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Watson, W. H.

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SOME ASPECTS OF THE THEORY OF COUPLED (MULTIPLE) R.F. TRANSMISSION SYSTEMS

OTTAWA

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S E C R E T

SOME ASPECTS OF THE THEORY OF COUPLED (MULTIPLE) R.F. TRANSMISSION SYSTEMS

The present paper presents the results of studies suggested by experiments made in this laboratory on the feeding of arrays of end-fed dipoles and on the propagation of waves along slotted wave-guides. In the case of the former it was found that in addition to the intended propagation of energy inside the guide-feed to the dipoles, there was present a wave system on the outside of the guide travelling with approximately free-space velocity and produced by the currents in the feeder probes attached to the dipoles. In this instance, the coupling between the two systems of waves occurs at the feeder probes only. We shall refer to this type of coupling as discrete coupling. When waves are propagated along a guide with a slot cut parallel to its entire length, there exist in the vicinity of the slot, outside (and inside) the guide, waves propagated approximately with free-space velocity. In this case, the coupling between the two wave systems occurs all along the transmission path. We shall refer to this as continuous coupling. Evidently we can regard continuous coupling of a given type as the limit of the corresponding discrete coupling when the number of coupling points per unit length of multiple line is increased indefinitely according to the appropriate law.

There is an essential physical difference between these two varieties of coupling which is not obvious at first glance, namely, that in the discretely coupled system of two transmission lines, each capable of propagating only one type of wave when uncoupled, the propagation between coupling points takes place by means of the respective wave systems for the uncoupled lines: whereas in each of the continuously coupled lines there will appear two types of waves which in the case of weak coupling are propagated roughly with the speeds of propagation for the uncoupled guides or transmission lines.

So far we have treated a guide as equivalent to a line transmitting in its principal mode, and this is quite allowable provided that the modes of possible transmission of order higher than the lowest mode selected for the transmission are physically insignificant in the propagation. If, for example, we had two wave guides of different dimensions coupled together along their length by a slot, and if for the radio-frequency used, the dimensions of each guide would tolerate the effective transmission of the lowest TE mode only, then as the result of the coupling, there would be present in both guides, waves travelling with two new velocities which depart more and more from the velocities for uncoupled propagation as the coupling is increased. This is not to say that in either guide the two waves have the same amplitude distribution across the guide section. Electromagnetic theory will not allow propagation with a modified velocity unless the amplitude distribution is also appropriately modified. It is not the purpose of this paper to examine these questions of amplitude distribution, which, of course, must be treated for a complete solution, but only to use the simple theory of transmission lines in order to examine the consequences of the multiple propagation, particularly when one line is loaded periodically.

Further, it may not be out of place to point out that essentially the present considerations would be relevant to propagation along a single guide capable of effectively transmitting two modes with different amplitude distributions, and therefore different velocities of propagation, where they are coupled together, either discretely by obstacles placed periodically in the guide, or continuously by some deformation of the cross-sectional shape of the guide.

Another practical problem touched by the present theory is that of altering the phase of excitation along an array of dipoles fed from a wave-guide by employing multiple transmission. This is discussed very briefly at the end of the paper.

The theory is presented in the following main sections.

1. Discrete coupling of two transmission lines.
   a) shunt coupling
   b) series coupling.

2. Continuous coupling derived from 1 a.

1(a) Discrete Shunt Coupling.

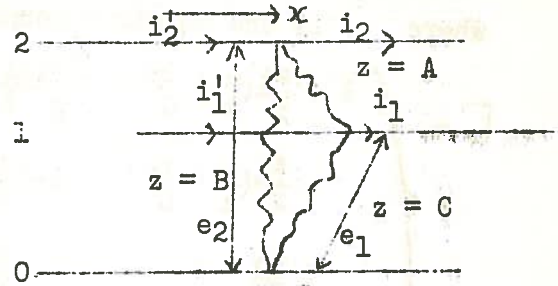


Figure 1.

The two transmission lines composed respectively of conductors 02 and 01 shown in Figure 1 are coupled at the point $x = x_k$ by the impedances A, B, C, as shown in the figure, and, of course this mesh could be replaced by its equivalent star if convenient. We assume that acting in the mesh ABC there is no voltage other than e_2 across (20) and e_1 across (10). i_1 and i_2 denote the currents in the lines to the right of the coupling point, and primed letters are used to denote the values of variables to the left of a coupling-point. Suffixes distinguish corresponding quantities on the two lines.

The following relations serve to connect variables to the left and right of the coupling point.

$$\left.\begin{aligned}
e_1' &= e_1 \qquad\qquad e_2' = e_2 \\
i_1' &= i_1 - \frac{1}{A} e_2 + \left(\frac{1}{C} + \frac{1}{A}\right) e_1 \\
i_2' &= i_2 - \frac{1}{A} e_1 + \left(\frac{1}{B} + \frac{1}{A}\right) e_2
\end{aligned}\right\} \qquad (1)$$

Let z_1 and z_2 denote the respective characteristic impedances of the two lines: then

$$\left.\begin{aligned}
a_1' &\equiv \tfrac{1}{2}(e_1' + z_1 i_1') = \left(1+\frac{z_1}{2}\left(\frac{1}{A} + \frac{1}{C}\right)\right) a_1 + \frac{z_1}{2}\left(\frac{1}{A} + \frac{1}{C}\right) b_1 - \frac{z_1}{2A} a_2 - \frac{z_1}{2A} b_2 \\
b_1' &\equiv \tfrac{1}{2}(e_1' - z_1 i_1') = -\frac{z_1}{2}\left(\frac{1}{A} + \frac{1}{C}\right) a_1 + \left(1-\frac{z_1}{2}\left(\frac{1}{A} + \frac{1}{C}\right)\right) b_1 + \frac{z_1}{2A} a_2 + \frac{z_1}{2A} b_2 \\
a_2' &\equiv \tfrac{1}{2}(e_2' + z_2 i_2') = \left(1+\frac{z_2}{2}\left(\frac{1}{A} + \frac{1}{B}\right)\right) a_2 + \frac{z_2}{2}\left(\frac{1}{B} + \frac{1}{A}\right) b_2 - \frac{z_2}{2A} a_1 - \frac{z_2}{2A} b_1 \\
b_2' &\equiv \tfrac{1}{2}(e_2' - z_2 i_2') = -\frac{z_2}{2}\left(\frac{1}{A} + \frac{1}{B}\right) a_2 + \left(1-\frac{z_2}{2}\left(\frac{1}{A} + \frac{1}{B}\right)\right) b_2 + \frac{z_2}{2A} a_1 + \frac{z_2}{2A} b_1
\end{aligned}\right\} \qquad (2)$$

We can therefore transform from the right to the left of a coupling point by means of the equation

$$\begin{pmatrix} a_1' \\ b_1' \\ a_2' \\ b_2' \end{pmatrix} = \Gamma \begin{pmatrix} a_1 \\ b_1 \\ a_2 \\ b_2 \end{pmatrix} \qquad (3)$$

where Γ is the matrix ('coupling transfer matrix'):

$$\Gamma = \begin{pmatrix} 1 + \frac{z_1}{2}(\frac{1}{A} + \frac{1}{C}) & \frac{z_1}{2}(\frac{1}{A} + \frac{1}{C}) & -\frac{z_1}{2A} & -\frac{z_1}{2A} \\ -\frac{z_1}{2}(\frac{1}{A} + \frac{1}{C}) & 1 - \frac{z_1}{2}(\frac{1}{A} + \frac{1}{C}) & \frac{z_1}{2A} & \frac{z_1}{2A} \\ -\frac{z_2}{2A} & -\frac{z_2}{2A} & 1 + \frac{z_2}{2}(\frac{1}{A} + \frac{1}{B}) & \frac{z_2}{2}(\frac{1}{A} + \frac{1}{B}) \\ \frac{z_2}{2A} & \frac{z_2}{2A} & -\frac{z_2}{2}(\frac{1}{A} + \frac{1}{B}) & 1 - \frac{z_2}{2}(\frac{1}{A} + \frac{1}{B}) \end{pmatrix} \qquad (4)$$

If k_1 and k_2 are the respective propagation constants for the two lines we can transform the values of a_1' b_1' a_2' b_2' to a point distant d to the left by means of the propagation transfer matrix:

$$P_{(d)} = \begin{pmatrix} e^{jk_1 d} & 0 & 0 & 0 \\ 0 & e^{-jk_1 d} & 0 & 0 \\ 0 & 0 & e^{jk_2 d} & 0 \\ 0 & 0 & 0 & e^{-jk_2 d} \end{pmatrix} \qquad (5)$$

Finally, if the line 01 is loaded by a shunt L, we can transform from the right to the left of the point of shunting by means of the shunt-loading transfer matrix:

$$\underline{L} = \begin{pmatrix} 1 + \frac{z_1}{2L} & \frac{z_1}{2L} & 0 & 0 \\ -\frac{z_1}{2L} & 1 - \frac{z_1}{2L} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (6)$$

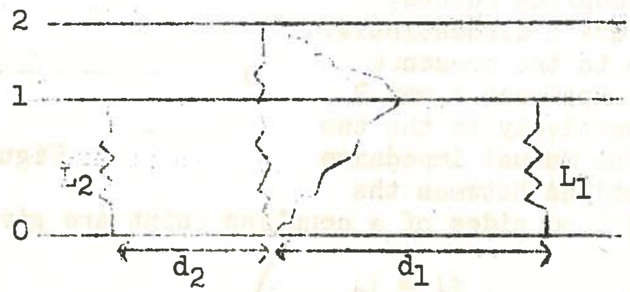
The corresponding series-loading transfer-matrix is

$$\underline{K} = \begin{pmatrix} (1 + \frac{K}{2z_1}) & -\frac{K}{2z_1} & 0 & 0 \\ \frac{K}{2z_1} & (1 - \frac{K}{2z_1}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (7)$$

where K is the series load impedance.

If we wish to transform the values of a_1 b_1 a_2 b_2 from the right of one shunt load L_1 to their values at the right of the load L_2 next on the left, we apply the matrix:

$$M = P(d_2) \,\Gamma\, P(d_1) \,\underline{L}_1 \tag{8}$$



$P(d_2) \,\Gamma\, P(d_1) \,\underline{L}_1$

Figure 2

where d_1 is the distance of the load L_1 from the coupling point between L_1 and L_2, and d_2 is distance from the same coupling point to L_2. By successive applications of matrices of the form M, it is possible to pass from the termination of the lines back to the generator and determine the current in each of the loads, when there is one coupling point between each successive pair of loading points. Other possible arrangements would obviously be covered by the same technique; further in order to apply the method, it is not necessary that all the loads or coupling nets be identical.

The voltage across and current through the shunt load L are given respectively by

$$\left.\begin{aligned} e_L &= a_1 + b_1 \\ i_L &= \frac{a_1 + b_1}{L} \end{aligned}\right\} \tag{9}$$

where a_1 and b_1 are reckoned at the position of the load. On the other hand for the series load K

$$\left.\begin{aligned} i_K &= (a_1 - b_1)/z_1 \\ e_K &= (a_1 - b_1)\,\frac{K}{z_1} \end{aligned}\right\} \tag{10}$$

(b) <u>Discrete Series Coupling.</u>

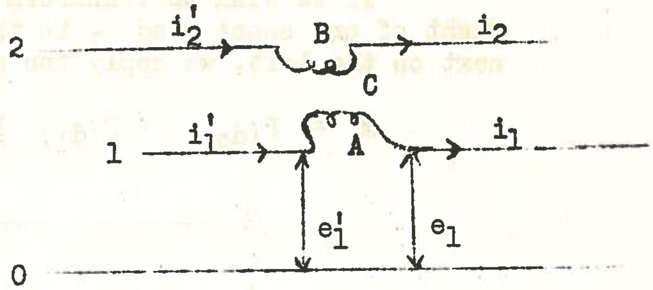


Figure 3.

In passing from shunt coupling to series coupling, the only change we need consider is to find the new matrix to replace Γ. At the coupling points, each line suffers a discontinuity in voltage due to the presence of the direct impedance A and B presented respectively to the two lines and of the mutual impedance C. The connections between the left and right hand sides of a coupling point are given by

$$\left.\begin{aligned} i_1' &= i_1 \qquad\qquad i_2' = i_2 \\ e_2' &= e_2 + Bi_2 - Ci_1 \\ e_1' &= e_1 + Ai_1 - Ci_2 \end{aligned}\right\} \tag{11}$$

These lead to the series-coupling transfer-matrix H

$$H = \begin{pmatrix} 1 + \frac{A}{2z_1} & -\frac{A}{2z_1} & -\frac{C}{2z_2} & \frac{C}{2z_2} \\ \frac{A}{2z_1} & 1 - \frac{A}{2z_1} & -\frac{C}{2z_2} & \frac{C}{2z_2} \\ -\frac{C}{2z_1} & -\frac{C}{2z_1} & 1 + \frac{B}{2z_2} & -\frac{B}{2z_2} \\ -\frac{C}{2z_1} & \frac{C}{2z_1} & \frac{B}{2z_2} & 1 - \frac{B}{2z_2} \end{pmatrix} \tag{12}$$

Although we have limited ourselves to the explicit treatment of shunt and series coupling, it is evident that the method we have used could be applied to any coupling whatever in which both the voltage and the current suffer discontinuities at the point of coupling, instead of only one of them as is the case for the two simple types.

The transfer matrices which we have introduced undergo great formal simplification if we regard them from the following point of view. Let us introduce the matrices of the second order

$$U_1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{pmatrix} \qquad\qquad U_2 = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \tag{13}$$

Then $U_1{}^2 = U_2{}^2 = 0$ (14)

$$U_1\,U_2 = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} = U_3 \quad\text{and}\quad U_2\,U_1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = U_4 \tag{15}$$

$$U_1 U_2 + U_2 U_1 = E_2 \quad \text{the unit matrix of order 2}$$

$$U_2 U_1 - U_1 U_2 = V_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad (16)$$

Hence U_3 and U_4 can be expressed in terms of E_2 and V_2. We may now write

$$\Gamma = \begin{pmatrix} E_2 + \alpha_{11} U_1 & -\alpha_{12} U_1 \\ -\alpha_{21} U_1 & E_2 + \alpha_{22} U_1 \end{pmatrix} \qquad (17)$$

where $\alpha_{11} = z_1 \left(\frac{1}{A} + 1\right)$, $\alpha_{12} = \frac{z_1}{A}$ $\quad \alpha_{21} = \frac{z_2}{A}$ $\quad \alpha_{22} = z_2\left(\frac{1}{A} + \frac{1}{B}\right)$

$$\underline{L} = \begin{pmatrix} E_2 + \beta U_1 & 0 \\ 0 & E_2 \end{pmatrix} \quad \text{where} \quad \beta = \frac{z_1}{L} \qquad (18)$$

$$\underline{K} = \begin{pmatrix} E_2 + \gamma U_1 & 0 \\ 0 & E_2 \end{pmatrix} \quad \text{where} \quad \gamma = \frac{k}{z_1} \qquad (19)$$

$$H = \begin{pmatrix} E_2 + \delta_{11} U_2 & -\delta_{12} U_2 \\ -\delta_{21} U_2 & E_2 + \delta_{22} U_2 \end{pmatrix} \quad \text{where} \quad \delta_{11} = \frac{A}{z_1} \quad \delta_{12} = \frac{C}{z_2} \quad \delta_{21} = \frac{C}{z_1} \quad \delta_{22} = \frac{B}{z_2} \qquad (20)$$

The propagation transfer matrix can be similarly simplified to the form

$$P(d) = \begin{pmatrix} \Omega_1(d) & 0 \\ 0 & \Omega_2(d) \end{pmatrix} \qquad (21)$$

where Ω_1 and Ω_2 are diagonal matrices and therefore commute with themselves.

Let us now evaluate the matrix M of equation (8)

$$M = P(d_2) \quad \Gamma \quad P(d_1)\, \underline{L} \qquad (8)$$

$$= \begin{pmatrix} \Omega_1(d_2) & 0 \\ 0 & \Omega_2(d_2) \end{pmatrix} \begin{pmatrix} E_2 + \alpha_{11}U_1 & -\alpha_{12}U_1 \\ -\alpha_{21}U_1 & E_2 + \alpha_{22}U_1 \end{pmatrix} \begin{pmatrix} \Omega_1(d_1) & 0 \\ 0 & \Omega_2(d_1) \end{pmatrix} \begin{pmatrix} E_2 + \beta U_1 & 0 \\ 0 & E_2 \end{pmatrix}$$

$$= \begin{pmatrix} \Omega_1(d_2) + \alpha_{11}\Omega_1(d_2)U_1 & -\alpha_{12}\Omega_1(d_2)U_1 \\ -\alpha_{21}\Omega_2(d_2)U_1 & \Omega_2(d_2) + \alpha_{22}\Omega_2(d_2)U_1 \end{pmatrix} \begin{pmatrix} \Omega_1(d_1) + \beta\Omega_1(d_1)U_1 & 0 \\ 0 & \Omega_2(d_1) \end{pmatrix}$$

$$= \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix} \qquad (22)$$

where

$$\left.\begin{aligned}
Q_{11} &= \Omega_1(d_1+d_2) + \alpha_{11}\Omega_1(d_2)U_1\Omega_1(d_1) + \beta\Omega_1(d_1+d_2)U_1 + \alpha_{11}\beta\Omega_1(d_2) \\
&\qquad U_1\Omega_1(d_1)U_1 \\
Q_{12} &= -\alpha_{21}\Omega_1(d_2)\,U_1\Omega_2(d_1) \\
Q_{21} &= -\alpha_{21}\left[\Omega_2(d_2)U_1\Omega_1(d_1) + \beta\Omega_2(d_2)U_1\Omega_1(d_1)U_1\right] \\
Q_{22} &= \Omega_2(d_1+d_2) + \alpha_{22}\Omega_2(d_2)\,U_1\Omega_2(d_1)
\end{aligned}\right\} \qquad (23)$$

We further note that if $\Omega = \begin{pmatrix} \omega & 0 \\ 0 & \omega^{-1} \end{pmatrix}$

$$\Omega U_1 = \tfrac{1}{2}\begin{pmatrix} \omega & \omega \\ -\frac{1}{\omega} & -\frac{1}{\omega} \end{pmatrix} \qquad U_1\Omega = \tfrac{1}{2}\begin{pmatrix} \omega & \frac{1}{\omega} \\ -\omega & -\frac{1}{\omega} \end{pmatrix} \qquad (24)$$

$$U_1\,\Omega\,U_1 = \tfrac{1}{2}(\omega - \omega^{-1})\,U_1 \qquad (25)$$

$$\Omega\,U_1\Omega' = \tfrac{1}{2}\begin{pmatrix} \omega\omega' & \omega/\omega' \\ -\omega'/\omega & -\frac{1}{\omega\omega'} \end{pmatrix} \qquad (26)$$

$$\Omega\,U_1\,\Omega'\,U_1 = \tfrac{1}{2}(\omega' - \tfrac{1}{\omega'})\,\tfrac{1}{2}\begin{pmatrix} \omega & \omega \\ -\frac{1}{\omega} & -\frac{1}{\omega} \end{pmatrix} \qquad (27)$$

It is now possible to read off the voltage across the load L_2 in terms of $a_1\ b_1\ a_2\ b_2$ to the right of the load L_1 and hence to deduce the connection between successive loads as regards amplitude and phase. Add the rows of Q_{11} and Q_{12} keeping the columns separate and multiply the result for each column by the corresponding a or b. Thus we obtain for shunt coupled lines and shunt-loading the voltage across L_2 in terms of a and b values to the right of L_1:

$$\begin{aligned}
&e^{jk_1(d_1+d_2)}a_1 + e^{-jk_1(d_1+d_2)}b_1 \\
&\quad + j\sin k_1 d_2\Big(\alpha_{11}\Big\{e^{jk_1d_1}a_1 + e^{-jk_1d_1}b_1\Big\} + j\alpha_{11}\beta\sin k_1d_1(a_1+b_1) \\
&\qquad\quad - \alpha_{21}\,(e^{jk_2d_1}a_2 + e^{-jk_2d_1}b_2)\Big)
\end{aligned} \qquad (28)$$

Many possible choices of the various parameters at once offer themselves as aids to simplification of this result so that it may be applied in practice. If, for example, $\alpha_{11} = 0$, secured by making A + C resonant, the expression above is very much simplified for loading points half wave apart in line 1, becoming

$$- (a_1 + b_1) - j\,\alpha_{21} \sin k_1 d_2 \left(e^{jk_2d_1}a_2 + e^{-jk_2d_1}b_2\right)$$

and leading to a relative phase shift of the voltages across two successive loads half-wave apart in the loaded line depending on the voltage and current in the unloaded line.

2. Continuous Coupling.

Based on the model 1(a) we proceed to the limit where A, B and C are continuously distributed.

The algebraic equations (1) are replaced by

$$- \frac{\partial i_1}{\partial x} = e_1(y_1 + y_A) - y_A e_2 \qquad - \frac{\partial e_1}{\partial x} = z_1' i_1 \tag{29}$$

$$- \frac{\partial i_2}{\partial x} = e_2(y_2 + y_A) - y_A e_1 \qquad - \frac{\partial e_2}{\partial x} = z_2' i_2$$

where z_1', z_2' are the series impedances per unit length and y_1, y_2 are the shunt admittance per unit length of the lines and y_A is the coupling admittance per unit length.

Thus, denoting differentiation of e re x by a prime

$$\left.\begin{aligned} e_1'' &= \alpha_1 e_1 - \beta_1 e_2 \\ e_2'' &= \alpha_2 e_2 - \beta_2 e_1 \\ \text{where} \quad \alpha_1 &= (y_1 + y_A)\, z_1' \qquad \beta_1 = y_A z_1' \\ \alpha_2 &= (y_2 + y_A)\, z_2' \qquad \beta_2 = y_A z_2' \end{aligned}\right\} \tag{30}$$

In the normal modes of propagation

$$e_1'' = - k^2 e_1 \qquad e_2'' = - k^2 e_2$$

where from above

$$(k^2 + \alpha_1)(k^2 + \alpha_2) = \beta_1 \beta_2 \tag{31}$$

$$\left.\begin{aligned} &\text{In the mode } k = k_1 \quad \text{let } e_2 = \lambda_1 e_1 \quad \text{i.e. } \lambda_1 = \frac{k_1^2 + \alpha_1}{\beta_1} \\ &\text{In the mode } k = k_2 \quad e_1 = \lambda_2 e_2 \quad \text{and} \quad \lambda_2 = \frac{k_2^2 + \alpha_2}{\beta_2} \end{aligned}\right\} \tag{32}$$

The expressions for the voltages across the two lines are

$$\left.\begin{aligned} e_1 &= A_1 e^{-jk_1x} + B_1 e^{jk_1x} + \lambda_2 (A_2 e^{-jk_2x} + B_2 e^{jk_2x}) \\ e_2 &= \lambda_1 (A_1 e^{-jk_1x} + B_1 e^{jk_1x}) + A_2 e^{-jk_2x} + B_2 e^{jk_2x} \end{aligned}\right\} \tag{33}$$

In obtaining the corresponding expressions for the current, we must keep in mind that the characteristic impedance will in general be different for each normal mode of propagation in the two guides. Accordingly we write

$$\left.\begin{aligned} i_1 &= y_{11}(A_1 e^{-jk_1x} - B_1 e^{+jk_1x}) + \lambda_2 y_{12} (A_2 e^{-jk_2x} - B_2 e^{jk_2x}) \\ i_2 &= \lambda_1 y_{21} (A_1 e^{-jk_1x} - B_1 e^{jk_1x}) + y_{22} (A_2 e^{-jk_2x} - B_2 e^{jk_2x}) \end{aligned}\right\} \tag{34}$$

Let

$$\begin{aligned} A_1 e^{-jk_1d} &= a_1 & \qquad A_2 e^{-jk_2d} &= a_2 \\ B_1 e^{+jk_1d} &= b_1 & \qquad B_2 e^{jk_2d} &= b_2 \end{aligned} \tag{35}$$

Where $x = d$ is a point of loading, line 1 being shunted by the admittance y. As before, we denote corresponding quantities to the left and right of a point of transfer by primed and unprimed letters respectively.

The transfer equations at the point of shunting are

$$\left.\begin{aligned} y\left\{a_1' + b_1' + \lambda_2(a_2' + b_2')\right\} &= y\left\{a_1 + b_1 + \lambda_2(a_2 + b_2)\right\} \\ &= y_{11}\left[a_1' - a_1 - (b_1' - b)\right] + \lambda_2 y_{12}\left[a_2' - a_2 - (b_2' - b_2)\right] \\ \lambda_1 y_{21}(a_1' - b_1') + y_{22}(a_2' - b_2') &= \lambda_1 y_{21}(a_1 - b_1) + y_{22}(a_2 - b_2) \\ \lambda_1(a_1' + b_1') + (a_2' + b_2') &= \lambda_1(a_1 + b_1) + (a_2 + b_2) \end{aligned}\right\} \tag{36}$$

Solving for the primed quantities, and putting

$$M = \lambda_1 \lambda_2 y_{12} y_{21} - y_{11} y_{22} \tag{37}$$

we have

$$\left.\begin{aligned} a_1' &= \left(1 - \frac{yy_{22}}{2M}\right)a_1 - \frac{yy_{22}}{2M} b_1 - \frac{\lambda_2 yy_{22}}{2M} a_2 - \frac{\lambda_2 yy_{22}}{2M} b_2 \\ b_1' &= \frac{yy_{22}}{2M} a_1 + \left(1 + \frac{yy_{22}}{2M}\right) b_1 + \lambda_2 \frac{yy_{22}}{2M} a_2 + \lambda_2 \frac{yy_{22}}{2M} b_2 \\ a_2' &= \frac{\lambda_1 yy_{21}}{2M} a_1 + \frac{\lambda_1 yy_{21}}{2M} b_1 + \left(1 + \frac{\lambda_1\lambda_2 yy_{21}}{2M}\right) a_2 + \frac{\lambda_1\lambda_2 yy_{21}}{2M} b_2 \\ b_2' &= -\frac{\lambda_1 yy_{21}}{2M} a_1 - \frac{\lambda_1 yy_{21}}{2M} b_1 - \frac{\lambda_1\lambda_2 yy_{21}}{2M} a_2 + \left(1 - \frac{\lambda_1\lambda_2 yy_{21}}{2M}\right) b_2 \end{aligned}\right\} \tag{38}$$

The equations lead at once to the shunt-load transfer matrix. It has the same form (17) as Γ above. The propagation-transfer matrix is identical with (5).

It is obvious that the continuous coupling corresponding to equations (11) can be similarly derived.

3. Phase-Shifting by Coupled Feed.

The use of multiple feed to produce phase-change along an array is a natural application of the idea that the phase of a voltage (or current) can be changed by adding to it a voltage (or current) in quadrature with it. The multiple feed would change progressively the phase of the voltage for shunted loads (current for series loads) at successive points of loading. Thus, by altering the gradient of phase change along the array, the direction of the main lobe would be shifted.

From the practical point of view, it would appear to be simpler to make coupling points also points of loading. Since, however, shunt coupling introduces no discontinuity in voltage, no progressive change in phase along a linear array of loads can be achieved by shunt coupling if they are shunt loads. For such loads placed at points of coupling, series coupling must be used and mutatis mutandis.

The practical problem is not merely one of securing the required phase-change but also of obtaining the proper power in the loads, and while coupling methods for parallel-wire transmission-lines are available, it cannot be said that present practical experience of suitable coupling methods for wave-guides is very wide. However, the possibility that the direction of a beam can be controlled at the input by suitable relative phase and/or amplitude variation between the two components of the coupled feed is sufficiently attractive to merit further investigation. The present brief report outlines methods for handling the fairly complicated calculations that will inevitably be involved in the proper design of any device working on this principle, if it prove practicable.

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W. H. Watson