# Data processing and harmonic analysis with a digital computer

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**Publisher's version / Version de l'éditeur:**
https://doi.org/10.4224/20338503
*Technical Note (National Research Council of Canada. Division of Building Research); no. TN-273, 1959-03-01*

NATIONAL RESEARCH COUNCIL OF CANADA

DIVISION OF BUILDING RESEARCH

No. 273

# TECHNICAL NOTE

NOT FOR PUBLICATION

FOR INTERNAL USE

PREPARED BY D.G. Stephenson CHECKED BY A.G.W. APPROVED BY R.F.L.

PREPARED FOR Record purposes DATE March, 1959.

SUBJECT Data Processing and Harmonic Analysis with a Digital Computer

This note is intended to record the data processing procedure being used with the periodic heat flow test cell results, and is for internal use only. This application of a digital computer may suggest other situations where a computer could be used in conjunction with the experimental work of the Division.

The availability of an automatic digital computer permits a new approach to the processing of experimental results. More than that, it permits a new approach to many problems because with it, methods requiring the analysis of large numbers of results are feasible, while without a computer such an approach would be impractical. The processing of results can be completely automatic if the apparatus used to record the results of an experiment produces a record on punched cards or punched paper tape. The results can then be processed by a computer without any transcription. This eliminates the possibility of an error in the transcribing and also reduces the time needed to handle the results. Furthermore, a fully automatic system can be operated by a non-professional person whereas most methods not using a computer require an expert to do the work.

These points are illustrated by the following particular problem. The periodic heat flow test cell apparatus requires the recording of periodically varying temperatures and the analysis of these records to find the amplitude and phase angle of a particular frequency component. The technique of Fourier analysis requires evaluation of the integrals.

$$\int_0^{2\pi} T(x).\sin x \,.\, dx$$

and

$$\int_0^{2\pi} T(x).\cos x \,.\, dx$$

When the function T(x) is known at N equally spaced values of x in the interval 0 to 2 , the integrals can be approximated by the sums

$$\frac{2\pi}{N} \sum_{n=1}^{N} T_n \cdot \sin\left(\frac{2\pi n}{N}\right)$$

and

$$\frac{2\pi}{N} \sum_{n=1}^{N} T_n \cdot \cos\left(\frac{2\pi n}{N}\right)$$

The problem is to convert the emf of a thermocouple to temperature, to calculate the sine and cosine for increments of $\frac{2\pi}{N}$ of the angle, and to form the sums

$$\sum_{n=1}^{N} T_n, \quad \sum_{n=1}^{N} T_n \cdot \sin\left(\frac{2\pi n}{N}\right) \text{ and } \sum_{n=1}^{N} T_n \cdot \cos\left(\frac{2\pi n}{N}\right)$$

These operations can be performed by the simplest of automatic computers. The following program is the one used with the ElectroData E101 machine at the Division of Radio and Electrical Engineering.

This machine uses external pinboards for the program. When the same program is to be used many times, templates can be prepared which greatly simplify the setting up of the program boards. Figure 1 is a copy of the template for pinboard one of this program.

Figure 2 is a sample of the computer result sheet. The sums for 125 steps are given in the final print out. The calculation of the complex ratio of the fundamental components of the two temperatures is given in the appendix.

This system is not now fully automatic since the experimental results are recorded on a strip chart by a 16-point millivolt recorder. This record has to be read and a table of results prepared and these numbers then put on a punched paper tape with a teletype machine. Finally, this tape is used to supply the data to the computer. The information can be entered manually from a key board on the computer but doing it this way has two serious disadvantages:

(i) The computer is not used at its maximum speed since it can operate faster than numbers can be entered.

(ii) If a wrong number is entered the cumulative sums will be wrong and it is troublesome to make the correction.

With the tape input the figures can be checked on the tape so there will be no mistake when using the computer.

Equipment is now on order which when added to the millivolt recorder will produce a punched tape record directly. The processing of results then will be a routine operation of setting up the computer program and starting the data tape into the reader. This will be done by one laboratory assistant.

## Conclusions

The use of a digital computer for analysis of experimental results greatly reduces the time and effort needed to perform the simple repetitive operations that are involved. A fully automatic data-processing system has the additional advantages of reduced chance for errors and better economy of computer and staff time.

**ElectroData 101 Program Sheet**

| Problem | Programmer | Date | Page No. |
|---|---|---|---|
| EMF to Temperature Conversion and Harmonic Analysis of Temperatures | Stephenson | Nov./58 | |

PB # 1

| | | | | |
|---|---|---|---|---|
| 0 | T | | 12 | |
| 1 | A | 1 | 7 | |
| 2 | B | | | |
| 3 | X | 4 | 1 | |
| 4 | + | 4 | 0 | |
| 5 | W | 4 | 2 | |
| 6 | X | 4 | 2 | |
| 7 | B | | | |
| 8 | U | 0 | 9 | * |
| 9 | + | 0 | 1 | |
| 10 | W | 0 | 1 | |
| 11 | X | 1 | 0 | |
| 12 | + | 1 | 1 | |
| 13 | W | 1 | 1 | |
| 14 | U | 2 | 0 | |
| 15 | | | | |

PB # 2

| | | | | |
|---|---|---|---|---|
| 0 | X | 2 | 0 | |
| 1 | + | 2 | 1 | |
| 2 | W | 2 | 1 | |
| 3 | U | 0 | 4 | * |
| 4 | T | | 12 | |
| 5 | A | 1 | 7 | |
| 6 | B | | | |
| 7 | X | 4 | 1 | |
| 8 | + | 4 | 0 | |
| 9 | W | 4 | 2 | |
| 10 | X | 4 | 2 | |
| 11 | B | | | |
| 12 | U | 0 | 13 | * |
| 13 | + | 0 | 2 | |
| 14 | W | 0 | 2 | |
| 15 | U | 3 | 0 | |

PB # 3

| | | | | |
|---|---|---|---|---|
| 0 | U | 0 | 1 | * |
| 1 | X | 1 | 0 | |
| 2 | + | 1 | 2 | |
| 3 | W | 1 | 2 | |
| 4 | U | 0 | 5 | |
| 5 | X | 2 | 0 | |
| 6 | + | 2 | 2 | |
| 7 | W | 2 | 2 | |
| 8 | U | 4 | 1 | |
| 9 | | | | |
| 10 | | | | |
| 11 | | | | |
| 12 | | | | |
| 13 | | | | |
| 14 | | | | |
| 15 | | | | |

PB # 4 Sine Cosine Sub Routine

| | | | | |
|---|---|---|---|---|
| 0 | | | | |
| 1 | R | 5 | 1 | |
| 2 | B | | | |
| 3 | X | 5 | 3 | |
| 4 | W | 5 | 4 | |
| 5 | X | 5 | 0 | |
| 6 | W | 5 | 5 | |
| 7 | R | 5 | 2 | |
| 8 | B | | | |
| 9 | X | 5 | 0 | |
| 10 | + | 5 | 4 | |
| 11 | W | 5 | 0 | |
| 12 | X | 5 | 3 | |
| 13 | - | 5 | 5 | |
| 14 | W | 5 | 3 | |
| 15 | U | 5 | 0 | |

PB # 5

| | | | | |
|---|---|---|---|---|
| 0 | W | 2 | 0 | |
| 1 | R | 5 | 0 | |
| 2 | W | 1 | 0 | |
| 3 | R | 3 | 2 | |
| 4 | - | 3 | 1 | |
| 5 | W | 3 | 2 | |
| 6 | C | 0 | 10 | |
| 7 | U | 1 | 0 | |
| 8 | | | | |
| 9 | | | | |
| 10 | R | 3 | 0 | |
| 11 | W | 3 | 2 | |
| 12 | U | 6 | 0 | |
| 13 | | | | |
| 14 | | | | |
| 15 | | | | |

PB # 6 Partial Print Out

| | | | | |
|---|---|---|---|---|
| 0 | H | 0 | 0 | |
| 1 | H | 1 | 0 | |
| 2 | R | E | F | |
| 3 | P | | | |
| 4 | S | 1 | 2 | |
| 5 | U | 0 | 2 | |
| 6 | P | 1 | 0 | |
| 7 | S | 0 | 2 | |
| 8 | U | 0 | 1 | |
| 9 | P | 3 | 0 | |
| 10 | U | 1 | 0 | |
| 11 | | | | |
| 12 | | | | |
| 13 | | | | |
| 14 | | | | |
| 15 | | | | |

PB # 7 Loading

| | | | | |
|---|---|---|---|---|
| 0 | H | 0 | 0 | |
| 1 | H | 1 | 0 | |
| 2 | T | | 12 | |
| 3 | W | E | F | |
| 4 | S | 1 | 2 | |
| 5 | U | 0 | 2 | |
| 6 | S | 0 | 4 | |
| 7 | U | 0 | 1 | |
| 8 | H | 1 | 0 | |
| 9 | T | | 12 | |
| 10 | W | E | F | |
| 11 | S | 1 | 3 | |
| 12 | U | 0 | 9 | |
| 13 | U | 8 | 0 | |
| 14 | | | | |
| 15 | | | | |

PB # 8 Full Print Out

| | | | | |
|---|---|---|---|---|
| 0 | H | 0 | 0 | |
| 1 | H | 1 | 0 | |
| 2 | R | E | F | |
| 3 | P | | | |
| 4 | S | 1 | 2 | |
| 5 | U | 0 | 2 | |
| 6 | P | 1 | 0 | |
| 7 | S | 0 | 4 | |
| 8 | U | 0 | 1 | |
| 9 | H | 1 | 0 | |
| 10 | R | E | F | |
| 11 | P | | | |
| 12 | S | 1 | 3 | |
| 13 | U | 0 | 10 | |
| 14 | P | 1 | 0 | |
| 15 | A | | * | |

* These steps can be replaced by print instructions

OPERATING INSTRUCTIONS SHEET

**Burroughs E 101**

PROBLEM: EMF to Temperature Conversion and Harmonic Analysis of Temperatures

PROGRAMMED BY: Stephenson

SET-UP — PINBOARDS:

PRINTING SCHEDULE:

SPECIAL INSTRUCTIONS: Prepare tape with initial values of a to s in alphabetical order with digits in correct positions. Load tape into reader.

START BUTTON: 7

| SIGNAL (K, H) | PINBOARD | STEP | PINBOARD CHANGES, MANUAL INSTRUCTIONS, ETC. | START BUTTON | KEYBOARD ENTRY X | KEYBOARD ENTRY Y | KEYBOARD ENTRY NUMBER | MOTOR BAR |
|---|---|---|---|---|---|---|---|---|
| | | | Initial values of constants a to s inclusive on loading tape in alphabetical order. Start button 7 loads these constants and prints back contents of these memory locations. | 7 | | | | |
| H | 8 | 15 | Start Data Tape into reader. | 1 | | | | |
| | | | Data will be read as required. Values of a through i are printed out at n = 1, 21, 41, 61, 81, 101, 121 Machine will halt after last data word has been processed. | | | | | |
| H | 1 | 0 | Start button 8 will print out a to s. | 8 | | | | |

Problem EMF to Temperature Conversion and Harmonic Analysis of Temperatures

E101 MEMORY MAP

F b

Page No.

Programmer D.G. Stephenson

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | a One | b $\sum_{n=1}^{N} T_n$ | c $\sum_{n=1}^{N} T_n'$ | | | | | | | |
| 1 | d $\sin\left(\frac{2\pi n}{N}\right)$ | e $\sum_{n=1}^{N} T_n \cdot \sin\left(\frac{2\pi n}{N}\right)$ | f $\sum_{n=1}^{N} T_n' \sin\left(\frac{2\pi n}{N}\right)$ | | | | | | | |
| 2 | g $\cos\left(\frac{2\pi n}{N}\right)$ | h $\sum_{n=1}^{N} T_n \cdot \cos\left(\frac{2\pi n}{N}\right)$ | i $\sum_{n=1}^{N} T_n' \cdot \cos\left(\frac{2\pi n}{N}\right)$ | | | | | | | |
| 3 | j ------ 19 | k ------ 1 | l ←------------ | | | These constants govern the frequency of the partial print out of memory | | | | |
| 4 | m $K_1$ | n $K_2$ | o ←------------ | | | Constants for emf to temperature conversion $T = K_1 e + K_2 e^2$ | | | | |
| 5 | p $\sin\left(\frac{2\pi n}{N}\right)$ | q $\sin\left(\frac{2\pi}{N}\right)$ | r $\cos\left(\frac{2\pi}{N}\right)$ | s $\cos\left(\frac{2\pi n}{N}\right)$ | | | | | | |
| 6 | | | | | | | | | | |
| 7 | | | | | | | | | | |
| 8 | | | | | | | | | | |
| 9 | | | | | | | | | | |

E a X

- 6 -

ED-1277

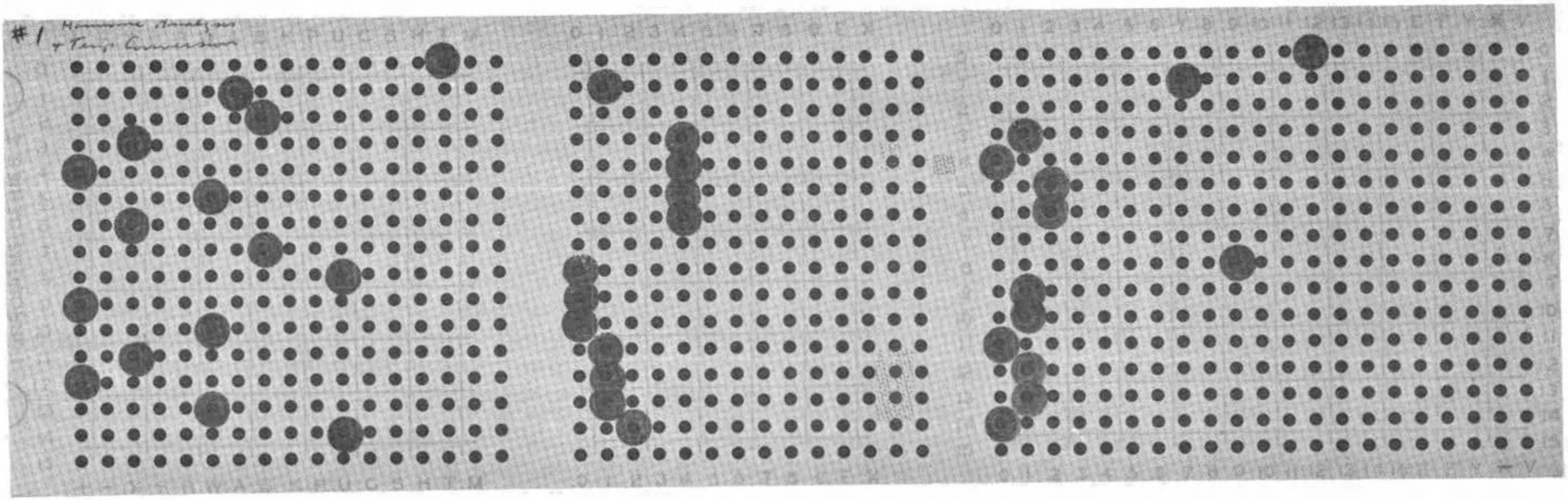

Figure 1  A copy of the template for pinboard #1

| n | | | | |
|---|---|---|---|---|
| | 0 999999 99999 | 0 000000 00000 | 0 000000 00000 | |
| | 0 000000 00000 | 0 000000 00000 | 0 000000 00000 | |
| 0 | 0 999999 99999 | 0 000000 00000 | 0 000000 00000 | |
| | 0 000000 00019 | 0 000000 00001 | 0 000000 00000 | |
| | 0 004414 70000 | 0 001324 00000- | 0 000000 00000 | |
| | 0 000000 00000 | 0 050244 31817 | 0 998736 95599 | 0 999999 99999 |
| | 0 999999 99999 | 0 000132 73382- | 0 000036 11151 | |
| 1 | 0 050244 31816 | 0 000000 00000 | 0 000000 00000 | |
| | 0 998736 95598 | 0 000132 73381- | 0 000036 11150 | |
| | 0 999999 99999 | 0 000938 05460 | 0 001112 87632 | |
| 21 | 0 870183 74347 | 0 001141 50503 | 0 000625 83860 | |
| | 0 492727 33491 | 0 000404 16016 | 0 000856 92368 | |
| | 0 999999 99999 | 0 006253 27242 | 0 004441 64938 | |
| 41 | 0 882291 20371 | 0 006239 94530 | 0 003822 77481 | |
| | 0 470703 92067- | 0 000476 72940 | 0 000821 26524 | |
| | 0 999999 99999 | 0 012465 90478 | 0 009002 67059 | |
| 61 | 0 075326 80179 | 0 009584 46302 | 0 006257 09903 | |
| | 0 997158 86251- | 0 004451 54105- | 0 002811 69525- | |
| | 0 999999 99999 | 0 016173 91223 | 0 013748 49456 | |
| 81 | 0 801566 94523- | 0 008817 38738 | 0 004537 08486 | |
| | 0 597904 95204- | 0 007997 28062- | 0 007031 19584- | |
| | 0 999999 99999 | 0 014955 65230 | 0 016378 08292 | |
| 101 | 0 934328 88330- | 0 009997 48082 | 0 002082 06979 | |
| | 0 356411 85774 | 0 007956 32102- | 0 007619 63222- | |
| | 0 999999 99999 | 0 012626 76296 | 0 017558 45268 | |
| 121 | 0 199709 96374- | 0 011448 66217 | 0 001278 58574 | |
| | 0 979854 97907 | 0 009654 51852- | 0 006821 89597- | |
| | 0 999999 99999 | 0 012106 16070 | 0 017716 95229 | |
| | 0 000000 00187 | 0 011513 50571 | 0 001258 44583 | |
| | 0 999999 92231 | 0 010170 23795- | 0 006664 93531- | |
| 125 | 0 000000 00019 | 0 000000 00001 | 0 000000 00015 | |
| | 0 004414 70000 | 0 001324 00000- | 0 004403 18120 | |
| | 0 000000 00187 | 0 050244 31817 | 0 998736 95599 | 0 999999 92231 |

Figure 2 Sample of computer result sheet

## Appendix

### Calculation of T/T' from Computer Results

Any periodic function with a fundamental frequency of $w/2\pi$ can be represented by

$$T(wt) = T_o + A \sin(wt + \delta_1) + B \sin(2wt + \delta_2) + \ldots.$$

where
$$T_o = \frac{1}{2\pi} \cdot \int_o^{2\pi} T(wt)\,.d(wt)$$

$$\tan \delta_1 = \frac{\int_o^{2\pi} T(wt)\ .\ \cos(wt)\ .\ d(wt)}{\int_o^{2\pi} T(wt)\ .\ \sin(wt)\ .\ d(wt)}$$

$$A = \frac{\sec \delta_1}{\pi} \int_o^{2\pi} T(wt)\ .\ \sin(wt)\ .\ d(wt)$$

and
$$\tan \delta_2 = \frac{\int_o^{4\pi} T(wt)\ .\ \cos(2wt)\ d(2wt)}{\int_o^{4\pi} T(wt)\ .\ \sin(2wt)\ d(2wt)}$$

$$B = \frac{\sec \delta_2}{2\pi} \int_o^{4\pi} T(wt)\ .\ \sin(2wt)\ d(2wt)$$

When the integrals are approximated by sums these expressions become

$$T_o = \frac{1}{N} \sum_{n=1}^{N} T_n$$

$$\tan \delta_1 = \frac{\sum_{n=1}^{N} T_n \cdot \cos\left(\frac{2\pi n}{N}\right)}{\sum_{n=1}^{N} T_n \cdot \sin\left(\frac{2\pi n}{N}\right)}$$

$$A = \frac{2 \sec \delta_1}{N} \sum_{n=1}^{N} T_n \cdot \sin\left(\frac{2\pi n}{N}\right)$$

$$\tan \delta_2 = \frac{\sum_{n=1}^{N} T_n \cdot \cos\left(\frac{4\pi n}{N}\right)}{\sum_{n=1}^{N} T_n \cdot \sin\left(\frac{4\pi n}{N}\right)}$$

$$B = \frac{2 \sec \delta_2}{N} \sum_{n=1}^{N} T_n \cdot \sin\left(\frac{4\pi n}{N}\right)$$

The final results ($n = 125$) shown in Fig. 2 give:

(1) Input Temperature T

$$\tan \delta_1 = -\frac{1017.02}{1151.35} = -0.883328$$

$$\delta_1 = -41.46°$$

$$\sec \delta_1 = 1.334369$$

$$A = \frac{2 \times 1.334369}{125} \times 1151.35 = 24.581$$

(2) Output Temperature T'

$$\tan \delta_1' = - \frac{666.494}{125.845} = - 5.29615$$

$$\delta_1' = - 79.31°$$

$$\sec \delta_1' = 5.39098$$

$$A' = \frac{2 \times 5.39098}{125} \times 125.845 = 10.855$$

Thus, $$\left(\frac{T}{T'}\right)_{\text{fundamental}} = \frac{24.581}{10.855} \angle\, - 41.46° - (-79.31°) - 0.72°$$

$$= 2.2645 \angle\, 37.13°$$

The 0.72° is the angle difference between the starting points of T and T'. For the machine analysis it was assumed that the values of the two functions were measured simultaneously, whereas all the values of T' were measured 0.008 hours before the corresponding values of T. This time interval equals 0.72° which should be subtracted from $\delta_1$ or added to $\delta_1'$ .