Analysis of errors in C.R.D.F. Adcock aerials

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ANALYSIS OF ERRORS IN C.R.D.F. ADCOCK AERIALS

OTTAWA

OCTOBER, 1943

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ANALYSIS OF ERRORS IN C.R.D.F. ADCOCK AERIALS

CONTENTS

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I. Introduction

The C.R.D.F. Adcock aerial is subject to a number of errors, some of which may be predicted from the design dimensions. Others may be found to exist in a particular installation due to faulty components. These errors show up in calibration curves, and it is the purpose of the following analysis to facilitate the isolation of these faults by examination of the calibration curves. Errors due to varying degrees of polarization of downcoming waves are not dealt with.

II. Error due to spacing of masts

Referring to figure 1, the phase angle between E_n and E_s is $\frac{2\pi d}{\lambda}\cos\theta$ and that between E_e and E_w is $\frac{2\pi d}{\lambda}\sin\theta$, whose d is the spacing, λ the wavelength of the incoming signal, and θ the true bearing.

From Figure 1(b) and (c),

$$E_{ns} = 2E\sin\left(\frac{\pi d}{\lambda}\cos\theta\right)$$

$$E_{ew} = 2E\sin\left(\frac{\pi d}{\lambda}\sin\theta\right),$$ where E is absolute value of e.m.f. in each mast.

If the bearing observed on the direction finder is ϕ, then

$$\tan\phi = \frac{E_{ew}}{E_{ns}} = \frac{\sin\left(\frac{\pi d}{\lambda}\sin\theta\right)}{\sin\left(\frac{\pi d}{\lambda}\cos\theta\right)}$$

It can readily be seen that unless d is small, making

$$\sin\left(\frac{\pi d}{\lambda}\sin\theta\right) = \frac{\pi d}{\lambda}\sin\theta,$$

and $$\sin\left(\frac{\pi d}{\lambda}\cos\theta\right) = \frac{\pi d}{\lambda}\cos\theta,$$

that ϕ differs from θ. It can also be seen by inspection that $\phi = \theta$ for any value of d when $\theta = 0, 45^\circ, 90^\circ, 135^\circ$, etc. Therefore, there are eight points at which the error is zero, and eight regions between these points where it is not. Let us assume the error is sinusoidal with respect to 4θ, then $\phi = \theta + K_1\sin 4\theta$, where K_1 is the maximum value of the error, occurring at $\theta = 22\frac{1}{2}^\circ, 67\frac{1}{2}^\circ$, etc.

Now we have $$\tan\phi = \tan(\theta + K_1\sin 4\theta)$$

$$= \frac{\tan\theta + \tan(K_1\sin 4\theta)}{1 - \tan\theta\tan(K_1\sin 4\theta)}$$

For $\theta < \frac{\pi}{4}$, $\tan \phi = \left[\tan\theta + \tan(K_1 \sin 4\theta)\right]\left[1 + \tan\theta \tan(K_1 \sin 4\theta) + \ldots\right]$

$$= \tan\theta + \tan^2\theta \tan(K_1 \sin 4\theta) + \tan(K_1 \sin 4\theta) + \tan\theta \tan^2(K_1 \sin 4\theta) + \ldots$$

$$= \tan\theta + \frac{\tan(K_1 \sin 4\theta)}{\cos^2\theta} + \frac{\tan\theta \tan^2(K_1 \sin 4\theta)}{\cos^2\theta}$$

If the error K_1 is small, $\tan(K_1 \sin 4\theta) \simeq K_1 \sin 4\theta$

Neglecting second order terms, $\tan\phi = \tan\theta + \dfrac{K_1 \sin 4\theta}{\cos^2\theta}$ (1)

Now we have $\tan\phi = \dfrac{\sin(\frac{\pi d}{\lambda}\sin\theta)}{\sin(\frac{\pi d}{\lambda}\cos\theta)}$

Expanding into series form, $\tan\phi = \dfrac{\frac{\pi d}{\lambda}\sin\theta - (\frac{\pi d}{\lambda})^3 \frac{\sin^3\theta}{6} + \ldots}{\frac{\pi d}{\lambda}\cos\theta - (\frac{\pi d}{\lambda})^3 \frac{\cos^3\theta}{6} + \ldots}$

Neglecting second order terms, $\tan\phi = \tan\theta \dfrac{1 - (\frac{\pi d}{\lambda})^2 \frac{\sin^2\theta}{6}}{1 - (\frac{\pi d}{\lambda})^2 \frac{\cos^2\theta}{6}}$

$(\frac{\pi d}{\lambda})^2 \frac{\cos^2\theta}{6}$ is less than 1 for values of $\frac{d}{\lambda} < .75$.

Therefore we may write

$$\tan\phi \simeq \tan\theta \left[1 - (\frac{\pi d}{\lambda})^2 \frac{\sin^2\theta}{6}\right]\left[1 + (\frac{\pi d}{\lambda})^2 \frac{\cos^2\theta}{6}\right]$$

$$\tan\phi = \left[1 + (\frac{\pi d}{\lambda})^2 \frac{\cos^2\theta}{6} - (\frac{\pi d}{\lambda})^2 \frac{\sin^2\theta}{6} - (\frac{\pi d}{\lambda})^4 \frac{\sin^2\theta \cos^2\theta}{36}\right]$$

Again neglecting second order terms,

$$\tan\phi = \tan\theta \left(1 + (\frac{\pi d}{\lambda})^2 \frac{\cos 2\theta}{6}\right)$$

$$= \tan\theta + (\frac{\pi d}{\lambda})^2 \frac{\cos 2\theta \sin 2\theta}{12\cos^2\theta}$$

$$\therefore \tan\phi = \tan\theta + (\frac{\pi d}{\lambda})^2 \frac{\sin 4\theta}{24\cos^2\theta} \qquad (2)$$

Comparing coefficients of (1) and (2),

$$K_1 = \left(\frac{\pi d}{\lambda}\right)^2 /24 \text{ radians}$$

$$= 23.6 \left(\frac{d}{\lambda}\right)^2 \text{ degrees}$$

Approximations made in the analysis give too low a value of K_1. The empirical formula $K_1 = 29 \left(\frac{d}{\lambda}\right)^2$ degrees is accurate to within 10% up to a spacing of $.6\lambda$. At $d = .75\,\lambda$ the formula gives an error 25% low. The error curve due to spacing will then be approximately $29 \left(\frac{\pi d}{\lambda}\right)^2 \sin 4\theta$. This is shown graphically in figure 2.
When applied to waves arriving at an angle of elevation, α, the error should be multiplied by the factor $\cos \alpha$.

III. Error due to pickup of one pair of masts being different from that of the other pair.

This error may be caused by cable mismatching on one pair or by difference in gain of one pair of base amplifiers. Let the pickup of the N-S pair be k times the pickup of the E-W pair. Then $\tan \phi = \frac{\tan \theta}{k}$ (3)

Assume the error is given by $\phi = \theta + K_2 \sin 2\theta$

$$\tan \phi = \frac{\tan \theta + \tan (K_2 \sin 2\theta)}{1 - \tan \theta \tan (K_2 \sin 2\theta)}$$

If K_2 is small, $\tan (K_2 \sin 2\theta) \approx K_2 \sin 2\theta$

$$\therefore \tan \phi = \frac{\tan \theta + K_2 \sin 2\theta}{1 - K_2 \tan \theta \sin 2\theta}$$

$$= \frac{\sin \theta + K_2 \sin 2\theta \cos \theta}{\cos \theta - K_2 \sin \theta \sin 2\theta}$$

$$\tan \phi = \frac{\sin \theta + 2K_2 \sin \theta \cos^2 \theta}{\cos \theta - 2K_2 \sin^2 \theta \cos \theta}$$

$$= \tan \theta \cdot \frac{1 + 2K_2 \cos^2 \theta}{1 - 2K_2 \sin^2 \theta} \qquad (4)$$

Fitting the curve at the point of maximum error, e.g. $\theta = 45^\circ$,

$$\left.\begin{aligned} \text{from (3), } \tan\phi &= \frac{1}{k} \\ \text{and from (4), } \tan\phi &= \frac{1+K_2}{1-K_2} \end{aligned}\right\} \text{at } \theta = 45^\circ$$

$$\therefore \frac{1}{k} = \frac{1+K_2}{1-K_2} \quad \text{or} \quad 1 - K_2 = k + kK_2$$

$$K_2 = \frac{1-k}{1+k} \text{ radians}$$

$$K_2 = \frac{1-k}{1+k}\,\frac{180}{\pi} \text{ degrees}$$

Therefore the error due to lack of gain on one pair is

$$\frac{1-k}{1+k}\,\frac{180}{\pi} \sin 2\theta \text{ degrees}$$

as shown in figure (3).

IV. Error due to phase shift between outputs of the two pairs of masts.

A phase displacement between E_{ns} and E_{ew} as they are applied to the direction finder causes a resultant field elliptical in form. If the major axis of the ellipse is taken as the observed bearing, an error is introduced which depends on the phase relation between E_{ns} and E_{ew} and also on their relative amplitudes.

Sinusoidal fields applied at right angles produce a resultant field elliptical in shape. Let $E_{ns} = E_2 \sin wt$

$$\text{and } E_{ew} = E_1 \sin(wt + \psi),$$

where ψ is the time phase displacement.

Refer to figure (4). The equation of the ellipse referred to the X' Y' axes is $\dfrac{X'^2}{a^2} + \dfrac{Y'^2}{b^2} = 1.$ Shifting the co-ordinate axes by $-(\frac{\pi}{2} - \phi)$

and referring to the XY axes, coaxial with the applied fields, we have

$$\frac{\left[X\cos(\frac{\pi}{2}-\phi) + y\sin(\frac{\pi}{2}-\phi)\right]^2}{a^2} + \frac{\left[y\cos(\frac{\pi}{2}-\phi) - X\sin(\frac{\pi}{2}-\phi)\right]^2}{b^2} = 1$$

$$\text{or} \quad \frac{(X \sin \phi + y \cos \phi)^2}{a^2} + \frac{(y \sin \phi - x \cos \phi)^2}{b^2} = 1$$

Now $x = E_1 \sin wt$ i.e. $\sin wt = \frac{x}{E_1}$

$$y = E_2 \sin (wt + \psi)$$

$$= E_2 (\sin wt \cos \psi + \cos wt \sin \psi)$$

$$= E_2 (\sin wt \cos \psi + \sqrt{1 - \sin^2 wt} \sin \psi)$$

$$= E_2 (\frac{x}{E_1} \cos \psi + \sqrt{1 - \frac{x^2}{E^2}} \sin \psi)$$

$$= \frac{E_2}{E_1} (x \cos \psi + \sqrt{E_1^2 - x^2} \sin \psi)$$

$$E_1 y = E_2 x \cos \psi + \sqrt{E_1{}^2 - x^2} \sin \psi)$$

$$(E_1 y - E_2 x \cos \psi)^2 = \sin^2 \psi (E_1{}^2 - x^2)$$

$$E_2{}^2 x^2 + E_1{}^2 y^2 - 2E_1E_2 xy \cos \psi = E_1{}^2 E_2{}^2 \sin^2 \psi \qquad (5)$$

For the ellipse,

$$b^2 (x \sin \phi + y \cos \phi)^2 + a^2 (y \sin \phi - x \cos \phi)^2 = a^2 b^2$$

$$(b^2 \sin^2 \phi)x^2 + (b^2 \cos^2 \phi)y^2 + (2b^2 \cos \phi \sin \phi)xy + (a^2 \sin^2 \phi)y^2$$
$$+ (a^2 \cos^2 \phi)x^2 - (2a^2 \sin \phi \cos \phi)xy = a^2 b^2$$

$$x^2 (b^2 \sin^2 \phi + a^2 \cos^2 \phi) + y^2 (a^2 \sin^2 \phi + b^2 \cos^2 \phi)$$
$$+ 2xy (b^2 - a^2) \sin \phi \cos \phi = a^2 b^2 \qquad (6)$$

Equate coefficients of (5) and (6) :

$$\frac{b^2 \sin^2 \phi + a^2 \cos^2 \phi}{a^2 b^2} = \frac{E_2{}^2}{E_1{}^2 E_2{}^2 \sin^2 \psi}$$

$$\text{or} \quad \frac{\sin^2 \phi}{a^2} + \frac{\cos^2 \phi}{b^2} = \frac{1}{E_1{}^2 \sin^2 \psi} \qquad (7)$$

$$\frac{a^2 \sin^2 \phi + b^2 \cos^2 \phi}{a^2 b^2} = \frac{E_1{}^2}{E_1{}^2 E_2{}^2 \sin^2 \psi}$$

$$\text{or} \quad \frac{\sin^2 \phi}{b^2} + \frac{\cos^2 \phi}{a^2} = \frac{1}{E_2{}^2 \sin^2 \psi} \qquad (8)$$

$$\frac{b^2 - a^2}{a^2 \; b^2} \sin \phi \cos \phi = - \frac{E_1 E_2 \cos \psi}{E_1{}^2 E_2{}^2 \sin^2 \psi}$$

$$\text{or} \quad \cos \phi \sin \phi \left(\frac{1}{a^2} - \frac{1}{b^2} \right) = - \frac{\cos \psi}{E_1 E_2 \sin^2 \psi} \qquad (9)$$

Using the property of the ellipse in which tangents which are perpendicular intersect on a circle of radius $\sqrt{a^2 + b^2}$, we have

$$E_1{}^2 + E_2{}^2 = a^2 + b^2 \qquad (10)$$

Add (7) and (8),

$$\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{\sin^2 \psi} \left(\frac{1}{E_1{}^2} + \frac{1}{E_2{}^2} \right)$$

From (10), $\dfrac{E_1{}^2 + E_2{}^2}{a^2 \; b^2} = \dfrac{1}{\sin^2 \psi} \; \dfrac{E_1{}^2 + E_2{}^2}{E_1{}^2 \; E_2{}^2}$

$$\therefore \quad a \, b = E_1 \, E_2 \sin \psi \qquad (11)$$

Using (10) and (11) solve for a and b :

$$a^2 + 2ab + b^2 = E_1{}^2 + E_2{}^2 + 2E_1E_2 \sin \psi$$

$$a + b = \sqrt{E_1{}^2 + E_2{}^2 + 2E_1E_2 \sin \psi} = \sqrt{A}$$

$$a^2 - 2ab + b^2 = E_1{}^2 + E_2{}^2 - 2E_1E_2 \sin \psi$$

$$a - b = \sqrt{E_1{}^2 + E_2{}^2 - 2E_1E_2 \sin \psi} = \sqrt{B}$$

Therefore $a = \frac{1}{2} (\sqrt{A} + \sqrt{B})$ = semi-major axis of ellipse,

and $b = \frac{1}{2} (\sqrt{A} - \sqrt{B})$ = semi-minor axis of ellipse.

The ratio of major to minor axes $= \dfrac{a}{b} = \dfrac{\sqrt{A} + \sqrt{B}}{\sqrt{A} - \sqrt{B}} = \dfrac{A + B + 2\sqrt{AB}}{A - B}$

Therefore by substitution,

$$\frac{a}{b} = \frac{2E_1{}^2 + 2E_2{}^2 + 2\sqrt{(E_1{}^2 + E_2{}^2)^2 - 4 \sin^2 \psi \; E_1{}^2 E_2{}^2}}{4 \, E_1 \, E_2 \sin \psi}$$

But $\tan \theta = \dfrac{E_1}{E_2}$ where θ is the true bearing,

$$\text{so} \quad \frac{a}{b} = \frac{1}{2 \sin \psi} \left[\tan \theta + \frac{1}{\tan \theta} + \sqrt{\left(\tan \theta + \frac{1}{\tan \theta}\right)^2 - 4 \sin^2 \psi} \right]$$

But $\tan \theta + \frac{1}{\tan \theta} = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}$

$$= \frac{2}{\sin 2 \theta}$$

Therefore $\frac{a}{b} = \frac{1}{2 \sin \psi} \left[\frac{2}{\sin 2 \theta} + \sqrt{\left(\frac{2}{\sin 2 \theta}\right)^2 - 4 \sin^2 \psi} \right]$

$$\frac{a}{b} = \frac{1}{\sin \psi \quad \sin 2 \theta} \left[1 + \sqrt{1 - \sin^2 \psi \quad \sin^2 2 \theta} \right]$$

At $\theta = 45^\circ$, $\frac{a}{b} = \frac{1}{\sin \psi} \left[1 + \sqrt{1 - \sin^2 \psi} \right] = \frac{1}{\sin \psi} \quad (1 + \cos \psi)$

This is the minimum ratio of major to minor axes, and at other bearings the ratio will be larger, reaching infinity at $\theta = 0, 90^\circ, 180^\circ$, and 270°.

Subtracting (8) from (7),

$$\frac{\cos^2 \phi - \sin^2 \phi}{b^2} + \frac{\sin^2 \phi - \cos^2 \phi}{a^2} = \left(\frac{1}{E_1{}^2} - \frac{1}{E_2{}^2} \right) \frac{1}{\sin^2 \psi}.$$

$$\cos 2 \phi \left[\frac{1}{b^2} - \frac{1}{a^2} \right] = \frac{1}{\sin^2 \psi} \left[\frac{1}{E_1{}^2} - \frac{1}{E_2{}^2} \right] \tag{12}$$

From (9) $\frac{\sin 2 \phi}{2} \left[\frac{1}{a^2} - \frac{1}{b^2} \right] = - \frac{\cos \psi}{E_1 E_2 \sin^2 \psi}$ (13)

Divide (13) by (12) :

$$\frac{\tan 2 \phi}{2} = \frac{\cos \psi}{E_1 E_2 \left(\frac{1}{E_1{}^2} - \frac{1}{E_2{}^2} \right)} = \frac{E_1 E_2 \cos \psi}{E_2{}^2 - E_1{}^2}$$

Then $\tan 2 \phi = 2 \cos \psi \left(\frac{1}{\frac{E_2}{E_1} - \frac{E_1}{E_2}} \right) = 2 \cos \psi \left(\frac{1}{\frac{1}{\tan \theta} - \tan \theta} \right)$

$$= \cos \psi \frac{2 \tan \theta}{1 - \tan^2 \theta} = \cos \psi \tan 2 \theta \tag{14}$$

From (14) it can be seen that when θ is $0^\circ, 45^\circ, 90^\circ, 135^\circ$, etc., the observed bearing $\phi = \theta$, but at other angles there is an error which is octantal. Assume the error is sinusoidal, then $\phi = \theta + K_4 \sin 4 \theta$ where K_4 is the maximum error, occurring at $\theta = 22\frac{1}{2}^\circ, 67\frac{1}{2}^\circ$, etc.

Now we have $\tan 2 \phi = \tan (2 \theta + 2 K_4 \sin 4 \theta)$

If K_4 is small $2K_4 \sin 4\theta \approx \tan(2K_4 \sin 4\theta)$ and

$$\tan 2\phi = \frac{\tan 2\theta + 2K_4 \sin 4\theta}{1 - \tan 2\theta \, 2K_4 \sin 4\theta}$$

From (14)

$$\cos\psi \tan 2\theta = \frac{\tan 2\theta + 2K_4 \sin 4\theta}{1 - 2K_4 \sin 4\theta \tan 2\theta}$$

$$\cos\psi \sin 2\theta = \frac{\sin 2\theta + 4K_4 \sin 2\theta \cos^2 2\theta}{1 - 4K_4 \sin^2 2\theta}$$

$$\cos\psi = \frac{1 + 4K_4 \cos^2 2\theta}{1 - 4K_4 \sin^2 2\theta}$$

The value of K_4 should be independent of θ in the approximate formula. Fit the curve at the point of maximum error,

i.e. at $\theta = \frac{\pi}{8}$, then $\cos\psi = \frac{1 + 4K_4(\frac{1}{2})}{1 - 4K_4(\frac{1}{2})} = \frac{1 + 2K_4}{1 - 2K_4}$

$$\cos\psi - 2K_4 \cos\psi = 1 + 2K_4$$

$$2K_4 = \frac{\cos\psi - 1}{\cos\psi + 1} = -\frac{1 - \cos\psi}{1 + \cos\psi}$$

$$K_4 = -\frac{1}{2}\,\frac{1 - \cos\psi}{1 + \cos\psi} \quad \text{radians}$$

The bearing error due to phase displacement is therefore

$$K_4 \sin 4\theta = -\frac{1}{2}\left(\frac{1 - \cos\psi}{1 + \cos\psi}\right) \sin 4\theta \quad \text{radians}$$

$$= -28.7\left(\frac{1 - \cos\psi}{1 + \cos\psi}\right) \sin 4\theta \quad \text{degrees}$$

The error curve is shown in figure 5.

V. Error due to output of one mast being different in amplitude to that of the other three.

Assume the output of the north mast is k times that of the other three. The vector diagram (figure 6) shows the phase and amplitude relations of E_{ns} and E_{ew}. It can be seen that errors will arise from two causes; the change of amplitude of E_{n-s}, and the shift of phase of E_{ns} with respect to E_{ew}.

$$E_{ns} = \sqrt{\left[(k+1)E\sin\left(\frac{\pi d}{\lambda}\cos\theta\right)\right]^2 + \left[(1-k)E\cos\left(\frac{\pi d}{\lambda}\cos\theta\right)\right]^2} \qquad (15)$$

$$E_{ew} = 2E\sin\frac{\pi d}{\lambda}\sin\theta$$

The phase displacement is given by

$$\tan\psi = \frac{(1-k)E\cos\left(\frac{\pi d}{\lambda}\cos\theta\right)}{(1+k)E\sin\left(\frac{\pi d}{\lambda}\cos\theta\right)} = \frac{1-k}{1+k}\cdot\cot\left(\frac{\pi d}{\lambda}\cos\theta\right)$$

When $\frac{\pi d}{\lambda}$ is small, $\tan\psi \simeq \frac{1-k}{1+k}\,\frac{\lambda}{\pi d\cos\theta}$ (16)

It can be seen from (16) that the phase shift varies with the bearing, θ, so that at $\theta = 0$, $\psi = \tan^{-1}\frac{1-k}{1+k}\,\frac{\lambda}{\pi d}$,

and at $\theta = 90^\circ$, $\psi = 90^\circ$.

This phase shift will contribute an error somewhat resembling that shown in figure (7).

The error due to amplitude difference will be zero at $\theta = 0^\circ, 90^\circ, 180^\circ$ and 270°, and is therefore quadrantal.

Neglecting the phase shift between E_{ns} and E_{ew} at $\theta = 45^\circ, 135^\circ, 225^\circ$ and 315° for practical values of $\frac{d}{\lambda}$ and k we have approximately

$$\tan\phi = \frac{E_{ew}}{E_{ns}} = \frac{2\frac{\pi d}{\lambda}\sin\frac{\pi}{4}}{(k+1)\frac{\pi d}{\lambda}\cos\frac{\pi}{4}} = \frac{2}{k+1}$$

i.e. $$\phi = \tan^{-1}\frac{2}{k+1}$$

Assume the error is approximately sinusoidal, then

$$\phi = \theta + K_5\sin 2\theta$$

$$\tan\phi = \tan\theta\cdot\frac{1+2K_5\cos^2\theta}{1-2K_5\sin^2\theta} \quad \text{as in (4).}$$

At $\theta = 45^\circ$ we then have

$$\frac{2}{k+1} \doteq \frac{1+K_5}{1-K_5}$$

$$2(1-K_5) = (1+K_5)(1+k)$$

$$K_5 = \frac{1-k}{3+k} \text{ radians } = \frac{180}{\pi}\;\frac{1-k}{3+k} \text{ degrees}$$

This gives an error curve resembling that shown in figure 8.

The combination of the error due to phase shift and that due to amplitude difference results in an error curve which is quadrantal but contains a relatively large octantal component of varying amplitude. See Figure 9.

The phase shift, ψ, produces an elliptical field whose ratio of major to minor axes varies with the bearing, θ, and also depends on k and $\frac{d}{\lambda}$. This ratio is a minimum at $\theta = 90^\circ$ and 270° and is infinite at $\theta = 0^\circ$, and 180° in the case analyzed. The minimum ratio is given by

$$R = \frac{E_{ew}}{E_{ns}} \quad (\text{at } \theta = 90)$$

$$\therefore \quad R = \frac{2\pi d}{\lambda(1-k)}$$

VI. <u>Error due to output of one mast being shifted in phase by an angle ψ</u>

For example let the output of the north mast be shifted in phase by an angle ψ so that it is lagging with respect to its correct phase. Referring to figure 10 we have

$$E_{n-s} = E_n - E_s$$

Assume E_{ns} is shifted in phase a negligible amount with respect to E_{ew} so that they may be considered in phase. Now,

$$E_{ew} = 2E \sin\left(\frac{\pi d}{\lambda} \sin\theta\right), \text{ and}$$

$$E_{ns} = 2E \sin\left(\frac{\pi d}{\lambda} \cos\theta\right) - \psi E \cos\left(\frac{\pi d}{\lambda} \cos\theta\right).$$

When $\frac{d}{\lambda}$ is small enough to neglect the effects of spacing error described in part II, we have the observed bearing ϕ given by

$$\tan \phi = \frac{E_{ew}}{E_{ns}} = \frac{\frac{\pi d}{\lambda} \sin \theta}{\frac{\pi d}{\lambda} \cos \theta - \frac{\psi}{2} \cos \left(\frac{\pi d}{\lambda} \cos \theta \right)}$$

The error curve given by this equation is shown in figure 11. The maximum error occurs at $\theta = 90^\circ$ and 270° in this case and its magnitude is given by

$$K_6 = \phi - \theta = \tan^{-1} \left(- \frac{2 \pi d}{\lambda \psi} \right) - 90^\circ$$

$$= \cot^{-1} \frac{2 \pi d}{\lambda \psi} \text{ radians}$$

$$= \frac{180}{\pi} \cot^{-1} \frac{2 \pi d}{\lambda \psi} \text{ degrees.}$$

VII. Conclusion

The array errors reviewed may occur singly or in combination. The array calibration curves will show the summation of these errors and the errors due to site and the direction finder. Site errors are generally irregular with respect to θ and frequency. Instrumental errors in the direction finder may be checked by reversing the feeder connections at the input. In any case these errors should be less than 4° and 1° respectively in the average H.F. D.F. installation.

Of the errors analized herein the simplest to separate out is the spacing error described in part II. This may be calculated if the spacing and frequency are known. Other errors must, in general, be isolated by adjustment and aligmment of the array. For instance that due to lack of gain on one pair of cables will be more or less independent of frequency if due to the base amplifier gain adjustments, but dependent on frequency if due to mismatching of the cables. Errors involving phase shift will vary with frequency on a given array. As an example, if a curve similar to that in figure 9 were obtained on calibration, inspection would reveal the presence of both quadrantal and octantal errors. With no other information this would be difficult to analyze, but if observations of the ellipticity of the trace on the C.R.D.F. are also taken, they will determine whether the octantal error is due to that described in part IV or that described in part V, or if to both, roughly what proportion is due to each.

OTTAWA

C.W. McLeish

November 1st, 1943.

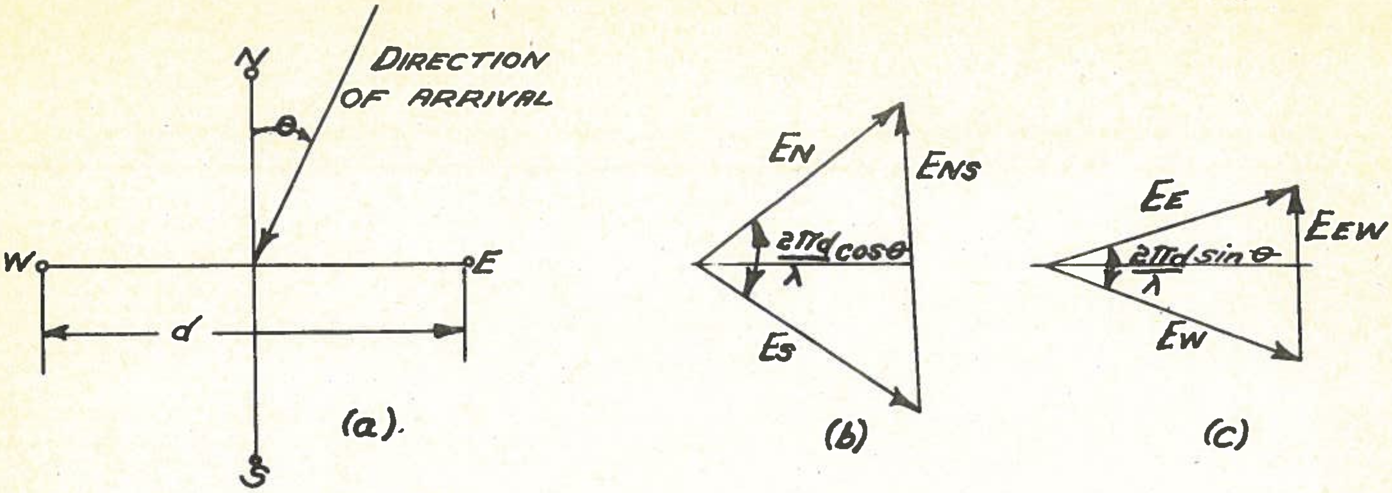

FIGURE ONE

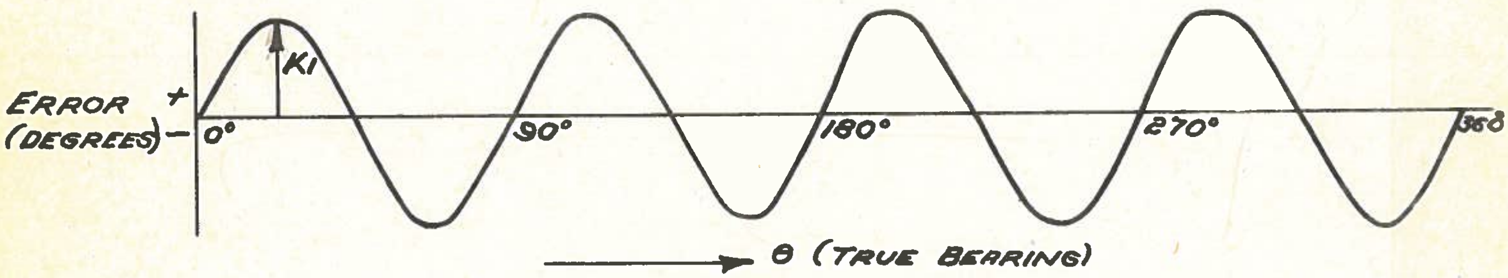

FIGURE TWO

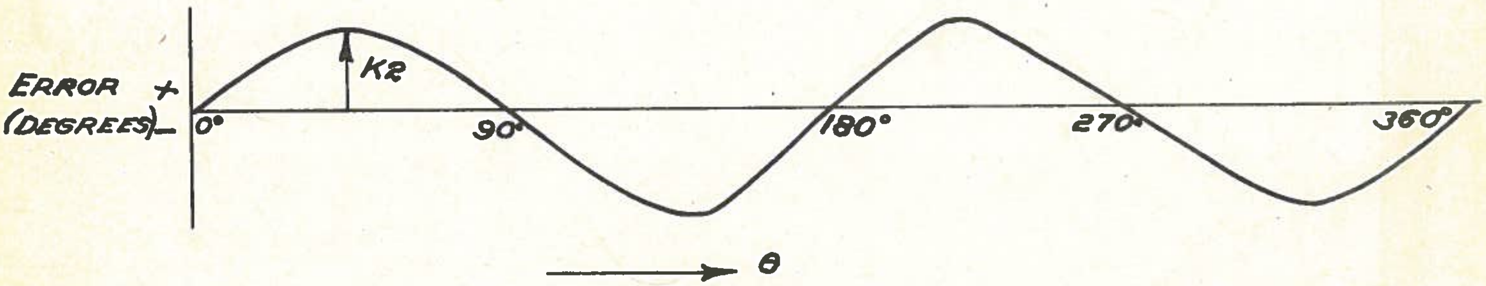

FIGURE ~~Two~~ Three

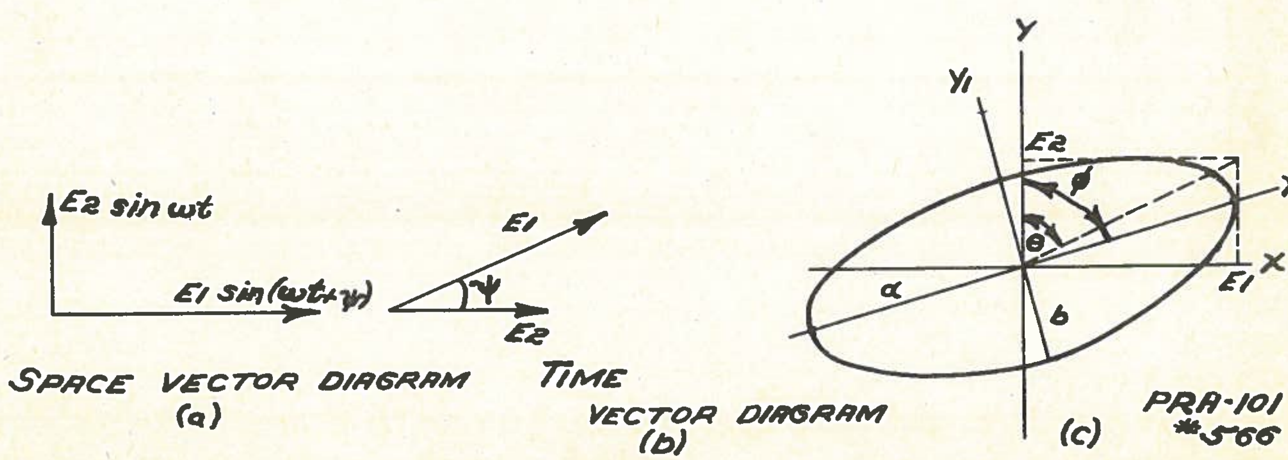

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FIGURE FOUR

RESULTANT FIELD

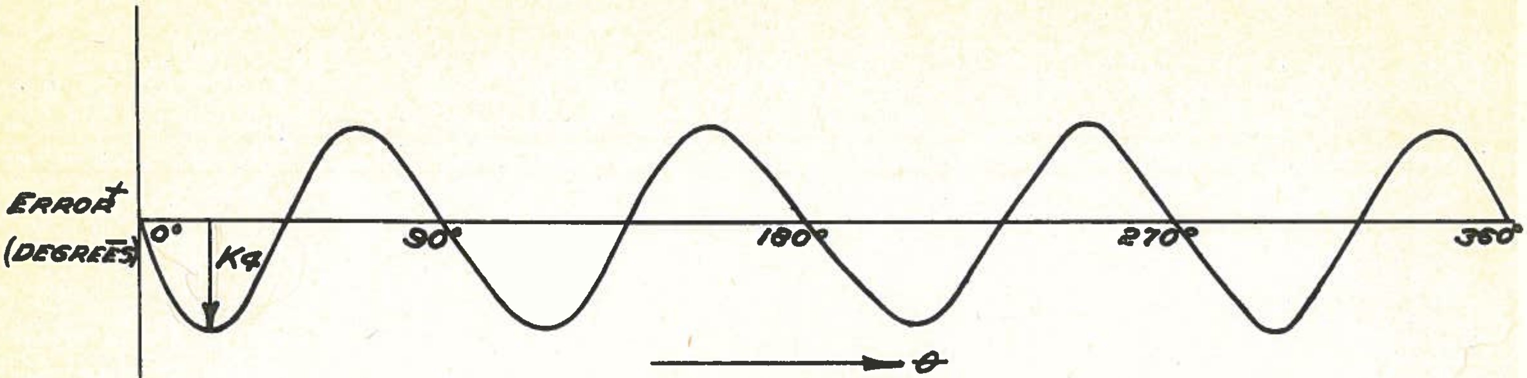

FIGURE FIVE

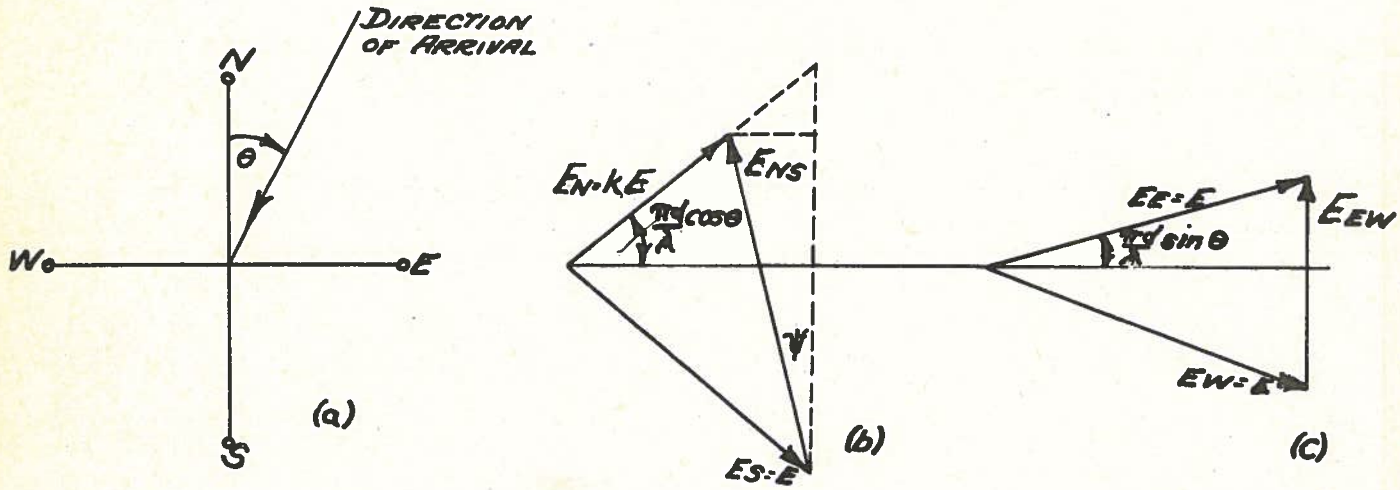

FIGURE SIX

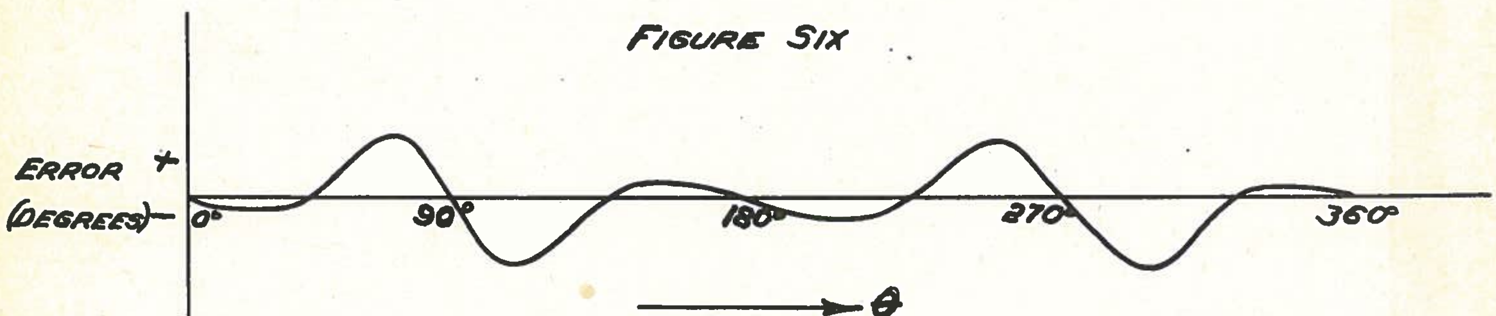

FIGURE SEVEN

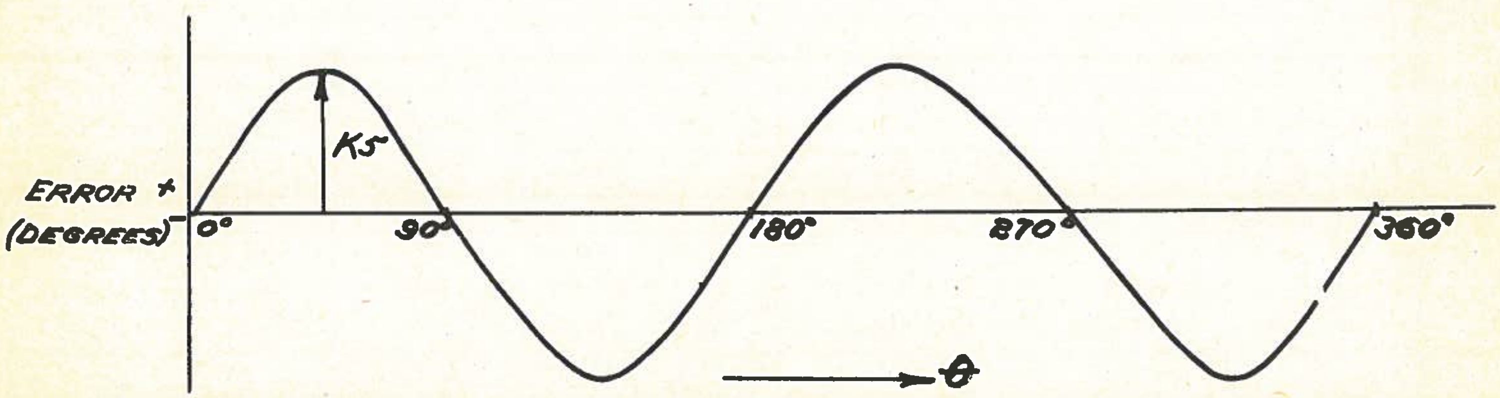

FIGURE EIGHT

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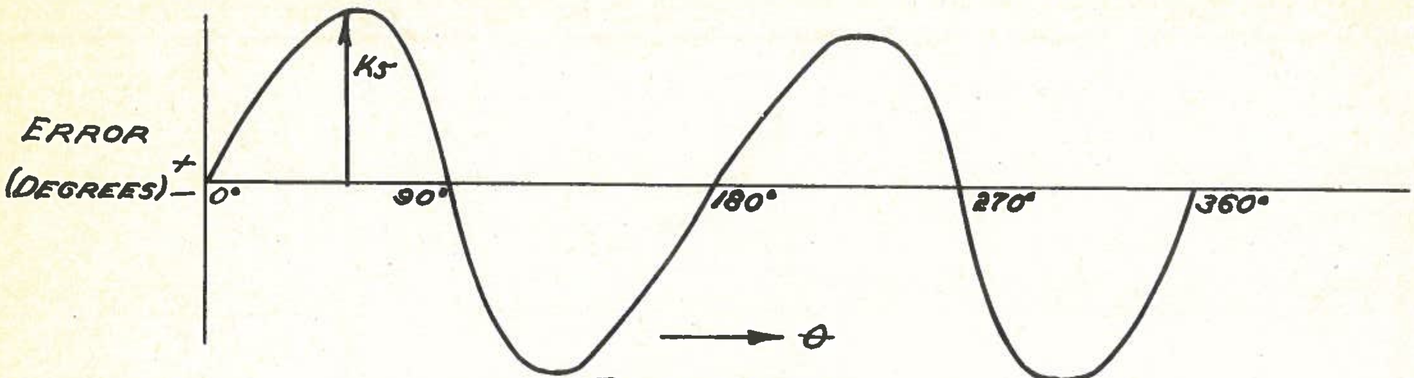

FIGURE NINE

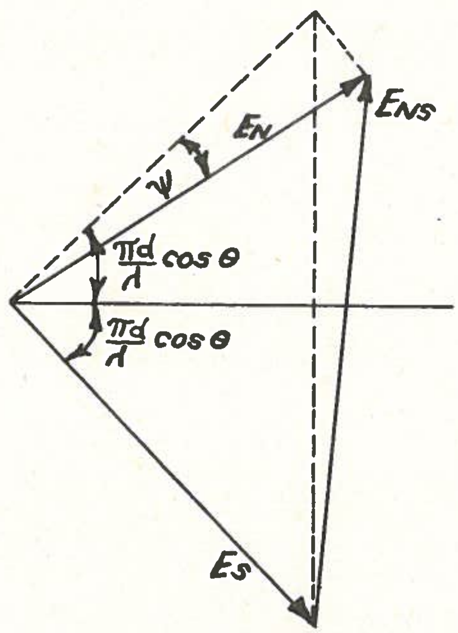

FIGURE TEN

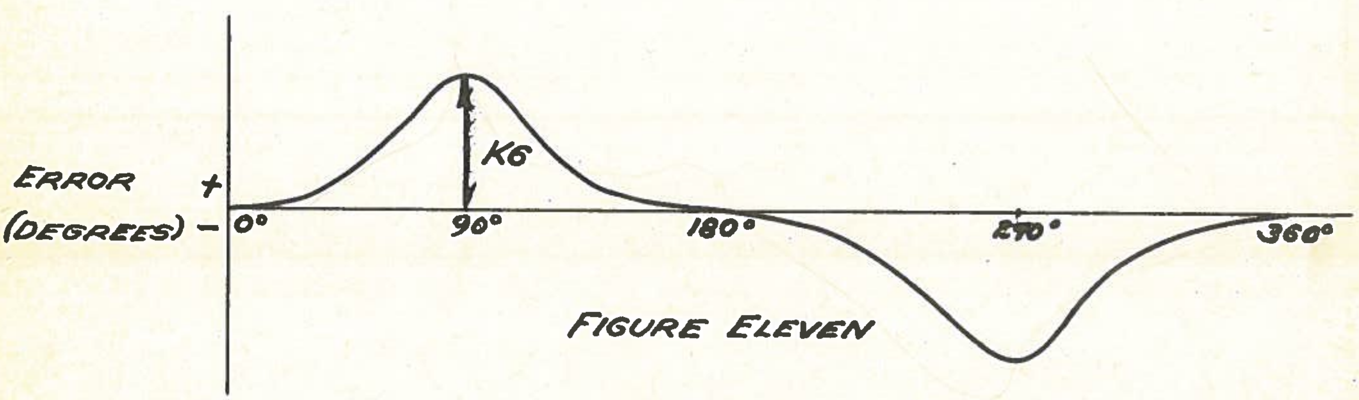

FIGURE ELEVEN

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