 \times 10^{10} \text{ kg m}^2}} = \sqrt{41.5} = 6.45 \text{ rad/sec}$$

$$f_\theta = 1.03 \text{ Hz}$$

or $T_\theta = 0.97$ sec.

Structural Frequency

$$T_o = 0.2 \text{ sec} = 5 \text{ Hz} = 31.4 \text{ rad/sec}$$

Combined Coupled Frequency

$$T^2 = T_o^2 + T_H^2 + T_\theta^2$$

$$= 0.2^2 + 0.433^2 + 0.97^2 = 1.17$$

$$T = 1.08 \text{ sec or } f = 0.925 \text{ Hz} = 5.92 \text{ rad/sec}$$

Remarks

As can be seen, the dynamic properties are completely dominated by the rocking motion, with the translation providing a small modification. For all practical purposes the structural deformability can be neglected for purposes of investigating the effect of ground-structure interaction.

A further consequence of this observation is that the rocking frequency should be determined to a higher degree of accuracy than was possible from the above data. A more accurate determination of the mass moment of inertia is required.

There are now three resonance frequencies, in accordance with the three degrees of freedom of rotation, base translation, and structural deformation. As an initial approximation it is sufficient to consider the first mode only, the one having the previously calculated period T = 1.08 sec.

2. Damping

An approximate damping coefficient can be computed from half-space theory.

$$a_o = \frac{\omega r}{V_s} \qquad V_s = \text{shear wave velocity of ground.}$$

assume $V_s = 1000$ fps (medium soil)

$$a_o = \frac{(5.92)\,(78)}{1000} = 0.46$$

The damping ratio in rocking is

$$\lambda_R = \frac{C_R}{2(B\eta K_R)^{1/2}}$$

$$B = \frac{M_1}{\rho r^3} = \frac{50{,}000{,}000}{1.95 \times 10^3 \times 23.7^3} = 1.93$$

$$\eta = \left(\frac{h}{r}\right)^2 = 1.12^2 = 1.25$$

$$\therefore\ \lambda_R = \frac{0.1}{2\,(1.93 \times 1.25 \times 2.7)^{1/2}} = 0.0195 \cong .02$$

$$\lambda_R = 2\% \text{ of critical}$$

Thus an approximate value for rocking damping is 2 per cent of critical due to radiation of energy into the half-space. To this can be added hysteretic material damping of the soil. At the moment this is difficult to evaluate, but with more investigation and literature search some justifiable value could be derived. A rough estimate of damping for soil material is between 2 and 5 per cent, additive to the radiation damping.

As the damping in the rocking mode is rather low, it becomes a dominant design parameter. Therefore for final design, a more thorough analysis of the frequency response curve for the entire system would be justified.

The motion of the structure in the horizontal direction is highly damped and this will somewhat influence the amount of over-all damping. A quantitative answer can be obtained only be evaluating the transfer function of the system. The contributions of the higher modes can also be obtained from a detailed study of the transfer function of the entire system.

Further information needed for more accurate evaluation of ground-structure interaction effects include:

1. A more accurate description of the mass distribution of structure.
2. More information about soil, especially modulus, type, etc.
3. Information of expected earthquake motion.
4. Detailed study of transfer function of the system.

3. Mode shapes

Using the approximations for an undamped system, the relative modal amplitudes are given by Balan et al.

$$X_B \cdot \omega_H^2 = (h\theta)\, \omega_\theta^2 = X \cdot \omega_o^2$$

$$\frac{X_B}{X} = \frac{\omega_o^2}{\omega_H^2} = \frac{31.4^2}{14.5^2} = 4.7$$

$$\frac{h\theta}{X} = \frac{\omega_o^2}{\omega_\theta^2} = \frac{31.4^2}{6.45^2} = 23.8$$

$$\frac{X_B}{h\theta} = \frac{4.7}{23.8} = 0.2$$

Modal ratios:

$$X_B : h\theta : X = 1:5:0.21$$

The mode shape is sketched in Figure 1. These modal ratios are approximate and are subject to inaccuracies because of non-proportional damping. More accurate answers can be obtained from the transfer function analysis.

APPLICATION IN SEISMIC DESIGN

As the system is dominated by the rocking mode, for preliminary design it is satisfactory to consider its effect alone. One can now treat the entire system as a single-degree-of-freedom (S.D.F.) oscillator with natural period $T = 1.08$ sec or frequency $f = 0.925$ Hz, and damping of λ = say 5 per cent (2 per cent radiation damping, 3 per cent hysteresis damping). The forces determined from response calculations or response spectra, using these S.D.F. values, are then the forces acting at the centre of gravity of the structure during that disturbance. The deformations determined from those response calculations are the total deformations at the C.G., i.e., having a modal amplitude of $1.0 + 5.0 + 0.21 = 6.2$ (relative).

1. Deformations of structure

For a spectral response value of

$$S_v = 25 \text{ in/sec}$$

$$S_D = 4 \text{ in.}$$

$$S_A = 0.40 \text{ g}$$

The following deformations are obtained (Figure 2):
Horizontal, at C.G.:

$$\frac{4}{6.2} = 0.65 \text{ in.}$$

$$\frac{4}{6.2} \times 5 = 3.2 \text{ in.}$$

$$\frac{4}{6.2} \times 0.21 = 0.13 \text{ in.}$$

Vertical movement at edge of mat:

$$3.2 \times \frac{23}{27} = 2.7 \text{ in.}$$

To reduce these deformations the following seem possible:

(a) refine the determination of damping; this may yield a higher value of λ;
(b) stiffen the soil foundation (piles, compaction, etc.);
(c) extend the base to provide greater foundation-soil stiffness.

2. Vertical interaction of structure with ground

Vertical natural frequency

$$\omega_Y = \sqrt{\frac{Kv}{M}} = \sqrt{\frac{13,900 \times 10^6}{50 \times 10^6}} = \sqrt{278} = 16.7 \text{ rad/sec}$$

$$f_v = 2.7 \text{ Hz}$$

This is a low vertical resonance and may play some role in the seismic design. The damping in the vertical direction is substantial, however. With the aid of machine vibration analogue (Richard, Hall & Woods, p. 204, etc.),

$$Bz = \frac{1-\nu}{4}\ \frac{M}{\rho r_o^3} \cong \frac{0.7}{4} \times \frac{50,000,000}{2 \times 10^3 \times 23.7^3} = 0.33$$

$$D = \frac{0.425}{\sqrt{Bz}} = \frac{0.425}{\sqrt{.33}} = 0.74$$

Therefore the damping coefficient λ = 74 per cent of critical, i.e., it is almost critically damped, considering only geometric damping. To this could be added material damping of the soil. It may therefore be concluded that vertical motion of the structure will not be amplified and, in fact, may be reduced as compared to the vertical motion of the ground.

ALTERNATIVE PROPOSAL: FRICTION PILE FOUNDATION

Reinforced concrete precast friction piles, 180 ft long, 16 in. diameter. Number of piles: 758; design load: 260 kips/pile.

An estimate of stiffness has to consider the whole pile group. The properties of the individual pile become rather secondary in character.

1. Assumptions

Rocking

Assume piles transfer load to tip of pile group. Soil stiffness increases by a factor of perhaps 5 to 10 due to overburden confinement. In addition, peripheral friction of pile group and soil contained therein will act against external soil. Assume that the vertical spring stiffness in friction is the same as the spring stiffness for the Winkler foundation in the horizontal direction at the surface, $k_H = 0.6\ kg/cm^3$ (Figure 3).

Horizontal translation

Assume that the entire soil cylinder within the perimeter piles acts against the exterior soil. In addition, frictional forces will be mobilized at right angles to direction of motion. Owing to pile deformations this pressure will be distributed in a triangular manner from the butt of the pile to, say, one half its length. In the direction of motion, assume the value for the Winkler-foundation constant in compression, $k_V = 0.8\ kg/cm^3$; in friction assume the constant in shear, $k_H = 0.6\ kg/cm^3$. The pressure distribution is assumed to vary sinusoidally from one end of a diameter to the other (Figure 4).

2. Calculation of Constants

Rocking Motion:

Moment of inertia of peripheral ring = 1/2 polar moment of intertia

$$= 1/2\ (\pi D\ r^2) = \frac{\pi r^3}{4}$$

Total rotational stiffness = $\frac{\pi r^3}{4}$. depth x 0.6 + 5 x (rotational plate stiffness)

$$= \frac{\pi(2375)^3}{4} \times 6000 \times 0.6 + 5\ (20 \times 10^{12})\ \text{kg cm/rad}$$

$$= 130 \times 10^{12}\ \text{kg cm/rad}$$

$$K_\theta = 13 \times 10^{12}\ \text{Nm/rad}$$

Revised moment of inertia of structure: centre of rotation is likely to shift downward from base to, say one third the pile length.

Old I = 4.7×10^{10} kg m^2

New I = 12.6×10^{10} kg m^2

Rocking frequency for multiplier of 5 for base stiffness:

$$\omega_\theta = \sqrt{\frac{13 \times 10^{12}\ \text{Nm/rad}}{12.6 \times 10^{10}\ \text{kg m}^2}} = \sqrt{100} = 10\ \text{rad/sec}$$

or $f_\theta = 1.6$ Hz

$T\theta = 0.62$ sec.

Horizontal motion:

Equivalent spring stiffness = 0.7 x 0.8 x 2r x depth/4

+ 2(0.7) x 0.6 x 2r x depth/4

$K_H = 10 \times 10^6$ kg/cm

or $K_H = 10 \times 10^9$ N/m

Additional soil inertia mass in horizontal direction:

$$= \frac{\pi D^2}{4} \times \text{depth}/4 \times 110\ \text{lb/ft}^3$$

$$= \frac{\pi(156)^2}{4} \times 180/4 \times 110 = 95{,}000{,}000\ \text{lb} = 40{,}000\ \text{tons}$$

Total mass = 40,000 + 50,000 tons = 90,000 tons

Whether this additional soil mass is fully active in the earthquake response is somewhat questionable. Consequently calculations will be performed with and without considering this soil mass.

Translational frequency:

$$\omega_H = \sqrt{\frac{K}{M}} = \sqrt{\frac{10 \times 10^9 \text{ N/m}}{90 \times 10^6 \text{ kg}}} = 10.5 \text{ rad/sec}$$

or $f_H = 1.7$ Hz

$T_H = 0.6$ sec with soil mass

$T_H = 0.44$ sec without soil mass (14.1 rad/sec).

Combined coupled period:

$$T^2 = 0.2^2 + 0.6^2 + 0.62^2$$

$$T = 0.88 \text{ sec or } f = 1.12 \text{ Hz}$$

The periods for the various assumptions are summarized in Table I.

Damping:

The over-all damping coefficient can probably be increased to 10 per cent because of the large surface areas over which relative deformation occurs and consequently where energy will be dissipated. 5% is certainly quite conservative.

3. Seismic Response

Using the same procedure as was used above, the seismic response for the various alternatives considered are presented in Table II.

DISCUSSION OF RESULTS

It is likely that the values of X_B are on the large side since higher lateral soil stiffness can probably be mobilized.

In comparing the results for the various assumptions it is evident that where the rocking component has been reduced, the horizontal motion has increased. This can be expected since the rocking stiffness has been increased relative to the horizontal stiffness. The total spectral displacement has to be accommodated by the available degrees of freedom.

Relatively rough assumptions have been made in arriving at the over-all pile foundation stiffness. DBR colleagues in the Geotechnical Section, supported the assumption of group action proposed in this Note as being reasonable. However, there is considerable latitude in the choice of the numbers used to describe the soil properties.

With the questionable improvement derived from a friction pile foundation and the other associated problems of settlement, has the possibility of partial submergence of the structure been investigated? This would be along the nature of a "floating" structure in the soil. The earthquake response of a structure substantially surrounded by proper backfill could then be greatly reduced. The contents of such a structure would then be subjected essentially to the ground motions.

One assumption that should be further investigated is the location of the centre of rotation and the subsequent calculation of I. Raising this centre of rotation would decrease the combined period, slightly reduce the total spectral displacement, and decrease the relative amount of rocking. The relative proportion of horizontal base movement however, would increase.

TABLE I

SUMMARY OF PERIODS, SECONDS

Case	Horizontal T_H, sec	Rotational T_φ, sec	Combined T, sec
1. Mat on soil	0.43	0.97	1.08
2. Friction Piles			
Multiplier 5			
a) with soil inertia	0.6	0.62	0.88
b) without soil inertia	0.44	0.62	0.72
Multiplier 10			
c) with soil inertia	0.6	0.44	0.77
d) without soil inertia	0.44	0.44	0.65

TABLE II

SUMMARY OF RESPONSE CALCULATIONS

Case	Description	Over-all Damping % Critical	Spectral Displacement S_D, in.	Base Displacement X_B, in.	Rocking Displacement $h\varphi$, in.	Structural Displacement X, in.	Modal Amplitude Ratios $X_B:h\theta:x$
1	Mat on grade T = 1 sec	5	4	0.65	3.2	0.13	1:5:0.21
2	Friction Piles						
	a) soil mass included T = 0.88 sec	5 10	3.5 2.5	1.65 1.2	1.65 1.2	0.18 0.13	1:1:0.11
	b) soil mass excluded T = 0.72 sec	5 10	3 2	1.0 0.65	1.9 1.3	0.1 0.06	1:2:0.1
	c) soil mass included T = 0.77 sec	5 10	3 2	1.9 1.3	0.9 0.6	0.19 0.13	1:0.5:0.1
	d) soil mass excluded T = 0.65 sec	5 10	2.5 1.7	1.15 0.8	1.15 0.8	0.23 0.16	1:1:0.2

BIBLIOGRAPHY

Balan, S., Ifrim, M., and Pacoste, C. Dynamic Equivalent of Antiseismic Structures Considering the Deformability of the Foundation Ground. Proc., Internat. Symp. on the Effects of Repeated Loading of Materials and Structures, RILEM, Mexico City. 15-17 Sept. 1966.

Rainer, J.H. Method of Analysis of Structure-Ground Interaction in Earthquakes, Nat. Res. Council of Canada, Div. Bldg. Res., Technical Paper No. 340, Ottawa, April 1971. (NRC 11920)

Rainer, J.H. Structure-Ground Interaction in Earthquakes. J. Eng. Mech. Div., ASCE, Vol. 97, No. EM5, Oct. 1971, p. 1431-1450. (NRC 12054)

Richart, F.E. Jr., Hall, J.R. Jr., and Woods, R.D. Vibrations of Soils and Foundations, Prentice Hall, 1970.

Roesset, J., Whitman, R.V., and Dobry, R. Modal Analysis for Structures with Foundation Interaction. J. Struc. Div., ASCE, Vol. 99, No. ST3, March 1973, p. 399-416.

Sarrazin, M.A., Roesset, J.M., and Whitman, R.V. Dynamic Soil-Structure Interaction. J. Struc. Div., ASCE, Vol. 98, No. ST7, July 1972, p. 1525-1544.

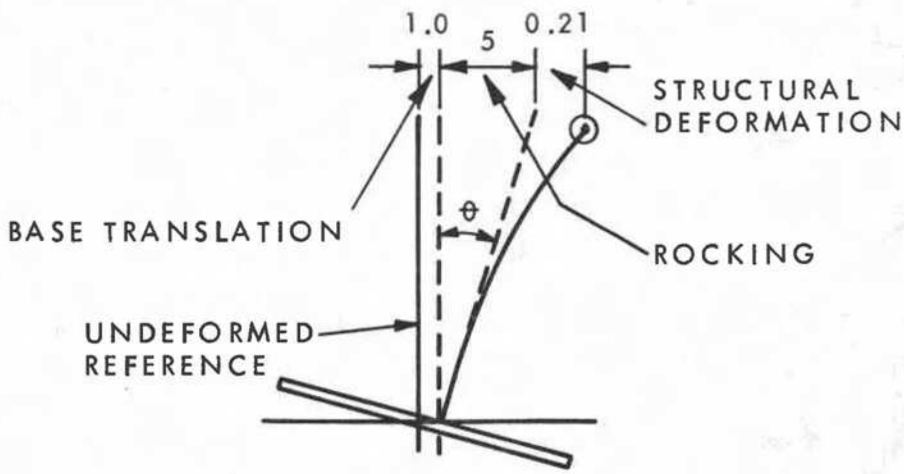

FIGURE 1 MODE SHAPE

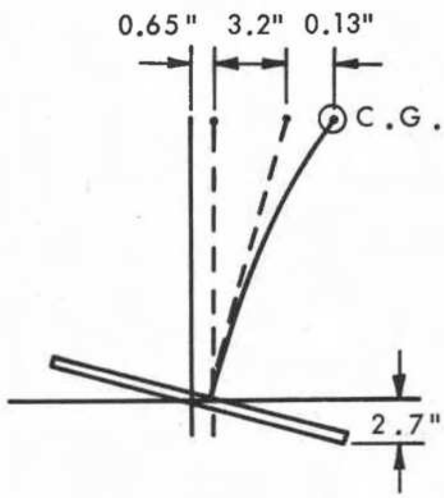

FIGURE 2 MAXIMUM SEISMIC RESPONSE OF STRUCTURE

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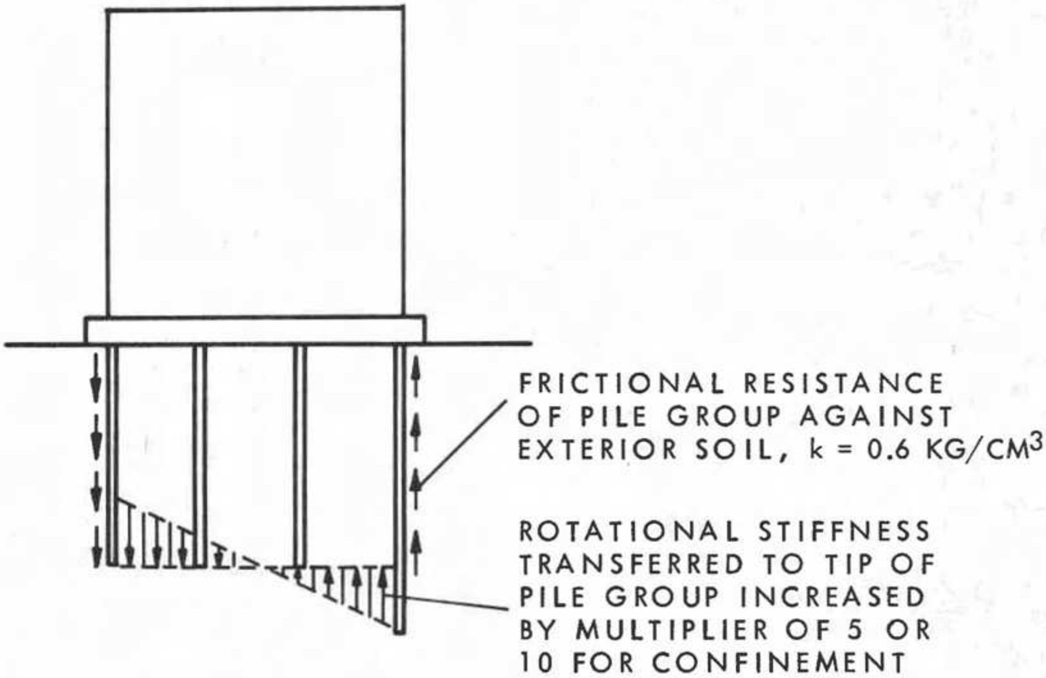

FIGURE 3 ROTATIONAL STIFFNESS ASSUMPTIONS

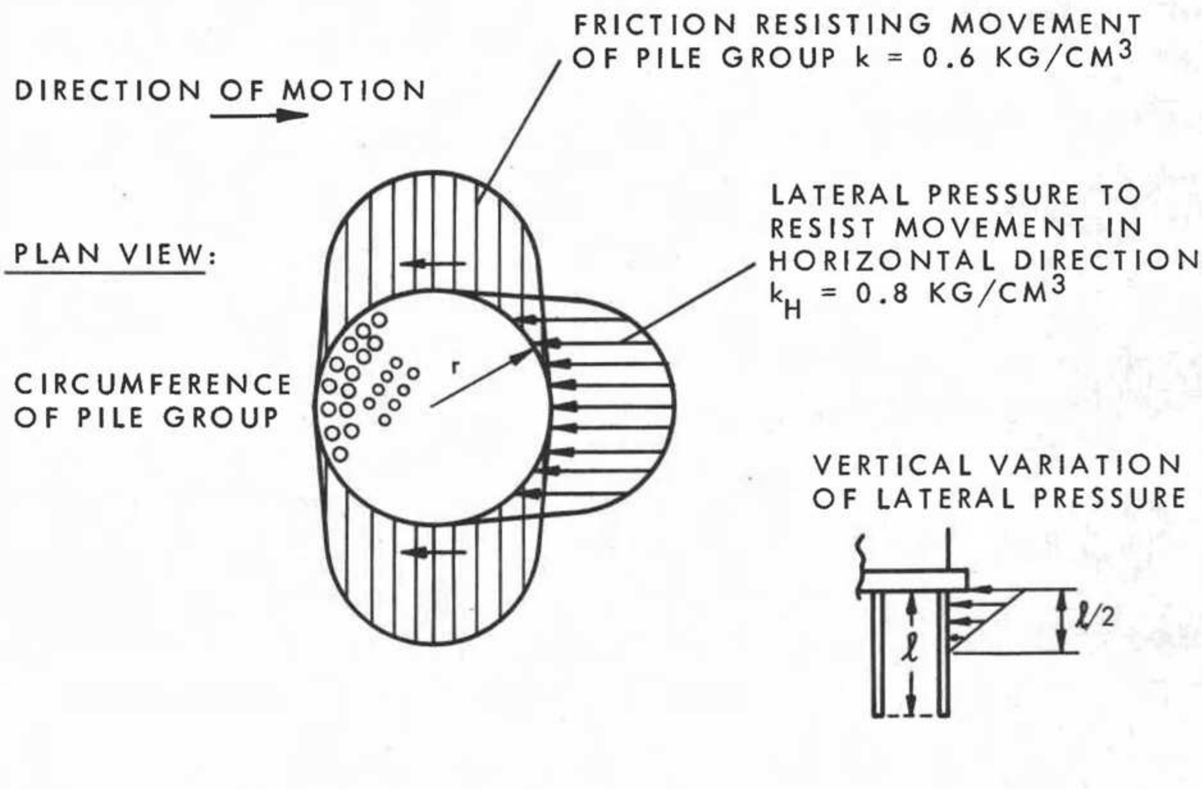

FIGURE 4 TRANSLATIONAL STIFFNESS ASSUMPTIONS

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