Considerations governing the feeding of antenna arrays by a single transmission line (or waveguide) with applications to 70 cm. variable frequency V.E.B.

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CONSIDERATIONS GOVERNING THE FEEDING OF ANTENNA ARRAYS BY A SINGLE TRANSMISSION LINE (OR WAVEGUIDE) WITH APPLICATIONS TO 70 CM. VARIABLE FREQUENCY V.E.B.

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I. FEEDING OF FIXED FREQUENCY ANTENNAS:

1. General Considerations

We will first discuss the factors involved in the design of a single line feed for a fixed frequency array and later the additional considerations introduced by requiring that the antenna operate over a frequency band of about 15%. The former problem is relatively simple, the latter more complex.

Let us consider the possible methods of feeding a number of dipoles by a single transmission line.

(1) Using complex impedance dipoles:-

(i) Space such dipoles $n\lambda/2$ cm. apart and start at the load end (end of the line remote from the generator) at a pure resistance point. The complex dipole load at a resistive point on the line will cause a shift of the standing wave along the line thus altering the voltage and phase of succeeding dipoles.

Examining such a case on an admittance chart will show that the total phase shift cannot be greater than 90° so that if the dipoles are highly reactive a large standing wave ratio (S.W.R. = $K = E_{max}/E_{min}$) will result at the input which may be difficult to match to the transmitter.

It must be noted also that each dipole will not absorb equal power unless G_D is varied as G_L changes.

Since: $$\frac{P_L}{P_D} = \frac{G_L}{G_D} \qquad (1)$$

where P_L, G_L refer to line on load side of junction

P_D, G_D refer to dipoles.

P = power
G = conductance

Feeding at a complex impedance point would not improve the situation.

(ii) Spacing the dipoles $(n\lambda/2 + \Delta\lambda)$ apart, however, would remove the difficulty of the large standing wave ratio at the input since the susceptances added at points of positive and negative line susceptance, would tend to cancel (this may be clearly seen from an admittance chart).

Each dipole, however, will still introduce a shift of the standing wave at its feed point and consequently a variation in phase and voltage along the array unless the ratio X_D/R_D is chosen so as to introduce no phase shift and the ratio G_L/G_D is made the correct value at each point such that each dipole absorbs equal power.

(2) Using pure resistance dipoles:-

From the above discussion of complex dipoles it seems evident that a considerable simplification could be realized by using a pure resistance dipole. It is obvious that with such dipoles the feed should be at pure resistance points, $(n\lambda/2$ apart) so that the loads will be in shunt.

To avoid too large a S.W.R. at the input end, the dipole conductances will have to be low, since S.W.R. = $\Sigma G_D/Y_o$ (where Y_o is the characteristic admittance of the line); but then to avoid too large a S.W.R. at the other end, the line must be terminated in a load of $G > G_D$. Then S.W.R. at input = $(G + \Sigma G_D)/Y_o$.

Now we note that the dipoles introduce no shift of the standing wave and, since they are effectively in shunt at the input end, they are driven with the same voltage, and if they are of the same conductance they will draw equal power. Hence we have a quite simple method for obtaining an array fed in phase and with constant voltage. Here also by varying G_D we have a simple method of introducing a tapered illumination function along the array, and of correcting for attenuation.

2. Design Procedure

It is evident that case (2) is the simplest approach to the problem and we shall now consider it in more detail.

Let us find the relationship between the dipole impedances, the load impedance and the power dissipated in this load on the condition that each dipole absorbs equal power P_o.

Let number of dipoles be n
total power be P
load impedance be yZ_o (Z_o is characteristic impedance of line)
fraction of P absorbed in load be $P_o/P \equiv 1/x$

(x and y are constants for a given array) let dipole impedances be

$R_1, R_2, \ldots R_m \ldots R_{n-1}, R_n$ (pure resistances and numbered from load end)

and impedances looking into junctions (1,2,3, ... n) of the dipoles and the line be

$Z_1', Z_2', Z_3' \ldots\ldots Z_m' \ldots\ldots Z_{n-1}', Z_n'$.

Then

$$\frac{P_o}{P_1} = \frac{R_1}{yZ_o} = \frac{\frac{P}{x}}{\frac{1}{n}(P - \frac{P}{x})} = \frac{n}{x-1} \tag{2}$$

$$Z_1' = \frac{R_1 yZ_o}{R_1 + yZ_o} = \frac{nyZ_o}{n+(x-1)} \tag{3}$$

Similarly it may be shown

$$R_m = \frac{nyZ_o}{x-1} \tag{4}$$

$$Z_m' = \frac{ny\,Z_o}{n+m(x-1)} \tag{5}$$

In particular

$$Z_n' = Z_{input} = \frac{yZ_o}{x} \tag{6}$$

Therefore the S.W.R. at the load end $= \frac{yZ_o}{Z_o} = y$ (7)

and S.W.R. at input end $= \frac{Z_o}{\frac{yZ_o}{x}} = \frac{x}{y}$ (8)

This conclusion is noteworthy; i.e., that the match at the input end, under the conditions of resistive dipoles at resistive points on the line drawing equal power, is independent of the number of dipoles and determined entirely by the prescribed conditions at the load end.

If we make S.W.R. at input = 1:1
case (1) $y = x$ (9)

If we make S.W.R. at input end = S.W.R. at load end
case (2) $y = x/y$ or $x = y^2$ (10)

The larger the x the smaller the power dissipated in the load and the larger the S.W.R. But if this array is to operate on a fixed frequency we see that this "dissipated" power may be taken through a $\lambda/4$ transformer and used to drive a second portion of the array and only a small fraction of the total power (or none at all) need actually be dissipated. But in choosing x we must also note the smaller the x the larger the Z_m hence we must make a compromise between the maximum S.W.R. allowable and the value of Z_m.

Choosing between case (1) and case (2) will depend chiefly on the length of antenna and the power to be put into it. Case (1) leads to a higher value of Z_m than case (2) but case (2) gives its maximum S.W.R. at the input end (as well as load end) where a large S.W.R. is most likely to promote a voltage breakdown.

Taking a specific example let us tolerate a S.W.R. of 2:1 and consider a design for a 100 element array on the basis of case (2).

For first section, $y = 2$, $x = y^2 = 4$.

Then percentage of power "dissipated" at end of section (1) = 1/4 x 100 = 25%. Therefore we will include 75 dipoles in section (1) and their impedance will be

$$Z_{1m} = \frac{nyZ_o}{x-1} = \frac{75(2)Z_o}{4-1} = 50Z_o$$

For second section again $y = 2$, $x = 4$

And percentage of power "dissipated" at end of section (2) = 25% of input
= 6.25% of total.

Therefore section (2) contains 19 dipoles of impedance

$$Z_{2m} = \frac{19(2)Z_o}{4-1} = 12.67Z_o$$

For third section again $y = 2$, $x = 4$

And percentage of power dissipated in load = 25% of input
= 1.55% of total.

Therefore section (3) contains about 5 dipoles of impedance

$$Z_{3m} = \frac{5(2)Z_o}{4-1} = 3.33Z_o$$

By tolerating a larger S.W.R. the dipole impedance may be reduced and it must be emphasized that the reasons for keeping the S.W.R. down are losses and voltage breakdown since the large S.W.R. in this case does not lead to any variation in the voltage and phase of the dipoles. The voltage and phase remain constant since each dipole is fed at the maximum or minimum of a sine wave as shown in Figure 1.

II. FEEDING OF VARIABLE FREQUENCY ANTENNAS:

1. Factors affecting the power distribution on the array

Now let us consider a similar antenna to operate over a frequency band such as a variable frequency VEB antenna with a $\pm 7\%$ bandwidth. Referring to Figure 1 we see that "off" the mean frequency each element will no longer be fed at a voltage maximum or minimum but at a point $(m-1)\Delta\theta$ degrees from a maximum or minimum ($\Delta\theta$ is phase shift corresponding Δf frequency shift). The voltage along the dipoles for off-frequency operation is shown in Figure 2.

It is of interest to note in passing that Figure 2 has been confirmed experimentally on a resistor loaded line - see Section III (1).

Returning to figure 2 let us define certain voltages:

(1) Considering the actual S.W. curve of voltage, E, (r.m.s. value of the instantaneous voltage), the power along the line is given by

$$P = \frac{E_{max} \times E_{min}}{Z_o} = \frac{(E')^2}{Z_o}$$

where $E' = \sqrt{E_{max} \times E_{min}}$ = mean voltage on line.

(2) Considering the S.W. curve formed by the voltage at the dipoles, E_d, i.e. ignoring the fluctuations between dipoles we may say that

$$\frac{E_{d\,max} \times E_{d\,min}}{Z_o} = \frac{(E_d')^2}{Z_o} \approx P$$

where $E_d' = \sqrt{E_{d\,max} \times E_{d\,min}}$ = mean voltage on dipoles.

The validity of this approximation is given by the following: If $E_{d\,max}$ and E_{max} are measured at or nearly at the same point on the line

$$E_{d\,max} \cong E_{max}$$

but $E_{d\,min}$ must be measured some distance along the line from E_{min} therefore if $E_{min} = \frac{E_{max}}{K}$ (where $K \equiv$ S.W.R. of E at section of line where E_{max} and E_{min} are measured)

since K is a slowly changing function along the line,

then $$E_{d}'{}_{min} = \frac{E_{max}}{K+\Delta K} \approx E_{min}$$

and $$P \approx \frac{E_{d}\,max \times E_{d}\,min}{Z_c} = \frac{(E_d')^2}{Z_o}$$

For the remainder of this section, we shall write simply E for E_d'.

Now considering the dipoles to be driven by this new voltage E, let us find the nature of this curve when each dipole is radiating equal power off frequency $\equiv P_o$. Let the dipoles be at

$$m = 0, 1, 2 \ldots\ldots (n-1)$$

$$\frac{E^2(m)}{Z_o} = P \text{ (passing } m+1^{th} \text{ dipole)}$$

$$= \left[n - (m+1)\right] P_o$$

or $$E^2(m) = Z_o P_o \left[n-1-m\right] \qquad (11)$$

a parabola in the (m,E) plane with vertex at (n-1,0).

The average power into each dipole = $G(m) \quad E^2(m) = P_o$

$$\therefore \quad G(m) = \frac{P_o}{E^2} = \frac{P_o}{Z_o P_o (n-1-m)}$$

or $$\frac{G(m)}{Y_o} = \frac{1}{n-1-m} \qquad (12)$$

Such values of G(m) would lead to a large variation in the feed "on" frequency because each dipole is fed with the same voltage. Therefore let us find the curves E(m) and P(m)($\equiv$ power into mth dipole), for any given curve of G(m). This should enable us to choose the best G(m) for the antenna.

We shall change from the discreet variable m to the continuous variable x letting one unit of x correspond to the distance between two dipoles (i.e. dipoles at $x = 0, 1, 2 \ldots \overline{n-1}$) and set up a differential equation for the antenna.

Let $G = G(x)$ be the conductances per unit of x then

$$d\left(\frac{E^2}{Z_o}\right) = -GE^2\,dx \tag{13}$$

$$\text{or} \quad \frac{d\,E^2}{E^2} = -\frac{G}{Y_o}\,dx$$

$$\text{or} \quad E^2 = c\,e^{-\int \frac{G}{Y_o}dx} \tag{14}$$

Thus for any function $G(x)$ we know the curve of $E(x)$ and consequently also the curve of $P(x) = E^2(x)\,G(x)$. It is obvious from Eq. 12 that if there is to be no load at the end of the antenna the variation in $G(x)$ for a long array is prohibitive for on-frequency operation and therefore a load is required which in effect replaces the last T dipoles and therefore x does not become greater than n-1-T. We shall want to know then the power in this load P_L and the power into the dipoles $P(x)$. Since it is simpler to apply experimentallly let us consider the case where Y_o/G decreases linearly from input end to load end.

Let $R = R(x) = \frac{1}{G}$

$$\frac{R_B}{Z_o} = A, \quad \frac{R_o}{R_B} = K$$

$B + 1 = n =$ no. of dipoles

Then $\frac{R_o}{Z_o} = KA$

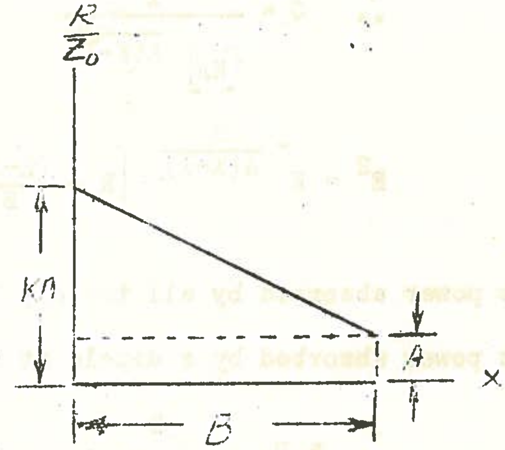

FIGURE 3

And from Figure 3

$$\frac{R}{Z_o} = KA - \frac{A(K-1)x}{B} \tag{15}$$

$$\text{Therefore at resonance } \frac{G}{Y_o} = \left(\frac{R}{Z_o}\right)^{-1} = \left[KA - \frac{A(K-1)x}{B}\right]^{-1} \tag{16}$$

which is the more general form of the hyperbolic curve(12).

It is important to note that over a 15% frequency band for most dipoles the conductances remain substantially constant (see Section (3)) hence Eq. 16 holds "off" frequency as well as "on".

$$\therefore -\int \frac{G}{Y_o}\,dx = -\int \frac{dx}{\left[KA - \frac{A(K-1)x}{B}\right]} = \frac{B}{A(K-1)} \log \left[KA - \frac{A(K-1)x}{B}\right]$$

$$\therefore \quad E^2 = C\, e^{+\log\left[KA - \frac{A(K-1)x}{B}\right]^{\frac{B}{A(K-1)}}}$$

$$= C \left[KA - \frac{A(K-1)x}{B}\right]^{\frac{B}{A(K-1)}}$$

For convenience set

$E^2 = 1$ when $x = 0$ or $P_{in} = \frac{E^2}{Z_o} = \frac{1}{Z_o}$ (P_{in} = input power to line)

$$\therefore \quad C = \frac{1}{[KA]^{\frac{B}{A(K-1)}}}$$

$$E^2 = K^{-\frac{B}{A(K-1)}} \left[K - \frac{(K-1)x}{B}\right]^{\frac{B}{A(K-1)}}$$

Let power absorbed by all the dipoles = P_S

Let power absorbed by a dipole at $x = P(x) = G\,E^2$

$$\therefore \quad Z_oP_S = \int_0^B Z_oP(x)\,dx = \int_0^B \frac{G}{Y_o}\,E^2\,dx$$

$$= K^{-\frac{B}{A(K-1)}} \int_0^B \frac{\left[K - \frac{x(K-1)}{B}\right]^{\frac{B}{A(K-1)}}}{A\left[K - \frac{x(K-1)}{B}\right]}\,dx$$

Solving we have

$$Z_oP_S = 1 - K^{-\frac{B}{A(K-1)}}$$

But $Z_o P_{in} = 1$

$$\therefore \quad Z_o P_{load} \equiv Z_o P_L = Z_o(P_{in} - P_D) = K^{-\frac{B}{A(K-1)}}$$

$$\text{i.e. fraction of power in load} = K^{-\frac{B}{A(K-1)}}$$

We will work out several other useful values

$$Z_o P(x) = \frac{G}{Y_o} E^2 = \frac{K^{-\frac{B}{A(K-1)}}}{A} \left[K - \frac{x}{B}(K-1)\right]^{\frac{B}{A(K-1)} - 1}$$

$$= \text{const. when } \frac{B}{A(K-1)} = 1$$

which from Eq. 15 gives us as the condition for constant power into the dipoles that the slope of the R/Z_o vs. x curve should = -1 which checks with Eq. 12.

$$Z_o P_o = \frac{1}{KA}$$

$$Z_o P_B = \frac{K^{-\frac{B}{A(K-1)}}}{A} = \frac{1}{A\, K^{\frac{B}{A(K-1)}}}$$

$$\frac{P_B}{P_o} = K^{1 - \frac{B}{A(K-1)}}$$

Total conductance G_S at input due to the dipoles "on" frequency.

$$\frac{G_S}{Y_o} = \int_o^B \frac{G(x)}{Y_o}\, dx = \int_o^B \frac{dx}{A\left[K - \frac{(K-1)x}{B}\right]}$$

$$= \frac{B \log_e K}{A(K-1)} = \frac{2.3\, B \log_{10} K}{A(K-1)}$$

$$\frac{G_{in}}{Y_o} = \frac{G_S + G_L}{Y_o}$$

where G_L is the load conductance
G_{in} is the conductance at input due to dipoles and load.

Fraction of power in load $= \frac{G_L}{G_{in}}$

Summing up

$$E^2 = K^{-\frac{B}{A(K-1)}} \left[K - \frac{x}{B}(K-1)\right]^{\frac{B}{A(K-1)}}$$

Fraction of power in load

$$\text{off frequency} = K^{-\frac{B}{A(K-1)}}$$

$$\text{on frequency} = \frac{G_L}{G_{in}}$$

$$G_{in} = G_L + G_S$$

$$\frac{G_S}{Y_o} = \frac{2.3\ B}{A(K-1)} \log_{10} K$$

$$\frac{P_B}{P_o} = K^{1 - \frac{B}{A(K-1)}}$$

If we make percentage of power in load the same "on" and "off" frequency then the load impedance is given by

$$\frac{R_L}{Z_o} = \frac{A(K-1)\left(\frac{1}{\text{Fraction of } P_L} - 1\right)}{2.3\ B \log_{10} K}$$

2. Optimum Design of Variable Frequency Array

In order to facilitate the proper choice of the function G(x) to be used we have computed the following table from the above formulae. The values in this table are for a thirty element array.

Percent Total Power in Load	K $(\frac{R_o}{R_B})$	A $(\frac{R_B}{Z_o})$	Max. & Min. Dipole Z $(Z_oKA)(Z_oA)$	Max. Power Variation between Dipoles off freq. (P_o/P_D)	Max. P. Variation between Dipoles on freq. (K)	R_L	Load End SWR on & off freq. (R_L/Z_o)	Input End SWR on freq. $(\frac{R_L\ P_L}{Z_o\ P_{in}})$
1. 16.7	1.0	16.1	$16.1Z_o$-$16.1Z_o$	6:1	1:1	$2.79Z_o$	2.79	2.15
2. 16.7	2.0	11.2	$22.4Z_o$-$11.2Z_o$	3:1	2:1	$2.79Z_o$	2.79	2.15
3. 16.7	4.0	7.49	$30Z_o$ - $7.5Z_o$	1.5:1	4:1	$2.79Z_o$	2.79	2.15
4. 16.7	6.0	5.8	$34.8Z_o$-$5.8Z_o$	1:1	6:1	$2.79Z_o$	2.79	2.15
5. 25	1.0	20.8	$20.8Z_o$-$20.8Z_o$	4:1	1:1	$2.16Z_o$	2.16	1.85
6. 25	2.0	14.5	2[illegible]Z_o - $14.5Z_o$	2:1	2:1	$2.16Z_o$	2.16	1.85
7. 25	3.0	11.5	$34.5Z_o$-$11.5Z_o$	1.33:1	3:1	$2.16Z_o$	2.16	1.85
8. 25	4.0	9.67	$38.7Z_o$-$9.7Z_o$	1:1	4:1	$2.16Z_o$	2.16	1.85
9. 37.5	1.0	29.3	$29.3Z_o$-$29.3Z_o$	2.67:1	1:1	$1.70Z_o$	1.70	1.57
10. 37.5	2.0	20.4	$40.8Z_o$-$20.4Z_o$	1.34:1	2:1	$1.70Z_o$	1.70	1.57
11. 37.5	2.67	17.37	$46.4Z_o$-$17.4Z_o$	1:1	2.67:1	$1.70Z_o$	1.70	1.57

As a compromise between a large power loss on the one hand and large power variations on the other, cases 5 - 8 were considered. We chose case 8 for further investigations because operating over a frequency band the antenna will be operated most of the time under off-frequency conditions.

Therefore, taking a 30-element array as a sample sufficiently long to indicate the actual conditions on still longer arrays, the following values have been calculated - on frequency, 1.7% higher in frequency and 7% higher in frequency:

1. S.W.R. and electrical degrees of the line impedance relative to a voltage maximum at each feed point - by means of an Admittance Chart. The S.W.R. along the line vs. dipole number (or distance along array) is shown in fig. 4.

2. The voltage at each dipole - by using a second chart (Chart 2) of voltage vs. electrical degrees from a maximum with S.W.R. as parameter. These voltages multiplied by the respective dipole conductances give the power radiated by each dipole. The square root of this curve (or the distribution of current) is shown in figs. 5 and 6.

3. The phase at each dipole - by using a third chart (chart 3) of phase deviation from the electrical degrees from a voltage maximum vs. electrical degrees, with S.W.R. as parameter. See fig. 7.

The assumption has been made in these calculations that 7% off frequency the dipole X = dipole R which is approximately correct - in any case the important value, dipole G, remains constant (see section III, 3).

Charts 2 and 3 were computed from the following equation for voltage on a transmission line.

$$E = E_{min}\sqrt{\cos^2\psi + K^2 \sin^2\psi}\ e^{j\tan^{-1}(K\tan\psi)}$$

where E is voltage at any point on the line
ψ is electrical degrees of **same** point from E_{min}, measured positive towards the generator.
$K \equiv S.W.R. = E_{max}/E_{min}$.

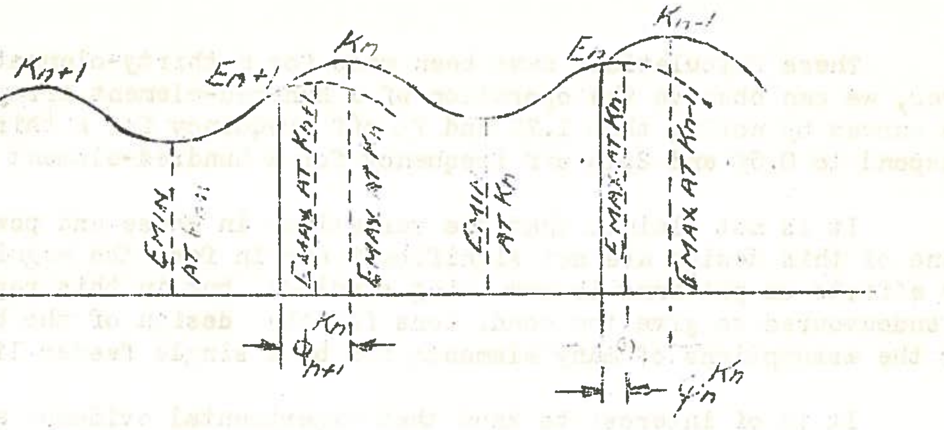

FIGURE 8

K_n = S.W.R. looking into junction of nth dipole (obtained from Admittance Chart).

$\phi_n^{K_n}$ = electrical degrees from E_n to E_{max} at S.W.R. = K_n (obtained from Admittance Chart).

$\phi_{n+1}^{K_n}$ = electrical degrees from E_{n+1} to E_{max} at S.W.R. = K_n (obtained from Admittance Chart).

Hence from voltage, electrical degree, and chart (2) we may obtain

$$\frac{E_n}{E_{n\,min}} \quad \text{and} \quad \frac{E_{n+1}}{E_{n\,min}} \quad \text{and} \therefore \quad \frac{E_{n+1}}{E_n}$$

If E_1 is taken = 1 all the E's may be found. From chart 3 we obtain α_n and α_{n+1} corresponding to $\phi_n^{K_n}$ and $\phi_{n+1}^{K_n}$, where $\alpha_n = \theta_n - \phi_n^{K_n}$. θ_n is the actual phase at that point in the wave relative to the phase at E_{max}.

$$\therefore \alpha_{n+1} - \alpha_n = (\theta_{n+1} - \theta_n) - (\phi_{n+1}^{K_n} - \phi_n^{K_n})$$

$$= (\theta_{n+1} - \theta_n) - \Delta\phi$$ where $\Delta\phi$ is the electrical degree difference between each dipole (i.e. 360° on frequency, 385° or 335° at limit of frequency band).

Hence taking $\theta_1 = 0$ and adding each value of $\alpha_{n+1} - \alpha_n$ we obtain the phase deviation from the desired value at the frequency considered.

These calculations have been made for a thirty-element array. However, we can observe the operation of a hundred-element array from these curves by noting that 1.7% and 7% off frequency for a thirty-element correspond to 0.5% and 2.1% off frequency for a hundred-element array.

It is not claimed that the variations in phase and power in an antenna of this design are not significant and in fact the magnitude of these effects on patterns is now being studied; but in this report we have endeavoured to give the conditions for the design of the best antenna under the assumptions of many elements fed by a single feeder line.

It is of interest to know that experimental evidence available to date appears to confirm the above considerations. Our own work on 90 mc/s and the simple resistor loaded line on 400 mc/s is in accord with this theory. Further, evidence available from the array work on 3000 mc/s (on fixed frequencies) seems also to bear out the above analysis.

III. DESIGN OF A 70 CM. VARIABLE FREQUENCY ARRAY:

1. General Considerations

For accurate height finding on enemy aircraft, an antenna is required to give a variable elevation beam (VEB) which is narrow in the vertical plane to avoid ground interference. The array to be described in this report is designed for that purpose. It consists of a vertical stack of one hundred horizontal end-fed, half-wave dipoles in front of a reflecting sheet. The stack is fed from one end with a variable frequency to give the required beam swing electrically. The antenna is fed by a single coaxial line and, for reasons to be given shortly, operates on a mean frequency of 400 mc/s.

No experimental results are included in this report as the antenna described is still being constructed and has therefore not been tested. However, the variable frequency beam swinging principle has been completely verified experimentally by an earlier V.E.B. operating on 90 mc/s, and the voltage distribution calculated for the array on and off frequency has been checked on a long open wire transmission line periodically loaded with resistors.

A 130 foot line (Z_o = 100 ohms) was set up and loaded at $5\lambda_o/2$ points with stubbed resistors of approximately (1000 + j0) ohms at f_o (where $f_o = c/\lambda_o$). The voltage across the resistors was measured when the line was fed with power at f_o = 406 mc/s, and f = 435 mc/s. The results are shown in Figure 9. It was not attempted here to obtain the best distribution but merely a rough correlation between theory and practice.

The earlier variable frequency V.E.B. referred to above consists of a vertical stack of twenty-four horizontal folded dipoles, spaced $\lambda/2$ from one another. The array was grouped into eight bays of three elements each, the bays being matched over the frequency band to eight shielded twin conductor transmission lines. The lines differed in length successively by $5\lambda/2$ instead of by the bay separation of $3\lambda/2$, so that a $\pm$ 7% frequency deviation gave a 13-1/2° beam swing.

It is in order to obtain greater accuracy of height finding and to operate closer to the horizon that the new antenna on a higher frequency is being designed. If the transmitted power and receiver gain are assumed the same for the two systems and, if the array lengths are the same, it can readily be shown that the range and coverage of the higher frequency system will be the same as that of the low frequency system. It was for this reason primarily that 400 mc. was adopted as the new frequency. Any substantially higher frequency would mean less receiver gain and probably a shorter antenna, (assuming one hundred dipoles a practical upper limit), resulting in either reduced range or horizontal coverage. If a sacrifice in either or both is acceptable, of course a much higher frequency might be more suitable.

2. General Construction

In order to obtain the proper phasing, the dipoles are spaced 0.7λ apart, and the coaxial line is zig-zagged so that the electrical length of line between dipoles is exactly λ, - see figure 10. A + 7% frequency variation will give therefore an 11° beam swing. End fed half wave dipoles, closely spaced to a reflecting screen are used to give the high impedance needed for a continuously fed array. (See Section II.) They are attached to the inner conductor of the coaxial feedline (Z_0 = 45 ohms) at alternate elbows, passing through polystyrene bushings as shown. As the feed line is mounted immediately behind the sheet metal reflecting surface, the dipoles are bent at right angles about $\lambda/12$ from their feed point so as to be parallel to the screen. Control of the individual impedances to maintain the proper voltage distribution along the array is obtained by varying the dipole diameters.

3. Design of Dipoles

In the last section it was shown that the dipoles of a thirty-element array to be evenly fed must have conductances from $1/38.7\ Z_0$ at the input end to $1/9.7\ Z_0$ at the load end. Any distribution which is not uniform will require these conductances to be changed. For example, consider a symmetrical tapered distribution with the power on the end dipoles 50% of the power on the centre ones. Their conductance must be 50% of the value at the centre. Or if the average dipole conductance is to remain the same to satisfy the analysis in Section II, the conductance of the centre dipole will have to be increased 41% and the end dipole decreased 29%. Thus the conductances for the array with a tapered distribution become $1/55\ Z_0$ for the input end and $1/13.7\ Z_0$ for the load end. These are very approximate figures only, as must be for an array subject to the large voltage variations shown in Figures 5 and 6.

All impedance calculations thus far are based on the assumption of a lossless transmission line. In actual practice there will be considerable attenuation along its 70λ length, so they will have to be modified. Keeping the input end dipole impedance constant, it will be necessary to reduce progressively the value of all others. As the attenuation constant for the line is not known exactly, this can best be done experimentally. The dipole conductance will then range from the 1/55 Z_o given above for the first one, to well below the 1/13.7 Z_o given for the thirtieth: 1/34 Z_o if a 6 db attenuation is assumed. It should be noted that much of the attenuation loss is at the expense of the power into the load, so it is not as serious a loss as might at first be expected.

For a one hundred-element array these values must be multiplied by 30/100. Thus the actual dipole impedances for a one hundred-element array using a 45 ohm coaxial line will vary from 510 to 8250 ohms, a variation of 16:1. Now the impedance of a full wave dipole is a function of the ratio of length to diameter. The free space resistance when the dipole has been adjusted to resonance varies from 105 ohms for L/D = 10 to 1750 ohms for L/D = 400.[1,2] (Half wave end fed dipoles are then 52 ohms for L/D = 5 and 875 for L/D = 200). This is the 16:1 variation required but the magnitudes are 1/10 the required values.

As in practice, however, the dipoles are not isolated in space but grouped in line in front of a reflecting screen, the mutual impedances must be considered. Replacing the screen by the dipole images, the problem is to find the mutual impedances between dipoles:

D_o', D_1, D_1', D_2, D_2'..... D_n, D_n' and D_o.

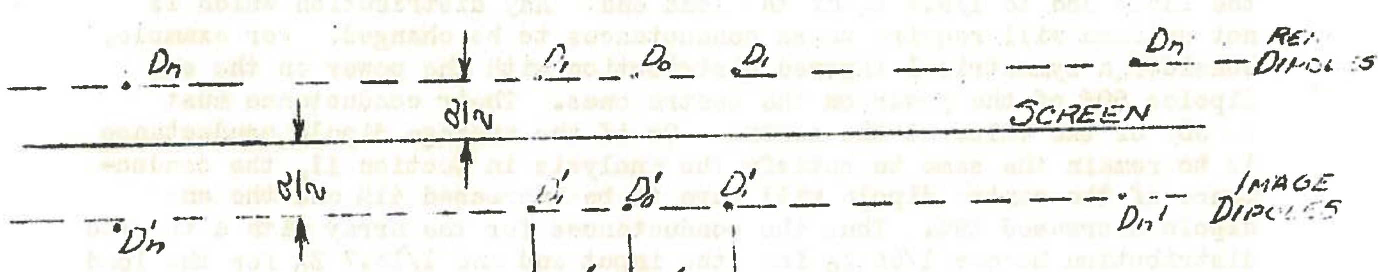

FIGURE 11

1. J.M.C. Scott, "Some Results of Modern Antenna Theory".
2. Schelkunoff, "Theory of Antennas of Arbitrary Size and Shape" Proc. I.R.E., Vol. 29, pp. 494-521, Sept. 1941.

When the dipoles are λ/4 or closer to the screen it is found that all dipoles beyond D_2 and D_2' may be omitted without a substantial loss in accuracy. The resultant admittance of dipole D_o then becomes:

$$Y_o = Y_{oo} - Y(D_oD_o') + 2\left[Y(D_oD_1) + Y(D_oD_2) - Y(D_oD_1') - Y(D_oD_2')\right]$$

$$= Y_{oo} - Y(a) + 2\left[Y(b) + Y(2b) - Y(\sqrt{a^2+b^2}) - Y(\sqrt{4a^2+b^2})\right]$$

where Y_o = total admittance of D_o

Y_{oo} = self admittance of D_o

$Y(D_pD_q)$ = mutual admittance between dipoles D_p and D_q

The values of the mutual admittances for resonant full wave dipoles driven in phase, as a function of dipole spacing are given by Scott.[1] For half wave dipoles they must be divided by two. Values of the resultant impedance of a half wave resonant dipole of L/D = 200 (which in free space has an impedance of 830 + j0 ohms) in a stack spaced 0.7 λ from one another are shown below as a function of spacing from the screen.

Note: a 70 cm. L/D = 200 corresponds to a 1/16" D, λ/2 dipole.

Spacing from Screen	Z_o (L/D = 200)
0.250 λ	870 + j 93 ohms
0.187	1030 - j 250 ohms
0.125	1370 - j 650 ohms
.093	2830 - j 1660 ohms
.075	4840 - j 350 ohms
.062	6900 + j 2700 ohms

It is believed in practice the reactance can be tuned out by adjusting the dipole length.

Schelkunoff[2] shows in graph form the resistance and reactance of a full wave cylindrical dipole as a function of frequency. These are for dipoles in free space, so are only self impedance values. However, it is believed that an approximate picture of the array dipoles off frequency will be had by considering the behaviour off frequency of a free space dipole with the same on frequency resistance. The following

[1,2] loc. cit.

table is given therefore showing the approximate resistance, reactance and conductance 7% off frequency for dipoles having on frequency resistance of 600 to 10,000 ohms.

Resistance on Freq.	Conductance on Freq.	Resistance 7% off Freq.	Reactance 7% off Freq.	Conductance 7% off Freq.
600 ohms	1.67×10^{-3} mhos	550 ohms	170 ohms	1.66×10^{-3} mhos
1,000 "	1.00×10^{-3} "	860 "	320 "	1.02×10^{-3} "
3,000 "	3.33×10^{-3} "	2400 "	1400 "	3.10×10^{-3} "
6,000 "	1.67×10^{-4} "	3400 "	3500 "	1.43×10^{-4} "
10,000 "	1.00×10^{-5} "	4500 "	5500 "	0.89×10^{-5} "

The table shows clearly that although the resistance varies widely over the frequency band, the conductance remains fairly constant. It is this together with its high impedance which makes the half wave end fed dipole so suitable for the element of a long continuously fed variable frequency array.

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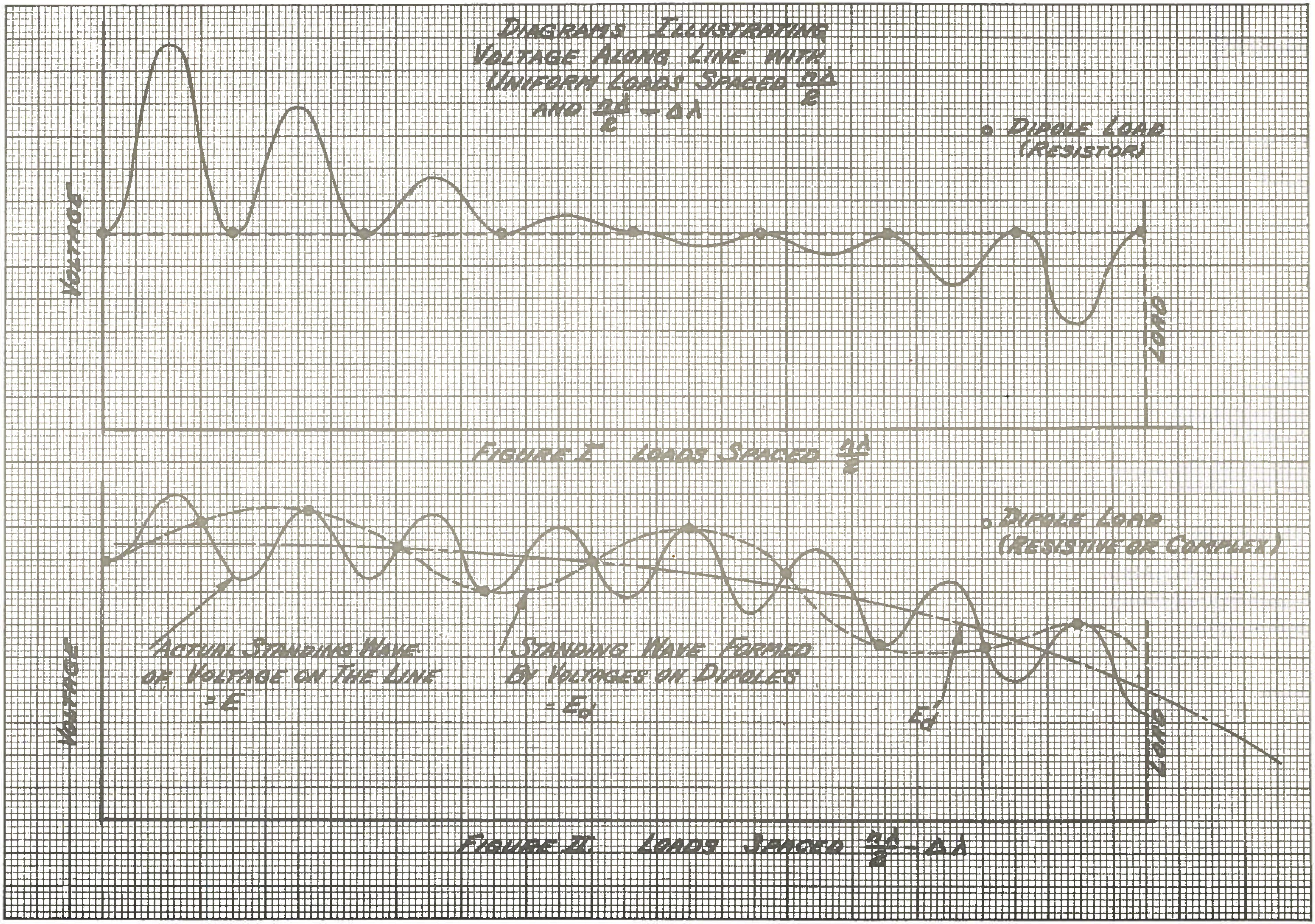
N. Z. Alcock



DIAGRAMS ILLUSTRATING
VOLTAGE ALONG LINE WITH
UNIFORM LOADS SPACED $\frac{n\lambda}{2}$
AND $\frac{n\lambda}{2} - \Delta\lambda$
DIPOLE LOAD
(RESISTOR)
VOLTAGE
LOAD
FIGURE I LOADS SPACED $\frac{n\lambda}{2}$
DIPOLE LOAD
(RESISTIVE OR COMPLEX)
ACTUAL STANDING WAVE
OF VOLTAGE ON THE LINE
= E
STANDING WAVE FORMED
BY VOLTAGES ON DIPOLES
= E_d
E_d'
VOLTAGE
LOAD
FIGURE II. LOADS SPACED $\frac{n\lambda}{2} - \Delta\lambda$

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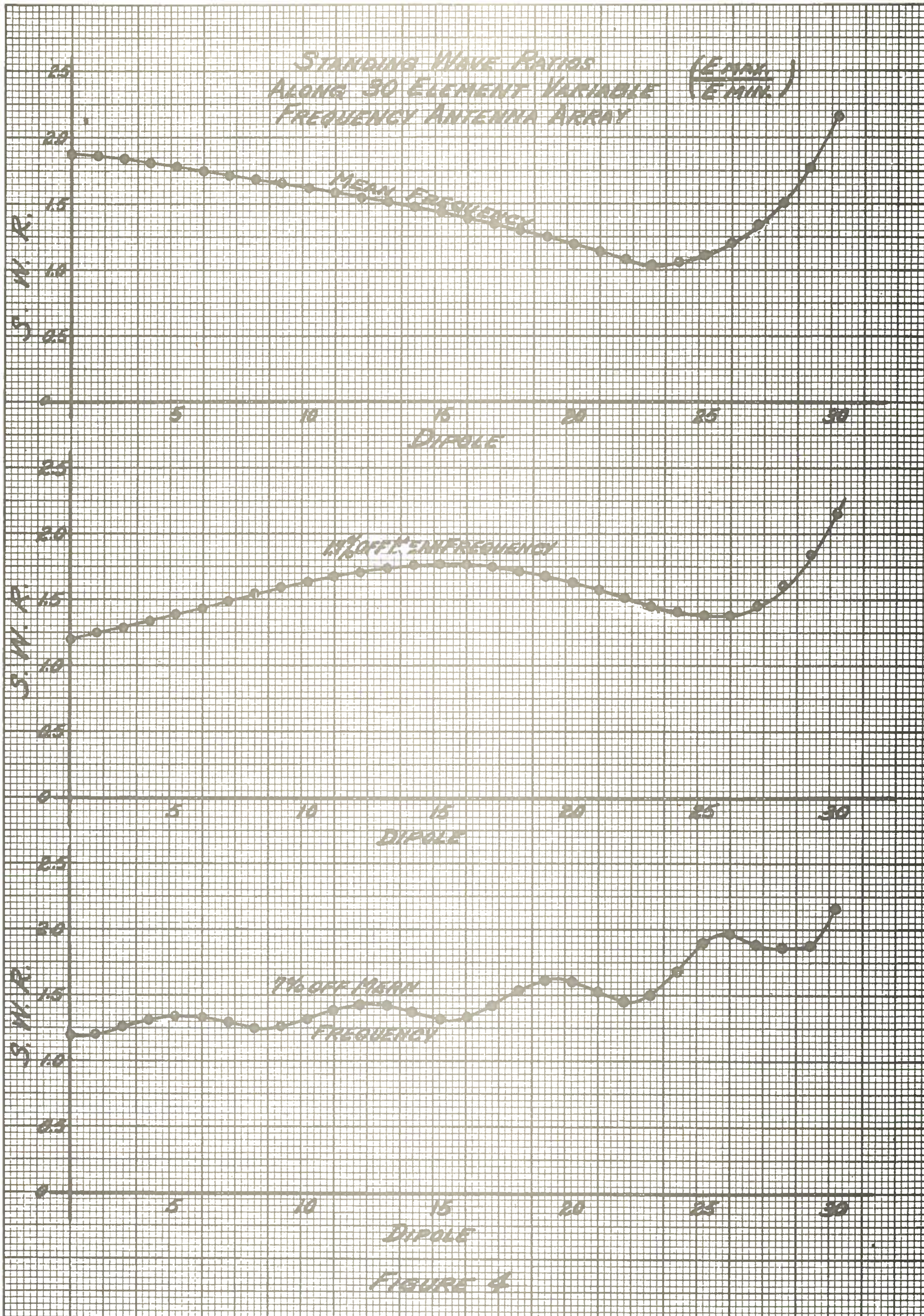

FIGURE 4

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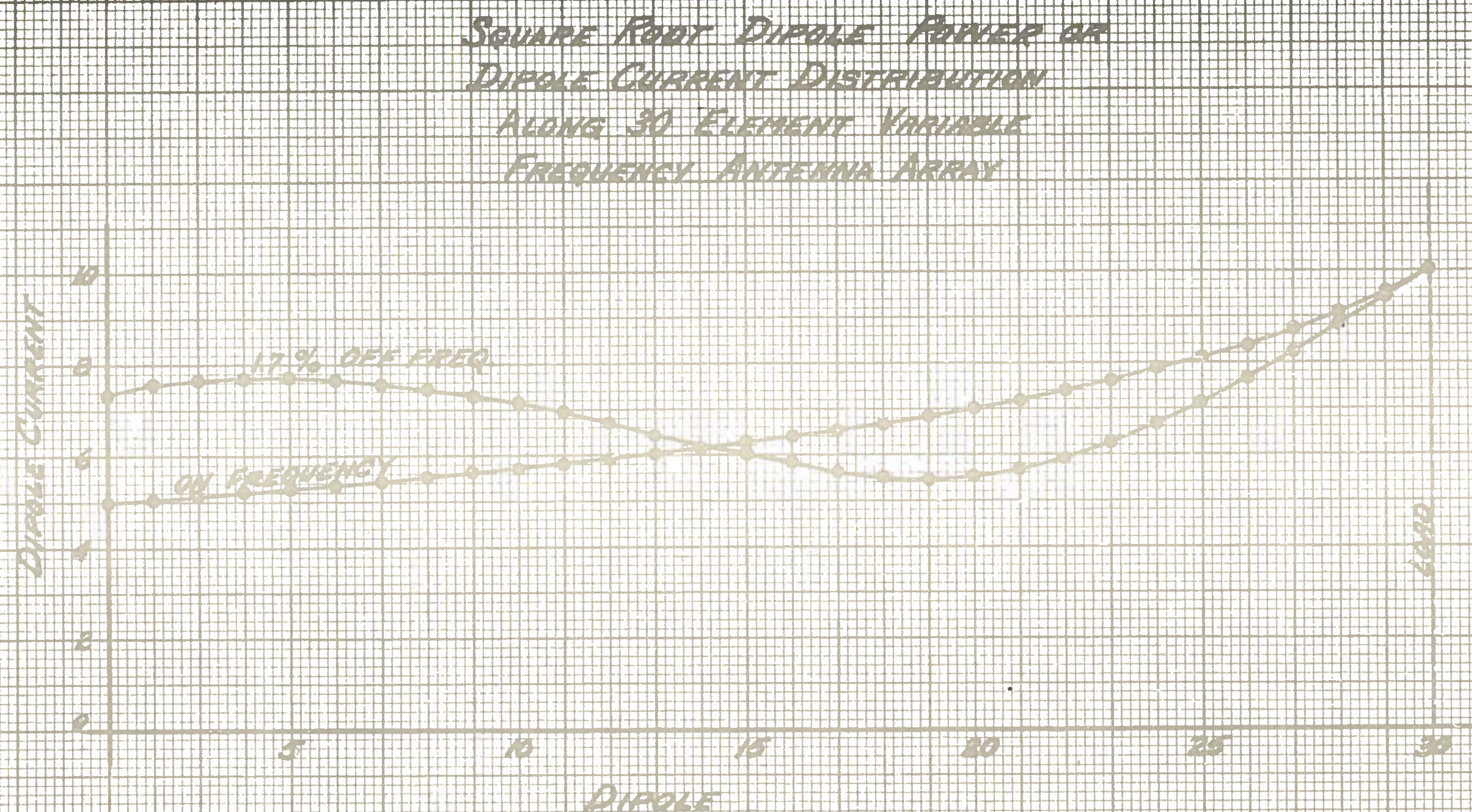

FIGURE 5

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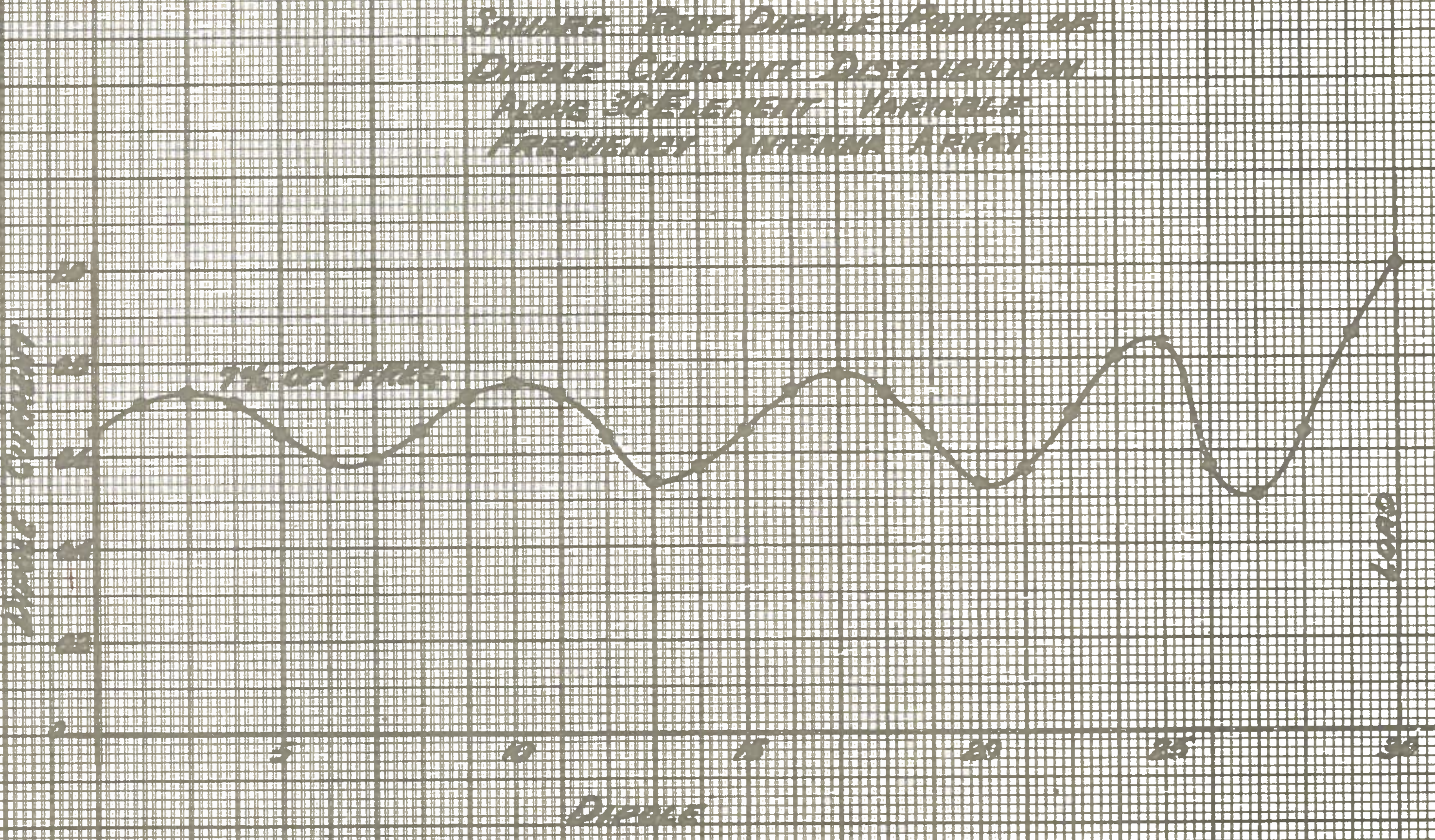

FIGURE 6

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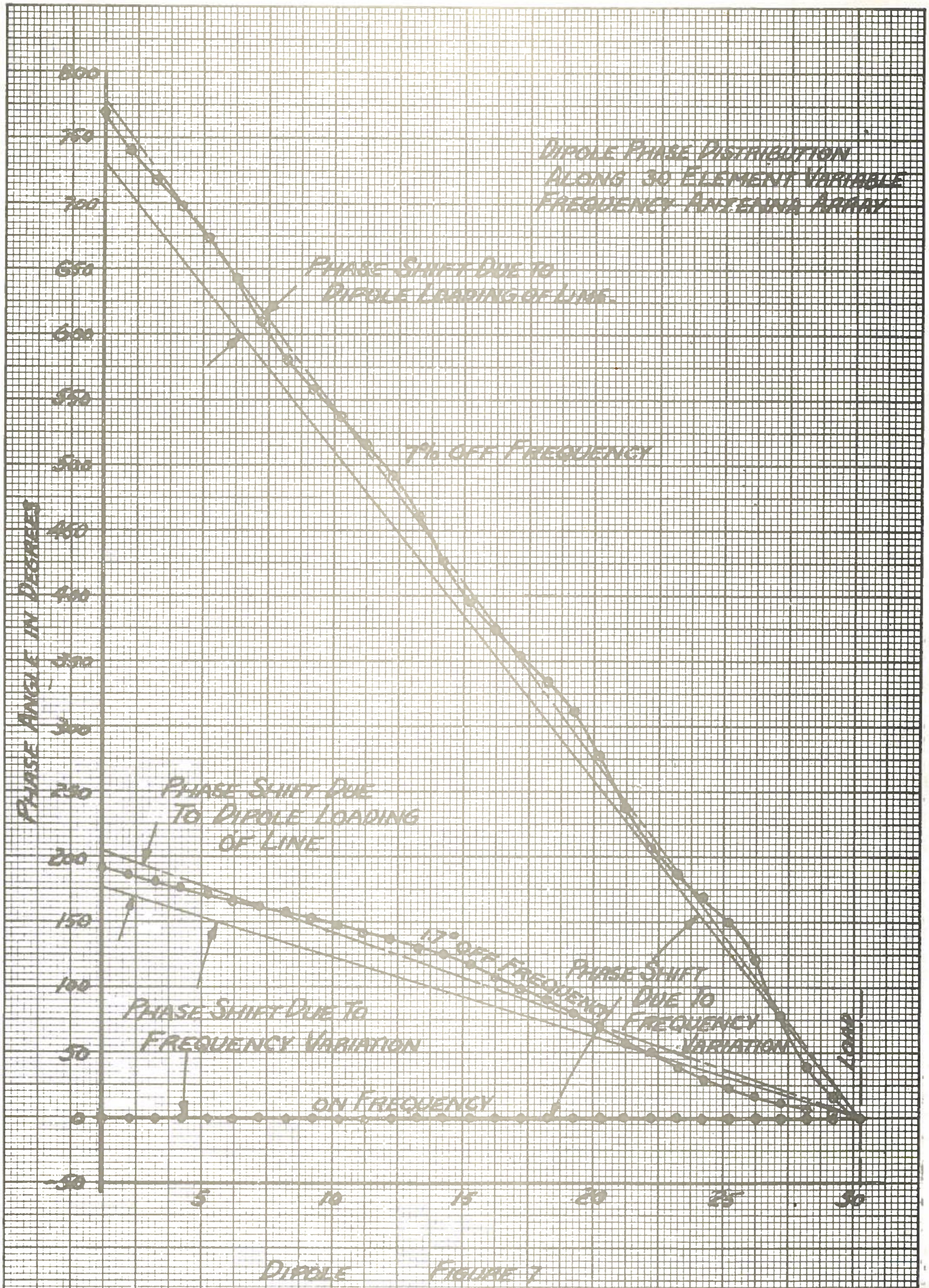

DIPOLE PHASE DISTRIBUTION
ALONG 30 ELEMENT VARIABLE
FREQUENCY ANTENNA ARRAY
PHASE SHIFT DUE TO
DIPOLE LOADING OF LINE
7% OFF FREQUENCY
PHASE ANGLE IN DEGREES
800
750
700
650
600
550
500
450
400
350
300
250
200
150
100
50
0
-50
PHASE SHIFT DUE
TO DIPOLE LOADING
OF LINE
1.7% OFF FREQUENCY
PHASE SHIFT
DUE TO
FREQUENCY
VARIATION
PHASE SHIFT DUE TO
FREQUENCY VARIATION
ON FREQUENCY
LOAD
5
10
15
20
25
30
DIPOLE

FIGURE 7

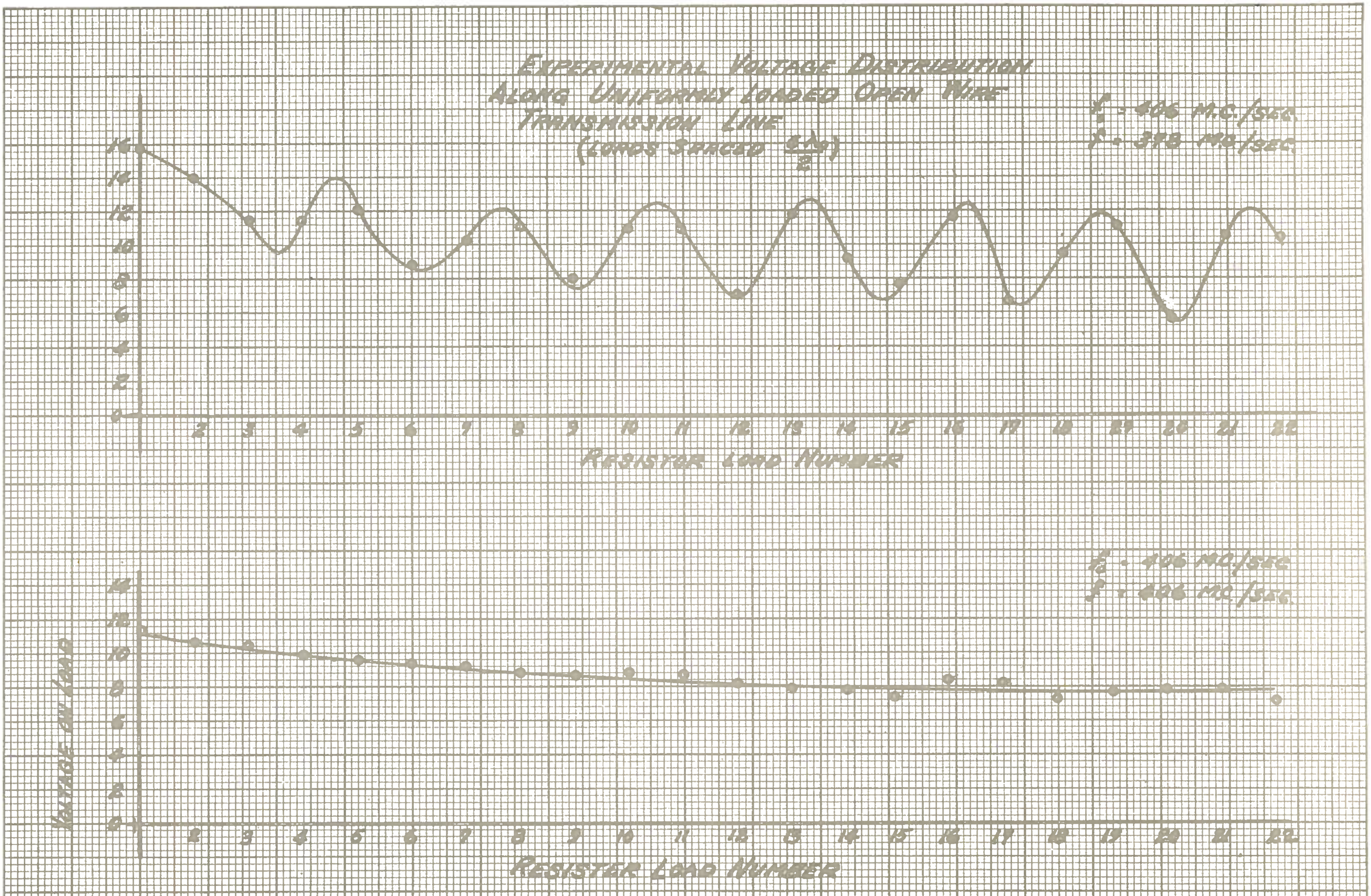

FIGURE 9

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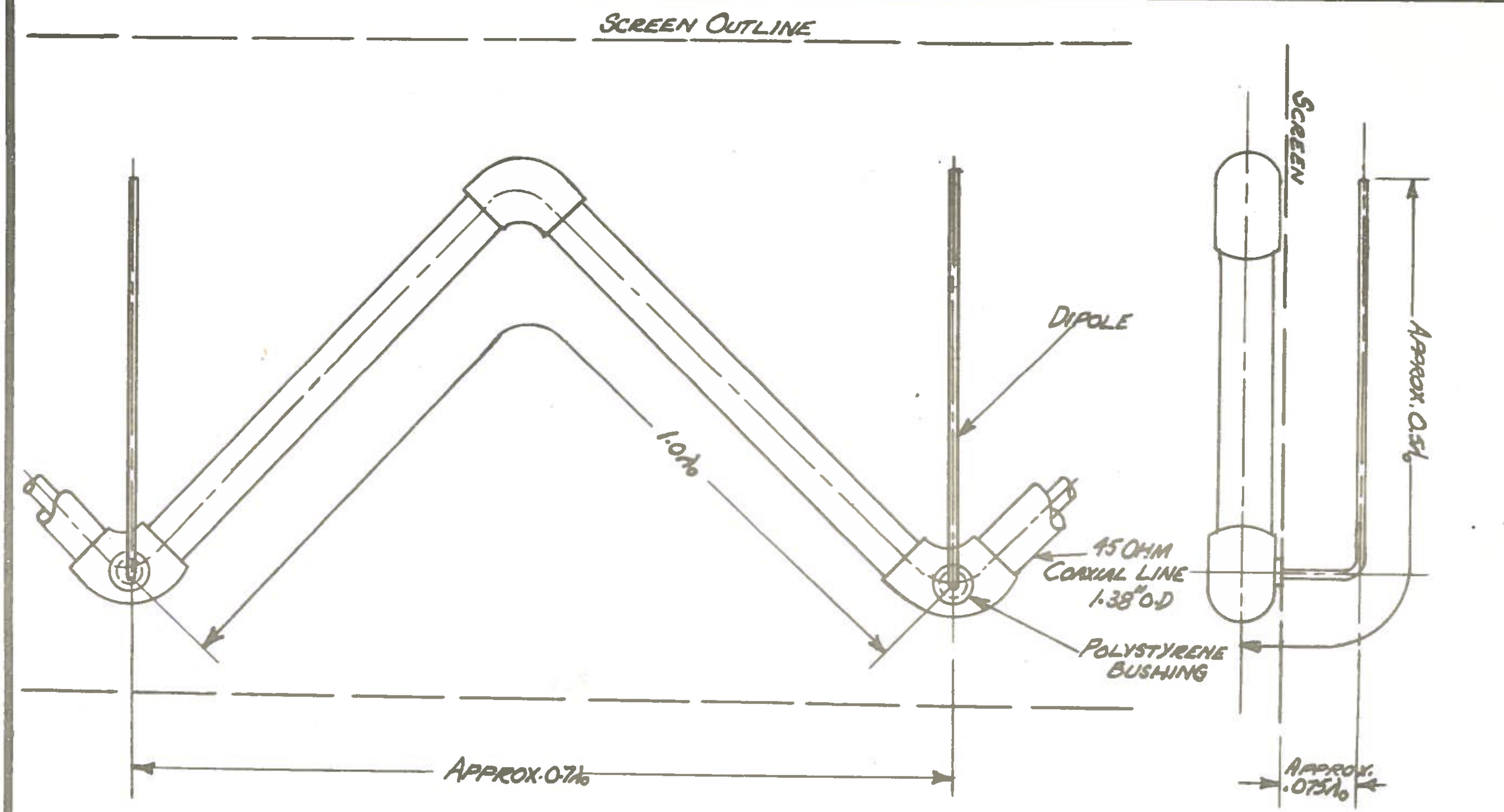

NOTE: λ_0 = MEAN WAVE LENGTH

FIGURE 10.

ITEM	PART NO.	QUAN.	MAT'L	DESCRIPTION

DRAWN BY D.C.	DATE 14-1-43	SUPERSEDES
CHECKED	DATE	SCALE 1/4 SCALE
ENG. APPROV. nza	DATE	FINISH.

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NAME FIGURE 10.	DWG. NO. PRA-72 388

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