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DETERMINATION OF THE TIME INTERVAL BETWEEN INTERCEPTS ON A TWO-BEAM COUNTER-MORTAR RADAR FROM A STORAGE TUBE DISPLAY

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ABSTRACT

A method is described for approximating the time interval between intercepts on a two-beam counter-mortar radar from previously stored trajectory information. Tests on simulated trajectories indicate that the accuracy is adequate. Some methods of mechanization are discussed.

CONTENTS

FIGURE

DETERMINATION OF THE TIME INTERVAL BETWEEN INTERCEPTS ON A TWO-BEAM COUNTER-MORTAR RADAR FROM A STORAGE TUBE DISPLAY

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On any projectile in flight, the two-beam radar locates two points on the ascending (or descending) portion of the trajectory and if a third parameter such as time between intercepts is available, a location can be made. When locations are made in real time, the operator may conveniently measure the time as the beam intercepts are being obtained, and the computation can be completed immediately.

However, in the case of multiple fire it may be quite impossible for the operator to measure and store all the information. A storage tube has been suggested which will store positional information quite adequately but tends to lose (or obscure) time information. It may be argued that position is more important but it should be pointed out that time between intercepts assumes an importance similar to that of the second intercept position since, with only two of the three parameters, large location errors are likely.

The storage tube display so far proposed presents a picture of the ground projection of the beam intercepts. That is, the display consists of the horizontal components of the path of the projectile as it traverses each beam. A method of determining the time between intercepts from these horizontal components (as displayed on a storage tube) has been devised and is given by:

$$\Delta T_4 = \left(\frac{2\Theta}{g} R \; \frac{\Delta x_U - \Delta x_L}{\Delta x_U + \Delta x_L} \right)^{\frac{1}{2}} \qquad (1)$$

where Θ = beam split
g = gravitational constant
R = range to intercepts
Δx_U = upper beam horizontal component
Δx_L = lower beam horizontal component

Derivation of the formula is given in Appendix A.

To check the validity of equation (1) a digital computer program simulating a wide range of trajectories was prepared. Parameters were varied as follows:

Gravitational constant = g = 9.8 m/sec^2
Air-drag constant = k = 0.01, 0.02, 0.03
Velocity vertical component = V_{yo} = 100, 200, 300, 400 m/sec
Quadrant elevation of weapons = QE = 45.0°, 53.2°, 63.5°
Radar lower-beam angle of sight = A/S = 32, 48, 64, 80 milliradians
Radar beam split = Θ = 40 milliradians
Radar beam thickness = α = 16 milliradians
Radar range to intercepts = R = 5, 10, 15, 20 km

Note that radar range and angle of sight as well as projectile air-drag, velocity, and quadrant elevation were varied. The air-drag was taken as being proportional to velocity although each type of projectile is different and may have components proportional to the square or cube (or some intermediate power) of velocity. The variation of the drag constant from 0.01 to 0.03 is representative of that found in practice, and is considered to be adequate to test the validity of equation (1).

Of the 576 possible combinations of the parameters, about one-third provide trajectories which do not pass completely through the upper beam. These were ignored leaving 381 trajectories on which the time calculations were made.

Times were calculated to the lower and upper edges of each beam and time differences were tabulated for lower-edge intercepts, ΔT_2, upper-edge intercepts, ΔT_1, and centre-of-beam intercepts ΔT_0, as shown in the figure. All of these were compared with the value derived from the upper- and lower-beam horizontal components as given by equation (1).

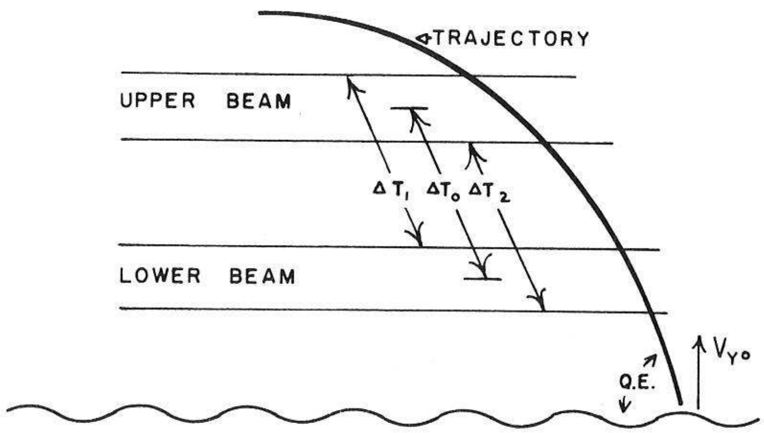


Fig. 1 Geometry of intercepts showing calculated time differences

RESULTS

Differences between ΔT_4 and the calculated times are shown in Table I. The table shows that ΔT_4, as derived by the storage-tube formula, gives an error with respect to ΔT_2 (as would be measured in real-time operation) of less than 0.2 second for 87% of trajectories. It has been shown previously [1] that the timing error contribution to location error is quite negligible for errors of this order.

The formula given appears to be quite adequate for the purpose and should not be too difficult to mechanize in a semiautomatic manner.

TABLE I

Differences between ΔT_4 and calculated times*

	Sign	Less than 0.1	0.1-0.2	0.2-0.5	0.5-1.0	Over 1.0
$\Delta T_4 - \Delta T_1$	all (-)	249 (65%)	60 (16%)	42 (11%)	24	6
$\Delta T_4 - \Delta T_2$	most (+)	294 (77%)	39 (10%)	27 (7%)	15	6
$\Delta T_4 - \Delta T_0$	all (-)	372 (98%)	6	3	None	None

* Times given in seconds

APPLICATION

Design of a device to mechanize equation (1) in a two-beam radar such as the AN/MPQ-501 may be considered in two stages.

a) The provision of transducers to measure Δx_U, Δx_L, and range as voltages or resistances. It is noted that the first two can be obtained directly from the difference easting and/or difference northing normally provided as artillery corrections on the computer output. In addition both easting and northing are available on potentiometers and the appropriate quantities can be easily stored in simple track-hold circuits. Range is also available as a voltage, or a shaft rotation, and, of course, appears on a counter.

b) The solution of equation (1). This equation may be written as:

$$\Delta T_4{}^2 = 8R \frac{\Delta x_U - \Delta x_L}{\Delta x_U + \Delta x_L} \qquad (2)$$

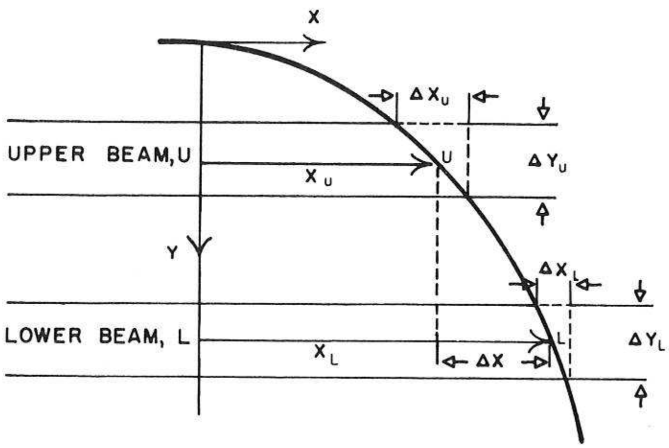
where $\Theta = 40$ milliradians
$g = 9.8$ metres/sec^2

which can be solved conveniently either by a Wheatstone bridge circuit or by a feedback amplifier, or by a combination of the two circuits. For minimum complexity, a manually obtained solution is suggested, or it may be semiautomatic with very little extra equipment.

CONCLUSIONS

Time between intercepts is given with adequate precision by the equation investigated and this equation should not be difficult to mechanize in conjunction with the computer of a two-beam radar.

APPENDIX A



For any parabola, $x^2 = 2py$ and $\frac{dy}{dx} = \frac{x}{p}$

Then to a first approximation $\frac{\Delta y_L}{\Delta x_L} = \frac{x_L}{p}$ and $\frac{\Delta y_U}{\Delta x_U} = \frac{x_U}{p}$

So that $\Delta y_L \cdot p = \Delta x_L \cdot x_L$ and $\Delta y_U \cdot p = \Delta x_U \cdot x_U$

But $\Delta y_L = \Delta y_U$

$$\therefore \frac{\Delta x_L}{\Delta x_U} = \frac{x_U}{x_L} = \frac{x_U}{x_U + \Delta x}$$

$$\therefore\ \Delta x = x_U \left(\frac{\Delta x_U}{\Delta x_L} - 1 \right) \qquad (1)$$

Since horizontal velocity is assumed to be constant, we have from (1)

$$\Delta T = T_U \left(\frac{\Delta x_U}{\Delta x_L} - 1 \right) \qquad (2)$$

where T_U = time to point U
ΔT = time between beams

Also, the average velocity of the projectile as it travels between the beams is

$$V_y = g\left(T_U + \frac{\Delta T}{2} \right) = \frac{R\Theta}{\Delta T}$$

where R = range in km
Θ = beam split in milliradians
g = gravitational constant

$$\therefore\ T_U = \frac{R\Theta}{g\Delta T} - \frac{\Delta T}{2} \qquad (3)$$

From (2) and (3) $\Delta T = \left(\frac{R\Theta}{g\Delta T} - \frac{\Delta T}{2} \right)\left(\frac{\Delta x_U}{\Delta x_L} - 1 \right)$

Solving for ΔT, $\Delta T = \left(\frac{2\Theta R}{g} \ \frac{\Delta x_U - \Delta x_L}{\Delta x_U + \Delta x_L} \right)^{\frac{1}{2}}$

Reference

1. Letter to Army Development Establishment, Department of National Defence, Ottawa, Ont. 26 May, 1958.