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MEASUREMENT OF LOW POWER MICROWAVE PULSES BY COMPARATIVE METHODS

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ABSTRACT

Three methods of measuring the peak power of low-level microwave pulses are compared, the tangential signal method, the notch method, and the heterodyne method. The tangential signal method gives power level in terms of calculable thermal noise, and the result is therefore absolute. The other two methods give the peak pulse power in terms of a measureable average power level. An outline of the theory, actual measurements, and some of the difficulties encountered are given for each method.

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TEXT

PLATES

MEASUREMENT OF LOW POWER MICROWAVE PULSES BY COMPARATIVE METHODS

- L.K. Anderson -

INTRODUCTION

The work described in this paper was carried out in an effort to find a quick, convenient, and accurate laboratory method for measuring low-level peak pulse powers. Such a measurement is necessary in determining the sensitivities of crystal detectors over a frequency range. In this laboratory, such measurements have been carried out using commercial signal generators operated on the assumption that the peak pulse power available at any frequency was the same as the c-w power. However, various anomalies in the results obtained suggested that this assumption was not always valid. Accordingly, some direct method of measuring these pulse power levels was sought.

For our purposes, low-power pulses may be defined as those pulses whose peak power cannot be obtained from direct average power measurements using r-f bridges or calorimetric means. Assuming that average powers down to 0.1 mw may be measured with fair accuracy, and that duty cycles of 1/1000 are typical, then any peak power below 100 mw would fall in this category. Of course independent measurement of higher-power pulses might also be desirable in some cases, so as to eliminate the need for measurements of pulse repetition rate and pulse shape.

The methods of power measurement investigated were:

1) Reference of pulse directly to receiver noise.

2)(a) Reference of pulse to a c-w or average power level by (i) notch method, or (ii) heterodyne method, followed by

(b) Measurement of c-w or average power level by (i) reference to receiver noise, or (ii) direct measurement on an r-f power bridge.

SOME THEORETICAL CONSIDERATIONS OF THE VARIOUS METHODS

1) In the first method, in which pulse power is referred to receiver noise, a visual display corresponding to a known signal-to-noise ratio is set up, and then, from a knowledge of the receiver noise figure the peak pulse power may be calculated. This method, then, involves two distinct steps: the visual setting, and measurement of the receiver noise figure. We shall consider them separately.

i) Visual Setting

There are several visual signals which may be obtained by pulse-modulating a signal generator, applying this signal to a receiver, and looking at the output on an A-type oscilloscope (triggered linear sweep). According to a Radiation Laboratory survey [1], the most reproducible of these visual signals is the "tangential signal", which may be defined as a setting in which "the bottom of the noise in the presence of signal is level with the top of the noise in the absence of signal". Its appearance is roughly as shown in Plate I. One must say "approximately", since a camera does not see noise, which is a constantly changing phenomenon, in the same way as the naked eye. The setting is to a certain extent subjective since noise has no definite peak value, but rather a statistical distribution in amplitude. Thus each observer will differ slightly in what he considers the "top" or "bottom" of the noise. Nevertheless, the tangential signal has proved to be a very valuable setting. Experiments conducted by Williams [2] indicate that a tangential signal corresponds to a signal-to-noise ratio of 8.5 db, and that the reproducibility from observer to observer, as indicated by the standard deviation of a number of observers, is about 1.1 db.

ii) <u>Measurement of Receiver Noise Figure</u>

The sensitivity of <u>any</u> receiver is limited by the thermal noise generated in the input termination. In any practical receiver there will also be additional noise generated inside the receiver itself. Thus there will appear at the output noise due to the two sources. The ratio of the total noise appearing at the output to the noise that would appear if the receiver itself were completely noise-free is called the average noise figure, or simply the noise figure, F, of the receiver*. There are several ways to measure this noise figure, but probably the most accurate is to use a gas discharge tube as a noise source. Mumford has demonstrated [3] that the noise power available from an argon gas discharge tube mounted in a waveguide is 15.28 ± 0.1 db above the thermal noise at the input to the receiver. To make use of this known power level, we may proceed as follows:

The noise power at the receiver output is measured with the input connected to a matched load. The noise source is then connected to the receiver input through a variable attenuator, and the attenuator is adjusted so that the noise power in the output is exactly doubled. Under these circumstances, the noise figure is given by:

$$F = 15.28 - \alpha \text{ db}, \qquad (1)$$

where α is the attenuator setting in db (see Appendix I).

In this method, we have assumed that the noise figure of the receiver is less than 15.28 db, that the receiver has no spurious responses, and is linear, at least over a dynamic range of 2 to 1. In particular, if the receiver is a superheterodyne, allowance must be made for the image response if this has not been

* See "IRE Standards on Receivers: Definitions of Terms", 1952.

suppressed. If the image response equals the signal response, then the noise figure as given by Eq. (1) must be corrected by adding to it 3 db, since the receiver accepts twice as much power from the noise source as the receiver pass band would indicate.

Having obtained the receiver noise figure, the power level corresponding to any particular visual signal may be determined by means of the following formula, which is derived in Appendix I.

$$P_s = (S/N)_o + F + 10 \log kTB + 30.0 \text{ dbm}, \qquad (2)$$

where P_s = peak pulse power in dbm (db below 1 mw),

$(S/N)_o$ = signal-to-noise ratio of visual setting in db (8.5 for tangential signal),

F = receiver noise figure in db,

kTB = thermal noise, in watts, where

k = Boltzman's constant = 1.38×10^{-23} joules/°K,

T = absolute temperature in degrees Kelvin,

B = bandwidth in cps.

Once F has been determined the peak pulse power may be obtained if the receiver bandwidth B is known.

2. Notch Method

The notch method of measuring peak pulse powers has been well covered in the literature [4, 5], and so only a brief outline of the technique will be given here. In this method an auxiliary signal generator is pulsed off for the duration of the positive pulse whose amplitude is to be measured. When detected the result is a negative video pulse or "notch". If the signal level is adjusted so that the notch depth is equal to the positive pulse height, then the peak power outputs of the two sources are equal. This is shown diagrammatically in Fig. 1, where for clarity the notch has been made wider than the pulse.

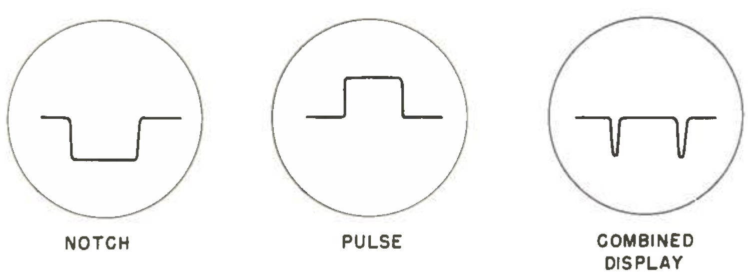


FIG. 1

COMBINATION OF VIDEO PULSES TO PRODUCE A NOTCHED DISPLAY

The average power output of the "pulsed-off" generator is usually sufficiently great to be measured by conventional means. Furthermore, since duty cycles of the order of 1/1000 are usual, there is negligible error in assuming that the peak power output of the pulsed-off generator is equal to the average power. Hence the peak power of the pulse may be determined.

3. Heterodyne Method

The heterodyne method appears to be much less well known than the notch method, although there is some mention of it in the literature [6, 7]. In this method, the output of a c-w generator, operating on the same frequency as the pulse source, is mixed with the r-f pulses and then detected. The resulting video output will look similar to that shown in Plate II.

To explain this phenomenon, we may proceed as follows:

Let the instantaneous c-w radio-frequency voltage be given by

$$e_{cw} = \hat{E}_{cw} \cos w_1 t.$$

Let the instantaneous pulsed r-f voltage, for the duration of the pulse, be given by

$$e_p = \hat{E}_p \cos w_2 t.$$

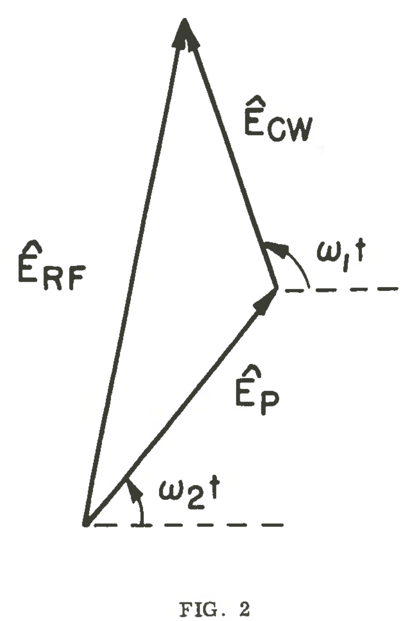


FIG. 2
COMBINATION OF R.F. VOLTAGES

The net r-f voltage from the mixer is then

$$e_{rf} = e_{cw} + e_p = \hat{E}_{cw} \cos w_1 t + \hat{E}_p \cos w_2 t.$$

The peak value of this r-f voltage is

$$\hat{E}_{rf} = \left[\hat{E}_{cw}^2 + \hat{E}_p^2 + 2 \hat{E}_{cw} \cos (w_1 - w_2) t \right]^{\frac{1}{2}},$$

as seen from Fig. 2, where $\hat{E}_{rf}$ is the peak value of the r-f voltage applied to the detector.

Now if $w_1 - w_2$ is small, i.e., if the two frequencies are almost exactly equal, then $\frac{2\pi}{w_1 - w_2}$ will be much greater then the pulse length, t_o, and consequently for the duration of the pulse, $\cos (w_1 - w_2)t$ will be constant, and may be looked upon as a phase factor which may, for a particular pulse, have any value from +1 to -1. If the two signal sources do not remain coherent (i.e., in the same relative phase) from pulse to pulse, as is generally the case, then in a period of time, $\cos (w_1 - w_2)t$ will have taken on all possible values. Thus the peak r-f voltage applied to the second detector, $\hat{E}_{rf}$, will take on many random values between $(\hat{E}_{cw} + \hat{E}_p)$ and $(\hat{E}_{cw} - \hat{E}_p)$. The net effect is that the video display will consist of a series of horizontal lines which appear to "fill in" a portion of the pulse more or less completely. The density of the filling in will depend on the pulse repetition rate and the persistence of the screen of the cathode-ray tube.

It is shown in Appendix I that for the case where $\hat{E}_p = 2 \hat{E}_{cw}$, the pulse appears to be filled in down to the baseline, as shown in Fig. 3. This corresponds to a peak pulse power equal to four times the c-w power. As shown in the appendix, other c-w to pulse power ratios also give reproducible displays. However, their nature depends on the law of the detector (the relation between r-f power in and video power out), and are consequently not as useful.

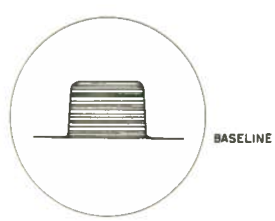
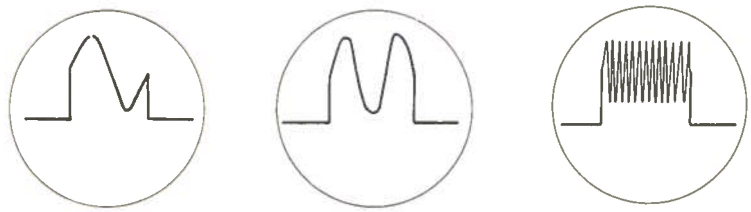


FIG. 3
INCOHERENT HETERODYNE VIDEO DISPLAY
WHEN PEAK PULSE POWER IS FOUR TIMES C.W. POWER



$\frac{2\pi}{\omega_1 - \omega_2} = t_0$ $\frac{2\pi}{\omega_1 - \omega_2} = \frac{2}{3} t_0$ $\frac{2\pi}{\omega_1 - \omega_2} << t_0$

FIG. 4
HETERODYNE DISPLAY FOR VARIOUS BEAT FREQUENCIES
UNDER CONDITIONS OF INITIAL COHERENCE

Once the peak pulse power has been referred to the c-w level, its absolute value may be obtained from average power measurements of the c-w power, just as in the notch method.

If the c-w and pulsed signal sources do remain coherent, then for $\frac{2\pi}{W_1 - W_2} >> t_0$ only one line will appear on the oscilloscope screen, i.e., a conventional looking pulse. If the two sources have the same relative phase at the beginning of each pulse, and have a constant frequency difference, the two signals will beat, and the video display will be as shown in Fig. 4. As in the case previously discussed, when the beat frequency waveform extends down to the base of the pulse, the pulse power is four times the c-w power. This is illustrated in Fig. 5. However, care must be taken that the beat frequency does not exceed the upper limit of the video amplifier passband.

The degree of stability required in each signal source, and in the pulse repetition rate, in order to achieve coherence is completely unachievable in practice. It therefore seems probable that coherence can only be due to the pulsed oscillator "locking" with the c-w oscillator. The author has witnessed the phenomenon of coherent beating only on one occasion. This occurred while measuring the power output of a breadboard model of a simple test oscillator. By increasing the isolation between the pulse and c-w sources, it was possible to render the beating incoherent.

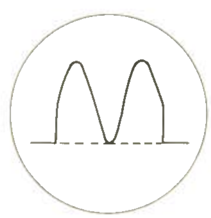


FIG. 5
HETERODYNE DISPLAY FOR CONDITIONS OF INITIAL COHERENCE
WHEN PULSE POWER IS FOUR TIMES C.W. POWER

4. Measurement of Average Power

Once the peak pulse power has been determined in terms of a c-w or "notched c-w" level, this average power must be measured. For average powers greater than about 0.01 mw, well-known calorimetric and r-f power bridge techniques are available [8]. These will not be discussed here.

At very low levels, other techniques must be used. The method investigated by the author made use of the diode detector in a receiver. Burgess has demonstrated [9] that under certain circumstances either a square law or exponential law detector may have the same calibration for noise, a c-w signal, and mixtures of c-w and noise. Thus a receiver which has such a characteristic, at least over the power range to be investigated, may be calibrated using the known output of a noise source, and then used to measure c-w power, using the rectified second detector voltage as the basis of indication.

EXPERIMENTAL RESULTS

In the preliminary experimental work, a radar receiver was used as the detector, because of its low noise figure (8.3 db for the receiver used). However, the results obtained, particularly in the measurement of low level c-w power, were not consistent. Investigation showed that the response of the receiver was logarithmic, even at low levels, with the result that noise and c-w power were not treated similarly by the detector.

Finally a General Radio intermediate-frequency amplifier was found with suitable detector characteristics and a built-in precision 3 db intermediate-frequency attenuator. The response of the unit used was accurately exponential at low levels and linear at higher levels. Accordingly a receiver was set up using an Empire Devices crystal mixer and a General Radio unit klystron oscillator as the local oscillator.

1. Noise Figure Measurement

The noise figure of this receiver at 3000 mc/s was then determined in the manner outlined previously. In order to avoid actual measurement of power in the intermediate-frequency amplifier, the following procedure was adopted.

With the input terminated in a flat load, the second detector current was noted. Then the 3-db attenuator was inserted in the amplifier, following the first stage. Next the noise source was connected to the receiver input through a variable radio-frequency attenuator and the noise level set so as to return the second detector current to its former value. Under these circumstances, the noise power in the output was doubled, subject only to the linearity in the first intermediate-frequency stage, and the additional assumption, almost invariably

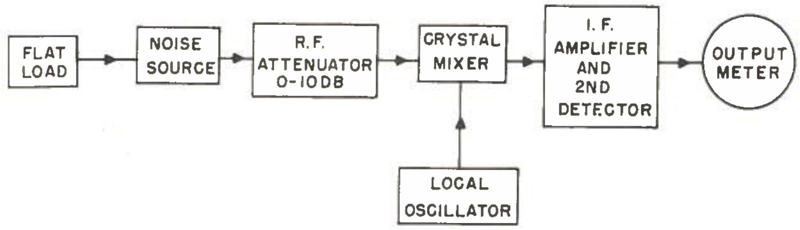


FIG. 6

BLOCK DIAGRAM OF APPARATUS FOR NOISE FIGURE MEASUREMENT

justified, that any noise generated in the remaining intermediate-frequency stages is swamped by the noise generated in the mixer and first intermediate-frequency stage. The main advantage of the method is that it does not depend on the law of the detector. A block diagram of the apparatus used is shown in Fig. 6. At 3000 mc/s the receiver was found to have a noise figure of 13.3 db. This figure was reproducible from day to day within better than 0.1 db when the mixer crystal current and temperature were maintained constant.

2. Tangential Signal

Using a Hewlett-Packard signal generator as a source, and a triggered oscilloscope as an indicator, a tangential signal was set up according to the definition given earlier. A pulse length of 10 μsec was used. On the basis of six trials the tangential level, according to the signal generator calibration, was -92.0 dbm, with a standard deviation of only 0.22 db.

The bandwidth of the intermediate-frequency amplifier was found to be 0.785 mc/s, probably within 5% (see Appendix II).

Using Equation 2, the theoretical value for the tangential signal level, at the standard temperature T = 290°K, is

$$P_s = 8.5 + 13.3 + 10 \log (1.38 \times 10^{-23} \times 290 \times 0.785 \times 10^6) + 30.0 = -93.2 \text{ dbm}.$$

3. Comparison of Notch and Heterodyne Methods

It has been demonstrated that the notch method provides a visual signal in which $P_{av} = P_s$ where P_{av} is the average power of notched c-w and P_s = peak pulse power. In the case of the heterodyne method, the usual visual display corresponds to $4 P_{av} = P_s$. Thus, if the same average power is used in each case, the readings should differ by a factor 4, or 6.0 db. Using the experimental setup shown in Fig. 7, this 6 db figure was verified.

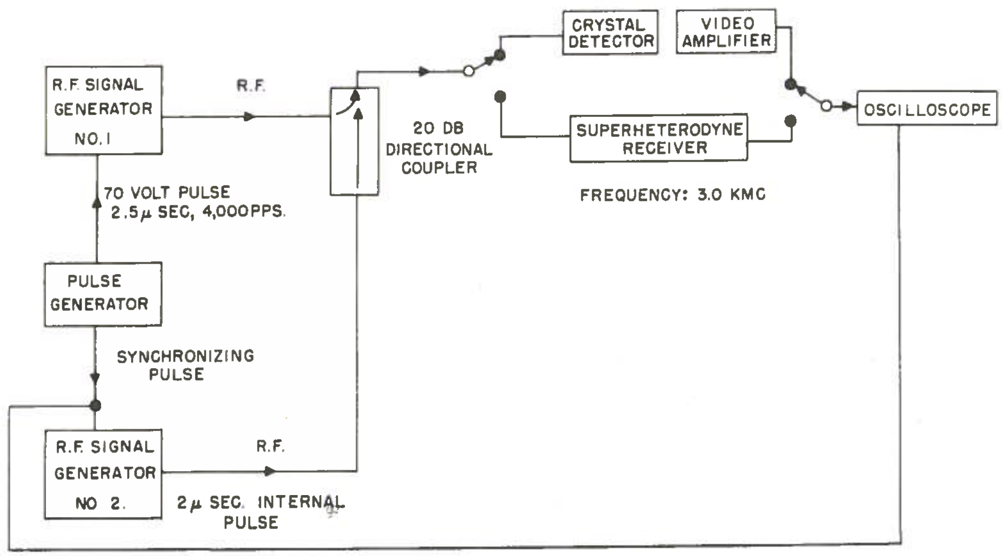


FIG. 7

BLOCK DIAGRAM OF APPARATUS FOR NOTCH AND HETERODYNE METHODS

Hewlett-Packard generator No. 1 was "pulsed off" according to the method suggested by Hewlett-Packard [5]. This involves pulsing the generator with a large positive pulse when the input selector is set to the negative external pulse position. This pulses the generator off completely. Both the pulse generator and the radio-frequency generators have pulse delay controls which greatly facilitate setting the notch and pulse in coincidence. Although the visual display, in the case of the notch method, does not depend on coincidence of pulsed and notch generator frequencies, it is nevertheless important that they be sufficiently close that any variation of detector sensitivity with frequency be negligible. The apparatus of Fig. 7 was modified for the heterodyne method by operating generator No. 1 on "c-w" and making the two frequencies exactly equal (within about 500 kc/s).

Variations in the notch visual setting from observer to observer were found to be less than the reading accuracy of the attenuator used, or less than 0.1 db. The reproducibility of the heterodyne method was somewhat poorer, with a standard deviation of 0.2 db. The greater sensitivity of the notch method is demonstrated in Plate III, which shows the change in the visual display corresponding to a 1 db change in pulse power for both methods.

The mean difference between the notch and heterodyne settings, as determined by experienced observers, was found to be 5.8 db. It is not likely that the difference from 6.0 db is significant in view of the small number (8) of samples taken. At low levels, in the vicinity of signal-to-noise ratio = 10 db, a tendency was noted for all observers to "overfill" the pulse, resulting in a difference between the two settings of as little as 5.0 db. This is understandable when one considers the effect of the noise. Each horizontal trace which goes to fill in the pulse will have noise superimposed upon it. Since this trace has a random position, the effect is for the noise to acquire a greater amplitude, and hence for the baseline of the pulse to become more indefinite — the noise peaks showing up less intensely on the screen. As a result the observer, to be sure that the pulse is filled in, sets the c-w level too high. Although the degree of error is somewhat subjective, an observer who is aware of the difficulty tends to set the correct level.

Some of the experimental difficulties encountered with the heterodyne method are shown in Plate IV. Plate IV (a) shows the broad trailing baseline caused by video amplifier overshoot at high power levels. Plate IV (b) illustrates the effect of having frequency modulation on the pulse, but no amplitude modulation. In both cases, the peak pulse power is four times the c-w power.

4. Average Power Measurements

Careful measurements were made of the average power output of the signal generator used as the c-w source, using both the radio-frequency bridge and noise comparison techniques.

i) Radio-frequency Bridge Measurements

These measurements were carried out at a relatively high level (-10 dbm), using two different types of bolometer and two different radio-frequency bridges. The measurements, which were repeated several times with each combination, agreed within 0.05 db, and indicated that the actual power output was 1.3 db higher than the indicated power output.

ii) Noise Comparison Measurements

The power output of the c-w generator was also measured at a low level (-100 dbm) by comparing it with the noise source. To do this the noise source was coupled directly to the receiver input and the resulting second detector current noted. The noise source was then replaced by the c-w signal generator and its power output adjusted to give the same second detector current. The signal generator reading was -98.4 dbm. Now the power from the noise source reaching the receiver was (15.3 + 3) db above kTB, or -96.7 dbm for B = 0.785 mc/s. The 3 db must be added to the noise source output of 15.3 db to allow for receiver image response, as noted earlier. Thus the noise comparison measurement indicates that the actual output of the generator was 1.7 db greater than the indicated level, and 0.4 db greater than the power output as given by the radio-frequency bridge. This difference of 0.4 db between the two determinations is well within the stated accuracy (± 1.5db) of the piston attenuator used, and furthermore is in the right direction to be accounted for by leakage.

It is difficult to determine absolutely whether any unsuspected systematic error is present in any of the methods. However, it is possible to compare the tangential-signal method with the heterodyne plus c-w measurement method. The results of this comparison are shown in Table I.

TABLE I

COMPARISON OF POWER MEASUREMENTS

QUANTITY	dbm
Indicated output of c-w generator when heterodyned with pulse	-81.5
Directional coupler attenuation	-20.0
$10 \log \frac{P_{pulse}}{P_{cw}}$ for heterodyne setting	+ 6.0
Correlation to c-w general reference level on basis of r-f bridge measurement	+ 1.3
Peak pulse power	-94.2
Calculated pulse power for a tangential signal	-93.2
Relative error of two determinations	+ 1.0

This relative error of 1 db is within the overall experimental error.

ESTIMATION OF PROBABLE EXPERIMENTAL ERRORS

The probable errors shown in the following tables are based, for the most part, on the manufacturer's stated accuracy or on experimental data, for the laboratory equipment used, and should be representative of the accuracy to be expected from any careful laboratory measurement.

a) Tangential-Signal Method

TABLE II

ERRORS IN TANGENTIAL-SIGNAL METHOD

Source of Error	Error (db)	Error (%)	Source of Data
Noise source 15.3 db	0.1	2.3	Manufacturer's stated accuracy
I.F. attenuation (p-1)	0.2 db error in 10 log P	9.4	See Appendix I
kTB	-	5.0	Rough estimate only
$(S/N)_o$	1.2	31.8	See Ref. (2)
α	0.3	7.1	Manufacturer's stated accuracy
Maximum error		56.0	
RMS error		34.0 = 1.3 db	

b) Notch and Heterodyne Methods

The error in referring to an average power level will arise from two main sources — error in the visual setting, and error in the directional coupler. These are summarized below:

TABLE III

ERROR IN NOTCH AND HETERODYNE METHODS

Source of Error		Error (db)	Error (%)	Source of Data
	Directional coupler	0.2	4.7	Manufacturer's stated accuracy
Reproducibility	Notch setting	0.1	2.3	Experimental
	Heterodyne setting	0.2	4.7	Experimental

In addition there will be an error in the measurement of the average power level.

c) Measurement of Average Power

i) Receiver Noise Method

TABLE IV

ERROR IN RECEIVER NOISE METHOD

Source of Error	Error (db)	Error (%)	Source of Data
Noise source	0.1	2.3	Manufacturer's stated accuracy
kTB	-	5.0	Rough estimate
Reproducibility	0.2	4.7	Experimental
Maximum error		12.0	
RMS error		7.2 = 0.3 db	

ii) R.F. Power Bridge Method

TABLE V

ERROR IN R.F. POWER BRIDGE MEASUREMENT

Source of Error	Error (db)	Error (%)	Source of Data
Bridge	—	5	Manufacturer's stated accuracy
Mount efficiency	—	3	See Ref. 4.
Mismatch	—	2	See Ref. 4.
Maximum error		10%	
RMS error		6.2% = 0.25 db	

TABLE VI

TABULAR SUMMARY OF POWER MEASUREMENT METHODS

POINT OF COMPARISON	DIRECT MEASUREMENT OF PULSE	REFERENCE OF PULSE TO AVERAGE POWER		MEASUREMENT OF AVERAGE POWER	
	TANGENTIAL-SIGNAL METHOD	NOTCH METHOD	HETERODYNE METHOD	R.F. BRIDGE METHOD	NOISE COMPARISON
Lowest measurable power	(F + 8.5) db above kTB	F db above kTB	About (F + 10) db above kTB	- 20 dbm	About F db above kTB
Power levels at which no calibrated attenuators are required	Auxiliary calibrated attenuator always required	—	—	Above - 20 dbm	External attenuator always required
Equipment required	(1) Good superheterodyne receiver with measurable noise figure (2) Calibrated R.F. attenuator (3) Oscilloscope To measure noise figure (a) Noise source (b) I.F. attenuator	(1) R.F. signal source capable of being pulsed off (2) Pulse generator (3) Directional coupler, magic tee or other mixer (4) Detector, either a crystal detector with video amplifier or a superheterodyne receiver (5) Oscilloscope	(1) C-W signal source (2) Mixer) as for) notch (3) Detector) method (4) Oscilloscope	(1) Bolometer (2) R.F. power bridge	(1) Noise source (2) Calibrated R.F. attenuator (3) Superheterodyne receiver
Maximum probable errors	33% + attenuator error. This large error is due to uncertainty in tangential visual setting	5%	7%	6%	7% + attenuator error
Main possible sources of systematic error	None	Failure to pulse notched generator completely off	Failure to allow for noise (low level) or video amplifier overshoot	Mismatch in bolometer mount	Detector characteristic not accurately exponential or square law
Main disadvantages of method	(1) Inaccurate (2) Requires knowledge of receiver bandwidth	(1) Requires relatively elaborate equipment (2) Requires synchronizing pulse signal source	Not as accurate as notch method, especially at very low level where noise is important	Requires large known attenuation if very low-level c-w reference is required	Depends on law of detector. Requires knowledge of receiver bandwidth
Main features of method	(1) Requires relatively simple equipment — especially does not require auxiliary R.F. signal source of any kind. (2) Gives absolute power directly	High accuracy	Inexpensive equipment; a simple c-w source may be used	(1) Direct reading (2) Quick and convenient	Usable with low attenuation at low levels

SUMMARY

The most important features of each of the methods investigated are shown in Tables VI and VII.

TABLE VII

MOST ACCURATE METHOD AT VARIOUS POWER LEVELS

Power Level	Reference Method	C.W. Measurement
Above -20 dbm	Notch or heterodyne	R.F. bridge
-20 dbm to -60 dbm	Notch	R.F. bridge
Below -60 dbm	Notch	Noise comparison

CONCLUSIONS

The tangential-signal method of measuring peak pulse power has a large random possible error. However the relative simplicity of the method, together with a freedom from systematic error, particularly a systematic error which depends on frequency, may make this method desirable in certain instances.

The notch and heterodyne methods both furnish accurate means for determining pulse power in terms of an average power level. The notch method is slightly more accurate, but the heterodyne method requires less equipment.

In order to obtain absolute pulse powers by means of the notch or heterodyne method, an average power level must be measured, and it is in this measurement that the largest errors, particularly systematic errors, are liable to occur.

ACKNOWLEDGEMENT

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APPENDIX I

DERIVATION OF NOISE REFERENCE FORMULAE

1) Derivation of Fundamental Formula, $F = 15.28 - \alpha$

Let N = noise power at output due to noise generated within receiver,
and N_O = noise power at output with matched load.

$$\text{Then } N_O = N + kT_OBG, \text{ where}$$

$$T_O = \text{temperature of load,}$$

$$B = \text{bandwidth of receiver,}$$

$$G = \text{gain of receiver.}$$

N_1 = noise power in output of receiver with noise source on, but with r-f attenuation between noise source and receiver of $1/\alpha'$.

$$\text{Then } N_1 = N + kT_OBG + \frac{kBG}{\alpha'}(T_2 - T_O),$$

where T_2 = effective temperature of noise source.

$$\text{Then } N_1 - N_O = \frac{kBG}{\alpha'}(T_2 - T_O),$$

$$\text{or } G = \frac{(N_1 - N_O)\,\alpha'}{kB\,(T_2 - T_O)}.$$

$$\text{Now, by definition } F' = \frac{N_O}{kT_OBG} = \frac{N_O}{kT_OB} \times \frac{kB\,(T_2 - T_O)}{(N_1 - N_O)\,\alpha'}.$$

$$\text{Therefore } F' = \frac{1}{\alpha'}\left[\frac{T_2 - T_O}{T_O} \Big/ \frac{N_1 - N_O}{N_O}\right]$$

$$= \frac{1}{\alpha'}\left[\frac{\frac{T_2}{T_O} - 1}{\frac{N_1}{N_O} - 1}\right].$$

For the argon tubes in question, $T_2 = 10{,}050\,^\circ K$*, whence $\frac{10{,}050}{290} - 1 = 35.2$.

The noise power input to the main intermediate-frequency amplifier is maintained constant by the insertion of 1/p of attenuation. Thus $\frac{N_1}{N_o} = p$, and so

$$F = \frac{35.2}{\alpha' \; (p - 1)} \;, \qquad (1)$$

or

$$F = 15.28 - 10 \log \alpha' - 10 \log (p - 1) \;,$$

$$= 15.28 - \alpha - 10 \log (p - 1) \text{ db} \;, \qquad (2)$$

where α = attenuation of r-f attenuator in db.

p = attenuation of i-f attenuator as a power ratio.

For $p = 2$, this gives

$$F = 15.28 - \alpha \;. \qquad (3)$$

2) Estimation of Errors

a) From (1), $\frac{dF}{F} = \frac{p}{p-1} \frac{dp}{p}$.

Thus near $p = 2$ (nominal value of i-f attenuator)

$$\frac{dF}{F} = \frac{2\,dp}{p} \;,$$

i.e., near $p = 2$, percent error in F is double percent error in p.

b) From (1), $\frac{dF}{F} = \frac{d\alpha'}{\alpha'}$,

i.e., percent error in F = percent error in α'.

---

* This is a theoretically derived value, and the assumptions used in its derivation have recently been challenged by workers whose experimental values have deviated from this figure. For example, see Baron and Trudgill, "The Absolute Power Output of a Gas Discharge Noise Source", RRDE Technical Note No. 73, July, 1952.

3) <u>Derivation of Formula: $P_s = (S/N)_o + F + 10 \log kTB + 30.0$ db</u>

Signal power at output = GP_s, noise power at output = N_o.

Thus
$$(S/N)'_o = \frac{GPs}{No} .$$

But, by definition
$$F' = \frac{N_o}{kT_oBG} ,$$

Therefore
$$(S/N)'_o = \frac{P_s}{kT_oBF'} ,$$

$$\text{or } P_s = (S/N)'_o \, kT_oBF' .$$

Converting to decibels, we get:

$$P_s = (S/N)_o + F + 10 \log k\, TB \text{ db w.r.t. 1 watt}$$

or
$$P_s = (S/N)_o + F + 10 \log k\, TB + 30.0 \text{ dbm}.$$

APPENDIX II

DERIVATION OF HETERODYNE METHOD RESULT

As before, $\hat{E}_{cw}$ = peak c-w r-f voltage.
$\hat{E}_p$ = peak pulsed r-f voltage.
$\hat{E}_{rf}$ = peak r-f voltage applied to detector.

We shall consider a square law detector, although the method can be applied to a detector having any law. For a square law detector, we may assume that:

$E_d = k\,\hat{E}^2_{rf}$, where E_d is the average d-c detector voltage.

With no pulsed r-f present, $E_d = k\,\hat{E}^2_{cw}$.
With the pulsed and c-w r-f in phase $E_d = k\,(\hat{E}_{cw} + \hat{E}_p)^2$.
With the pulsed and c-w r-f out of phase $E_d = k\,(\hat{E}_{cw} - \hat{E}_p)^2$.
These levels are shown in Fig. 8.

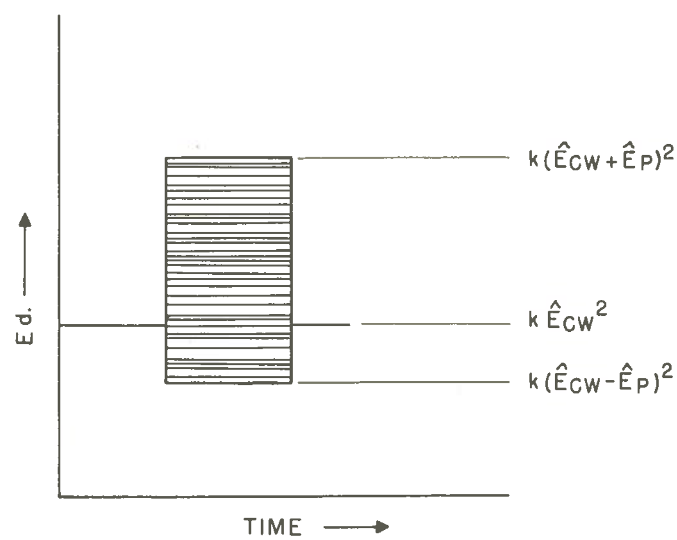

FIG. 8

DIAGRAM OF D.C. LEVELS
AT OUTPUT OF A SQUARE LAW DETECTOR

Since the video amplifier is a-c coupled, the baseline on the cathode-ray display will be the line corresponding to $E_d = k\,\hat{E}^2_{cw}$. From the figure it is evident that the condition for the pulse to be completely filled in is

$$k\,(\hat{E}_{cw} - \hat{E}_p)^2 = k\,\hat{E}_{cw}^2, \text{ i.e. } \hat{E}_p = 2\,\hat{E}_{cw}.$$

From this same figure the display produced by other simple ratios, such as $P_p = P_{cw}$ may be determined, and some of these are presented in Fig. 9. Photographs which show the corresponding experimental results are also included, in Plate 5. The actual pulse has been superimposed on the photographs for scaling purposes. The setting shown in Fig. 9 (b), in which $P_p = 4P_{cw}$, is the most satisfactory, because the appearance of the display is independent of the detector law, although the amplitude is not.

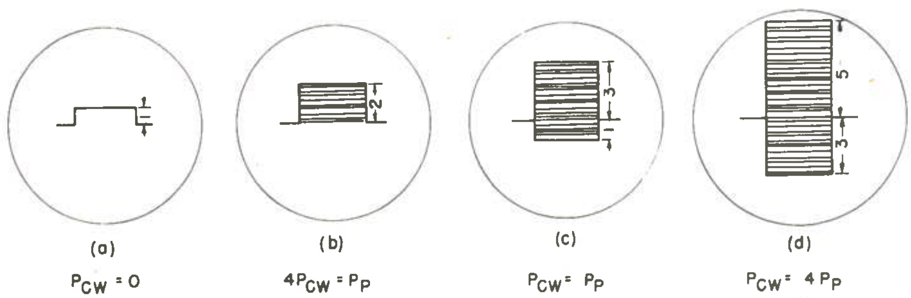

FIG. 9

HETERODYNE DISPLAYS FOR VARIOUS POWER RATIOS WITH A SQUARE-LAW DETECTOR

APPENDIX III

MEASUREMENT OF BANDWIDTH

Since the amount of noise energy present at the second detector is proportional to the area under the "power gain vs. frequency" characteristic of the receiver i-f amplifier, we may define the bandwidth by means of the following equation:

$$B\,G\,(f_o) = \int_o^\infty G\,(f)\,df$$

where B = bandwidth,

$G(f)$ = receiver power gain as a function of frequency, and

f_o = frequency of maximum gain.

In the present experiment $G(f)$ was plotted point by point and the area under the curve determined with a planimeter.

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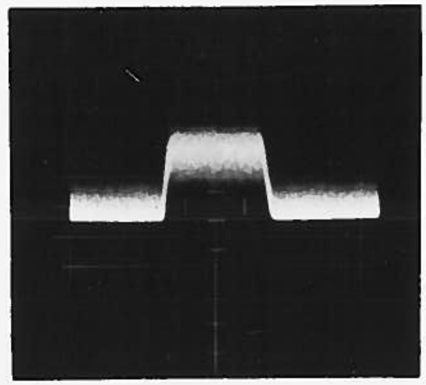


PLATE I
TANGENTIAL SIGNAL

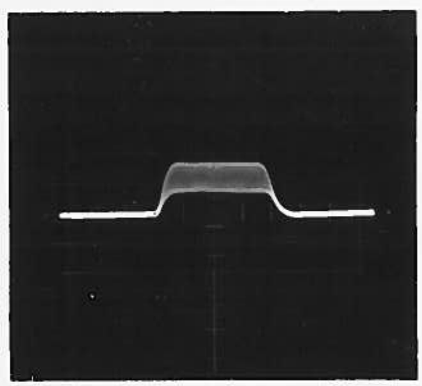
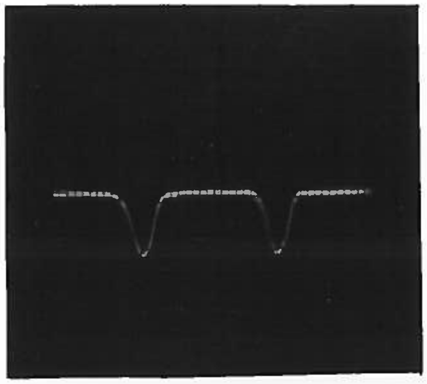
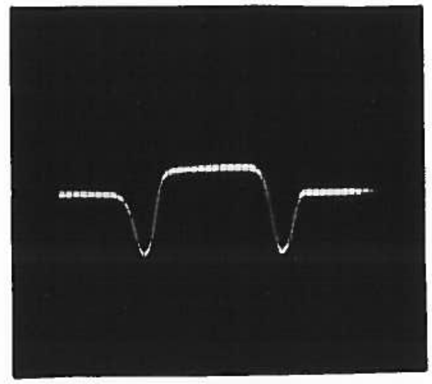


PLATE II
HETERODYNE-METHOD VIDEO DISPLAY



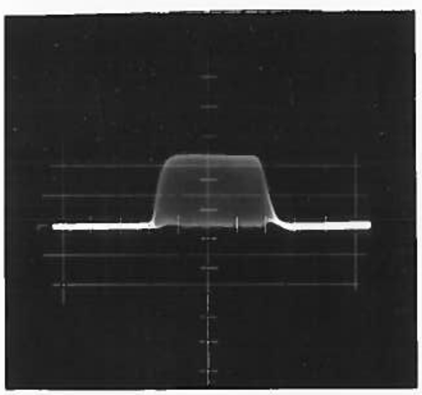
P_p = -30 dbm



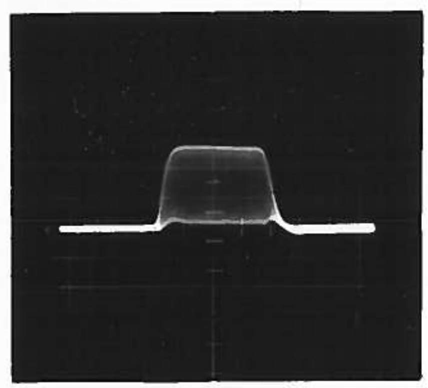
P_p = -29 dbm

NOTCH DISPLAY

(Notched Generator Output Constant)



P_p = -25 dbm



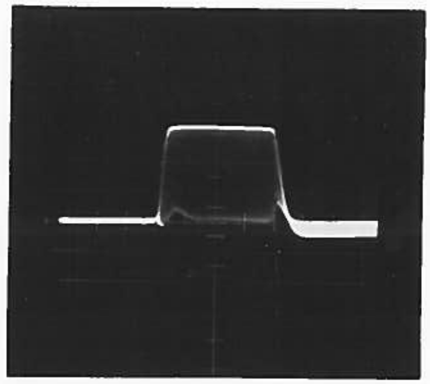
P_p = -24 dbm

HETERODYNE DISPLAY

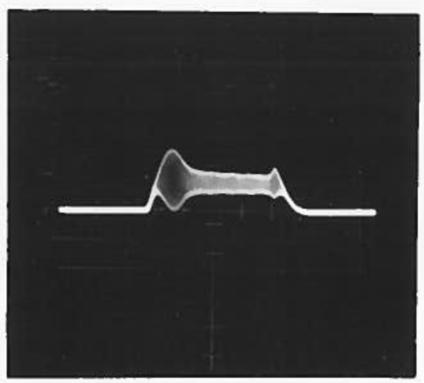
(C.W. Generator Output Constant)

PLATE III

CHANGE IN VIDEO DISPLAY FOR ONE DB CHANGE IN PULSE POWER



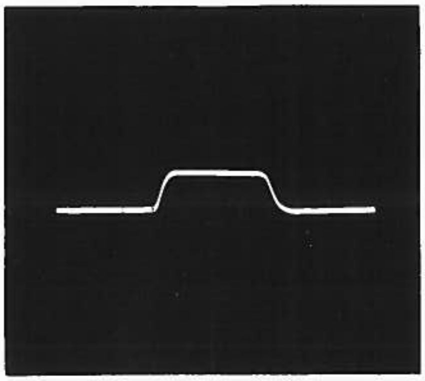
EFFECT OF VIDEO AMPLIFIER OVERSHOOT



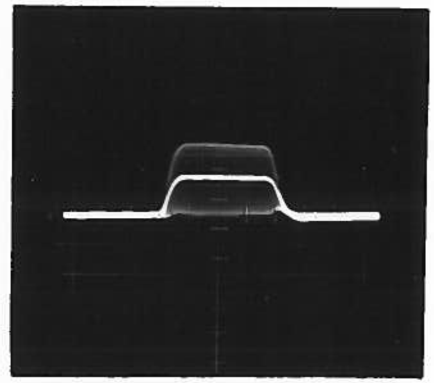
EFFECT OF FREQUENCY MODULATION ON PULSE

PLATE IV

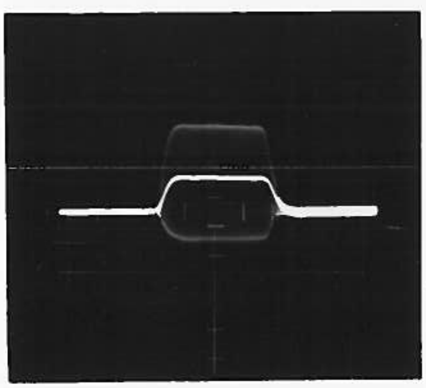
EXPERIMENTAL DIFFICULTIES ENCOUNTERED
WITH HETERODYNE METHOD



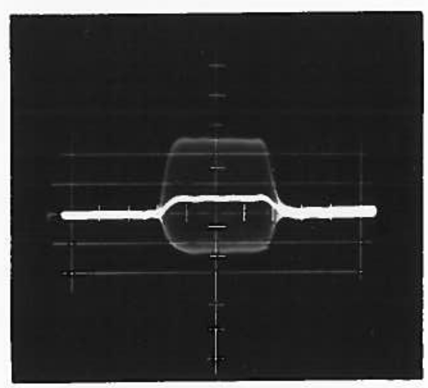
$P_{cw} = 0$



$4\ P_{cw} = P_p$



$P_{cw} = P_p$



$P_{cw} = 4\ P_p$

PLATE V

HETERODYNE VIDEO DISPLAYS OBTAINED WITH SQUARE-LAW DETECTOR