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PREFACE

This translation is the eighth from the Russian permafrost publication "Principles of Geocryology", Part II (Engineering Geocryology). The first translation in this group was Chapter I entitled, "Principal Aspects of Engineering Geocryology (Permafrost Studies)" by N.I. Saltykov (TT-1215). The second was Chapter VII "Particular Aspects of Mining in Thick Permafrost" by V.P. Bakakin (TT-1217). The third was Chapter II "Deformation of Structures Resulting from Freezing and Thawing" by A.I. Dement'ev (TT-1219). The fourth was Chapter VIII "Beds for Roads and Airfields" by G.V. Porkhaev and A.V. Sadovskii (TT-1220). The fifth was Chapter IX "Underground Utility Lines" by G.V. Porkhaev (TT-1221). The sixth was Chapter XI "Specific Features of the Maintenance of Structures in Permafrost Conditions" by A.I. Dement'ev (TT-1232). The seventh was Chapter III "Basic Mechanics of Freezing, Frozen and Thawing Soils" by N.A. Tsytovich et al. (TT-1239).

This translation of Chapter IV by G.V. Porkhaev discusses the thermal regime of the ground in permafrost regions in relation to engineering structures. The chapter begins with a review of the heat exchange between the ground and its surroundings prior to construction. This is followed by a description of the thermal properties of the ground and the possibility of controlling them. Factors affecting the ground thermal regime are also discussed. The chapter concludes with an account of currently available mathematical methods for analyzing the influence of engineering structures on the ground thermal regime in the permafrost region.

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Ottawa — R.F. Legget

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Title: Thermal physical principles of controlling the interaction between structures and frozen soils

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THERMAL PHYSICAL PRINCIPLES OF CONTROLLING THE INTERACTION BETWEEN STRUCTURES AND FROZEN SOILS

Introduction. 1. Heat exchange between the ground and its surroundings. 2. Thermal physical properties of the ground and the possibility of controlling them. 3. Natural thermal state of the ground and its variation. 4. Changes in the surface of interaction of the soil and its surroundings. 5. Interaction of various factors in the formation of the ground thermal regime. 6. Principal methods of thermal engineering calculations.

Introduction

It was shown in the previous chapter that the physico-mechanical properties of frozen soils (temporary strength, modulus of elasticity, creep, etc.) depend on temperature. Particularly acute changes take place in soil properties during the transition from the frozen to the thawed state. Therefore the stability of a structure can be ensured only by establishing a specific thermal regime in the foundation soils. Moreover, such specific effects as naled formation, heaving, fissuring, etc., resulting in deformation of structures and hindering their normal function, are likewise caused by thermal processes occurring in the soil. Finally, the thermal state of the soil affects the heat losses from the structure which has an effect on operating expenses. At times if the heat losses are unpredictably large a failure in the structure may occur (for example, freezing of the water pipes or sewage system, etc.). Thus, the establishment of the specific thermal regime in the soil cannot only decrease operation expenses and prevent deformation of the structure but in some cases is an absolutely necessary condition without which one cannot erect a structure on permafrost.

The thermal physics of frozen soil being a discipline studying the control of the thermal and moisture regimes of the soil, is closely involved with the mechanics of frozen, freezing and thawing soils. Problems of the mechanics of frozen soils cannot be solved without considering thermal physics since thermal physics here plays an auxiliary role. On the other hand the temperature field of the soil under the usual pressures does not depend on the stressed state which makes it possible to consider thermal physics independent of mechanics.

Thermal physical processes taking place during the freezing and thawing of soils are the result of a complex interaction of various aspects of the geological-geographical environment. These processes include heat and moisture exchange occurring within the soil itself and the heat and moisture exchange occurring within itself and the heat and moisture exchange occurring between the soil and the atmosphere. In engineering geocryology it is customary to consider the system soil-atmosphere-structure.

To obtain quantitative mechanisms in the form of mathematical relationships it becomes necessary to make diagrammatic representations of complex thermal physical processes occurring during the freezing and thawing of soil.

Heat may propagate in soil by conduction as well as by convective transfer by water migration or filtration. In the solutions of geocryological problems available at the present time it is customary to take into account only conductive heat transfer. This is due on the one hand to the complexity of obtaining solutions taking into account heat transfer by convection and on the other hand to the fact that heat propagation by conduction predominates in the overwhelming majority of cases encountered in construction practice on permafrost. Filtration may be of great importance in the thermal regime of soil but it is eliminated, as a rule, by construction practices (provisions for clay seals, belts of frozen soil, etc.). The migration of water, because of its slow rate, usually has little effect on the temperature regime of the soil. Taking migration into consideration becomes necessary for calculations of changes in soil volume on heaving. Thus the determination of the temperature field in the soil by taking into account heat transfer through conduction only is in many practical cases sufficiently accurate. Exceptions include practices of thawing coarse-grained soils by jet filtration, drainage filtration and other methods where most of the heat is transferred by convective heat exchange (Chapter V). The mechanism of convective heat exchange for this case is still in the process of being derived and this question is not considered in this chapter.

Regardless of the fact that the calculation methods are only approximate, the possibilities derived on the basis of these formulae are not yet being used to the full since the initial data has to be selected very arbitrarily. At the present time there are no sufficiently complete and accurate tables of the thermal physical characteristics of soils. The most important defect is that only a very approximate consideration is given to boundary conditions or, in other words, conditions of heat exchange in the system soil-structure-atmosphere. As a rule, in the many formulae intended for determining the temperature field of the soil, only air temperature of all the external factors affecting the heat processes is considered. Therefore, using the existing formulae with the same thermal physical properties of the soils, the same temperature field is obtained for the foundations of buildings with ventilated crawl space, under road pavement, under natural cover, etc.

Attempts to introduce into the formula the surface temperature of the soil or the temperature of the soil at some depth instead of air temperature is no solution to the problem. This temperature for each particular case will be determined by heat exchange conditions in the system soil-structure-atmosphere and the thermal physical properties of the soil.

This problem can be solved successfully only by considering jointly the processes of heat transfer in the soil and the processes of heat exchange between the soil and its surroundings as a single system soil-structure-atmosphere.

The temperature field of any system is determined not only by the mechanism of heat transfer but also by the following factors:

(1) conditions of heat exchange with the surrounding medium;

(2) thermal physical characteristics;

(3) the quantity of heat and its distribution throughout the system before the beginning of the process;

(4) the shape and dimensions of the system as a whole as well as its separate parts.

These factors formulated mathematically comprise unique conditions which, along with differential equations indicating the mechanism of the heat transfer, make it possible to obtain a solution describing the temperature field in a given system.

By changing the conditions of heat exchange between the soil and its surroundings, its thermal physical characteristics, the initial thermal state, and the shape and dimensions of the surface of interaction, one can control the thermal regime of the soil, i.e. its temperature field and consequently its thawing and freezing.

1. Heat Exchange Between the Soil and Its Surroundings

(a) Contact layer

The temperature field of any solid body placed in a liquid or gaseous medium is formed under the influence of two different types of processes. The first type includes processes of energy exchange between the solid and the surrounding medium accompanied by the conversion of one form of energy into another (for example, radiant energy is converted into heat energy and vice versa); the second type includes processes of heat propagation to the solid from the surrounding medium. Processes of the second type take place within the body itself whereas processes of the first type occur in a layer at the boundary between the body and the surrounding medium. The dimensions of this layer depend on the composition of the body as well as the composition of the medium. As a result of processes occurring in this layer there is a flow of heat from the boundary into the body (or vice versa) and the value of this flow is determined not only by processes occurring in the surface layer but by the thermal state and properties of the body. Herein is the connection between the two types of processes.

These two types of processes occur also in the formation of the temperature field of the soil. This was mentioned by A.V. Voeikov: "The upper layer of the earth's crust which receives heat directly from the sun and transmits it by radiation into interplanetary space or clouds I call the external active layer" (cited from a publication in 1952, page 186); "the upper surface of the earth's crust receives heat from solar radiation and since the land surface is not transparent to heat it undergoes a considerable temperature increase if the solar rays strike at a large angle and the air is sufficiently transparent to heat. Hence heat propagates into the earth mainly by conduction and into the air by conduction and radiation" (ibid, page 202).

Thus in the "active layer", according to Voeikov, energy is exchanged between the soil and its surroundings; further energy propagation into the soil in the form of heat occurs by conduction. Since in permafrost studies the term "active layer" has a completely different meaning (Sumgin et al., 1940) we shall henceforth refer to Voeikov's "active layer" as "contact layer". The term "contact layer" suggested by N.S. Ivanov is more appropriate than terms suggested by other authors (active layer, forming layer, surface layer, etc.).

In the contact layer which is formed at the interface between the soil and its surroundings (atmosphere, structure, etc.) the following processes occur:

(1) energy exchange in the form of heat not accompanied by changes in the mechanism of transfer (heat transfer by conduction in the contact layer and in the surrounding medium);

(2) energy exchange in the form of heat accompanied by changes in the transfer mechanism; heat transferred by convection in the surrounding medium is transferred through the contact layer of the soil and further by conduction;

(3) conversion of various forms of energy into heat, for example the conversion of radiant, chemical and other forms of energy;

(4) conversion of energy not accompanied by conversion to heat energy (photosynthesis);

(5) phase transformations of matter (water) accompanied by the liberation or absorption of heat (condensation and evaporation of water, melting and evaporation of snow, formation of hoarfrost, etc.).

Thus processes occurring in the contact layer are very complex and their intensity may vary over a wide range. For example over motor road with an asphalt surface, evaporation and condensation are almost absent, whereas on a moss covered bog the heat of evaporation and condensation is an important component of the heat exchange.

A characteristic feature of processes occurring in the contact layer is their direct relationship to processes occurring in the surrounding medium and any change in the latter has an effect almost immediately on the trend and intensity of these processes. For example, the absence of solar energy at night brings about condensation which is replaced during the day by evaporation. In comparison with the underlying soil the properties of the contact layer are incomparably more variable. The occurrence of precipitation in the form of rain or snow immediately changes the properties of the contact layer. There are also changes in the properties and processes occurring in the contact layer during the course of the year, resulting from changes in vegetation, freezing and thawing of this layer, etc.

The conversion of energy and direct heat and moisture exchange with the surrounding medium, which are dominant in the contact layer, are rapidly attenuated with distance from the interface and undergo a transition to heat propagation within the soil.

The depth of penetration of radiant energy, convective heat exchange, evaporation, etc., are different. Therefore, the lower boundary of the contact layer will be determined by those processes which penetrate deepest into the soil. There is no clear boundary between the contact layer and the underlying soil, since the attenuation of processes in the contact layer occurs gradually. However, in most cases this boundary can be determined with required practical precision. For example, in the case of grass cover the mineral soil surface under the sod can be taken as this lower boundary.

In this case the soil in the sod layer will comprise part of the contact layer. If the surface is devoid of any cover, the contact layer is the upper layer of soil where conversion of one form of energy into another occurs. Therefore, in our further discussion, in many cases the division between soil and contact layer is quite conventional (with respect to types of processes).

It is more difficult to define the upper boundary of the contact layer. In the first approximation one can assume that it is the surface evenly encompassing all irregularities of the microrelief and vegetation (a small plot not exceeding several hundred square metres is being considered here).

As applied to engineering structures the contact layer will in most cases be very small and the transformation of energy within it will be negligible. For example, for a concrete cover the exchange of energy with the surrounding medium will take place in a layer having a depth of several millimetres. This, however, does not mean that in investigating heat exchange one can neglect the physical existence of processes occurring in the contact layer, since its role in the formation of the thermal regime of the soil does not decrease because of this.

Thus, in the formation of the temperature field there is either the flow of energy from the surrounding medium into the contact layer, its subsequent conversion into heat energy and then propagation of the heat into the soil, or the flow of heat from the soil to the contact layer, partial transformation of it into other forms of energy and transfer of it to the surrounding medium.

The simplest way of controlling the thermal regime of the soil is by changing the properties of the contact layer and controlling processes occurring in it. Herein is the great importance of the contact layer for engineering geocryology.

(b) Equating the energy balance in the soil and contact layer

As mentioned above, energy is transformed from one type to another in the contact layer. Therefore from a thermal physical point of view the fundamental aspect of studying the contact layer is the investigation of the energy balance in it, since only then can one make a quantitative comparison of qualitatively different forms of energy (radiant energy, heat energy, etc.) by reducing them to one form of energy. i.e. heat energy. Therefore the basic thermal physical characteristics of the contact layer is the quantity of heat; temperature is used as a supplementary characteristic.

Before dealing with the energy balance in the contact layer we will consider possible methods of defining the energy balance for purposes of engineering geocryology.

The law of the conservation of energy for any body can be written in the form

$$\int_S q_f dS = \int_V \frac{\partial q}{\partial t}\, dV. \tag{4.1}$$

The integral on the left side of this equation represents the total flow of energy through the surface S, limiting the volume V, in a unit time, and the inegral on the right side of the equation indicates the change in the internal energy of the body with volume V in a unit of time. q_f is the heat flow through a unit area perpendicular to the surface S;

$\frac{\partial q}{\partial t}$ - change in the specific energy in a volume dV.

In most cases the equation for the conservation of energy is taken for a certain period of time. Then the law for the conservation of energy is expressed by the equation

$$\int_{t_1}^{t_2} dt \int_S q_f dS = \int_{t_1}^{t_2} dt \int_V \frac{\partial q}{\partial t} dV. \tag{4.2}$$

For the volume V in equations (4.1) and (4.2) one can take:

(a) the entire volume of soil under investigation excluding the contact layer;

(b) the entire volume of soil under investigation including the contact layer;

(c) only the contact layer.

Each of these variants has its advantages and disadvantages.

This can be illustrated by a specific example. Let us assume that we are interested in the temperature regime of soil in some plot under natural conditions to a depth of h.

Let us construct the equation for the conservation of energy for the soil without the contact layer,

$$\int_{S_1} q_{1f} dS + \int_{S_2} q_{2f} dS + \int_{S_3} q_{3f} dS = \int_V \frac{\partial q}{\partial t} dV. \tag{4.3}$$

In equation (4.3) the first term on the left side represents the heat flow from the contact layer through the upper surface of soil. If it is assumed that heat propagation in soil takes place only by conduction, this flow of heat will be equal to λ grad ϑ_1, where grad ϑ_1 is the temperature gradient at the lower surface of the contact layer.

The second term of the left side of the equation expresses the heat flow through the lower surface of a soil column from the lower lying layers of soil and is equal to λ grad ϑ_2, where ϑ_2 is the temperature gradient at the lower surface of the soil column.

If the plot has an even surface, and if the soil and vegetation cover are sufficiently uniform, there would be practically no heat flow in a horizontal direction. Thus, instead of considering the entire volume of soil within the limits of the plots one can take a column of soil of height h with an area equal to a square unit. Since the thermal state of such a column of soil would not differ in any manner from any other column within the limits of the plot, it is sufficient to consider only one such column in order to obtain the characteristic of change in heat content of the entire volume of soil in the plot. The third term of equation (4.3) is equal to zero because of the assumed conditions (even surface of plot and uniform soil). Finally, the right side of the equation represents the change in the heat content of the soil column.

For a soil column $S_1 = S_2 = 1$ and the flow q_{1f} and q_{2f} are constant along the surfaces S_1; $S_2 q_{3f} = 0$.

Taking into account all that has been said above, equation (4.3) can be written in the form

$$\lambda \operatorname{grad} \vartheta_1 - \lambda \operatorname{grad} \vartheta_2 = \int_0^h c_{ef}(\vartheta)\gamma \frac{\partial \vartheta}{\partial t}\, dh, \tag{4.4}$$

where $c_{ef}(\vartheta)$ is the effective heat capacity by means of which the heat of phase transformation of water is taken into account (Part I, Chapter VI). If one proceeds from equation (4.2) in deriving the energy balance for the column of soil under consideration, equation (4.4) changes to

$$\int_{t_1}^{t_2} \lambda \operatorname{grad} \vartheta_1 dt - \int_{t_1}^{t_2} \lambda \operatorname{grad} \vartheta_2 dt = \int_{t_1}^{t_2} dt \int_0^h c_{ef}(\vartheta)\frac{\partial \vartheta}{\partial t}\, dh. \tag{4.5}$$

The change in heat content of the entire column of soil can be established from either the left or right side of equation (4.5), but one cannot determine the temperature field of the soil and consequently the position of the boundary of its thawed or frozen state which is of primary interest for engineering geocryology. The solution to these questions can be obtained only if one considers the simple heat balance within the body, i.e. if one uses differential equations of heat conductivity.

Let us assume that the height of the soil column h is equal to the depth to which seasonal variations in temperature penetrate. Then the second term of the left side of equation (4.5) can be neglected in many cases. For natural conditions (without artificial heating or cooling) for a period of time equal to one year one can write

$$\int_{t_1}^{t_2} \lambda \operatorname{grad} \vartheta_1 dt = \int_{t_1}^{t_2} dt \int_0^h c_{ef}(\vartheta)\gamma \frac{\partial \vartheta}{\partial t}\, dh = 0, \tag{4.6}$$

since the change in heat content of the column for a year is almost equal to zero. This is explained by the fact that during the warm period of the year the heat flows into the soil, and during the cold period of the year the heat flows out of the soil. The difference between these values in many cases is less than the inflow or outflow of heat and with modern methods of measurement the inflow and outflow of heat is less than the accuracy of measurements.

Let us write equation (4.6) in the form

$$\int_{t_1}^{t^*} \lambda \operatorname{grad}\vartheta_1 dt + \int_{t^*}^{t_2} \lambda \operatorname{grad}\vartheta_1 dt = \int_{t_1}^{t^*} dt \int_0^h c_{ef}(\vartheta)\gamma \frac{\partial\vartheta}{\partial t} dh + \int_{t^*}^{t_2} dt \int_0^h c_{ef}(\vartheta)\gamma \frac{\partial\vartheta}{\partial t} dh,$$

where t^* is a moment of time when the average heat flow through the surface S changes direction.

The quantity

$$Q_T = \int_{t_1}^{t^*} \lambda \operatorname{grad}\vartheta_1 dt = - \int_{t^*}^{t_2} \lambda \operatorname{grad}\vartheta_1 dt = \int_{t_1}^{t^*} dt \int_0^h c_{ef}(\vartheta)\gamma \frac{\partial\vartheta}{\partial t} dh = - \int_{t^*}^{t_2} dt \int_0^h c_{ef}(\vartheta)\gamma \frac{\partial\vartheta}{\partial t} dh \tag{4.7}$$

is customarily called the annual heat cycle of the soil. It is usually determined by changes in the heat content of the soil during the period of heating or cooling since it is exceedingly difficult to determine it from the amount of heat flowing through the surface because of the daily variation.

Similarly one can introduce the concept of diurnal heat cycle.

Heat cycles can be used for characterizing the intensity of thermal processes under natural conditions for agronomic purposes, etc.

The energy balance can also be derived for the entire volume of the soil under investigation including the contact layer. However, in this case difficulties are encountered making it necessary to approach the solution of the problem in a somewhat different way. In the contact layer, as was noted above, there are very complex processes differing from those occurring in the soil itself. The mechanism of processes occurring in the contact layer are just being established at the present time and a quantitative solution, particularly for engineering purposes, can be obtained experimentally, whereas the temperature field of the soil in its interaction with the structure is determined primarily by analytical means. It is therefore advisable to consider thermal processes in the soil and the transformation of various forms of energy in the contact layer separately and the connection between them is reflected by the heat flow at the boundary between these two media.

Thus the heat flow through the contact layer will be the boundary condition for the remaining volume of soil and hence one can obtain a solution for the temperature field of the soil in a much simpler form. The separate consideration of processes of energy exchange in the contact layer and the heat transfer within the soil facilitates the elucidation of quantitative and qualitative mechanisms, and makes it possible to obtain finite results in a very simple form. Nevertheless, the mechanisms of change in heat flow at the boundary between the contact layer and the soil can be elucidated only with a joint consideration of the energy balance of the contact layer and equations of heat conductivity for the soil, since the heat flow in the soil depends not only on the energy exchange with the surrounding medium but also on the thermal physical properties of the soil itself.

The periods of time for which an energy balance should be constructed for the contact layer depend on processes forming the temperature regime of the soil.

Industrial development and the construction of various types of buildings results in changes in individual components of the heat exchange between the atmosphere and the soil, which in turn brings about a change in the natural temperature regime of the soil. At first the formation of a new temperature regime of the soil is characterized by a clearly evident instability of the thermal state. This process finally tends towards a new state of dynamic equilibrium similar to steady state thermal processes.

Depending on the type of structure, its interaction with the soil may result in steady state thermal processes in which the heat flow will be unidirectional, or it may result in periodic steady state thermal processes, when from season to season there will be a repetition of the same processes which are completely alike quantitatively as well as qualitatively.

Steady state thermal processes in which the heat flows in one direction correspond to relatively stable temperature fields in the soil, which remain practically unchanged throughout the year. Such fields are established after prolonged service of structures (10 - 30 years) under buried structures, under the central part of surface structures, etc. (Fig. 19).

Periodic steady state thermal processes are formed a relatively short time after the structure is put to use (3 - 5 years) and are characterized by harmonic oscillations of temperature within the layer having annual variations in soil temperature. Such temperature fields are established in the soils under buildings with ventilated crawl spaces, in the ground beneath earth dams, near the foundations of surface structures, around shallow underground utilities, etc. (Fig. 20).

In correspondence with processes occurring during the thermal interaction between the structure and the soil one should consider the energy balance of the contact layer for various periods of time. In the interaction of the system structure-soil-atmosphere, resulting in steady state processes in one direction for the period of the formation of the temperature field as well as for its steady state, one should consider the balance of the contact layer for a year. In the case for the energy balance of the contact layer we have the expression,

$$R + P + E + A + M + K = 0, \quad (4.8)$$

where R - the radiation component of heat exchange,
P - convection component of heat exchange,
E - consumption of heat for evaporation,
A - heat entering the soil or leaving it,
M - consumption of heat on melting snow,
K - consumption of heat on biochemical and other processes.

Equation (4.8) is written in the general form. Some of its components, depending on the type of structure, may be equal to zero or may have various energy sources (for example, radiation energy of the sun, radiation from high temperature internal heat sources, etc.).

In contrast to the equation for energy balance used in climatology for a period of time equal to a year, in equation (4.8) there are such components as the heat flow in the soil, consumption of heat to melt snow, etc. In natural conditions these components are much less in value than the radiation component of the energy balance, the consumption of heat on evaporation and turbulent heat exchange with the air. The basic unknown value in this equation as far as geocryology is concerned is the flow of heat in the soil (A). The small value of this heat flow makes it necessary to consider small variations introduced into the energy balance by the last two terms of equation (4.8). However, the extent to which the heat exchange components have been investigated and the accuracy of their measurement makes it impossible to use equation (4.8) in general geocryology for estimating the trend and intensity of thermal processes under natural conditions. Thus, for example, the accuracy of measuring the radiation component of the energy balance, which for the central part of the Yakut Autonomous Soviet Socialist Republic is approximately 150,000 kcal/m^2/year (Atlas of Heat Balance, 1955), using present-day apparatus is 5 - 10%. Thus the value of the mean annual heat flow in the soil, which under natural conditions is only several hundred kilo calories per m^2 per year, is beyond the limits of measurement accuracy of the other heat exchange components.

Engineering structures introduce important changes in the heat flow in the soil, which for purposes of engineering geocryology makes it possible to use equation (4.8). Thus the annual heat flow from the soil to the atmosphere near a heated building in the absence of a ventilated crawl space will comprise several tens of thousands of kilo calories per m^2.

For a periodic thermal state the energy balance should be constructed separately for periods of heating of the soil and for periods of cooling. It should be noted that the period of soil warming does not coincide with the beginning of thaw under natural conditions, but begins when the soil is still in a frozen state (in March-April); the period of cooling also begins long before the soil freezes.

For the period of cooling and for the period of warming the equations for energy balance of the contact layer are written in the following form, respectively,

$$R_c + P_c + E_c + A_c = 0; \tag{4.9}$$

$$R_h + P_h + E_h + A_h + M + K = 0. \tag{4.10}$$

The notations in equations (4.9) and (4.10) are the same as in equation (4.8) and differ only by the indices c - cooling and h - heating (warming).

(c) Components of external heat exchange

The elucidation of mechanisms forming the components of the heat exchange in equations (4.8), (4.9) and (4.10), particularly for natural conditions, are very difficult. Only by investigating the formation of individual components of the heat exchange, their variation and interrelationship can one reveal all aspects of the physical phenomenon and correspondingly control the thermal regime of soils in the required direction.

This approach to the solution of problems of engineering geocryology was initiated only during the past decade and only for the purpose of thawing soils to facilitate their workability (Bakakin, 1955).

At the present time there is still no sufficiently reliable method of determining individual components of the heat exchange with which one could elucidate the relationship through various natural factors. In determining the components of the heat exchange, the processes occurring in the contact layer are usually considered conventionally for the upper surface (convection of heat exchange and evaporation) or to a conventional surface at some distance from the contact layer in the surrounding medium (heat exchange by radiation). Thus one can obtain the numerical value for energy entering the contact layer or lost by it. However, in order to influence the formation of the heat flow in the contact layer one must investigate the conversion of various forms of energy in the layer.

Let us consider individual components of the heat exchange appearing in equations (4.8), (4.9) and (4.10). We shall consider them as applicable to two systems: atmosphere-soil and structure-soil. In the heat exchange for the system atmosphere-soil one should include the heat exchange occurring outside the structure but influencing the thermal regime of its foundation soils (heat exchange between the atmosphere and the soil near the building, above underground utilities, etc., and also heat exchange between the atmosphere and the base course of roads, airdroms, dams, etc., for earthworks and open pit mining operations, etc.). The heat exchange in the system structure-soil is that between the structure itself and the soil. In this consideration the first term of equations (4.8), (4.9) and (4.10) - a radiation component - should be regarded as the result of two sources of energy: natural for the system atmosphere-soil and artificial for the system structure-soil.

For the system atmosphere-soil the resulting radiant energy exchange, as we know, is determined by the expression

$$R = (Q + q)\ (1 - a) = I, \qquad (4.11)$$

where $Q + q$ - the sum of direct Q and scattered q short-wave radiation,

a - reflective capacity of the contact layer (albedo),

I - effective radiation (the difference between long-wave radiation of the contact layer and long-wave counter radiation of the atmosphere).

The total short-wave radiation $Q + q$ for engineering purposes, if there are no observation data, can be determined by calculation (Savinov, 1933). The basic measures taken to reduce the amount of radiation are: the construction of shade awnings; ventilated ground decking or shading by planting shrubs and trees, etc. The result of this type of shading requires study; it cannot be considered simply a change in albedo since radiant energy within shrubs and trees, being in this case part of the contact layer, is consumed on transpiration, photosynthesis, etc., and a small part of it reaches the surface of the soil.

Changes in the natural conditions result in changes in albedo of the contact layer* and its long-wave radiation. The albedo value for some types of contact layers are given in Table XXI.

As follows from Table XXI the albedo varies over a very wide range: from 0.05 for freshly ploughed dark moist soil to 0.85 for fresh snow.

Changes in the natural conditions of the surface usually result in a decrease in albedo. Whereas the albedo of vegetation cover is 0.19 - 0.28, the albedo of various types of other cover and exposed soil varies from 0.05 to 0.25. At the same time there is an increase in the effective radiation owing to an increase in the temperature of the contact layer. Therefore decreasing the albedo as a method of controlling the thermal regime of the

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* In this case the thickness of the contact layer is determined by the depth of penetration of the radiation.

soil has not been applied in engineering practice. A few suggestions have been made which involve methods of lightening the colour of the soil to increase its reflectivity. The technical advantage of such measures requires rigorous proof.

Controlling the thermal regime of soils by artificially changing the albedo with the simultaneous control of long-wave radiation may find extensive application in improving the thermal regime of the soil (Chapter V).

Finding the last term of equation (4.11) - effective radiation - presents certain difficulties. Devices for direct determination of long-wave radiation during the day are still in the development stage (Khvoles, 1952). For engineering purposes one can use computation methods (for example Berlyand, 1952).

It should be noted that if the computation methods of determining total radiation and effective radiation are to be used for engineering purposes, it would be necessary to construct detailed tables for the entire permafrost region.

The control of effective radiation or more precisely long-wave radiation of the contact layer is reduced to changing two basic values on which this radiation depends: the temperature of the contact layer and its radiation properties. In this respect the best method of decreasing long-wave radiation is the use of heat transparent films which intensify the hot-house effect (Fedorov, 1935). Snow is a good means of decreasing long-wave radiation. It has a lower temperature than the soil and has a small radiation coefficient; exposed soil on the other hand is subject to the strongest radiation. Some practical measures for controlling long-wave radiation are given in Chapter V.

Heat exchange by radiation in the system structure-soil (ventilated crawl space, conduits holding water pipes and hot water heating pipes, etc.) is determined by the usual formulae in heat engineering (Machinskii, 1949; Mikheev, 1949; Fokin, 1953, and others). As we know, the coefficient of heat exchange α usually takes complex heat exchange into account, i.e. simultaneous heat exchange by radiation and convection. As a rule, in the heat exchange between engineering structures and the soil it is sufficient to determine the convective component of the heat exchange (ventilated crawl spaces, mining excavations, etc.). Only in some cases is it necessary to take into account radiation separately (for example if in a building there is a source of heat with very high temperature or a high temperature water-heating pipe laid in a conduit).

The next term in equations (4.8), (4.9) and (4.10) is the turbulent heat exchange (P) of the contact layer with the surrounding medium (primarily with air).

In present-day methods of determining turbulent heat exchange there is extensive use of the hypothesis of turbulent diffusion. According to this hypothesis turbulent heat exchange is expressed by the relationship

$$P = -\rho c_p k \frac{\partial \vartheta}{\partial z}, \qquad (4.12)$$

where ρ - density of the air,

c_p - heat capacity of the air at constant pressure,

k - coefficient of turbulent diffusion (analogous to the diffusivity coefficient),

$\frac{\partial \vartheta}{\partial z}$ - vertical temperature gradient.

Integration of equation (4.12) gives the relationship

$$P = \rho c_p D(\vartheta_s - \vartheta_a), \qquad (4.13)$$

where $D = \frac{1}{\int_0^z \frac{dz}{k}}$ - the integral coefficient for tubulent diffusion,

ϑ_s - temperature of the surface,

ϑ_a - air temperature at height z above the surface.

Equation (4.13) is analogous to the known relationship widely used in the analytical theory of heat conductivity and in engineering calculations (the so-called boundary conditions of the third type),

$$q = \alpha(\vartheta_s - \vartheta_a), \qquad (4.14)$$

where α is the heat exchange coefficient.

The rather simple relationships (4.13) and (4.14) greatly facilitate the mathematical solution of a number of problems although this approach is purely formal since mathematical difficulties are replaced by physical difficulties. Thus the heat exchange coefficient α characterizes the complex heat exchange with the surrounding medium occurring in the surface layer of a body and the highest value in this heat exchange under usual conditions is that of convection. Because of this the heat coefficient α depends not only on the nature of the surface layer but also on the shape of the body, the hydrodynamic conditions under which a liquid or gas flows over the body, on the properties of the surrounding medium, temperature, etc.

Analytical determination of the coefficient α and also the coefficients k and D appearing in the formulae (4.12) and (4.13) is possible only by using a number of simplifying suppositions for some particular cases. Thus the above-mentioned coefficients are determined mainly by empirical means.

For the system structure-soil when heat exchange occurs through an air medium the coefficient of convective heat exchange α depends basically on the velocity of the air and the structure of the surface. As applied to engineering structures a number of empirical formulae have been obtained which in some particular cases can be used for determining the coefficient of convective heat exchange in the system structure-soil (Mikheev, 1949; Machinskii, 1949, and others).

The dependence of convective heat exchange on two factors (velocity of the air and structure of the surface) predetermines the measures for control. If it is necessary to accelerate convective heat exchange these measures are reduced to increasing the surface and its roughness and also to increasing the air velocity. To decrease convective heat exchange measures are correspondingly taken to decrease air velocity and to decrease the surface of interaction and its roughness.

It is much more involved to determine the coefficients of turbulent diffusion (k and D) for the system soil-air which change with respect to time and depend on many factors which are difficult to take into account. In particular, by contrast to the coefficient of heat exchange the coefficient of turbulent diffusion depends to a large extent on the temperature of the contact layer. According to some data (Budyko, Zubenok, Strokina, 1956) the integral coefficient of turbulent diffusion depends little on wind velocity since air causing dynamic turbulence decreases thermal turbulence at the same time. It should be noted that these investigations were carried out on soil covered with vegetation. For exposed soil surfaces and particularly for artificial cover one should expect mechanisms analogous to the dependence of the coefficient of convective heat exchange α on roughness and velocity of the air. Here the heat exchange can be compared to heat exchange under conditions of forest convection, since the wind regime (if one excludes large cities) will be formed basically outside the limits of the area under consideration.

The comparatively small number of determinations of coefficients of turbulent diffusion do not as yet permit the use of expressions (4.12) and (4.13) for engineering calculations. For the system atmosphere-soil one has to use equation (4.14), using the coefficient of convective heat exchange obtained for engineering structures. As will be shown below, the value of α has little influence on the heat flow in the soil if one takes into account only convective heat exchange. If one takes into account other components of the heat exchange (for example, radiation) the error in calculation may reach substantial values. At the same time for the system structure-soil when convective heat exchange predominates (pipes and conduits at the base of structures, ventilated galleries of dams, etc.), a rather precise determination of the coefficient of heat exchange α is required, since this determines the dimension of the cooling surface and flux of the cooling agent.

The greatest amount of energy in the consumption part of the balance of a contact layer, if one excludes deserts, is attributed to evaporation. Detailed observations of evaporation under various natural conditions in permafrost regions have not been carried out. Some idea of the consumption of heat on evaporation is given by the Novgorod Station given in Table XXII (Sapozhnikova, 1950).

As follows from Table XXII, radiant energy in many cases is almost completely consumed by evaporation. Here evaporation could occur at such a rapid rate only by receiving additional heat from advective transfer.

The engineering development of a region is in most cases accompanied by the destruction of the vegetation cover, which consumes a great quantity of heat on evaporation, and by drying of the area. Naturally these measures result in a substantial decrease in the consumption of heat on evaporation. Very substantial changes in the moisture regime occur when various types of cover are constructed along with drainage. It can be assumed that on road and airdrome covers there is virtually no evaporation.

The influence of evaporation on heat exchange for ventilated crawl spaces, mining excavations, the rate of ice accretion and the process of ice accumulation, etc., require special study.

The process opposite to evaporation, i.e. condensation, is accompanied by the liberation of heat which is much less than that consumed by evaporation even under natural conditions (Sapozhnikova, 1950). Moreover, one cannot expect any substantial condensation of moisture on artificial surfaces (roofs, covers, etc.) since their temperature in the summertime is usually higher than the temperature of the vegetation cover. One can expect that the heat liberated during condensation or consumed on evaporation will have some influence on the thermal regime of fill consisting of coarse material permitting relatively free air circulation.

The study of evaporation and condensation as heat factors influencing the temperature regime of the soil for conditions of construction, fill for railroad beds, etc., has been completely neglected up to the present time.

(d) Control of heat exchange between the soil and the surrounding medium

The temperature field of the soil almost under any structure or around structures depends on the heat exchange occurring in the system structure-soil and in the system atmosphere-soil. For example between a water duct laid in the soil and the soil there is heat exchange in the system structure-soil and simultaneously there is heat exchange between the soil and the atmosphere. As a result, the temperature field of the soil around the ground duct is formed under the influence of heat exchange in the system atmosphere-soil and structure-soil. A building transfers heat to the soil under it. Simultaneously there is heat exchange between the soil outside the building and the atmosphere. The temperature field of the soil under the building depends on the interaction of these two types of heat exchange. Only for structures that are laid deep in the ground does heat exchange between the soil and the atmosphere have little influence on the temperature field of the soil surrounding the structure.

Heat exchange between the soil and the surrounding (with respect to the soil) medium occurs by three types of heat transfer: heat conductivity, convection and radiation. Moreover, as was shown above, external factors of heat exchange include the consumption of heat on evaporation and melting of snow. Correspondingly one can point to the following basic methods of controlling the heat exchange between the soil and the surrounding medium:

(1) increasing or decreasing individual components of the heat exchange without changing the mechanism of heat transfer;

(2) changing the mechanism of heat transfer;

(3) using additional sources or dissipators of heat.

The first method of controlling the external heat exchange is used primarily in improving the heat conditions for the system atmosphere-soil (during preliminary preparation of building sites, mine drainage conduits, sites of airports, highways and railroads, etc.). The possibility of controlling the basic component of heat exchange between the soil and the surrounding medium has been considered briefly above; more detailed measures of controlling the external heat exchange is given in Chapter V.

The second method of controlling heat exchange is changing the mechanism of heat transfer from the surrounding medium to the soil. Changing the mechanism of heat transfer is used basically only for the system structure-soil.

Changing the mechanism of heat transfer by heat conductivity is reduced to the elimination of direct contact between the heat source or the absorber of heat with the soil by construction measures. For example in providing for a crawl space, heat transfer from the building to the soil will be by convection. A similar result will be attained by laying underground utilities in underground conduits.

In conversion of radiant energy to heat energy in some cases one should also change the mechanism of subsequent heat propagation. The construction of decking with an air space under it can change the transfer of heat from heat conductivity to convection.

However, the possibility of controlling external heat exchange by changing the mechanism of heat transfer is limited and in most cases is of little effect. As a means of decreasing the depth of thaw, this method can be used only where the temperature of permafrost is low and where the heat source is of small dimension. In this case a great deal of importance is attached to heat exchange in the system atmosphere-soil. Low temperature of the permafrost is an indirect sign of increased heat exchange between the soil and the atmosphere under changes in natural conditions.

The most effective and widely used method of controlling heat exchange is the use of various sources and dissipators of heat. In this case cooling and heating are accomplished by natural as well as artificial sources and dissipators of energy.

This method is widely used for thawing permafrost with water, steam and electric energy. The thawing of permafrost occurs very rapidly if the supply of heat is accompanied by some influence on the mechanism of heat transfer (thawing by means of exposed water jets, filtration-drainage method, etc.).

Supplementary sources of cooling as a method of controlling heat exchange is used most extensively to conserve the soil in the frozen state during its interaction with structures of various types. This method consists in the transfer of heat from the soil or structure by a heat conductor with subsequent dissipation of the heat into the atmosphere or some other medium. This method was first used to conserve the soil in the frozen state at the base of buildings by providing for a ventilated crawl space.

Subsequent development of the method of supplementary cooling of soil led to the construction of cooling devices in the form of conduits in contact with the structure or a system of tubes laid in the soil near the structure. The tube type system of cooling has the advantage that the cooling occurs within the soil mass. Moreover in such structures as earth dams, when their base and core are to be preserved in the frozen state, the tube system is the only possible method. As an agent for removing heat one can use air or brine; which one is used is not important. The small heat capacity of air can be successfully compensated by increasing the cooling surface of the system and the volume of air passing through it.

The use of air cooling is particularly expedient because under permafrost conditions as a rule it is sufficient to carry out cooling only during the winter time by using cold outside air. Artificial cooling may be used only under exceptional conditions (to prevent accidental failure, to eliminate the effect of accelerated filtration of groundwater, etc.).

The cooling effect of ventilation systems using natural low temperature air during the winter can be intensified by the provisions of reservoirs filled with cryohydrates - salt solutions which freeze at low temperatures. During the winter, when outside air is passed through the cooling system, the cryohydrates should freeze; in the summer, however, when the ventilation system is switched off, the cryohydrates melt at temperatures below 0°C thus preserving the surrounding soil in the frozen state.

The cooling effect of ventilation systems can be used also in summer. For this purpose the ventilation system is disconnected from the outside air source and connected to auxiliary conduits laid in frozen soil. This measure can be effective only if the frozen material surrounding the conduit has a large moisture content: if around the auxiliary conduits there are reservoirs of cryohydrates the efficiency will be greater. Measures for intensifying the effect of cooling devices have not been taken in practice and require further theoretical development and subsequent checking.

Cooling and heating sources are used for various purposes during construction on permafrost. In this chapter we are touching on only the fundamental aspect of the problem. Particular cases and individual solutions applied to construction are considered in subsequent chapters.

2. Thermal Physical Properties of the Ground and the Possibility of Controlling Them

Regulation of the heat exchange between the soil and the surrounding medium is the most effective means of controlling the temperature field of the soil. However, the external heat exchange does not determine the temperature field in any unique way. Under otherwise equal conditions it is formed by the interaction of the soil with the surrounding medium and from thermal processes in the soil itself depending on its thermal physical properties.

All forms of heat exchange occur in the soil (conduction, convection and radiation). For practical purposes heat transfer by convection is reduced to two basic cases.

(1) Heat transfer by convection occurs only within isolated pores; interpore convection is secondary to heat conductivity.

(2) Heat transfer by convection occurs between pores (transfer of heat by water, vapour and air at times accompanied by evaporation and condensation) and has an important influence on the temperature field of the soil.

In engineering calculations of temperature fields of the soil one can include in the first case all frozen soils and also the majority of thawed soils in which there is no filtration. In such cases the temperature field of the soil can be found by using equations of heat conductivity and will depend on the following thermal physical characteristics: heat conductivity coefficient λ, heat capacity coefficient c, unit weight of the soil γ and moisture content W.

In the second case the transfer of heat is described by equations of heat and moisture conductivity (Lykov, 1956). This case is of great interest for solving problems of heaving, structure formation, etc. However, at the present time there are no solutions for equations of heat and moisture conductivity which can be used in engineering geocryology and for this reason this case is not considered.

Thus the temperature field of the soil, in addition to external heat exchange, will be determined by thermophysical characteristics: λ, c, γ and W. The first two characteristics λ, c depend on its unit weight and moisture content (see for example Chudnovskii, 1954). Correspondingly one can use the means of influencing the thermal physical characteristics which consists of changing the unit weight of the soil by compacting or loosening it and by changing the moisture content by draining or adding moisture.

Although the mechanisms of changing the thermal characteristics, unit weight and moisture content of the soil have been extensively investigated by laboratory methods they are not used to any extent in practice in controlling the thermal regime of the soil. This is explained by the fact that for fine-grained soils, changing the moisture content and density to any extensive depth is accompanied by considerable technological difficulties. This method of controlling thermal processes is used effectively only in improving the thermal conditions of the soil, since the surface layers under the influence of natural conditions become compact, dry out or become more moist. Therefore it is necessary to intensify artificially the heat transfer in the required direction which in this case is readily attainable (Chapter V).

A more effective and technologically possible method of regulating the heat transfer in the soil is the partial replacement of the soil by other material or types of soil possessing the required thermal physical properties. These methods include the construction of various types of heat insulating layers laid on the surface of the soil.

In engineering geocryology thawing is retarded, localized or completely eliminated by using insulating material having a low coefficient of heat conductivity or by using material with a high heat capacity and moisture content.

If the flow of the heat is constant in one direction material with a low conductivity coefficient should be used. The rate of thawing will then be decreased. Moreover, with sufficiently low temperatures of the permafrost and limited dimensions of the structure the final depth of thaw of the soil in its natural state can be decreased by using material with a low coefficient of conductivity; in some cases when the temperature of the permafrost is very low and the structure is small thawing may be completely eliminated.

With steady state periodic thermal processes, when the heat flow changes direction, it is convenient to use material with a large moisture content. If placed on the surface of undisturbed ground or used to replace the upper soil surface, moisture saturated material displaces the heat cycle closer to the surface and thus facilitates a decrease in depths of thaw.

One of the methods of changing the thermal physical properties of soil is by changing the mechanism of heat transfer. This method is reduced to the substitution of one type of soil by another or the appropriate working of the soil to eliminate interpore convection. Measures for reducing convective heat transfer in the soil have received little development. At the same time extensive use is made of measures to intensify convective heat exchange (thawing of the soil by filtration of water and flooding of coarse-grained soils).

3. Natural Thermal State of the Ground and Its Alteration

As a result of thermal interaction between the structure and the soil, one of two states of dynamic equilibrium in the soil may be formed: steady state or periodic steady state. In the soil beneath or around the structure a thermal regime should be created which has the minimum harmful influence on cryogenic processes and ensures the required strength characteristics of the soil.

In many cases the strength of the soil under a natural thermal regime ensures the stability of the structure and cryogenic processes have little influence on the function of the structure. The heat exchange regime undergoes changes after the structure is built; therefore under the new conditions provision should be made for ensuring the natural thermal regime of the soils or something close to it.

In other cases the soil under natural conditions does not satisfy the requirements as a foundation base or medium in which the structure is to be placed, and the properties of the soil have to be improved before the structure is erected. One of the methods of changing the physico-mechanical properties of the soil and also decreasing the influence of cyrogenic processes on the structure is changing the natural thermal regime of the soil before erection of the structure, i.e. creating a thermal regime which is required for normal operation of the structure. Later when the structure is put to use, the problem is reduced to retaining the newly created thermal regime of the soil. It should be noted that such a thermal regime, if measures are provided for conserving it, would be formed during the operation of the structure. However, the new thermal regime would be formed over a prolonged period of time and during this process the properties of the soil would not satisfy the designed requirements and the structure would suffer deformation.

With controlled change in the thermal regime of the soil the primary factor is the physico-mechanical properties under natural conditions and the possibility of changing these properties depending on the climate of the construction site, the type of structure, hydrological conditions, etc. Correspondingly, the change in natural thermal state is reduced to freezing of the soil to create a frozen bearing medium of uniform strength when the depth of the permafrost table is not uniform; to reducing the temperature of the permafrost in order to increase the permissible pressure; to preliminary thawing of the soil to prevent thaw settlement if it is impossible or economically inadvisable to retain the soil in the frozen state.

Preliminary change in the initial thermal state is accomplished by controlling the external heat exchange and using supplementary sources or dissipators of heat.

Preparatory measures to change the natural state usually must be done in a short period of time. Without changing the general principles of controlling the thermal regime this requires intensification of the processes. For further conservation of the newly formed regime during operation of the structure, measures may be used which differ from those used during its formation.

4. Changing the Surface of Interaction Between the Soil and Its Surroundings

The temperature field of the soil depends on the shape and size of the surface on which thermal interaction between the soil and the surrounding medium takes place. The possibility of controlling the thermal regime by changing the surface of interaction is extremely limited.

For the system atmosphere-soil this method of controlling heat exchange is not used to any extent. One can only mention deep ploughing where the surface is given a saw-tooth form as a method of improving thermal conditions (Chapter V).

For the system structure-soil, the change in dimensions and the shape of the surface of interaction is primarily limited to the dimensions and shape of the structure which are determined as a rule by the purpose of the structure. Some possibilities of controlling the surface of interaction are provided by supplementary heat sources and dissipators of heat. If it is necessary to decrease the depth of seasonal thaw at the outside walls of buildings with ventilated crawl spaces, the surface of interaction can be increased by building ventilated decking around the perimeter of the building. This ventilated decking will be equivalent actually to extending the ventilated crawl space. In this respect buried cooling conduits or pipes present great possibilities. By selecting the dimensions and number of conduits or pipes the surface of interaction can be changed. A similar change in the surface of interaction can be achieved for example by thawing the soil using covered jets, the heat source being steam, water, etc.

5. Interaction of Various Factors in the Formation of the Ground Thermal Regime

The thermal regime of the soil is formed from the interaction of all the factors considered above; external heat exchange, heat transfer in the soil, thermophysical characteristics, natural thermal state which on changing initiates the formation of the temperature field, the dimensions and shape of the surface of interaction of the soil with the surrounding medium. During the formation of the temperature field the majority of these factors reacts reciprocally.

The mechanism of heat transfer in the soil and also the thermal physical characteristics of the soil depend on heat exchange between the soil and the surrounding medium. For example, during rapid freezing of the ground caused by large losses of heat into the surrounding medium, there is practically no redistribution of moisture in the soil during the freezing process, i.e. there is almost a complete absence of convective heat transfer; on the other hand slow freezing caused by small heat losses can greatly increase migration and consequently convective heat transfer.

Processes of evaporation and condensation in the contact layer may on the one hand bring about a movement of moisture in the soil, i.e. facilitate heat transfer by convection; and on the other hand they influence the thermal physical characteristics of the soil changing its moisture content. In turn the thermal physical characteristics of the soil to a considerable extent determine the rates of evaporation and condensation. These processes are interrelated with the formation of the radiation component of the energy balance; with changes in the moisture content of the contact layer which depends on the moisture content of the soil and on evaporation or condensation; the albedo of the contact layer also changes as well as its temperature and radiation properties, and consequently the radiation component of the energy balance.

Turbulent heat exchange depending on the temperature of the contact layer also affects its temperature, the rates of evaporation and condensation, radiation balance, etc.

Changes in the heat exchange in the system structure-soil has an influence on the heat exchange in the system atmosphere-soil.

The initial thermal state of the soil, and the shape and size of the surface of interaction of the soil with the surrounding medium, influence the rate of establishment of the state of dynamic equilibrium and also the temperature of the contact layer and consequently all other factors determining the temperature field of the soil.

Thus all factors determining the temperature field are interdependent. At the present time it is impossible to take its interdependence into account in engineering calculations. Therefore, it must be assumed that thermal physical characteristics of the soil and the heat transfer mechanism do not depend on heat exchange with the surrounding medium. Some possibilities of taking into account the interrelationship between the external heat exchange and heat transfer within the soil, while determining the temperature field, occur when equations of energy balance (4.8), (4.9) and (4.10) are used as boundary conditions in thermal calculations.

6. Principal Methods of Thermal Engineering Calculations

(a) The general position

The problem of thermal engineering calculations is to determine the temperature field since the physico-mechanical characteristics of the soil depend essentially on temperature, and the intensity of geocryological processes depend on the rate of change of the temperature field (Chapter III). Taking into account particularly rapid changes in the physico-mechanical characteristics of the soil during changes in the phase composition of the water, the first thing to be done is to determine the boundary between the frozen and thawed state of the soil, i.e. the position of one isotherm - thawing or freezing; the relatively small pressures exerted by present-day structures on permafrost make it possible for the most part to neglect other factors.

The freezing and thawing of the moisture in fine-grained soils occurs in a layer of freezing or thawing soil having a wide range of below freezing temperatures (Chapter III). Correspondingly Kolesnikov (1952) obtained a differential equation which takes into account the evolution or absorption of heat of crystallization in the layer of freezing or thawing soil. This equation can be solved only by numerical integration which makes it impossible for the time being to use the mechanism developed by Kolesnikov for solving engineering problems.

To simplify the problem at the present time it is still assumed that the heat of crystallization for fine-grained soils as well as for coarse-grained soils is evolved or absorbed on a mobile interface between the thawed and frozen soil and not in a layer of soil. Then the problem of determining the temperature field during thawing of the soil in the permafrost region can be formulated for uniform soils in the following manner. The temperature fields in the regions of thawed and frozen soil, respectively, are described by the Fourier equations (for the sake of brevity a unidimensional problem is considered):

$$\left.\begin{aligned} \frac{\partial \vartheta_T}{\partial t} &= a_T \frac{\partial^2 \vartheta_T}{\partial x^2}, \quad 0 \le x \le h \\ \frac{\partial \vartheta_{fr}}{\partial t} &= a_{fr} \frac{\partial^2 \vartheta_{fr}}{\partial x^2}, \quad h \le x \le \infty \end{aligned}\right\} . \qquad (4.15)$$

The initial conditions:

$$\text{when } t = 0;\ h = 0;\ \vartheta_{fr} = f(x). \qquad (4.16)$$

The boundary conditions:

$$\text{when } x = 0,\ R(t) + P(t) + E(t) + \lambda_T \left[\frac{\partial \vartheta_T}{\partial x}\right]_{x=0} = 0; \qquad (4.17)$$

$$\text{when } x = h,\ \vartheta_T = \vartheta_{fr} = \vartheta_0 .$$

The conditions of union at the boundary of the thawed and frozen soils,

$$\lambda_T \left[\frac{\partial \vartheta_T}{\partial x}\right]_{x=h} - \lambda_{fr} \left[\frac{\partial \vartheta_{fr}}{\partial x}\right]_{x=0} = q_0 w \frac{\partial h}{\partial t}; \qquad (4.18)$$

where ϑ_T - temperature of the thawed layer of soil,

ϑ_{fr} - temperature of frozen soil,

ϑ_0 - temperature of the melting of ice in the soil,

a_t, a_{fr} - coefficients of diffusivity of the soil in the thawed and frozen state,

λ_t, λ_{fr} - coefficients of heat conductivity of the soil in the thawed and frozen states,

h - depth of thaw,

w - quantity of ice in the soil,

R(t); P(t); E(t) - heat exchange components, the same as in equation (4.8),

q_o - melting point of ice.

Present-day mathematics cannot produce a precise solution to this problem. Moreover, great difficulties are involved in producing a sufficiently simple analytical expression for the functions of R(t), P(t) and

E(t) in condition (4.17). The analytical solutions obtained even for the simplest boundary conditions (Rubinshtein, 1947; Datsev, 1947; and, Volokhonskii, 1950; Shekhter, 1955, and others) cannot be used for engineering purposes.

The mathematical difficulties in solving equations (4.15) with the condition of the union of temperature fields at the boundary between the frozen and thawed soil (4.18) has forced the search for a simplified solution.

The greatest consumption of heat during the thawing of moist soil goes to change the phase composition of water and the interface between the two states of the soil (thawed and frozen) is usually displaced very slowly. As a result, the temperature field in the thawed or frozen layer of soil at any moment of time is close to the steady state, whereas the non-steady state process can be regarded as a continuous transition from one steady state to another. This specific feature of freezing and thawing of moist soils has been used as the basis for a number of solutions. In these solutions it is assumed that in frozen or thawed layers the temperature of the soil varies according to the law of a steady thermal state or is approximated by some function satisfying the boundary conditions. This method is formulated most fully in papers of L.S. Leibenzon, (1931, 1939). Moreover, in such analytical solutions simplified initial and boundary conditions are used.

For the initial conditions usually one of the following variants is given: (a) the temperature of the soil is equal to 0°C; (b) the temperature of the soil is different from zero but constant with depth, (c) the temperature of the soil varies linearly with depth.

As boundary conditions it is assumed that the source of heat is generated instantaneously and furthermore the temperature either at the surface of the soil or in the surrounding medium remains constant.

The components of the external heat exchange can quite readily be introduced in analogous solutions if one takes turbulent heat exchange into account according to equations (4.3) or (4.14). In this case the boundary condition (4.17) is written in the form

$$R(t) + \alpha[\vartheta_a(t) - \vartheta_s(t)] + E(t) + \lambda_T\left[\frac{\partial \vartheta_T}{\partial x}\right]_{x=0} = 0.$$

Introducing the equivalent temperature ϑ_e corresponding to the total effect of the components of the external heat exchange on the thermal regime of the soil we obtain

$$\alpha[\vartheta_e(t) - \vartheta_s(t)] + \lambda_T\left[\frac{\partial\vartheta_T}{\partial x}\right]_{x = o} = 0, \qquad (4.19)$$

where

$$\vartheta_e(t) = \vartheta_a(t) + \frac{R(t)}{\alpha} + \frac{E(t)}{\alpha} \quad .$$

If one assumed that ϑ_a, ϑ_s (see equation 4.13), R and E are constant, the radiation component of heat exchange and the consumption of heat on evaporation or its evolution during condensation can be taken into account in any of the approximate solutions.

For an approximate estimate of the influence of the radiation component of the heat exchange and the consumption of heat during evaporation on the equivalent temperature, we use the observations of heat exchange components carried out on experimental plots in the vicinity of the Aldan River (Bakakin, 1955). The data of observations are given in Table XXIII.

In thermal engineering calculations of the thawing and freezing of soils, the coefficient of heat exchange α is usually used which is obtained for engineering structures within the range of 10-30 (Iskrin, Luk'yanov and others), since there are no special investigations of this problem. For a natural plot where turbulent heat exchange is retarded by vegetation cover, we assume the coefficient of heat exchange to be 15; for coarse gravelly deposits which have the greatest similarity to the appropriate surfaces of engineering structures $\alpha = 20$ and for artificially blackened gravelly deposits where one can expect accelerated thermal turbulence, $\alpha = 25$.

The equivalent temperatures calculated from the data of Table XXIII and the assumed heat exchange coefficient are given in Table XXIV.

From the date in Table XXIV one can estimate the influence of the radiation component and heat consumed by evaporation on the value of the equivalent temperatures. These data are only approximate since at the present time the coefficient of heat exchange cannot be determined with sufficient accuracy. Nevertheless one can conclude that in the majority of cases it is necessary to take into account simultaneously both the consumption of heat on evaporation and the radiation heat exchange component. Taking into account only the radiation component gives an excessive value to the equivalent temperature. Thus for a plot with natural cover the mean equivalent temperature in July taking only radiation into account will be

$$\vartheta_e = \vartheta_a + \frac{R}{\alpha} = 17.0 + 7.2 = 24.2°,$$

whereas if consumption of heat on evaporation is also considered ϑ_e = 18.8°C (Table XXIV). Therefore as a first approximation, if there are no data on evaporation, it is more reliable under natural conditions to consider that freezing and thawing is due only to heat exchange between the soil and the air for determining the depth of freezing and thawing. The results given in Table XXIV also permit the conclusion that in calculating the thermal regime for the fill of a railroad bed and dry slopes of dams one should take as the boundary conditions not only the heat exchange with the air but also the radiation component of the heat exchange and the consumption of heat on evaporation, or in the first approximation consider only the heat exchange with the air.

Only in the determination of the depth of freezing or thawing of earth roadbeds with hard surfaces where evaporation is practically excluded can the radiation component of heat exchange obviously be considered.

Thus a precise consideration of heat exchange between the soil and the atmosphere is difficult at the present time since there are no reliable data with which to determine the loss of heat on evaporation and the coefficient of heat exchange α. The heat exchange coefficient α varies over a wide range; however, if one takes into consideration only the heat exchange with the air it has little influence on the heat flow within the soil.

A more precise determination of the heat exchange coefficient is required when the calculations take into account such components of the heat exchange as the consumption of heat on evaporation and radiation.

(b) Approximate analytical solutions

Approximate analytical formulae are used extensively in calculations during the design stage for the thermal interaction of various structures and soils.

Moreover, in a number of engineering calculations they ensure the required accuracy at the present time (preservation of the frozen condition of the foundation soils under buildings with ventilated crawl space, determination of the depths of thaw at the base of roadbeds with hard surfaces, etc.).

Many of the problems in engineering geocryology can be classified as unidimensional in which the temperature field depends on a single coordinate (determination of the depth of thaw: under natural conditions, in earth roadbed constructed in opened excavations, at ground level or on low embankment, problems of thermal improvement, etc.). Some problems which are essentially two dimensional can also be considered approximately as being unidimensional (determination of the depth of thaw under buildings with ventilated crawl spaces, under the central part of buildings covering a large area, etc.).

On the basis of the simplification considered above, a large number of calculation formulae have been obtained for determining the depth of freezing or thawing for unidimensional cases (Krylov, 1934; 1940; Leibenzon, 1939; Luk'yanov, 1946; Saltykov, 1944; Khakimov, 1952, etc.).

The basic difference between these formulae is the method of taking into account boundary conditions, heat capacity of freezing or thawing soil and heat flow from the underlying thawed soil in the case of freezing or heat flow into the underlying frozen soil during thawing.

The boundary conditions are usually given in the form of the surface soil temperature or the temperature at a certain depth in the form of the temperature of the surrounding medium. In any of the formulae, taking into account the temperature of the surrounding medium, the temperature of the medium can be replaced by the equivalent temperature and in this way all the components of the external heat exchange can be accounted for.

Below, the most widely used formulae are given which differ in the quantity and methods of taking into account various factors influencing the depth of freezing and thawing of the soil.

The formula for determining the depth of freezing was first derived by Zaal'shyuttse; later a similar formula was derived by Stefan (Golovko, 1949) and it was named after him:

$$h = \sqrt{\frac{2\lambda_T \vartheta_s t}{q_o w}} . \qquad (4.20)$$

This formula is based on the consideration that all the heat entering the soil on thawing or eliminated on freezing goes to change the physical state of the water in the soil. This formula does not consider either the heat capacity of the soil or heat flow to the underlying frozen soil from the thawing boundary. In engineering geocryology it can be used only for very rough calculations of the depth of thaw of permafrost if the temperature at the depth of annual amplitude is close to 0°C.

The heat capacity in formula of the type (4.20) was first taken into account by Stefan (Golovko, 1949),

$$h = \sqrt{\frac{2\lambda_T\vartheta_s t}{q_o w + \frac{C_T\vartheta_s}{2}}}, \tag{4.21}$$

where C_T - volumetric heat capacity of thawed soil.

The introduction into the formula of the additional factor of heat capacity does not increase the accuracy of the formula to any extent and the possibilities of its use in calculations are the same as for formula (4.20).

It should be noted that the consumption of heat on heating thawing soil or heat flow from freezing soil in the form in which it was done by Stefan can be introduced in any of the unidimensional formulae given below (for example, in the formulae of Krylov, 1934, 1940, and others).

The perfection of formulae of the type (4.20) and (4.21) is reduced mainly to taking into account heat exchange with the surrounding medium and heat flow from the thawing boundary into the frozen soil or heat flow from unfrozen soil to the freezing boundary.

Krylov (1934) suggested defining the freezing of soil as the result of two independent processes: the freezing of soil from above and the thawing of soil from below considering the heat flow from the unfrozen soil q as being constant. Then the depth of freezing according to Krylov would be,

$$h = \sqrt{\frac{2\lambda_T\vartheta_s t}{q_o w}} - \frac{qt}{q_o w}. \tag{4.22}$$

Formula (4.22) is semi-empirical since the single process of freezing and the heat flow to the freezing boundary are considered separately. With it one can calculate freezing over a relatively short period of time.

Later Krylov (1939) derived a formula suitable for any interval of time,

$$t = q_o w\left(\frac{\lambda_{fr}\vartheta_s}{q^2} \ln \frac{\lambda_{fr}\vartheta_s}{\lambda_{fr}\vartheta_s - qh} - \frac{h}{q}\right). \tag{4.23}$$

A great contribution to the development of approximate methods of calculating the depth of freezing was made by V. F. Luk'yanov (1946). In the formula he derived, the most complete consideration is given for factors influencing the depth of freezing,

$$t = \left(q_o w + \frac{C_{fr}\vartheta_a}{2}\right)\left[\frac{\lambda_{fr}\vartheta_a}{q^2} \ln \frac{\lambda_{fr}\vartheta_a - qS}{\lambda_{fr}\vartheta_a - q(h + S)} - \frac{h}{q}\right], \qquad (4.24)$$

where
$$S = \lambda_{fr}\left(\frac{1}{\alpha} + R\right);$$

R - the thermal resistance of insulation on the soil surface.

The instructions developed by V. S. Luk'yanov and N. V. Golovko (1957) in which a basis is given for the selection of initial data including the heat flow q permits effective use of formula 4.24 for calculating the depth of seasonal freezing of the soil.

The determination of the heat flow q is of course difficult. Therefore attempts were made to determine it analytically. Kh. R. Khakimov (1952) assumes that at some distance from the freezing boundary or thawing boundary the temperature of the soil remains practically constant. The distribution of temperatures in the soil below the freezing or thawing boundary is approximated by him to the function

$$\vartheta(x) = \frac{\vartheta_e}{\ln \frac{L}{h}} \ln \frac{x}{h}, \; h \leq x \leq L, \qquad (4.25)$$

where ϑ_e - the natural temperature of the soil at a distance L;
L - the distance from the surface to the depth where the influence of freezing or thawing at a given moment of time is practically absent.

On the basis of observations of the distribution of temperatures during freezing, Khakimov considers that the ratio L/h can be assumed constant and approximately equal to 5.

If one considers heat insulation at the soil surface and asumes that the distribution of temperatures in frozen soil is analogous to expression (4.25) the depth of thaw will be

$$h = -S + \sqrt{S^2 + \frac{2(\lambda_T\vartheta_a + m\lambda_{fr}\vartheta_e)t}{q_o w + \frac{C_T\vartheta_a}{2}}}, \qquad (4.26)$$

where
$$m = \ln \frac{h + S}{L + S}.$$

The value of coefficient m in formula (4.26) depending on the thermo-physical characteristics of frozen soil can be taken as 0.5 to 0.6. Formula (4.26) is valid when $\lambda_T \vartheta_a > -m\lambda_{fr}$. In formula (4.26) the heat flow from the thawing boundary into the frozen soil is variable and the zone of constant temperature ϑ_e during thawing continually moves away from the thawing boundary, which is the main advantage of the formula. The temperature gradient varies usually within a narrow range (0.02 - 0.03°C/m which makes it possible to assume in calculations that the temperature ϑ_e is constant.

In the derivation of a number of formulae (Leibenzon, 1939; Iskrin, 1952; Voitkovskii, 1954, and others) it was assumed that the temperature of the soil at the beginning of the process was constant with respect to depth; then in the frozen soil on thawing or in unfrozen soil on freezing it varies according to equation

$$\vartheta(x,\ t) = \vartheta_e \operatorname{erf}\left(\frac{x - h}{2\sqrt{at}}\right). \tag{4.27}$$

Of the formulae derived using temperature distribution according to (4.27), the formula of Iskrin (1952) takes the most complete account of factors on which the depth of freezing depends and is a further development of the Leibenzon formula (1939),

$$h = -S + \sqrt{S^2 + b^2 t}, \tag{4.28}$$

where

$$b = -M + \sqrt{M^2 + \frac{2\lambda_T \vartheta_a}{q_o w + \frac{C_T \vartheta_a}{2}}};$$

$$M = \frac{\vartheta_e}{q_o w + \frac{C_T \vartheta_a}{2}} \sqrt{\frac{\lambda_{fr} C_{fr}}{\pi}}.$$

Formula (4.28) has a rather complex form and at the same time is no more precise than, for example, (4.26).

Formulae of the type of (4.26) and (4.28) can be used for determining the depth of thaw under the central part of a heated structure for a time period of 4 - 6 years.

The heat flow from the unfrozen soil to the freezing boundary during seasonal freezing is usually a small value and can be averaged for the entire period of freezing. The same can be said of the heat flow into frozen soil from the thawing boundary in the southern part of the permafrost region.

In regions where the temperature of the permafrost is low during the winter there is intensive cooling of the upper soil strata and at the beginning of the thawing period the temperature of the soil is subject to considerable variation with depth (Figure 21). The heat flow from the thawing boundary into the frozen soil is substantial at the beginning of the thaw and subsequently decreases.

In determining the depth of thaw under structures, this temperature distribution will have some influence on the depth of thaw only during the first year after erection of the structure and only if the erection of the structure coincides with the beginning of thaw under natural conditions. Therefore in determining the depth of thaw under structures the temperature of the frozen soil (ϑ) can be taken as constant.

If a determination is to be made of the depth of thaw under natural conditions or at the base of an earth roadbed one cannot regard summer thawing separately from winter cooling. However, the consideration of winter cooling at the present time can only be done very approximately from observation data under natural conditions. On the basis of these data a determination is made of the heat flow q separately for each month which requires observations throughout the entire summer period. Therefore, the measurements are restricted usually to the distribution of temperatures at the beginning of the thawing period and it is then averaged to a depth at which it remains practically constant during the summer, i.e. a depth of 6 - 10 m (Figure 21). This technique permits the use of formulae of the type (4.26) and (4.28) for determining the depth of thaw under natural conditions and at the base of earth roadbeds.

In a manner differing from those considered above heat flow in frozen soils is taken into account in the formula of Redozubov, first published by N. I. Saltykov (1944),

$$h = -S + \sqrt{S^2 + \frac{2\lambda_T \vartheta_a t}{nq_o w}}, \qquad (4.29)$$

where n - an empirical coefficient which takes into account the consumption of heat on warming up the frozen soil.

The determination of coefficient n in formula (4.29) presents considerable difficulties and as a result this consideration of heat flow in frozen soil will not be widely used. An attempt was made to determine coefficient n theoretically by S. V. Tomirdiaro (1957) which uses a linear temperature distribution in the frozen zone but has no advantage over the consideration of heat flow in the frozen zone in formulae of the type (4.26) and (4.28) and at the same time tends to reduce the value of heat flow.

In the above formula the temperature field in the freezing of thawing soil and also in the underlying soils was approximate and an extremely primitive consideration was given to the influence of winter cooling. Therefore all these formulae are applicable mainly to determining the depth of thaw or freezing during a steady state periodic regime (at the base of buildings with ventilated crawl space, at the base of earth road-beds, etc.). They do not permit a consideration of the dynamics forming the depth of thaw or freezing after disturbance of the natural conditions.

Of particular interest is the use of these formulae for calculating multi-layered media such as fills consisting of mineral soils, a mantle consisting of several layers with different thermal physical properties.

In this case there are two methods of taking into account the thermal physical properties of individual layers of multilayer media. Using the first method one can determine the time of thawing of the first layer using the most suitable formula for calculation; then after including its thermal resistance in S one can determine the time of thawing of the second layer, etc. With this method it is difficult to take into account the thermal capacity of the thawed layers. At the same time the material used for the fill and the separate layers of the mantle usually has a very low moisture content and as a result its thermal capacity must be considered. Therefore, a multi-layer medium should be converted to a homogeneous medium which can be done if one uses the criteria Fo and Bi (Veinrub and others, 1934). Here, for the newly obtained equivalent layers having the same coefficient of heat conductivity, the moisture content must also be replaced by the equivalent moisture content (for more details see Chapter VIII).

The solution of two-dimensional problems and particularly three-dimensional presents great difficulties. Thus for engineering calculations there are only a limited number of formulae which are short of satisfying the requirements of engineering practice.

These formulae are derived on the basis of simplifications considered above for a unidimensional problem.

The greatest number of formulae have been obtained for determining the time of freezing of a cylinder, or radius of thawing and freezing around a pipe (Leibenzon, 1931; Charnyi, 1948; Khakimov, 1952, and others).

The most complete consideration is given to factors governing the radius of thawing or freezing around a cylinder in the formula of

Kh. R. Khakimov (1952) in which G. A. Martynov introduced the heat exchange coefficient α between the surface of the cylinder and the heat carrier flowing along it. This formula has the form

$$t = \frac{r_o S}{2\lambda_T \vartheta_a}\left[\left(1 - \frac{r_o}{2S}\right)(q_o w - mC_{fr}\vartheta_e) + \frac{C_T r_o \vartheta_a}{2S}\right](\xi^2 - 1) +$$

$$+ \frac{r_o^2}{2\lambda_T \vartheta_a}(q_o w - mC_{fr}\vartheta_a)\xi^2 \ln \xi - \left\{\frac{C_T r_o^3}{4\lambda_T}\frac{\xi^2 - 1}{2S + r_o \ln \xi} +\right.$$

$$\left. + \frac{C_T r_o S}{2\lambda_T}\left(1 - \frac{r_o}{2S}\right)\ln\left(1 + \frac{r_o}{S}\ln \xi\right)\right\}, \qquad (4.30)$$

where r_o - radius of the cylinder,

$\xi = \frac{r(t)}{r_o}$ - relative radius of thawing,

$r(t)$ - radius of thawing;

$$m = \frac{a^2 - 1}{2 \ln a} \approx 5 \div 8.$$

In deriving formula (4.30) an assumption was made that there is a mobile radius of influence R(t) beyond which the temperature is practically constant. According to data of Khakimov (1952) the empirical coefficient $a = \frac{R(t)}{r(t)}$ can be assumed to be 4 - 5. Numerical calculations according to formula (4.30) indicate that if the term in braces is neglected the error does not exceed 5 - 10%.

For determining freezing or thawing around a pipe or a system of pipes taking into account heat exchange at the soil surface there is only one formula, derived by A. G. Kolesnikov in 1944, which has not yet been published. In deriving formula (4.31) Kolesnikov assumed:

1. Steady state of the temperature fields in the frozen and thawed zones.

2. Constant temperature of the heat carrier in the pipes and that this temperature is equal to the temperature of the surface of the pipe (ϑ_T).

A diagram explaining the application of Kolesnikov's formula is shown in Figure 22.

$$t = \frac{q_o w r_o^2}{2\lambda_T \vartheta_T q} \left\{ \xi^2(\ln \xi^2 - 0.5) + 0.5 - \left(1 - \frac{1}{q}\right) B(\xi^2 - 1) + \right.$$

$$\left. + 2B^2\left(\frac{1}{q} - \frac{1}{q^2}\right) \exp\left(\frac{2B}{q}\right) \left[- Ei\left(- \frac{2B}{q} + 2\ln\xi\right) - Ei\left(- \frac{2B}{q}\right)\right] \right\}, \quad (4.31)$$

where

$$q = 1 + \frac{\lambda_{fr}\vartheta_a}{\lambda_T \vartheta_T}.$$

$B = \ln \frac{2h}{r_o}$ - for a single pipe,

$B = \frac{2\pi h}{b} + \ln \frac{b}{2\pi r_o}$ - for a system of pipes,

$h = h_o + \lambda_{fr}R$ - depth in the ground of the pipe or system of pipes taking into account the heat insulation.

$Ei(-x) = \int_{\infty}^{-x} \frac{e^{-z}}{z} dz$ - tabulated function.

The radius of freezing ($r(t) = r_o\xi$) obtained from equation (4.31) is an average value since the boundary formed by the frozen body around the pipe is not circular. Therefore with some relations of values in equation (4.31) $r(t) > h_o$ may be obtained which indicates complete freezing of the soil above the pipes.

It should be noted that for a system of pipes in the case of $r(t) \geqslant \frac{b}{2\pi}$ the upper boundary of the frozen soil can be considered practically horizontal.

To determine the thaw basin in the soil beneath the foundation of heated buildings with respect to time as a two dimensional problem there is as yet only one analytical solution, that of S. S. Kovner (1933). In this solution it is assumed that the initial temperature of the soil is equal to the thawing temperature (ϑ_o). This temperature is maintained throughout the entire period of thawing at the ground surface outside the building (Figure 23). The solution does not take into account thermal insulation of the floor: a constant temperature is assumed for the ground surface under the building (ϑ_s).

The Kovner solution reduced to a very simple form by N. N. Saltykov has the form

$$\frac{\vartheta_s - \vartheta_o}{q_o w l^2} t = I(\xi), \quad (4.32)$$

where $I(\xi)$ - a tabulated function.

The value of ξ is found from the graph shown in Fig. 23.

The radius of the thaw basin is determined by the relation

$$R = \sqrt{1 + \xi^2}. \tag{4.33}$$

It should be kept in mind that in deriving formula (4.32) half the width of the building is taken as the unit of measurement. Therefore, the values of ξ and R according to formulae (4.32) and (4.33) are obtained in conventional units and for conversion to metres they should be multiplied by half the width of the building (l).

Formula (4.32), by virtue of the suppositions taken in deriving it (temperature of the soil and temperature at its surface outside the building is equal to ϑ_0), can be used to determine the thaw basin only for southern parts of the permafrost region where the temperature of the permafrost is close to 0°C. Moreover the formula does not include the thermal resistance of the floor of the first storey which further limits its application.

K. F. Voitkovskii (1954), by generalizing individual solutions carried out on the hydrointegrator, obtained a semi-empirical formula for determining the position of the freezing boundary beneath the centre of an ice storehouse. This formula can be used for determining the depth of thaw of the foundation soil of a building taking into account lateral heat flow. In this case the formula has the form

$$h_0 = \frac{h}{1 + k\sqrt{t}} - S, \tag{4.34}$$

where h_0 - depth of thaw beneath the centre of the building,

t - time in hours.

The empirical coefficient k is determined by the formula

$$k = 0.14\left(1 + \sqrt{\frac{b^3}{l^3}}\right)\sqrt{\frac{\vartheta_e \lambda_T}{b^3 w}}, \tag{4.34a}$$

where b - half the width of the building,

l - half the length of the building,

ϑ_e - mean annual ground temperature outside the building.

The value of h in formula (4.34) can be found from one of the above formulae for solving the unidimensional problem.

The determination of steady temperature fields in the foundation soils of various structures is greatly simplified if one uses the method of S. G. Gutman (1952) who suggested introducing the reduced temperature in the form of a boundary condition.

$$\vartheta_{re} = \frac{\lambda}{\lambda_{re}} \vartheta.$$

In this way the nonhomogeneous thermal physical characteristics of thawed and frozen soil can be reduced to a homogeneous medium and for separate zones we will have, respectively,

$$(\vartheta_T)_{re} = \frac{\lambda_T}{\lambda_{re}} \vartheta_T.$$

$$(\vartheta_{fr})_{re} = \frac{\lambda_{fr}}{\lambda_{re}} \vartheta_{fr} .$$

If one takes as the reduced medium the thawed zone ($\lambda_{re} = \lambda_T$), the below freezing temperature at the boundary of the frozen zone will be:

$$(\vartheta_{fr})_{re} = \frac{\lambda_{fr}}{\lambda_T} \vartheta_{fr}$$

In the region of thawed soil the temperature field will correspond to the true value and in the region of frozen soil the temperature will vary by the factor of λ_f/λ_T.

The method suggested by Gutman permits summing and expansion of temperature fields. Using known solutions G. V. Porkhaev in 1957 with this method obtained the temperature field in the foundation soil of a building having n rooms with different inside temperatures and with different thermal resistance on the floors. The formula is rather complex (Figure 24),

$$\vartheta(x, y) = \frac{\lambda_{fr}}{\lambda_T} \left[Gy + \Theta_2\left(0.5 - \frac{1}{\pi} \arctan \xi_2\right) + \Theta_1\left(0.5 - \frac{1}{\pi} \arctan \xi_1\right)\right] -$$

$$- \frac{1}{\pi} \sum_1^n \vartheta_s [\arctan\xi_2 - \arctan \xi_1 + \exp Y (\sin X_2 ciX_2 - \sin X_1 ciX_1 -$$

$$- \cos X_2 siX_2 + \cos X_1 siX_1) - I_2(X_2, Y) + I_1(X_1, Y), \qquad (4.35)$$

where $\xi_1 = \frac{x}{y}$; $\xi_2 = \frac{l_n - x}{y}$;

$$I_1(X_1, Y) = X_1 \int_0^1 \frac{\exp(-Yz)}{z^2 + X_1^2} dz;$$

$$X_1 = \frac{x - l_n}{\lambda_T R_n} ; \quad X_2 = \frac{x - l_{n-1}}{\lambda_T R_n} ;$$

$$I_1(X_1, Y) = \frac{X_1}{Y} \int_0^1 \frac{\exp(-Yz)}{z^2 + \left(\frac{X_1}{Y}\right)^2} dz$$

$$I_2(X_2, Y) = \frac{X_2}{Y} \int_0^1 \frac{\exp(-Yz)}{z^2 + \left(\frac{X_2}{Y}\right)^2} dz;$$

$$Y = \frac{y}{\lambda_T R_n} ;$$

G - mean annual temperature gradient of the permafrost.

For practical purposes formula (4.35) is reduced to the form

$$\vartheta(x, y) = \frac{\lambda_{fr}}{\lambda_T} [Gy + \Theta_2 f(\xi_2) + \Theta_1 f(\xi_1)] -$$

$$- \sum_1^n \vartheta_n [F(X_1, Y) - F(X_2, Y)], \qquad (4.36)$$

where $f(\xi)$, $F(X, Y)$ - tabulated functions.

An example of calculation and graphs of $f(\xi)$ and $F(X, Y)$ are given in Chapter VI.

The above analytical solutions can be used to determine only the position of the interface between the frozen and thawed soil. The temperature field is distorted.

A more precise temperature field and also the interface between the frozen and thawed soil for any boundary conditions and any initial temperature distribution can be obtained by solving equation (4.15) numerically or using methods of analogy and computers.

(c) Numerical methods (method of finite differences)

Difficulties arising in analytical solutions of heat conductivity equations led to the development of methods of numerical analysis. Using the method of finite differences one can solve any problem of heat conductivity. This method does not impose any limitations on the choice of initial temperatures, boundary conditions and thermal physical characteristics. For variable thermal physical characteristics it has been worked out in greatest detail by A. P. Vanichev (1946) but the use of this method even for solving unidimensional problems involves a great many calculations. The computation of two-dimensional or three-dimensional temperature fields, taking into account changes in the physical state, involves a great deal of work which make this method of little use in solving problems of engineering geocryology.

(d) Methods of analogy and computers

The analytical solution of problems of heat conductivity, particularly with complex boundary conditions and variable thermal physical characteristics, involves insurmountable difficulties. The method of finite differences involves a great deal of calculation work.

The simulation of a particular phenomenon with the help of models of the same physical nature as that of the object under investigation involves many difficulties (high expense in preparing the model, large consumption of time, complexity of measurements, etc.). This makes the method of little use in solving practical problems and as a result there is a trend to reproduce the given phenomenon by analogues which are described by the same mathematical relationships but much more easily carried out. This led to the construction of devices for solving differential equations of heat conductivity based on analogues.

For solving problems of heat conductivity the most widely used are devices of electric and hydraulic analogues. The first of these are based on analogies between the distribution of temperature in a solid and a distribution of voltages in an inductionless conductor; the second is based on the analogies between the mathematical relationships describing the distribution of heat in a solid and the motion of water in a hydraulic circuit with a laminar regime.

For solving differential equations by the method of electric analogues Academician N. N. Pavlovskii developed a device which has become widely used for solving planar problems of filtration of groundwater. The development of this method has led to the creation of three-dimensional models. The propagation of heat in a solid and the filtration of water in a fine-grained medium are described by the same equations; therefore the method of N. N. Pavlovskii was successfully used for solving problems of heat conductivity. An advantage of this method is the possibility of a comparatively simple determination of the temperature fields with a complex profile of the surface of interaction between the soil and the surrounding medium taking into account changes in the thermal physical characteristics of the soil for any stratification. However this method can be used for solving only problems of steady state heat propagation which greatly limits its application.

Further development of the method of electric analogues was made by L. I. Gutenmakher (1943). The design principles developed by him of unique electro-integrators and devices built under his direction can be used for effectively solving various problems on the propagation of heat for steady state as well as non-steady state conditions.

However the electro-integrators being built at the present time cannot take into account changes in the physical state of substances which greatly limits the possibility of using them for engineering geocryology. At the same time the accounting for changes in the physical state of substances is possible in principle. With such a modification electro-integrators could find extensive use for calculating temperature fields of freezing and thawing soils.

Particularly great advances have been made in solving problems of heat conductivity taking into account changes in the physical state after the invention by V. S. Luk'yanov in 1934 of a device of hydraulic analogues (Luk'yanov et al. 1957). With this device one can determine the temperature field under any initial and boundary conditions taking into account the changes in the physical state of the substance. A great advantage of the device is that transitional processes can be reproduced for practically any period of time. This makes it possible to change the parameters of the system during the process of solution and also makes it possible to observe graphically the entire process of the formation of the temperature field. However for solving two dimensional problems a considerable amount of time is required (primarily for assembling and regulating the device). For example, in determining the temperature field at the base of buildings, present-day devices require about two weeks for one particular solution taking into account assembly and control. When the device has been adjusted approximately one day is required to solve each subsequent problem. Unfortunately the theory of errors for this device has not been developed with which one could, in conjunction with the required accuracy of solution, break down the field of investigation into sections to establish permissible errors in calibrating two different parts of the device, etc.

Other defects of the device are its awkwardness and the possibility of dividing the field of investigation into only a small number of units. For solving a three-dimensional problem in engineering geocryology it is necessary to divide the field under investigation into not less than 800 - 1000 units which makes it exceedingly difficult to solve this type of problem with existing devices.

At the present time, for solving equations of mathematical physics successful use is made of high-speed electronic computers of the BESM type. The use of present-day computers would make it possible to set any initial and boundary conditions, to consider any detail of either two-dimensional or three-dimensional problems, thus giving a maximum approximation to reality. However, for using modern electronic computers, solutions must be found for the problems at least in the most general form. Such solutions have been worked out only for the unidimensional problem with simplified boundary conditions (Rubinshtein, 1947; Melamed, 1957).

The working out of solutions for two-dimensional and three-dimensional cases, as well as for unidimensional but with boundary conditions of the third order, is the problem to be solved in the near future.

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Table XXI

Albedo (in decimal fractions) for some types of contact layers

Type of contact layer	Albedo	Source
Snow, freshly fallen	0.85	Sapozhnikova, 1950
Snow, dirty and thawing	0.40	
Green grass	0.28	Kalitin, 1938
Dry grass	0.19	
Swamp with shrubs (mar')	0.20-0.25	Bakakin, 1955
Tundra	0.15-0.20	Budyko, 1956
Brush	0.10	
Deciduous forest	0.20	
Pine forest	0.10-0.15	Sapozhnikova, 1950
Light coloured bare soil	0.35	
Dark coloured dry bare soil.	0.15	
Dark coloured moist bare soil.	0.10	
Dark coloured freshly ploughed soil. .	0.05	
Concrete mantle	0.25	
Mantle consisting of optimum gravel mixture	0.12	Stepanov, 1955
Crushed stone mantle	0.14	

Table XXII

Consumption of heat on evaporation kcal/m^2 hour

Type of evaporating surface	Months				
	V	VI	VII	VIII	IX
Black fallow	74	84	105	66	35
Swamp peat	95	99	105	70	46
Winter rye	131	134	95	49	21
Oats	92	138	127	60	28
Artificial meadow.	109	127	144	74	46
Radiation component of the energy balance during daylight hours	138	138	138	96	42

Table XXIII

Radiation component of heat exchange (R) and consumption of heat on evaporation (E) for experimental plots, kcal/m^2 hour

Plot		Months VII	VIII	IX
Natural	R	107	81	37
	E	-81	-59	-20
Coarse gravel deposit, light yellow in colour	R	93	72	26
	E	-41	-38	-18
Same deposit artificially darkened	R	107	79	35
	E	-36	-33	-18

Table XXIV

Equivalent temperature ϑ_e on experimental plots (°C) by months

Plot	VII				VIII				IX			
	a	$\frac{R}{\alpha}$	$\frac{E}{\alpha}$	e	a	$\frac{R}{\alpha}$	$\frac{E}{\alpha}$	e	a	$\frac{R}{\alpha}$	$\frac{E}{\alpha}$	e
Natural	17.0	7.2	-5.4	18.8	13.3	5.4	-3.9	14.8	6.0	2.5	-1.3	7.2
Course gravel deposit, light yellow in colour	17.0	4.6	-2.0	19.6	13.3	3.6	-1.9	15.0	6.0	1.3	-0.9	6.4
Same deposit artificially darkened	17.0	4.3	-1.4	19.9	13.3	3.2	-1.3	15.2	6.0	1.4	-0.7	6.7

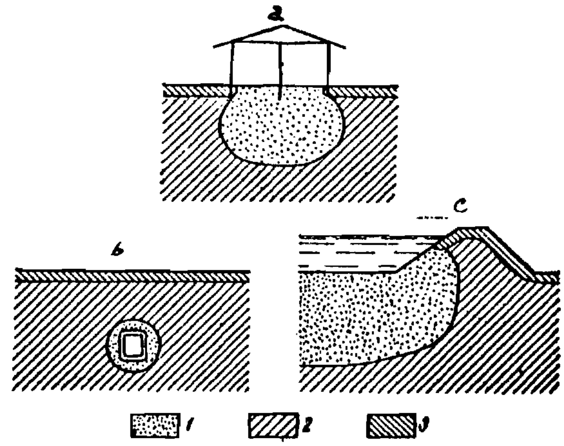

Fig. 19

Established thermal state with heat flow in one direction

a - thaw basin at the base of the building; b - underground duct around which no provision is made to preserve the permafrost; c - thaw basin under a reservoir. 1 - soil in a thawed state when the structure is in operation; 2 - soil in the frozen state; 3 - seasonally freezing and thawing soil

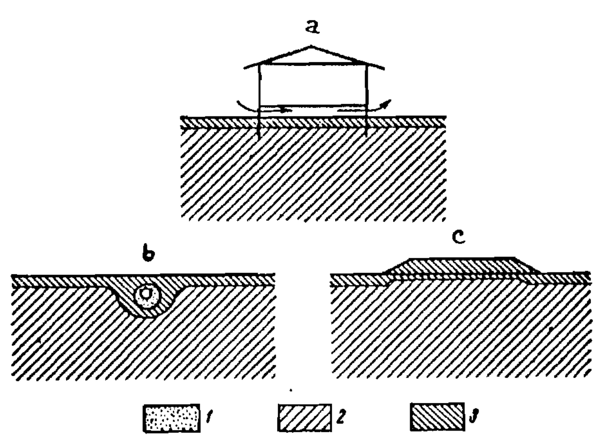

Fig. 20

Established periodical thermal state

a - building with ventilated crawl space; b - shallow water duct; c - road or airdrome mantle. 1 - soil in the thawed state when structure is in operation; 2 - soil in the frozen state; 3 - soil that freezes and thaws seasonally

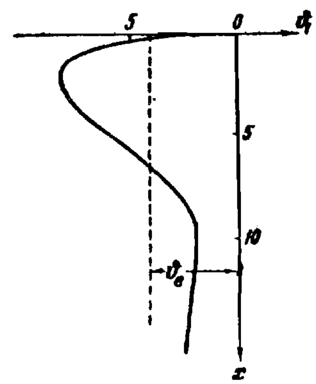

Fig. 21

Temperature distribution before the beginning of thawing in regions with low mean annual temperatures

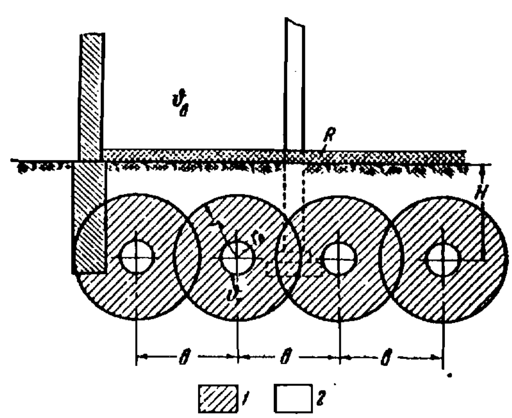

Fig. 22

Diagram explaining the application of formulae for calculating the freezing of soil around ducts (after A.G. Kolesnikov)

1 - soil in the frozen state; 2 - soil in the thawed state

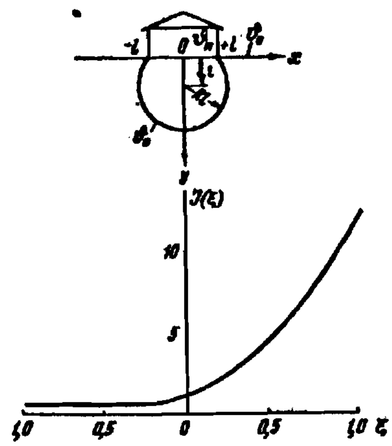

Fig. 23

On calculating the thaw basin at the base of a heated structure (after Kovner)

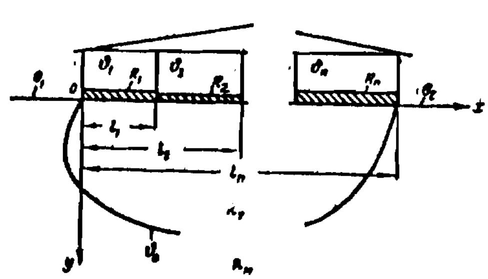

Fig. 24

Diagram explaining the calculation of the temperature field at the base of a structure with a steady state thermal regime (after G.V. Porkhaev)