Vertical distribution of solid flux in a snow-wind flow
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Preface

Snowdrifts can be a major source of disruption in the operation of transportation services and a general nuisance in the normal wintertime activity of a community. Such drifts are formed whenever a wind, strong enough to transport horizontally a significant amount of snow, encounters an obstacle which forces it to deposit some of this snow. The usual approach taken in defending an area or structure against snowdrifting has been to locate the structure properly so that the drift problem will be a minimum and to erect obstacles, such as snowfences, to control where the snow will be deposited. The approach taken in the development of these defences has been largely empirical. Attention has been directed primarily to the character of the air flow with little attention being given to the material transported. In some circumstances, it would be an advantage to have a more complete defence against snowdrifting than is now available. In their attempts to develop this defence, engineers are giving more consideration to the theoretical aspects of the problem and in particular to the relationships between the air flow and the snow being transported.

It is one of the responsibilities of the Snow and Ice Section of the Division of Building Research to collect and make available information required for the solution of snow and ice problems encountered in engineering practice. This report gives theoretical calculations and experimental observations on the transport of snow by wind. It presents formulae giving the dependence on wind speed and height above ground of the amount of snow moved in a given time. This is the second Russian paper on this subject that has been published in the National Research Council translation series. The first paper, also by A.K. Dyunin, is entitled "Fundamentals of the Theory of Snowdrifting" (NRC Technical Translation 952).

The paper was translated by Mr. G. Belkov of the Translation Section of the National Research Council Library, to whom the Division of Building Research wishes to record its thanks.

Ottawa,
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Robert F. Legget,
Director

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Technical Translation 999

Title:	Vertical distribution of solid flux in a snow-wind flow (O raspredelenii raskhoda snegovetrovogo potoka po vysote)
Author:	A.K. Dyunin
Reference:	Trudy Transportno-Energeticheskogo Instituta, (4): 49-58, 1954
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VERTICAL DISTRIBUTION OF SOLID FLUX IN A SNOW-WIND FLOW

Summary

In this paper an attempt is made to analyze theoretically the distribution with height of the solid flux in a snow-wind flow. The analytical results are compared with field observations.

The general expression for the solid flux of a snow-wind flow q* at a certain height above the surface y has the form

$$q_y = p_y \cdot \bar{v}_y \ g/m^2 sec. \tag{1}$$

where p_y is the weight of snow contained in a unit volume of air at height y, in g/m ,

$\bar{v}_y$ is the average translational velocity of snow particles in m/sec.

Let us define the value of p_y.

It has been experimentally proven that river silt and also sand, dust and snow in moving air(1,3,6) are raised in the form of "clouds". The "cloud" of particles is cast up by surface eddies and penetrates into the mean flow.

The dimensions l of these penetrating masses increase in approximate proportion to the height y which is confirmed by the observations of Prof. A.Ya. Milovich (I, page 33), i.e. we have the following dependence

$$l = \alpha y + \beta,$$

where α and β are constant parameters.

Assume the penetrating masses ("clouds") to be cylinders with a cross-sectional area of $\alpha\, l^2$ (see Fig. 1) and consider a plane

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* In rail transport literature the solid flux of a snow-wind flow is frequently called "intensity of snow transfer" and is measured in g/cm^2 min.

section.

Let p be the weight of snow particles in a unit of volume of the penetrating mass (at the height of its centre of gravity y).

Then the total weight of particles per unit of cylinder length is

$$P_y = p_y \alpha l^2.$$

When the height is increased by Δy, we have

$$P_{y+\Delta y} = (p_y + \Delta p)\alpha(l + \Delta l)^2.$$

Neglecting the small components of higher order we get

$$P_{y+\Delta y} = \alpha p_y l^2 + 2\alpha l p_y \cdot \Delta l + \alpha l^2 \cdot \Delta p.$$

Assuming that the fluid (air) outside the penetrating cloud has a negligible quantity of impurities, we find that the total weight of particles in the "cloud" does not change, i.e.

$$P_y = P_{y+\Delta y}$$

from which one can immediately obtain

$$2\alpha l p_y \cdot \Delta l = -\alpha l^2 \cdot \Delta p.$$

Passing to the limit when $\Delta l \to 0$, we get the differential equation

$$\frac{dp_y}{p_y} = -\frac{2dl}{l}.$$

But according to the foregoing $l = \alpha y + \beta$ and $dl = \alpha dy_2$. Consequently

$$\frac{dp_y}{p_y} = -\frac{2dy}{y + \frac{\beta}{\alpha}}$$

Integrating we get

$$p_y = \frac{c_0}{\left(y + \frac{\beta}{\alpha}\right)^2}$$

Let $\frac{\beta}{\alpha} = c$, where c is a linear magnitude. To define the arbitrary constant c_o we state the boundary conditions; $p_y = p_o$ when $y = 0$. Then

$$c_0 = p_0 c^2.$$

And finally

$$p_y = p_0 \frac{c^2}{(y + c)^2} \qquad (2)$$

This formula was derived by V.N. Goncharov (reference (1), page 194). Here it is given in a somewhat altered and simplified form.

The same author also derived the following phenomenalogical formula for determining v_y (reference (1), page 160).

Let a particle move at the velocity of v_y at the height y where the rate of flow is v_{y_1}. The aerodynamic forces F acting on the particle, as was shown theoretically by S.A. Chaplygin[5] and experimentally confirmed by A.I. Losievskii[2], depend on the square of the relative velocity $(v_{y_1} - v_y)^2$.

$$F = \alpha_1 (v_{y_1} - v_y)^2 \qquad (3)$$

where α_1 is a constant coefficient for the given particles and fluid.

If the mass of particles is equal to m, then according to the impulse theory

$$F dt = m . d v_y$$

Expressing F by (3) we get

$$\frac{\alpha_1}{m} dt = \frac{d v_y}{(v_{y_1} - v_y)^2}$$

After integration

$$v_y = v_{y_1} \cdot \frac{1 + \frac{c' m}{\alpha_1 s}}{1 + \frac{m}{\alpha_1 s}}$$

where $s = v_{y_1} t$, c' is an integration constant. We assume that during the period of time t there will be no change in the flow direction. In a quasi steady state turbulent flow the averaged magnitude $\overline{s} = \overline{v_{y_1} t}$ can be considered constant.

Then

$$v_y = \varphi v_{y_1} \qquad (4)$$

where

$$\varphi = \frac{1 + c' m/\alpha_1 s}{1 + \frac{m}{\alpha_1 s}} = \text{const}$$

Substituting (4) and (2) in (1) we have

$$q_y = p_0 \varphi \frac{c^2}{(y + c)^2} v_{y_1}$$

For the definition of v_{y_1} we use the logarithmic formula of Prof. S.A. Sapozhnikova (reference (4), page 72):

$$v_{y_1} = \overline{v}_1 \left(-1 \frac{\lg y}{\lg \delta}\right) \tag{5}$$

where $\overline{v}_1$ - the average velocity at unit height above the ground (for example 1 metre),

δ - a linear characteristic of the roughness of the surface.

Consequently

$$q_y = p_0 \varphi \overline{v}_1 c^2 \frac{\left(1 - \frac{\lg y}{\lg \delta}\right)}{(y + c)^2} \tag{6}$$

One should keep in mind that this formula is valid only for values of $y \geqslant \delta$. A schematic graph of the function (6) is shown in Fig. 2.

At a certain height y_o the solid flux of a flow reaches a maximum q_o and then decreases asymptotically approaching the Oy axis.

The magnitude y_o we find by taking the derivative with respect to y from $\frac{\left(1 - \frac{\lg y}{\lg \delta}\right)}{(y + c)^2}$ and equating it to zero

$$(y_0 + c)\frac{\lg e}{y} - 2(\lg y_0 - \lg \delta) = 0$$

Whence:

$$\frac{\lg e}{2}\left(1 + \frac{c}{y_0}\right) - \lg y_0 = -\lg \delta \tag{7}$$

The graph shown in Fig. 3 was constructed according to this formula. With this graph, knowing c and δ, we can determine y_o.

The value of a maximum flux q_o is

$$q_0 = p_0 \varphi \overline{v}_1 c^2 \frac{\left(1 - \frac{\lg y_0}{\lg \delta}\right)}{(y_0 + c)^2} \tag{8}$$

Dividing (6) by (8) we get

$$q_y = q_0 \left(\frac{y_0 + c}{y + c}\right)^2 \frac{\lg \frac{y}{\delta}}{\lg \frac{y_0}{\delta}} \tag{9}$$

For determining the total solid discharge Q_y in a wind-snow stratum from δy to y one must integrate equation (9) within this range:

$$Q_y = \frac{q_0 (y_0 + c)^2}{\lg \frac{y_0}{\delta}} \int_\delta^y \frac{\lg \frac{y}{\delta}}{(y+c)^2} dy = \frac{q_0 (y_0 + c)^2}{\lg \frac{y_0}{\delta}} \left| -\frac{\lg \frac{y}{\delta}}{c+y} + \frac{1}{c} \lg \left(\frac{\frac{y}{\delta}}{c+y} \right) \right|_\delta^y$$

Keeping in mind (7) one can write

$$Q_y = 2q_0 y_0 (y_0 + c) \left[\frac{1}{c} \ln \left(\frac{1 + \frac{c}{\delta}}{1 + \frac{c}{y}} \right) - \frac{1}{y+c} \ln \frac{y}{\delta} \right] \quad (10)$$

when $y \to \infty$

$$Q_\infty = 2q_0 y_0 (y_0 + c) \frac{\ln\left(1 + \frac{c}{\delta}\right)}{c} \; g/m/sec \quad (11)$$

We will show that for the purpose of simplifying calculations in processing the experimental data one can replace formula (9) with the following approximate formula which is structurally similar to expression (2):

$$q = q_0 \frac{c_1^2}{(y - y_0 + c_1)^2} \quad (12)$$

where

$$c_1 = 2y_0 (y_0 + c) \frac{\ln\left(1 + \frac{c}{\delta}\right)}{c} - \frac{y_0}{2} \quad (13)$$

Formula (12) is valid only under the condition $y_0 < y$. Let us assume that in the range of $0 < y < y_0$ the flux q depends linearly on height y,

$$q = q_0 \frac{y}{y_0}$$

Then the total flux Q_y in the stratum 0 - y is

$$Q_y = \frac{q_0}{y_0} \int_0^{y_0} y dy + q_0 c_1^2 \int_{y_0}^{y} \frac{dy}{(y - y_0 + c_1)^2} = q_0 \left[\frac{c_1 (y - y_0)}{(y - y_0 + c_1)} + \frac{y_0}{2} \right] \quad (14)$$

When $y \to \infty$:

$$Q_\infty = q_0 \left(c_1 + \frac{y_0}{2} \right) \quad (15)$$

Substituting (13) in (15) we get (11).

Fig. 4 shows a comparison of graphs plotted from formulae (9) and (12) using various values of c and δ. Curves corresponding to formula (12) are plotted in dashed lines. It can be readily seen that the dashed line curves are very close to the solid line curves corresponding to formula (9). The value of c_1 when c and δ are known can be determined from the graph (Fig. 5) plotted from expression (13) and (7).

The experimental data of TsNII MPS (Central Research Institute of the Ministry of Railroads) and TEI ZSFAN (Transportation-Power Institute of the East Siberian Section of the Academy of Sciences) make it possible to find numerically the average value of c and δ for a snow-wind flow.

Fig. 6 shows the data of anemometer measurements of wind velocities at various heights. The circles with dots are plotted from the data of TsNII MPS (Orenburg Railway, February 1952) and the solid circles from the data of TEI ZSFAN (Tomsk Railway, December 1952 - January 1953). In each case the velocity at the height of 0.5 m is taken as unity. The absolute values of these velocities vary within the range of 4.4 - 10.7 m/sec. Fig. 6 shows that the wind velocity over a smooth snow surface has on the average a logarithmic dependence on the height y.

Substituting in formula (5) the dependence of wind speed on height given in Fig. 6, we find that $\log \delta = -4.85$, $\delta = 0.00141$ cm.

Blizzard data was used to determine c. The blizzard metre of the type VO, used by TsNII MPS* and TEI ZSFAN, has an entrance aperture 2 cm high. By placing the nozzle right on the snow surface ("surface blizzard metre") we do not consequently determine the maximum flux q_0 but the average flux q_{0-2}, which is determined theoretically from formula (10)

$$q_{0-2} = q_0 y_0 (y_0 + c) \left[\frac{1}{c} \ln \left(\frac{1 + \frac{c}{\delta}}{1 + \frac{c}{2}} \right) - \frac{1}{2 + c} \ln \frac{2}{\delta} \right] \qquad (16)$$

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* At the Vodenyapino Snowdrift Station

where y_o, c, δ are expressed in centimetres.

V.N. Goncharov found (reference (1), page 41) that the value of c in a liquid flow depends primarily on the depth of the liquid. Fig. 7 shows the data of V.N. Goncharov obtained by two different methods. The more reliable of the methods was that of suspended particles, the analysis of which will not be dealt with here.* The maximum depth of flow in the experiments of V.N. Goncharov was 44 cm. For this depth, c = 2.5 cm.

Fig. 7 also considers the dependence of c on the roughness of the surface. It is interesting to note that for a snow-wind flow, the calculations give c = 2.7 cm on the average, about the same value as for flow in a very deep fluid body.

In Table I the theoretically calculated values of $\frac{q_y}{q_{o-2}}$ are compared with the corresponding mean values of this ratio calculated from experimental data of TsNII MPS and TEI ZSFAN. For the theoretical calculation it was assumed that c = 2.8 cm; δ = 0.00141 cm. According to the graph in Fig.3 $y_o \cong 0.28$ cm. The corresponding value of c_1 = 4.7 cm.

According to formula (16) $q_{o-2} = 0.767\ q_o$.

These data are shown in Fig. 8. The results of observations agree satisfactorily with theory.

The value of c apparently does not depend in any essential way on the absolute value of the velocity of the snow-wind flow as can be seen from Fig. 9 where the theoretical curves $q = f\ (y, v_1)$ are compared with experimentally found points of single measurements of a solid flux at velocities of v_1 (at the height of 1 m) 6.3 - 12 m/sec. The blizzard measurements were carried out by TEI ZSFAN.

Conclusions

1. The distribution with height of the solid flux of a snow-

* The value of c, as shown by V.N. Goncharov, characterizes the change with height of a linear vertical pulsating component of the velocity of turbulent flow.

wind flow, as any other mixed flow, depends on the profile of the averaged translational velocities and the linear characteristic of turbulence C of the wind flow.

2. The precise expression for the total solid flux in a layer 0 - y has the form

$$Q_y = 2q_0 y_0 (y_0 + c) \left| \frac{1}{c} \ln \left(\frac{1 + \frac{c}{\delta}}{1 + \frac{c}{y}} \right) - \frac{1}{y + c} \cdot \ln \frac{y}{\delta} \right|$$

This formula can be replaced by the approximate formula

$$Q_y = q_0 \left[\frac{c_1 (y - y_0)}{y - y_0 + c_1} + \frac{y_0}{2} \right]$$

3. Curves constructed from these formulae agree satisfactorily with experimental data obtained under field conditions.

4. In using these formulae one can carry out the major part of the blizzard-measuring observations with the use of surface-blizzard measuring instruments and from these data one can precisely determine the total solid flux of a snow-wind flow.

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Table I

y, cm	No. of observations	$\frac{q_y}{q_{0-2}} \cdot 100$ Mean of experimental data	$\frac{q_y}{q_{0-2}} \cdot 100$ Theoretical calc.	Organizations carrying out observations
0 - 2		100	100	
6	43	26.6	25.2	TsNII MPS
6	6	24.2	25.2	TEI ZSFAN
11	47	11.55	10.9	"
16	43	5.54	6.1	TsNII MPS
21	10	4.57	4.0	TEI ZSFAN
31	4	2.1	2.0	"
41	4	0.49	1.2	"
51	4	0.45	0.8	"

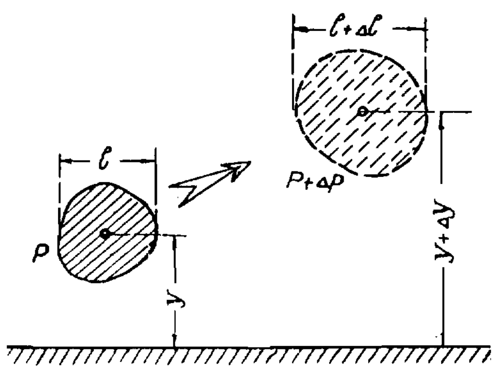


Fig. 1

The motion of penetrating masses saturated with admixtures

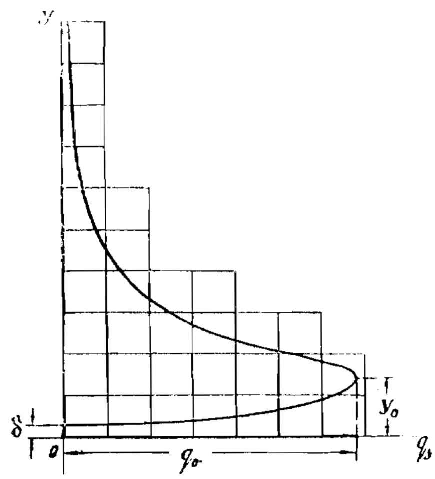


Fig. 2

A schematic graph of the function $q = f(y)$

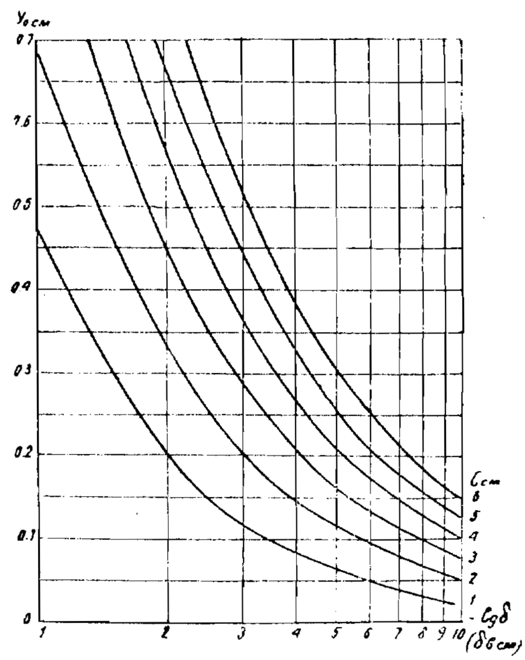


Fig. 3

A graph of the function $y_o = f(c, \delta)$

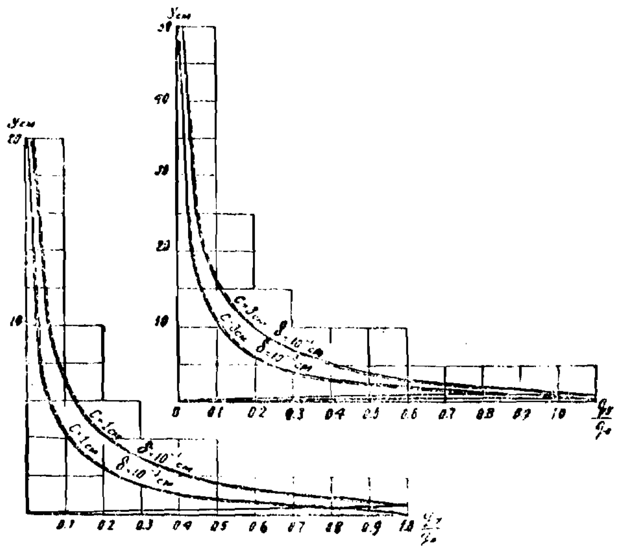


Fig. 4

A graph comparing formulae (9) and (12)

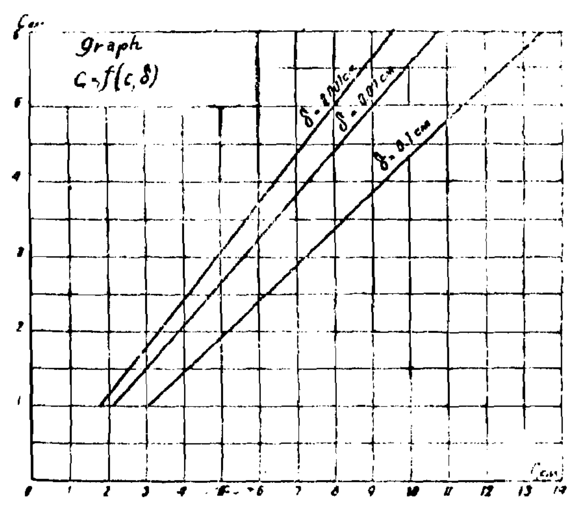


Fig. 5

A graph of the function $c_1 = f(c, \delta)$

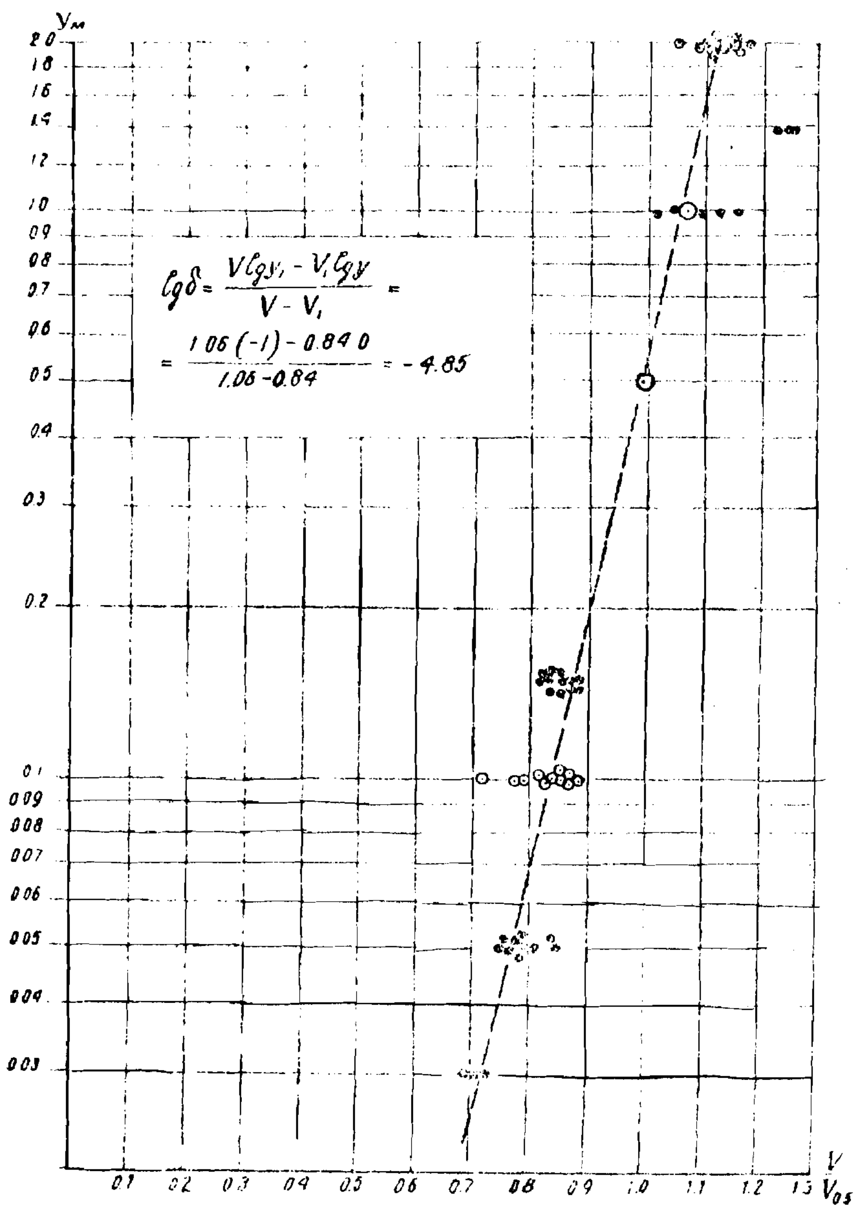
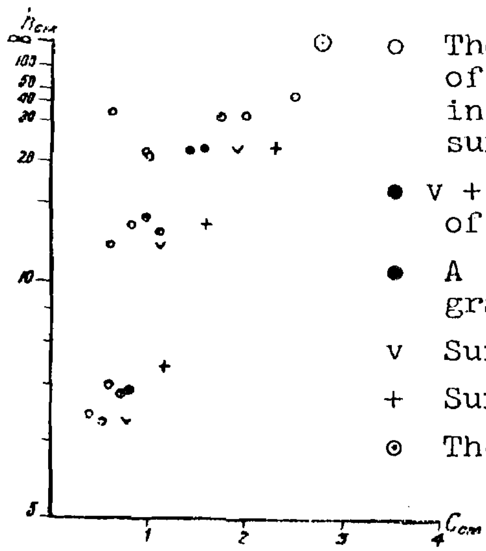


Fig. 6

A profile of average translational velocities of the snow-wind flow according to the data of TEI ZSFAN and TsNII MPS

Graph C = f(h)



o The value of C determined by the method of measuring the falling rate of particles in a flow. (On a smooth surface and on a surface made up of 5 - 7 mm grain sizes)

● v + The value of C determined by the method of suspended particles

● A smooth surface and a surface of 5 - 7 mm grain sizes

v Surface of 18 - 12 mm grain sizes

\+ Surface of 31 - 22 mm grain sizes

⊙ The average value of C for a snow-wind flow

Fig. 7

Value of c depending on the depth of flow
(according to the data of V.N. Goncharov)

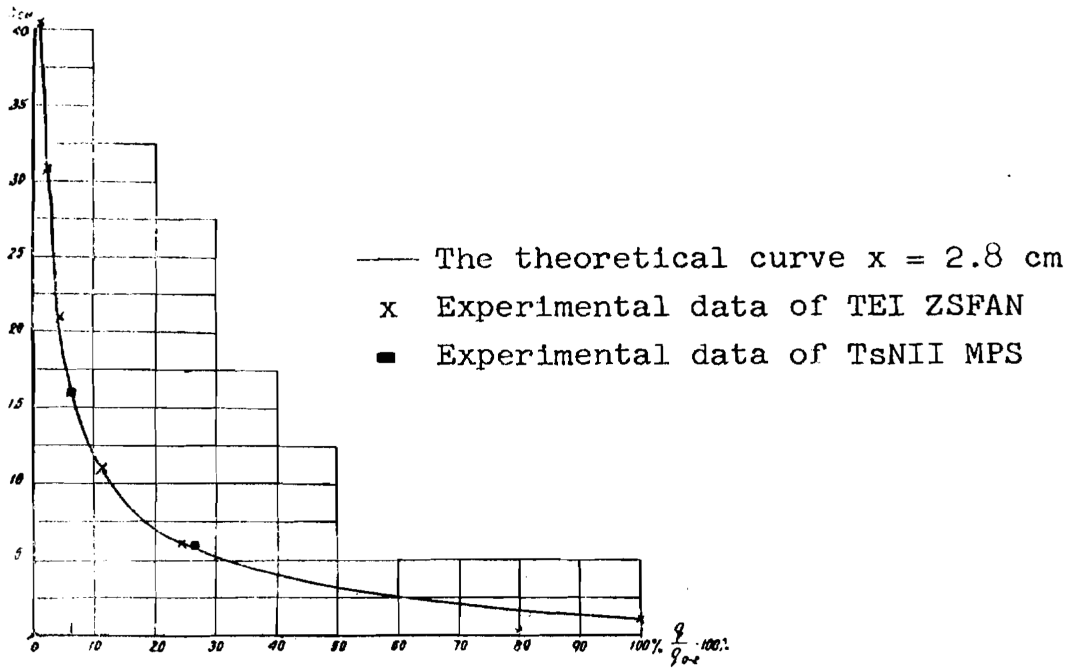


Fig. 8

Graph of the function $\frac{q}{q_{o-2}} \cdot 100 = f(y)$

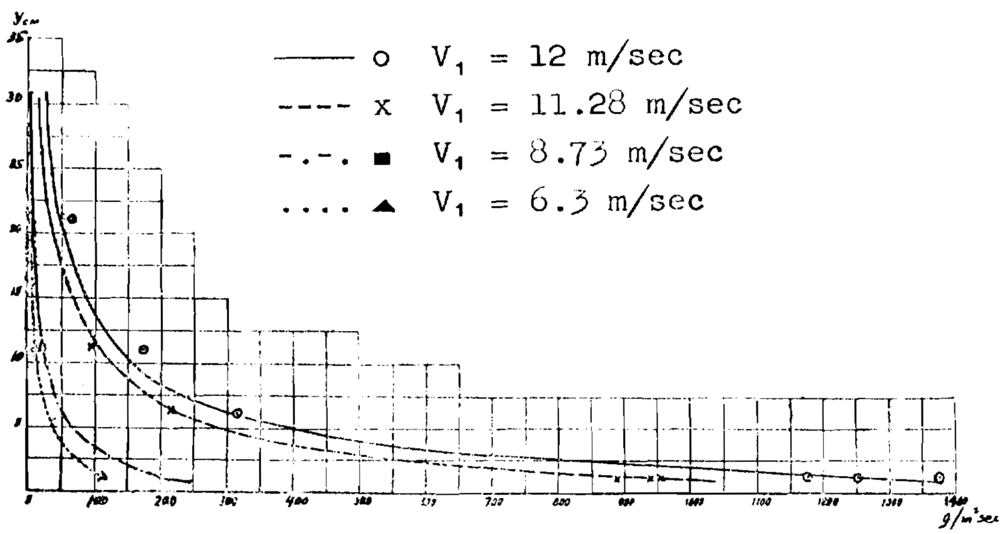


Fig. 9

Graph of the function $q = f(y \cdot v_1)$