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PREFACE

One of the long range interests of the Division of Building Research, through its Building Physics Section, is a study of vibrations in the ground. This is a matter of considerable practical significance and yet one upon which there exists very meagre literature.

The Division therefore welcomes the publication by the Council of this translation of an important European paper in this field and is grateful to Mr. Nathan of the Main Library Staff for the work which he has done in translating this somewhat specialized technical contribution.

The Division will be glad to hear from any who may have occasion to consult this translation, and who have mutual interest in the field of ground vibrations.

R. F. Legget.

NEW RESULTS OBTAINED FROM DYNAMIC FOUNDATION SOIL TESTS

A number of reports recently published deal with the method of dynamic foundation soil tests, on which work was begun at the Deutsche Forschungsgesellschaft für Bodenmechanik (Degebo) about three years ago(1). A study of the detailed report no. 1 of the Veröffentlichungen der Degebo will enable the reader to familiarize himself with the method itself and with the theory of the oscillating mass point on an elastic support, which was used for the evaluations.

A short summary of the results to date is given in the present paper. In addition, new tests with an improved experimental set-up and the results thus obtained are described with particular emphasis on practical application*.

Results Published to Date

The results of dynamic foundation soil tests published to date may be summarized as follows.

(1) Two constants, the natural frequency α and the damping factor λ, are determined by graphical evaluation of three curves obtained independently during one test (i.e., the amplitude curve, performance curve, and phase curve).

(2) In order to obtain comparable values for α and λ, the same experimental set-up must be used for all the tests.

(3) In terms of soil mechanics the two constants have the following meaning:

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* The limited scope of the present paper does not permit dealing with the results obtained from investigations of systems of oscillation with more than one degree of freedom. A report on this will probably be published in one of the next numbers of the Veröffentlichungen der Degebo.

α is a criterion of the bearing capacity of the soil,
λ is a criterion of the behaviour of the soil when the latter is subjected to dynamic pressure.

The following data refer to the new standard experimental set-up:

weight of vibrator 2,700 kgm., base area 1 sq. m.
eccentricity 10° (moment of inertia: eccentric weight × eccentricity = 30.4 × 1.02 = 31.0 kgm. per cm.).

In order to compare the results obtained from the old standard experimental set-up with those obtained from the new one, comparative tests were made with both set-ups at the same sites.

The values for the constant α which correspond to the individual soil types have been compiled in Table I. It becomes immediately evident that increasing bearing capacity corresponds to the soil types arranged according to increasing α values. The following values may be taken as safe limiting values:

Lower limit: peat soil, α = 12.5 cycles per sec.

Upper limit: closely packed medium gravel, α = 28.1 cycles per sec.

The tests on top of bedrock were carried out with the cooperation of the Geophysical Institute of the University of Göttingen in order to study vibrations in the soil. A preliminary report on these tests was published by Köhler and Ramspeck[(2)]. The standard set-up was used to determine the α values for shell lime and mottled sandstone (30.0 and 34.0 cycles per sec., respectively). Although the plotted resonance curves show several maxima, the α values could be uniquely defined by taking into account the fact that the maximum value which depends on the vibration of the machine on an elastic support becomes displaced on the curve as the experimental conditions (eccentricity, weight of vibrator, position of the machine, etc.) change, while a maximum value which is not displaced on the curve, even when the experimental conditions are changed, depends on the properties of the soil through which elastic waves pass.

Table I is well suited to determining the bearing capacity of soils to be investigated since a higher value for α must be assigned to more compact soils.

In the last column of Table I, the safe soil pressures obtained from the tentative standards of the Ausschuss für einheitliche technische Baupolizeibestimmungen* (DIN tentative sheet 2, E 1054) are shown. The listed values for the safe pressures on the soil do not include the respective values for cohesive soils since the value α cannot be a sole criterion of the safe pressure on the soil here; however, the duration of load application is the determining factor. Furthermore, another fact emerges from Table I. The manner in which the admissible soil pressures have been assigned to different types of soil in the tentative standard sheet mentioned above is not yet quite satisfactory. For example, this is shown by the results for fine sand containing 30% of sand of intermediate grain. The standard sheet specifies that this sand should not be loaded with more than 1.5 kgm. per sq.cm.

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* Committee on uniform industrial building regulations.

Table I

The natural frequency α as a function of the soil type (determined by means of the standard experimental set-up: weight of vibrator = 2,700 kgm., base area = 1 sq. m., eccentricity = 10°)

Test No.	Soil type	c.p.s.	Safe soil pressure kgm./sq.cm.
179	1.5 m. peat bog on sand	12.5	-
582	1.5 m. old filling: medium sand* with peat residues	19.1	1.0
472	Gravelly sand with clay lenses	19.4	-
255	Old, stamped-down slag filling	21.3	1.5
468	Very old, stamped-down filling of loamy sand	21.7	2.0
392	Tertiary clay, wet	21.8	-
525	Lias clay, wet	23.8	-
428	Very homogeneous, yellow medium sand*	24.1	3.0
458	Fine sand containing 30% of medium sand (so-called Stettin sand)*	24.2	1.5
329	Homogeneous gravel	26.2	4.5
260	Nonhomogeneous, closely packed sand	26.7	4.5
348	Absolutely dry tertiary clay	27.5	-
475	Closely packed medium gravel	28.1	4.5
546	Shell lime (bedrock)	30.0	2/3 of the admissible compression strain σ_D
563	Mottled sandstone (bedrock)	34.0	2/3 σ_D
	* i.e., sand of intermediate grain		

By comparing the α values with the safe soil pressures a relationship is established between the results obtained from dynamic foundation soil tests and building practice. It is merely necessary to check the accuracy of α values for soil types not listed in the standard draft by means of soil analyses in the laboratory, possibly by test loads and eventually by observation of the settlement of a completed building.

With respect to the damping factor λ it is not possible, unfortunately, to assign a definite soil property to each λ value. This is due to the fact that the damping depends on the energy transformed during the production of vibrations and that the damping factor does not indicate whether the energy has been consumed for either plastic or elastic deformation. However, the dynamic settlement curve, i.e., the plot of the permanent settlements of the vibrator as a function of the exciter frequency, should be of assistance here. This curve indicates whether the soil investigated underwent extensive permanent deformation. However, if a large value for λ($\lambda \geqq 3$ cycles per sec.) is obtained from a test on a noncohesive soil and if the dynamic settlements ε measured with the standard experimental set-up are also large ($\varepsilon \geqq 10$ mm.), then it is certain that the soil will tend towards large settlements when it is subjected to any dynamic strain. This prediction may be supported by a grain analysis of the soil. Such analyses always showed that inequigranular soils tend towards large settlement as a result of vibration.

Application of the Results to Actual Cases

The most important results for practical application may be summarized as follows.

(1) It is evident from the values listed in Table I that the safe soil pressure may be determined from the result of a dynamic, foundation-soil test with the standard experimental set-up.

(2) By means of dynamic, foundation-soil tests it is now possible to predict the expected differences in settlement of a planned building with a sunken-shaft foundation.

Example: Figs. 1 and 2 show a tunnel underneath all the tracks of a railway station that carry passenger trains other than local ones. At the end of the tunnel a shaft, extending approximately 2 m. below the floor of the tunnel, is to be sunk. It is assumed that the tunnel has been built on a foundation of old fill and that the shaft is to be sunk into the underlying soil (gravelly sand with clay lenses). Dynamic tests were made at two points, one on the floor of the tunnel (position I) and the other on the floor of the shaft (position II).

Comparison with the results obtained from the standard experimental set-up show, surprisingly enough, that the value α and thus the bearing capacity of the fill, is higher (21.3 cycles per sec.) than that of the underlying soil (19.4 cycles per sec., cf. Table II). Furthermore, it should be remembered that the tests on the floor of the shaft had to be carried out in a timbered trench. It is a known fact that, because of the fixity resulting from the high loads, higher values for α are obtained from the tests at this location than from tests on the same soil at ground level. Furthermore, at both points of testing the same value (approximately 3 cycles per sec.) was obtained for the damping factor λ and a value of 0.15 for λ/α, whereas the value for the dynamic settlement obtained in the standard test at position II was almost ten times as high as that for the settlement at position I. These results justify the assumption that at identical soil pressure the settlement of the shaft differs greatly from that of the tunnel. It is safe to assume that this settlement is a multiple of that of the tunnel. On the basis of empirical values the safe soil pressure for positions I and II was established as 1 to 1.2 kgm. per sq. cm. Since these predictions indicate that the tracks carrying main line passenger trains are endangered by settlement of the tunnel, a construction is recommended in which the tunnel is not affected by settlement of the shaft.

Table II

Comparison of the results obtained from the standard experimental set-up at two positions as shown in Figs. 1 and 2

Eccentricity °	Position I Natural frequency α c.p.s.	Position I Damping factor λ c.p.s.	Dynamic settlement ω cm.	Position II Natural frequency α c.p.s.	Position II Damping factor λ c.p.s.	Dynamic settlement ω cm.
10	21.7	3.3	0.25	19.4	2.8	1.91
20	17.4	3.0	0.61	16.0	2.0	3.63
30	15.5	2.6	1.85	15.8	2.2	2.58
40	13.5	2.8	2.15	-	-	-

New Tests - Change of the Constants for the Apparatus

It was explicitly stated that the values for α in Table I are only comparable for tests made with the standard experimental set-up.

However, the apparatus in its present form permits changes in the eccentricity of the working loads, weight of the vibrator, and the base area within wide limits. There are various reasons in favour of this system. With the standard experimental set-up, curves which are comparatively difficult to evaluate are obtained from some soil types (e.g. gravelly sand). However, it is much easier to evaluate the curves if certain constants for the apparatus are changed, for example, by increasing the weight of the vibrator. This adaptability of the apparatus makes it possible to obtain favourable experimental conditions for any type of soil and thus enables clear records to be made. However, one single test, as in the case of the standard experimental set-up, is no longer sufficient here; at least three tests are required. The conversion of the value α obtained from the new experimental set-up to that obtained from the standard experimental set-up requires a curve for the effect which the respective constant for the apparatus has on the value α. From this curve the value converted to standard experimental conditions is then obtained by interpolation or extrapolation.

Change in Weight of the Vibrator

Furthermore, an insight into the behaviour of the soil, which is excited by periodic forces, may be obtained from a study of the resonance curves. For example, the resonance of the soil may be determined from the effect caused by a change in the weight of the vibrator (identical base area assumed).

In Fig. 3, the natural frequency α has been plotted on the abscissa and the weight of the vibrator, G, on the ordinate. The measured value α = 23.8 cycles per sec. has been plotted as the initial point (indicated by two circles) for the weight of the vibrator G = 1800 kgm., the eccentricity E = 10° and the base area F = $\frac{1}{4}$ sq.m. If under otherwise identical conditions (soil type, eccentricity, base area) only the weight of the vibrator is increased, then the value α must decrease in the following manner:

$$2\pi\alpha = \sqrt{\frac{c}{m}} = \sqrt{\frac{cg}{G}} ,$$

where c = elasticity constant, m = vibrator mass, g = acceleration due to gravity. The decrease of α must be parabolic since G is a radicand. The curve obtained by determining the value α belonging to different values of G from the above relation has been denoted as $G_B = 0$. The decrease in the frequency values on increasing the weight of the vibrator should be concordant with this curve, if merely the mass of the vibrator itself acted as the vibrating mass. If it is assumed that a certain weight of the soil, G_B, is participating in addition to the weight of the vibrator, G, and must be regarded as part of the vibrating mass, then the following relation is obtained:

$$2\pi\alpha = \sqrt{\frac{cg}{G + G_B}} .$$

If the initial point (α_1 = 23.8 cycles per sec., G_1 = 1.8 metric tons) is retained, the relation for the decrease of the value α becomes

$$\alpha = \alpha_1 \sqrt{\frac{G_B + G_1}{G_B + G}} ,$$

i.e., a family of curves with the parameter G_B in the system of coordinates α_1, G.

In Fig. 3 a network of the curves $\alpha = f(G)$ has been plotted for assumed values of G_B*. The measured values of α for the weights of the vibrator, G = 1.8, 2.35, 2.7, 3.05 and 3.4 metric tons, have been plotted in this network. The curve drawn through these points falls well on this network, i.e., it falls approximately midway between the curves G_B = 10 metric tons and G_B = 15 metric tons, and thus indicates that the vibrating soil has the weight $G_B \approx 12.5$ metric tons and that it is independent of the weight of the vibrator G applied (at least within the limits $0.14 < G/G_B < 0.26$).

Change in the Value of the Base Area

The experimental set-up makes it possible to use three different base areas for a fixed weight of the vibrator, i.e., F = 1, $\frac{1}{2}$ and $\frac{1}{4}$ sq. m. Here the relation $2\pi\alpha = \sqrt{\frac{cg}{G}}$, or if the

* G_B = 0, 2, 5, 10, 15, and 30 metric tons.

dynamic ground constant c'(kgm./cc.) multiplied by the base area F(sq.cm.) is substituted for the elasticity c (kgm./cm.) i.e., $2\pi a = \sqrt{\frac{c' F g}{G}}$, states that the value a must increase in a parabolic manner when the area F is increased. On the strength of the same consideration, which resulted in the determination of G_B, this quantity may again be obtained from the effect of the area F (or the static soil pressure $\sigma = G/F$, as the case may be) on the natural frequency a for the same weight of vibrator, G. In Fig. 4 the value a has been plotted on the abscissa, the area F on the ordinate, and the soil pressure σ for G = 2,700 kgm. The initial point is $a_1 = 22.85$ cycles per sec. here, measured on gravelly soil for a base area of 2500 sq.cm. (σ = 2,700/2,500 = 1.08 kgm./sq.m.). If it were possible to consider the mass of the vibrator alone as the vibrating mass, then the value a on the flattest parabola ($G_B = 0$) should increase as the area increases. Here, too, the measured curve fits closely to the network of the family of curves obtained for different values of G_B and falls between the curves for G_B = 12 metric tons and G_B = 15 metric tons. Hence, the agreement between the two values for G_B, which had been obtained in an entirely different manner by means of two completely independent test series at the same site, is also very good. Here again the tight fit of the measured curve into the theoretical network proves once more that the weight G_B is largely independent of the value of the soil pressure σ, (or base area of the vibrator).

In order to utilize the foundation soil tests for predicting the safe soil pressures of structures to be erected and the vibration frequencies of the machine foundations to be constructed (whose base area is to be a multiple of the base area of the vibrator) on the soil investigated, the effect of the base area alone, i.e., the effect of various values of the base area at identical soil pressure on the value a, must be investigated. Unfortunately, the experimental set-up does not permit the use of the available three area values (1, $\frac{1}{2}$, $\frac{3}{4}$ sq. m.) at identical surface pressure. In this connection it must be remembered that since the lowest weight (vibrator alone) is 1800 kgm. it would be necessary to increase the weight of the vibrator to $G = \sigma F = 0.72 \times 10{,}000 = 7{,}200$ kgm. in order to obtain the same pressure for 1 sq. m. area as for $\frac{1}{4}$ sq. m., i.e., $\sigma = 1800/2500 = 0.72$ kgm./sq.cm. However, this has not been possible as yet due to transportation difficulties. Therefore, it must suffice for the time being to investigate, at any one time, only two successive area values at identical soil pressure for their effect on the value α and to make a comparison with the third value for the area by extrapolating the curve $\alpha = f(\sigma)$. In Fig. 5 the effect of the three area values 1, 0.5 and 0.25 sq. m. on the value α at identical soil pressure is shown in this manner. It can be seen that this effect is exceedingly slight. This is a noteworthy result. However, if the effect of various area values on the static bedding coefficient C at identical soil pressure is considered, (as obtained by Kögler and Aichhorn(3) from static load experiments, cf. Fig. 5), then the results may be applied for practical purposes as follows.

The results of dynamic foundation soil tests are better suited to make a deduction from small areas to large ones than the results obtained from static soil investigations. In order to explain this physically, the following statement is made. (The observation that for a constant soil pressure the settlement increases as the area increases may be explained by the fact that an increase in the depth of soil affected results from the increase in the loaded area (i.e., the settlement increases similarly to the volume of affected soil, as the third power, whereas the area increased as the second power.*) However, in the dynamic foundation soil test the

* Translator's Note: The meaning of the statement in parentheses is not clear. The conventional concept is to consider a "bulb of constant pressure" in the soil, whose depth is proportional to the diameter of the loaded area. Settlement is proportional to the depth of the affected soil, and hence proportional to the diameter of the loaded area.

soil affected is largely independent of the static initial loading σ, which is only a fraction of the total load. The soil affected depends rather on the centrifugal forces (eccentricity of the working loads) and the stratification of the soil. Since this resonant soil is a multiple of the working load, it is easy to understand that even considerable changes in the base area of the vibrator have only an unimportant effect on the soil and thus on its elastic properties.

Change in the Eccentricity of the Working Loads

The eccentricity of the working loads is the third constant whose effect on the natural frequency α is to be investigated. This constant is expressed in radians because the eccentricity is given by the relative position of two masses which are adjustable on a circumference. For example, eccentricity $E = 0°$ means that the two equal masses are opposed on a diameter so that the two centrifugal forces cancel each other.

If a clearly defined vibrating system (e.g. a beam on two supports loaded by a vibrator (cf. Fig. 6)) is examined, it can be seen that the natural frequency, α, of this system is completely independent of the eccentricity of the working loads and thus of the amplitude of the impulse of vibration.

The case under consideration, i.e., vibrations of a mass on the soil, is not a clearly defined vibrating system with given elasticity and mass but the size of the vibrating mass depends here on the magnitude of the impulse as shown in Fig. 7. This manner of representation serves the purpose of showing that the area of the soil participating in the vibration increases as the impulse increases until it attains its maximum at some limitation (on reaching a layer in the soil whose density varies considerably, e.g. a deep trench, etc.). The natural frequency, α, which decreases in the manner indicated as the mass of affected soil increases until it attains the limiting value corresponding to the maximum value for the soil, depends on the value for this area of resonant soil.

From the assumption that the affected soil depends on the adjusted eccentricity according to some function, it follows that the natural frequency α must decrease as the eccentricity increases. This phenomenon has been satisfactorily determined by tests. Only the extent of the decrease of α as the eccentricity increases (i.e., the shape of the decrement curve $\alpha = f(E)$ differs for different soil types). Therefore, it may be assumed that some data on the soil investigated may be obtained from this curve.

In order to clarify this question the resonant soil must be determined for various eccentricities. The experimental results represented by four curves in Fig. 8 are available for this purpose. Four experimental series for the effect of the eccentricity on the value α are dealt with here. These series differ merely by the soil pressure σ and the weight of the vibrator, G. By means of these curves the resonant soil may be determined, as in Fig. 3, for each of the three eccentricities 10°, 20° and 30°. The curve for the resonant load as a function of the eccentricity is thus obtained (cf. Fig. 9). From this curve it is evident that at first the extent of the soil affected increases as the eccentricity decreases and then tends to a limit (approximately 40 metric tons here). The assumption of Fig. 7 is confirmed; the limiting value for G_B and thus the lower limit of the natural frequency α ($\approx$ 18 cycles per sec. here) has been found.

The fact that the shape of the curve which represents the value α as a function of the eccentricity is characteristic for the soil investigated, is clearly evident from Fig. 8. The four curves, which were obtained independently from the same soil, have the same shape and tend to approach the same asymptote. However, when curves obtained from different soils (cf. Fig. 10) are compared, the effect of the soil type becomes evident. Curve "a" was taken on a layer of clay, approximately 1 m. thick, on top of tertiary fine sand. The curve indicates that, for an eccentricity E as low as 20°, the vibration in the soil has reached the fine sand and that a clearly defined system of vibration has formed which will not change even if the impulse increases. This also applies to curve "c", which was taken on a layer of sandy peat (1.5 m. thick) on grit. However, curve "b" is based on tests in a pit of clay of great thickness. The sharply broken profile producing reflections of the waves is presumably the reason why the limiting value for G_B is attained so early here, despite the great thickness of the remarkably homogeneous soil. Curve "e" represents a practically homogeneous terrain with horizontal surface. In this case the tests were made in a gravel pit, approximately 20,000 sq.m. in area and at least 15 m. thick. This curve is the flattest of all the curves taken and its vertical asymptote does not become immediately evident.

Finally, curve "d" indicates that the difference in density of the old filling and that of the gravelly sand is small and that the very thin layer of clay (provided this is not merely a case where clay lenses had been struck) is not sufficient to confine vibrations in the soil. Curve "d" thus is very flat, indicating that here this is essentially a case of a fairly homogeneous soil.

Practical Applications

An example of an actual case is used here to show the advantage of these experimental series. For laying a machine foundation (propeller test stand) data were required on the expected vibration frequency and on the dynamic bedding coefficient of the foundation soil.

The results of the dynamic soil foundation test are shown in Table III. In the experiment with the standard set-up a value of 26.7 cycles per sec. was obtained for α. When these experiments were conducted the effect which the constants of the apparatus has on the value α was still unknown and no conclusions had been drawn at that time. Therefore, systematic test series did not seem justified. Two different weights of vibrator on the same base area (F = 1 sq. m.) had to suffice. However, the effect of the eccentricity was thoroughly investigated.

The vibration frequency and the dynamic bedding coefficient (c') are determined as follows. The vibration frequency α depends on the soil pressure σ and on the eccentricity. The effect which the extent of the base area has at identical soil pressure σ has been omitted here. This has been discussed in greater detail in the preceding paragraphs. If the eccentricity, or at least centrifugal force of the unbalance of the machine for which the foundation is to be laid, is known, then the experimental set-up can be adjusted to the respective value and from it the natural frequency α may be given within very narrow limits, taking into account the static soil pressure of the planned foundation. However, if for a planned machine foundation data on centrifugal force or eccentricity are not available, except for the rated full-load speed, then the lower limit for the natural frequency α, with the soil pressure given, is determined by the vertical asymptote to the respective curve $\alpha = f(E)$ (Fig. 11).

Table III

Results of the dynamic soil test of a machine foundation (propeller test stand)

Weight of vibrator G_S kgm.	Soil pressure σ kgm./sq. cm.	Frequencies α when E = 10° c.p.s.	12° c.p.s.	16° c.p.s.	30° c.p.s.	Lower limit for α c.p.s.	Lower limit for c' kgm./cc.	Upper limit for c' kgm./cc.
2060	0.27	29.0	26.9	24.6	23.2	23	4.4	~ 6
2700	0.27	26.7	25.4	23.77	22.3	22	5.2	~ 6

The dynamic bedding coefficient c' is obtained from the vibration frequency α as follows:

$$c' = \frac{c}{F} = 4\pi^2\alpha^2 \frac{G}{Fg} = 4\pi^2\alpha^2 \frac{\sigma}{g} \ .$$

The lower limit for c' is accurately obtained from this relationship by using the lower limiting value of α and the smallest possible vibrating mass (mass of the vibrator alone, $G_B = 0$). However, the upper limit of c' cannot be determined with the same accuracy, since only two test series with two different weights of the vibrator are available here for the determination of the resonant soil. The calculation of the latter for any given eccentricity must then be carried out by means of the slightly less accurate equation

$$G_B = \frac{G_2\ \alpha_2^2 - G_1\ \alpha_1^2}{\alpha_1^2 - \alpha_2^2} \ .$$

The following values are thus obtained: $G_B = 1{,}000$ kgm., $G = G_S + G_B = 3{,}000$ kgm. and when $\alpha = 23$ cycles per sec. the dynamic bedding coefficient becomes $c' = 6.3$ kgm./cc.

All the above statements refer exclusively to vertical vibrations. It is briefly shown below that the experimental set-up permits not only the production of vertical vibrations but also horizontal and angular momenta.

Test with Torsional and Horizontal Excitation

The set-up of the eccentrics for torsional excitation is shown in Figs. 12-15. The equation of the vibrating mass also applies to this type of vibration. But the constants have a different meaning (cf. Figs. 12 and 13). It can be seen that the dynamic shear coefficient S of the investigated foundation soil can be obtained by torsional tests with given moments of inertia of the vibrator, i.e., Θ (inertia moment) and J_p (polar angular impulse), in the same manner as the bedding coefficient was obtained from the test with vertical excitation.

At the point mentioned above, where the value $\alpha = 26.7$ cycles per sec. was obtained with vertical excitation (standard experimental set-up), a test with torsional excitation resulted in a value of 14.0 cycles per sec. for the natural frequency α_τ. The polar inertia moment of the vibrator about its vertical centroidal axis was experimentally determined as $\Theta = 4{,}900$ kgm. cm. sec.^{2} (4) and the polar moment of inertia of the base area as $J_p = 16.7 \times 10^6$ cm.^{4}. Hence, the shear coefficient is obtained,

$$S = \frac{4\pi^2 14.0^2 \times 4{,}900}{16.7 \times 10^6} = \underline{2.3 \text{ kgm./cc.}}$$

The arrangement of the working loads for horizontal excitation is shown in Figs. 16 - 21. It should be noted that, in addition to the periodic horizontal force, a periodic angular momentum about the longitudinal axis of the vibrator is always excited. By using the dynamic bedding coefficient c' = 6.3 kgm./cc. and the shear coefficient S = 2.3 kgm./cc., which were obtained from tests with vertical and torsional excitation, the lower horizontal frequency α_{H_1} of the vibrator was computed for the test stand mentioned above. The vibrator mass was replaced here by two smaller masses m_1 and m_2, as suggested by Rausch(5). Each of these masses oscillates about their common centre of gravity. The natural frequency α_{H_1} of the pendulum with the greater mass is obtained as

$$(2\pi\alpha_{H_1})^2 = \frac{F}{m_1} \frac{1}{\left[\left(\frac{s - a_2}{i_F}\right)^2 \frac{1}{c'} + \frac{1}{S}\right]} \quad .$$

The meaning of the values s, a_1 and a_2 is evident from Fig. 21. Furthermore, i_F is the radius of gyration of the base area F about the centre of gravity in the plane of the projection, i_G is the radius of gyration of the total mass M about the horizontal centroidal axis in the plane of the projection. Then the following relationship applies:

$$a_{1,2} = a_0 \pm \sqrt{a_0^2 + i_G^2} \quad \text{and} \quad a_0 = \frac{s^2 + \frac{c'}{S} i_F^2 - i_G^2}{2\,s}$$

$$m_1 = \frac{G}{g} \frac{a_1}{a_1 - a_2} \quad .$$

From these equations the lower horizontal frequency of the vibrator is obtained with the following values:

$G = 2{,}000$ kgm. $\quad i_F^2 = 835$ sq. cm. $\quad c' = 6.3$ kgm./cc.

$F = 1$ sq. m. $\quad i_G^2 = 875$ sq. cm. $\quad S = 2.3$ kgm./cc.

$s = 34$ cm.

$$a_0 = \frac{1{,}156 + \frac{6.3}{2.3}\,835 - 875}{68} = 38 \text{ cm.}$$

$$a_{1,2} = 38 \pm \sqrt{38^2 + 875} = \begin{cases} +86 \\ -10 \end{cases}$$

$$a_1 - a_2 = 96$$

$$m_1 = \frac{1}{981} \times \frac{86}{96} \times 2{,}000 = 1.83 \text{ kgm. sec.}^2/\text{cm.}$$

as

$$\alpha_{H_1}^2 = \frac{10^4}{4\pi^2 1.83} \times \frac{1}{\frac{44^2}{835} \times \frac{1}{6.3} + \frac{1}{2.3}} = 173; \ \alpha_{H_1} = 13.1 \text{ cycles.}$$

The test with horizontal excitation actually resulted in a horizontal natural frequency of the vibrator at 13.5 cycles per sec. However, it was not possible in this test to attain the other higher frequency α_{H_2}.

Summary of Results for Practical Purposes

Results from Individual Tests with a Standard Experimental Set-up on Noncohesive Soils

1. Data on the admissible soil pressure.
2. Data on the expected differences in settlement.
3. Determination of the safety of the foundation soils against traffic vibrations.

Results from Test Series with the Constants of the Apparatus Changed

1. Determination of the soil mass participating in the vibration.
2. Data on the thickness of layer and homogeneity of the investigated soil.
3. Definition of the natural frequency of a machine foundation.
4. Reliable figure for the lower limit of the dynamic bedding coefficient.

Trend in Improving the Method of Dynamic Foundation Soil Tests

1. Extension of Table I so as to include the dependence of the α values on the soil type.
2. Completion of a curve for the determination of the admissible soil pressure from the value α.
3. Control of settlement of completed buildings.
4. Absolute prediction of the settlement for noncohesive soils.
5. Verification of the predetermined natural frequencies by measurements on the erected machine foundation, i.e., for vertical and horizontal vibrations.
6. Continuation of the investigation of the effect of dynamic stress on the damping constant and dynamic settlement curve.

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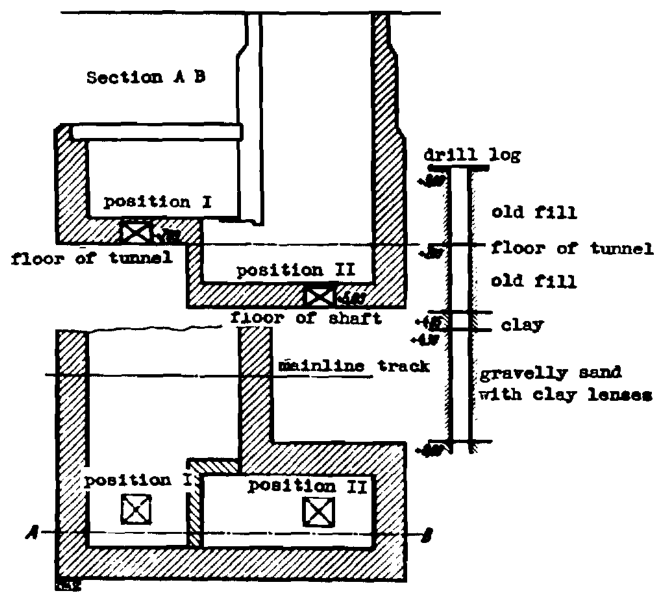


Figs. 1 and 2 Example for applying the dynamic soil foundation test. A shaft is to be sunk to a depth of approximately 2 m. at the end of a railway station tunnel underneath the tracks for main-line passenger trains.

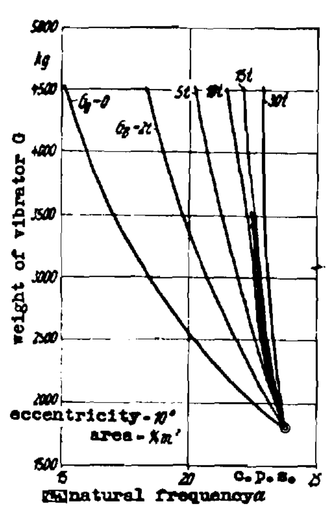


Fig. 3 Effect of the weight of the vibrator on the natural frequency (G_B = weight of the resonant soil). The thin curves were plotted on the basis of assumed values for the resonant soil. The thick curve connects the values of α measured for various weights of the vibrator.

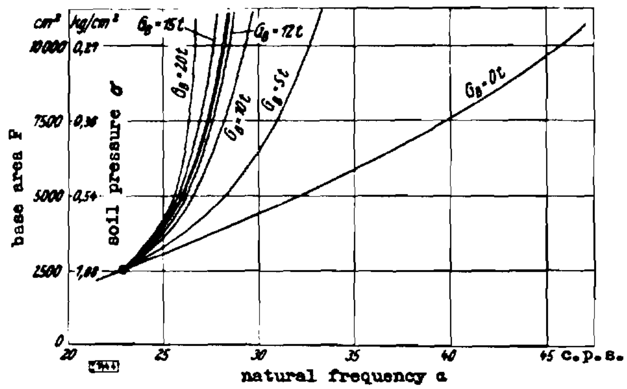


Fig. 4 Effect of both the extent of the base area and the static soil pressure on the natural frequency for a given weight of the vibrator (2,700 kgm.).

Cf. text of Fig. 3 regarding the thin lines and the thick line.

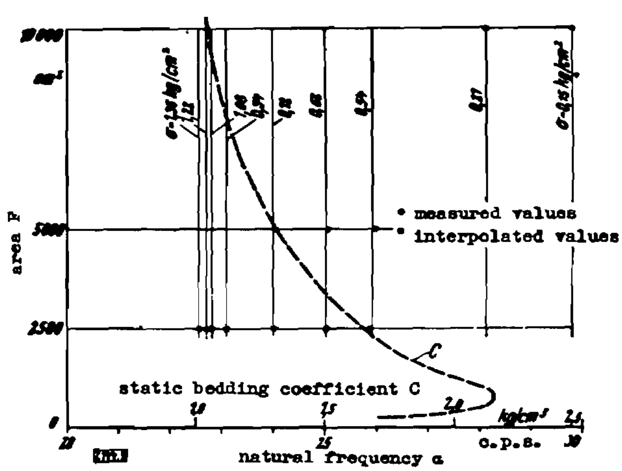


Fig. 5 Effect of area variations on the natural frequency and static bedding coefficient at identical soil pressure.

The straight lines designated by σ connect points of identical soil pressure. The fact that these straight lines are vertical indicates that the effect of varying the area on the natural frequency is slight. If, in static load tests, too, the static bedding coefficient C were independent of the area, the curve C would also be a vertical straight line.

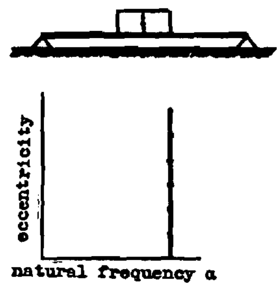


Fig. 6 Beam on two supports loaded with vibrators.

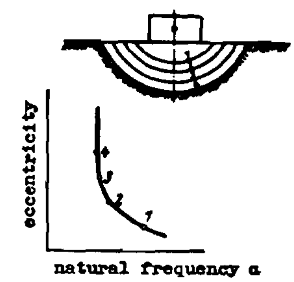


Fig. 7 Mass supported on the ground.

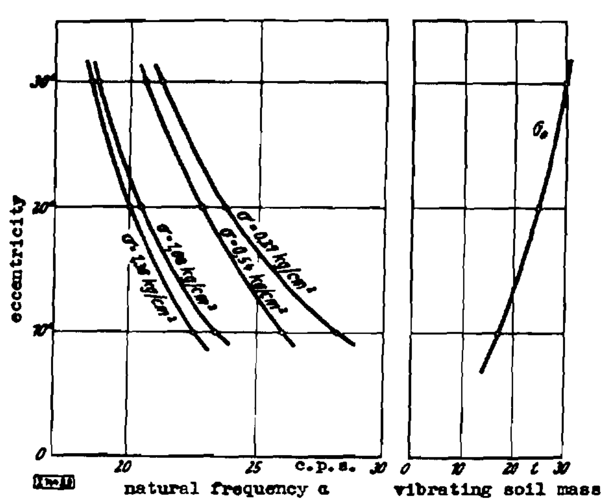


Fig. 8 (left) Behaviour of eccentricity and natural frequency at different soil pressures σ.

Fig. 9 (right) Effect of eccentricity variations on the weight of soil G_0, participating in the vibration.

Figs. 6 to 9 Effect of the eccentricity of the working load on the natural frequency $\alpha \left(\frac{1}{2\pi}\sqrt{\frac{c}{m}}\right)$.

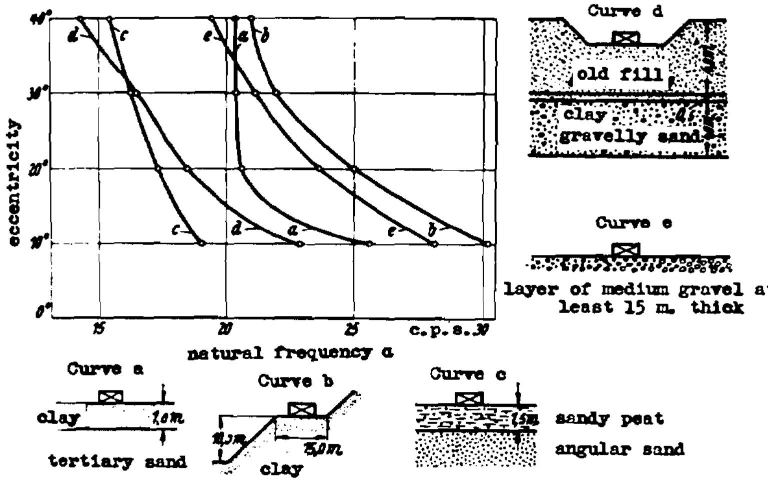


Fig. 10 Eccentricity and natural frequency for different soil types.
(The accompanying figures explain the curve designation).

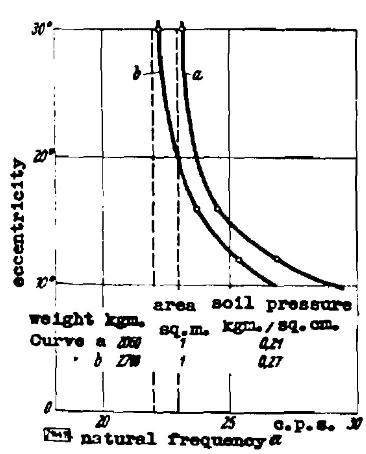
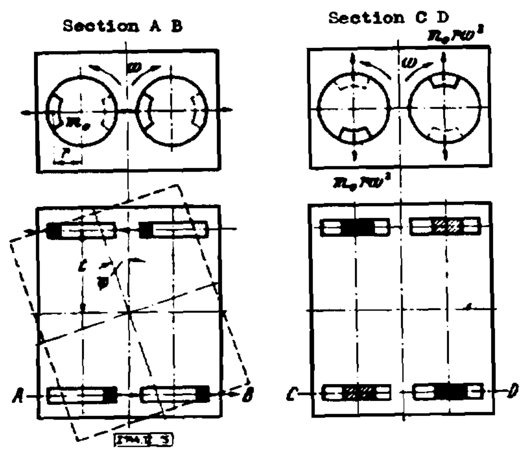


Fig. 11 Determination of the lower limit of the natural frequency.



Figs. 12 to 15 Set-up of the eccentrics for torsional excitation. Equation of motion of the torsional vibrator:

$$\Theta\ddot{\psi} + b\dot{\psi} + c_M\psi = M_D \sin \omega t,$$

where Θ = inertia moment relative to the vertical centroidal axis,

ψ = torsion angle with respect to the equilibrium position,

b = damping factor,

c_M = J_pS - elasticity constant,

J_p = polar angular impulse,

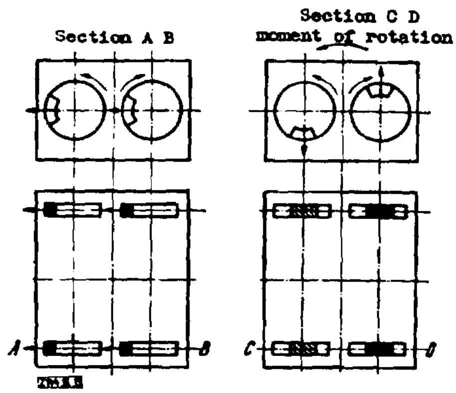
S = shear coefficient,

M_D = $m_o r\omega^2 t$ - moment of rotation of the exciter force about the vertical centroidal axis.

Solution of the differential equation $\psi = \Psi \sin(\omega t - \phi)$ equation of the amplitude

$$\Psi = \frac{m_o\ r\ \omega^2\ t}{\sqrt{(c_M - \Theta\omega^2)^2 + b^2\omega^2}} = \frac{\varkappa_\tau\ \omega^2}{\sqrt{(\alpha_\tau^2 - \omega^2)^2 + 4\lambda_\tau^2\omega^2}}$$

$$\alpha_\tau^2 = \frac{c_M}{\Theta} = \frac{J_pS}{\Theta} .$$



Figs. 16 to 21 Dynamic foundation soil tests by means of horizontal excitation.

Figs. 16 to 19 Set-up of the eccentrics.

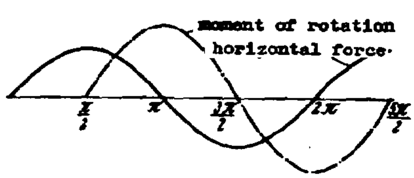


Fig. 20 Diagram showing the excitation curve.

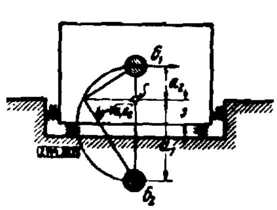


Fig. 21 Disk supported elastically (according to Rausch). Substitution of the vibrator masses by two smaller masses which oscillate about their common centre of gravity.