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Technical Translation TT-626

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PREFACE

This translation, dealing with the role of radiation in heat conduction, is of great interest to the Division of Building Research in its work on heat flow in building materials and constructions. The particular aspect of heat flow with which it deals is a specialized one but is most pertinent to the study of transient methods for the measurement of thermal properties which are currently of decided interest to a number of laboratories.

Thanks are due to Mr. H.A.G. Nathan of the Translations Section, National Research Council, who translated this article.

OTTAWA,
July 1956.

N.B. Hutcheon,
Assistant Director.

THE CONTRIBUTION OF RADIATION TO HEAT CONDUCTION

Summary

In this article an attempt is made to explain the processes involved in the contribution of radiation to the conduction of heat rather than to deduce exact formulae for this. Such formulae have been worked out extensively by the present author and have been published elsewhere*. The present article deals primarily with the non-stationary processes, showing that some of these make a smaller radiation contribution and some a larger one than the stationary process. The larger contribution is discussed here for the first time.

It is a known fact that radiation contributes to the heat transfer in solid substances (crystals, glass) and light-weight insulation materials (fibrous materials containing a great deal of air). In these latter materials the density may be easily varied, with the radiation decreasing as the density increases (Fig. 1). This had been calculated before but always for a linear temperature curve and then only for the stationary state. The greatest advance in this respect was made by the astronomers. However, the effect of radiation on non-stationary processes was unknown. Furthermore, thermal conductivity was first measured only in the stationary state, a tedious method. One or two days are required to establish this state. In order to reduce lateral flow heated guard rings are used with a view to simplifying the calculation. This heating must be adjusted until the lateral flow decreases to an admissible minimum. After each adjustment it is necessary to re-establish the stationary state. Consequently, such a measurement usually takes one week.

*van der Held, E.F.M. Appl. Sci. Res. A3: 237, 1952 and A4: 77, 1953. Allg. Wärmtechnik 4: 236, 1953.

Therefore, rapid methods of measurement had to be evolved and the non-stationary methods were thought to be the most suitable ones.

Many methods are applied as follows: heating with a heated wire, heating emitted from a flat surface, forcing a sinusoidal temperature variation on a flat plane or a cylindrical plane. Sometimes there was agreement with the results obtained from measurements with the stationary method, but in many cases this was not so. Moreover one non-stationary method showed greater deviations than another, sometimes even deviations with a different sign, depending on the method applied. When the accuracies attained by the experimentalists are examined it is found that the deviations are greater than the errors of measurement. The greatest deviations were obtained for substances which can easily be penetrated by radiation. It is thus clear that the deviations are due to the radiation contribution.

A number of questions arise here:

1. In which case do all the methods of measurement give the same result?
2. When there are deviations which method will give a useful result?
3. Which quantities must be known in order to calculate the deviations?
4. What is the form of this calculation?

These questions will be answered in order.

1. Only when the radiation contribution is sufficiently small. However, it will be shown that in some methods of measurement the radiation contribution is many times greater than that obtained from a stationary method. Therefore, the method of measurement giving the greatest deviations should be applied. Then, if no deviations are obtained for a particular substance, the

result may be relied upon. However, difficulties may be encountered for substances which do give deviations.

2. The frequently applied stationary method, in which a difference in temperature is produced across the specimen which is in the form of a flat plate and the heat flow is measured. This method may yield values that are too low if the sample investigated is too thin and a sufficiently thick sample is chosen as the standard. In order to explain this difference we examine an air space with walls having different temperatures T_1 and T_2 . If these temperatures do not differ too much, it may be assumed that the amount of heat transmitted by radiation is proportional to the difference in temperature, viz.,

$$Q_s = \alpha (T_1 - T_2) \text{ cal./sq.cm.sec.}$$

If the air is stationary, then the amount of heat transmitted by conduction is proportional to the difference in temperature and inversely proportional to the thickness d of the air space. The factor of proportionality λ is called the thermal conductivity of the material in question, in this case air. Thus

$$Q_c = \frac{\lambda}{d} (T_1 - T_2) \text{ cal./sq.cm.sec.}$$

Assuming now that the radiation is unknown, then an apparent thermal conductivity, λ_s , is obtained. It is related to the known quantities as shown by the formula

$$Q_{\text{total}} = \left(\frac{\lambda}{d} + \alpha \right) (T_1 - T_2) = \frac{\lambda_s}{d} (T_1 - T_2)$$

or

$$\lambda_s = \lambda + \alpha d.$$

Therefore, for very thin spaces $\lambda_s = \lambda$, but with increasing thickness the "radiation contribution" increases linearly with the thickness.

If the space is filled, e.g. with a lightweight insulation material, the radiation is partially suppressed, scarcely at all for very small thicknesses but if the thickness is great, the radiation is suppressed sufficiently for the apparent thermal conductivity to approach a constant value (Fig. 2). This maximum will now be calculated in a simple manner and a clear picture of the phenomenon will thus be obtained.

If a parallel bundle of monochromatic rays strikes perpendicularly a substance which absorbs homogeneous radiation and which has a flat surface, then a portion will be reflected and a portion I_0 will penetrate the material and will be absorbed in this process (absorption coefficient β). Then the intensity of the radiation at distance x from the surface is reduced to $I = I_0 \exp(-\beta x)$. The mean depth of penetration of the radiation is calculated next. The fraction $f dx = \exp(-\beta x) \beta dx$ is absorbed between x and $x + dx$. The mean depth of penetration d then is

$$\int_0^{\infty} x f dx = \int_0^{\infty} x \beta \exp(-\beta x) dx = 1/\beta.$$

In the case of diffuse irradiation of a medium with constant index of refraction throughout, the mean depth of penetration becomes $2/3\beta$, where the word 'mean' indicates the mean distance between the point where the radiation is absorbed and the surface.

In the case of radiative equilibrium an identical fraction $f dx$ from the layer between x and $x + dx$ participates in the radiation striking the surface perpendicularly from the inside. Therefore, the mean depth of emission, i.e., the point where on the average the radiation which eventually will leave the surface originates, is again $1/\beta$ for normal emission and $2/3\beta$ for diffuse emission.

In the absence of radiative equilibrium the depth of radiation remains the same, but the depth of emission now appears to be a function of the temperature distribution. Only in the presence of a constant temperature gradient does the emitted

radiation seem to have the intensity corresponding to the temperature at the point of an emission depth which equals the depth of penetration. The emission depth becomes greater or smaller than the depth of penetration, depending on whether the temperature over the first section lies above or below the tangent to the temperature curve at the surface.

The radiation phenomena within the material are now examined in greater detail. The radiation which at constant temperature gradient in the material passes an arbitrary surface will on the average come from a surface at a distance of $2/3\beta$ at the one side and will on the average be absorbed at a distance of $2/3\beta$ at the other. At the point of the imaginary plane it just seems as if exchange of radiation takes place here at a distance of $4/3\beta$ and between two absolutely black surfaces with temperatures equal to the temperatures at that point. The difference in temperature is $\frac{4}{3\beta} \frac{dT}{dx}$. The rate of flow is thus:

$$- \frac{4}{3} \int_0^\infty \frac{1}{\beta} \frac{\delta I}{\delta T} d\ell \frac{dT}{dx}.$$

The conduction alone contributes the portion $-\lambda \frac{dT}{dx}$ to the total rate of flow $-\lambda_s \frac{dT}{dx}$. Hence the radiation contribution to the heat conduction is

$$\frac{4}{3} \int_0^\infty \frac{1}{\beta} \frac{dI}{dT} d\ell.$$

Neither the change in the speed of propagation of the radiation in the material nor the dispersion has been taken into account as yet. If, according to Clausius, I is multiplied by n^2 (n = index of refraction) and β is replaced by $\beta + \sigma = \epsilon$ (σ = dispersion coefficient, ϵ = extinction coefficient), the correct formula for the radiation contribution in the stationary case is obtained:

$$\frac{4}{3} \int_0^\infty \frac{n^2}{\epsilon} \frac{\delta I}{\delta T} d\ell.$$

This rough argument gives some insight into the phenomenon but does not fit the non-stationary case. Furthermore, it is

not known how much the radiation contribution is if a tri-dimensional temperature gradient is involved. The deduction of the following formulae is given elsewhere* and has therefore been omitted here. It will suffice merely to mention and explain it. The differential equation of Fourier is now replaced by two formulae.

In the complete Fourier equation (heat balance):

$$C_p \rho \frac{\delta T}{\delta t} = \lambda \Delta T + 4 \int_0^\infty n^2 \beta (J - I) d\ell, \quad (I)$$

or, in words, the heat ($\lambda \Delta T$), supplied to an element of volume per unit of volume by the pure conduction, increased by the difference between the radiation absorbed and that emitted for this element of volume per unit of volume (integral term) is converted into a temperature increase ($C_p \rho \frac{\delta T}{\delta t}$).

In the above formula

C_p = specific heat at constant pressure,

ρ = specific mass,

T = temperature,

t = time,

λ = pure thermal conductivity (hence without radiation contribution),

Δ = Laplace's operator = $\frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2} + \frac{\delta^2}{\delta z^2}$,

n = index of refraction,

β = absorption coefficient,

$I d\ell$ = radiation emitted by an absolutely black surface per unit of surface and time between the wavelengths ℓ and $\ell + d\ell$ at an absolute temperature T ,

$4 I d\ell$ = omnidirectional radiation at temperature T ,

$4 J d\ell$ = present omnidirectional radiation for wavelengths between ℓ and $\ell + d\ell$.

*The answer to question 4 may be found in van der Held, E.F.M. Appl. Sci. Res. A3: 237, 1952 and A4: 77, 1953. Allg. Wärmetechnik 4: 236, 1953.

The second formula for the relationship between J and I is

$$J = \int_{(V)} (\beta I + \sigma J) \frac{\exp(-\epsilon r)}{4 \pi r^2} dV + \int_{(O)} \left\{ eI + (1 - e) J_1 \right\} \frac{\exp(-\epsilon r)}{4 \pi r^2} \cos \gamma dO, \quad (II)$$

or, in words, the omnidirectional radiation for wavelengths between ℓ and $\ell + d\ell$ ($4Jd\ell$) in an element of volume emanates from other elements of volume where radiation is being produced (βI part of the first integral) or being dispersed (σJ part of the first integral) and from surfaces where radiation is being produced (eI part of the second integral) or being dispersed ($(1 - e) J_1$ part of the second integral). These radiations are weakened by absorption and dispersion (factor $\exp(-\epsilon r)$) and are distributed over steadily increasing spheres,

$$\left(\frac{1}{4 \pi r^2} \right).$$

In the above formula

- σ = dispersion coefficient,
- $\epsilon = \beta + \sigma$ = extinction coefficient,
- r = distance between an arbitrary element of volume dV and that under consideration,
- e = emissivity of the adjacent surfaces,
- J_1 = radiation striking an element of the adjacent surface,
- r = distance between surface element dO and the element of volume under consideration,
- γ = angle which this r makes with the normal to the surface element dO .

The second integral of equation II makes no appreciable contribution to J for the phenomenon being considered at a distance from the surfaces. Then by means of equation (II), equation (I) may be expressed in the following form:

$$C_p \rho \frac{\delta T}{\delta t} = A_1 \Delta T + A_2 \Delta^2 T + A_3 \Delta^3 T + \dots$$

where

$$A_1 = \lambda + \frac{4}{3} \int_0^\infty \frac{n^2}{\varepsilon} \frac{\delta T}{\delta T} d\ell = \lambda_S ,$$

$$A_2 = \frac{4}{5} \int_0^\infty \frac{n^2}{\varepsilon^3} \left(1 + \frac{5}{9} \frac{\sigma}{\beta} \right) \frac{\delta I}{\delta T} d\ell ,$$

$$A_3 = \frac{4}{7} \int_0^\infty \frac{n^2}{\varepsilon^5} \left(1 + \frac{14}{15} \frac{\sigma}{\beta} + \frac{7}{27} \frac{\sigma^2}{\beta^2} \right) \frac{\delta I}{\delta T} d\ell ,$$

etc., and where Δp is the p times application of the Laplace operator.

At first glance this formula appears to be rather hopeless, but it seems possible to present a number of solutions, among others by the method of separation of variables.

For example, if it is a case of differences in cooling and if $T = G(x, y, z) \Theta(t)$, then the differential equation requires that $\frac{1}{\Theta} \frac{d\Theta}{dt}$ be constant, say $-k$ (negative because fading processes are involved). A solution may then be found by putting at the same time $\Delta G/G = -C$, a second constant. C must then satisfy the equation

$$C_p \rho k = \lambda C + 4 \int_0^\infty n^2 \frac{\beta \varepsilon \psi\left(\frac{C}{\varepsilon^2}\right)}{\beta + \sigma \psi\left(\frac{C}{\varepsilon^2}\right)} \frac{\delta I}{\delta T} d\ell ,$$

where

$$\psi(x) = 1 - \frac{\tan^{-1} x^{\frac{1}{2}}}{x^{\frac{1}{2}}} .$$

It will be noted that $0 \leq \psi \leq 1$ and $\psi = 0$ for $x = 0$ and $\psi = 1$ for $x = \infty$. From the limiting conditions eigenvalues of k and C are obtained and the problem may be solved with eigenfunctions thus obtained. As the rank increases k increases continuously and so does λC . At the same time the integral approaches a limiting value. As a result the radiation contribution for eigenfunctions with high ranks may be neglected compared with the pure conduction. These thus behave like the terms of a Fourier series but with a

lower heat conduction (i.e., the pure heat conduction) than those of the stationary process (Fig. 3).

An example in which the radiation contribution is greater than that in the stationary case is obtained if $\frac{1}{\Theta} \frac{d\Theta}{dt} = +k$ is chosen, e.g. accelerated heating. If now $\Delta G/G = +C$, then C must satisfy

$$C_p \rho k = \lambda C + 4 \int_0^\infty \frac{n^2 \beta \epsilon X \left(\frac{C}{\epsilon^2} \right)}{\beta - \sigma X \left(\frac{C}{\epsilon^2} \right)} \frac{\delta I}{\delta T} d\epsilon ,$$

where

$$X(x) = \frac{1}{2x^{\frac{1}{2}}} \ln \frac{1+x^{\frac{1}{2}}}{1-x^{\frac{1}{2}}} - 1.$$

It can be seen that $X \geq 0$ and $X(0) = 0$ and $X(1) = \infty$. Furthermore, X alone is real for $0 \leq x \leq 1$. As k is increasing the integral is assuming high values such that $X\left(\frac{C}{\epsilon^2}\right) < \frac{\beta}{\sigma}$ will be satisfied by this equation, while C will certainly be smaller than ϵ^2 .

Let us take the case that $\beta \ll \sigma$, thus a substance with small absorption and great dispersion (this need only be the case for a region of small wavelengths), then, for sufficiently large k , C approaches the constant value $3 \epsilon^2 \beta / \sigma \cong 3 \sigma \beta$ for the wavelength where this quantity is at a minimum. $C_p \rho k$ then is large with respect to λC and the radiation contribution is practically identical with $C_p \rho k / C = C_p \rho k / 3 (\sigma \beta)_{\min.}$. If this result is compared with the radiation contribution in the stationary case, it becomes evident that for this non-stationary process the radiation contribution may be many times that in the stationary case.

Comparison of two materials with identical pure conduction, and, in the stationary case, identical radiation contribution, shows that the material with low absorption but high dispersion has the greatest increase in the radiation contribution, i.e., in a measurement with definite non-stationary methods results are obtained which differ greatly from those obtained by means of a

stationary method. An example of this is a fine crystalline rock consisting chiefly of pure quartz. The difference will be great if the temperatures exceed 150°C.

Even by simple reasoning such a conclusion may be reached. For example, if the surface of such a piece of rock is suddenly subjected to a high temperature a radiation field will form directly in the material. This radiation field is far ahead of the pure conduction, and, immediately begins to heat the rock at a distance from the surface a long time before the pure conduction can take effect.

In addition to the solution discussed, it was possible to calculate the temperature distribution over a point source, a line source, and an extended source by means of the theory of perturbations. This will not be discussed in detail here, but it should be noted that this method, which depends on the non-stationary method of heating with an electric heating wire (method of Stålhane and Pyk), produces the same radiation contribution as the stationary method does for sufficiently thick layers.

It would be interesting to study the surface phenomena more closely. However, this will only be done in the stationary state in the unidimensional case. Therefore, it is assumed that the phenomena take place at an imaginary plane perpendicular to the direction of maximum temperature drop inside the material. It can be proved that $J = I$ at a distance from the surface. A greater amount of radiation reaches the plane from the region of higher temperature than from that of lower temperature. If the radiation from the region with the higher temperature be denoted by the index + and that from the region with the lower temperature by the index -, then $J^+ = I + \Delta I$ and $J^- = I - \Delta I$. If the temperature remains unchanged, then the rate of heat flow will be lower at the surface than below it. This is impossible in the stationary case and the temperature in the material is going to drop. In order to maintain the initial temperature distribution at a distance from the

boundary, the temperature of the boundary must be increased until radiation + conduction at the surface are equal to the rate of flow everywhere along the boundary and particularly to that at a distance from the boundary. This seems to indicate that there is additional resistance at the boundary which may be taken into account by introducing a transfer coefficient α such that the rate of flow, $-\lambda_s \frac{\delta\theta}{\delta x}$, at a distance from the boundary is equal to $\alpha\theta_0$, where θ_0 denotes the effective increase in temperature at the boundary; α is a complex function of β , σ , λ , T and ϵ (answer to question 3).

With this the stationary experiments may be corrected in order to calculate the total heat conduction in thick layers. The heat conduction in kapok with air and kapok with hydrogen (as a buoyant gas) as a function of the density is shown as an example in Fig. 4.

These investigations have shown that discretion must be used in selecting the method of measurement. It is suggested that the latter be adapted to the purpose for which the heat conduction is measured. If stationary processes are involved the methods employing flat plates (stationary) and the heating wire (Stålhane and Pyk, non-stationary) are suitable. For materials which are used at high temperatures and with variable temperature fields, other non-stationary methods may be considered with a view to obtaining an idea of the effect of radiation contribution. Metals are substances whose radiation contributions may be neglected under all circumstances. Therefore, measurements on metals may be carried out by any reliable method.

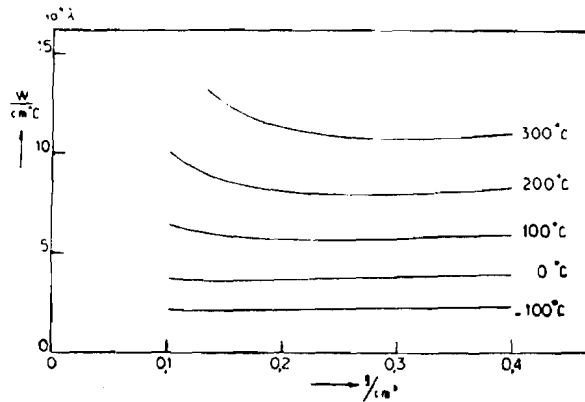


Fig. 1

The thermal conductivity of rock wool as a function of density and temperature.

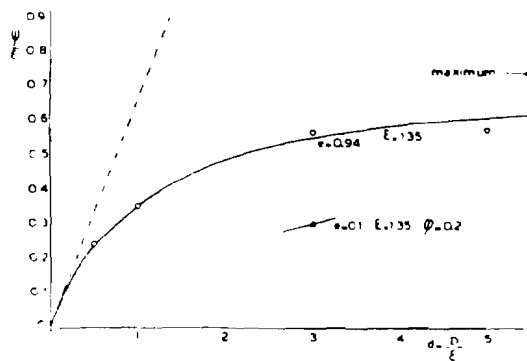


Fig. 2

The contribution of radiation to heat conduction as a function of layer thickness, d , for kapok plucked apart. O = measurements with "black" walls, emissivity 0.94. Δ = measurement with aluminium walls. ψ = proportionality factor of the radiation contribution to very thick layers.

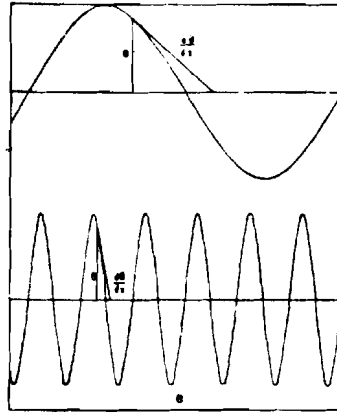


Fig. 3

A = Fourier term with low rank; B = Fourier term with high rank.
It is evident that the ratio of pure conduction (proportional to $d\Theta/dx$) to radiation exchange (proportional to Θ) is greater for case B than for case A.

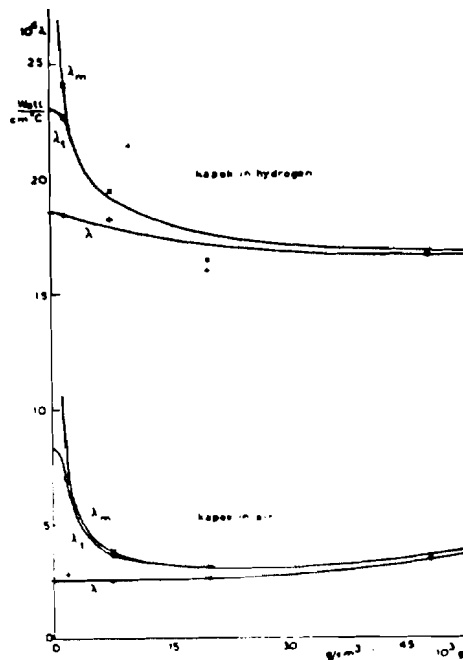


Fig. 4

The apparent thermal conductivity, (λ_t), the pure thermal conductivity (λ) and the maximum thermal conductivity (λ_m) of kapok in air and kapok in hydrogen as a function of the density; the first two for a layer of 3 cm. thickness.