Design of foundations, anchored in unfrozen ground, to resist heaving forces

Solov'ev, Y. I.; Puskov, V. I.; National Research Council of Canada. Division of Building Research

Publisher's version / Version de l'éditeur:
https://doi.org/10.4224/20359077
Technical Translation (National Research Council of Canada), 1970

PREFACE

The Division of Building Research is currently investigating the behaviour of a variety of small footings in frost-susceptible soils, with particular reference to heaving forces. The studies reported in the present translation will be helpful in assessing the results and will assist with practical problems of foundation design in areas of seasonal frost.

The Division makes available English translations of papers published abroad in order to disseminate useful scientific information in its particular field of interest.

The Division is grateful to Mr. G. Belkov of the Translations Section, National Research Council, for translating this paper and to Mr. E. Penner of this Division, who checked the translation.

Ottawa
September 1970

N.B. Hutcheon
Director

NATIONAL RESEARCH COUNCIL OF CANADA

Technical Translation 1424

Title: Design of foundations, anchored in unfrozen ground, to resist heaving forces

(Raschet fundamentov, zaankerennykh v talom grunte, na deistvie sil pucheniya)

Authors: Yu.I. Solov'ev and V.I. Puskov

Reference: Osnovaniya Fundamenty i Mekhanika Gruntov, (2): 15-17, 1961

Translator: G. Belkov, Translations Section, National Science Library

DESIGN OF FOUNDATIONS, ANCHORED IN UNFROZEN GROUND, TO RESIST HEAVING FORCES

In regions where there is deep seasonal freezing of fine-grained soils there is a serious problem of preventing heaving of foundations. One of the methods of providing stable foundations in such a case is anchoring the foundations in unfrozen ground by providing an anchor footing.

At the present time there is no satisfactory method of designing such foundations. Existing suggestions dealing with this problem(1) are to a considerable extent conventional since they do not consider the equilibrium of forces but rather the relationship of stresses at the point of intersection of the frozen and unfrozen interface with the axis of the foundation. Below a suggestion is made for designing such foundations to resist heaving using two limit states: the deformation limit and the strength limit.

Let us consider the performance of a foundation anchored in unfrozen ground in resisting heaving forces generated along the contact surface between the foundation wall and the layer of freezing soil (Fig. 1). We shall consider that up to the onset of freezing there has been no settlement of the foundation due to compaction of the foundation soil and also that the heaving forces are generated rather slowly and deformations at each moment of time are virtually stabilized. Because of this, at each moment of time one can apply known relationships between the pressure of the foundation on the soil and its final displacement.

In the absence of heaving forces T, the load of the structure, the weight of the foundation and the weight of the soil above it are equilibrated by the stress σ_o acting along the foot of the foundation. When the upper layer of soil freezes, heaving forces τ_Π are generated along the lateral surfaces of the foundation wall which partially or fully counteract the above-mentioned load, thus either decreasing or completely cancelling the pressure at the foot of the foundation. Here the performance of the foundation can be divided into two stages:

The first stage in the performance is defined by the boundaries of stress variation along the foot of the foundation σ within the range of σ_c to zero.

In the second stage the stresses along the foot of the foundation are absent.

Since compacting of the foundation soils is considered to be terminated, the decrease in pressure along the foot of the foundation will result in swelling of the foundation soils owing to elastic counteraction to compacting, and the foundation will receive an upward displacement S. As a result of this displacement an additional pressure p will be generated along the upper surface of the anchor footing.

If the displacement of the foundation S is less than the swelling deformation of the foundation soil S_H, which corresponds to the first stage of the performance of the foundation, the equation for the equilibration of the forces at some moment of time t will be

$$P + (\gamma_M h_M + \gamma_T h_T + p)(F - f) - T - \sigma F = 0, \tag{1}$$

where P - load from the structure including the weight of the foundation,

F - the area of the foot of the foundation,

f - the cross-sectional area of the foundation wall,

γ_M, γ_T - bulk density of the frozen and unfrozen ground, respectively,

h_M, h_T - the thickness of the layers of frozen and unfrozen soil above the anchor footing,

σ - stresses along the foot of the foundation, and

$\sigma_o \geqslant \sigma \geqslant 0$ when $0 \leqslant T \leqslant P + (\gamma_M h_M + \gamma_T h_T + p)(F - f)$.

In the subsequent moment of time t + dt the thickness of the frozen soil will increase by the value of dh_M and the thickness of the unfrozen soil will decrease by the same value; the result and heaving force will be increased by the factor dT and the stress a along the foot and upper surface of the foundation will vary respectively by the values $d\sigma$ and dp. Then the equation for equilibration of forces will have the form

$$P + [\gamma_M (h_M + dh_M) + \gamma_T (h_T - dh_M) + p + dp](F - f) - (T + dT) - (\sigma + d\sigma) F = 0. \tag{2}$$

Subtracting equation (1) from equation (2) we obtain the equation for equilibration in the differential form

$$[(\gamma_M - \gamma_T)\, dh_M + dp]\,(F - f) - dT - F d\sigma = 0. \qquad (3)$$

As is known[2], the relationship between the pressure of the foundation and its displacement S has the form:

$$q = \frac{SE}{b\,\omega\,(1 - \mu^2)}. \qquad (4)$$

Here q - the pressure resulting from the foundation; b - the shortest side of the rectangle in plan view or the radius of a circular foundation in plan view; E, μ - the deformation modulus and lateral expansion of the soil respectively; ω - the coefficient depending on the shape of the foundation in plan view and on its rigidity. If there is a rigid underlying layer the coefficient ω will also depend on the relative depth of this layer.

Neglecting the negligible deformation of the frozen layer which draws the foundation towards it, in comparison with the deformation of the unfrozen soil above the foundation, one can consider the frozen soil to be a rigid underlying layer and on the basis of the formula (4)

$$p = \frac{SE}{b\,\omega_0\,(1 - \mu^2)}. \qquad (5)$$

In this formula coefficient ω_0 depends not only on the shape of the foundation and its rigidity, but also on the thickness of the unfrozen layer of soil. On the other hand, from the same formula (4) we obtain

$$\sigma_0 - \sigma = \frac{S_1 E_H}{b\,\omega\,(1 - \mu^2)}, \qquad (6)$$

where E_H is the modulus of elastic deformation of the soil on swelling and S_1 is the displacement within the limits of the first stage of the foundation's performance.

There are special tables[2] for determining coefficients ω and ω_0.

The heaving force on the foundation can be expressed in the general form by the relationship

$$T = u\,\tau_n\,h_M, \qquad (7)$$

where u is the perimeter of the vertical member of the foundation; τ_{Π} is the specific heaving force, converted to a unit of adfreezing interface between the soil and the foundation wall.

From equation (3), keeping in view relationships (5), (6) and (7), one can obtain for the first stage in the foundation performance

$$S_1 = \frac{u\bar{\tau}_{\Pi} - (\gamma_M - \gamma_T)(F - f)}{E\,\omega(F - f) + E_H \bar{\omega}_0 F}\, b\,\omega\,\bar{\omega}_0 (1 - \mu^2)\, h_M, \qquad (8)$$

where $\bar{\tau}_{\Pi}$ and $\bar{\omega}_0$ are averaged values corresponding to values for the period of variation in depth of freezing from zero to h_M.

The last equation is valid under the condition that $S_1 \leqslant S_H$, where S_H is the full value of foundation soil swelling which can be determined from equation (6) if one puts in it $\sigma = 0$:

$$S_H = \frac{\sigma_0}{E_H}\, b\,\omega (1 - \mu^2). \qquad (9)$$

The depth of freezing corresponding to termination of the process of foundation soil swelling can be obtained from the relationships (8) and (9):

$$h_{MH} = \sigma_0 \frac{\frac{E\,\omega}{E_H \bar{\omega}_0}(F - f) + F}{u\bar{\tau}_{\Pi} - (\gamma_M - \gamma_T)(F - f)}. \qquad (10)$$

When freezing of the soil goes beyond the value of h_{MH}, which will correspond to the second stage in the foundation's performance, there is a further upward displacement of the foundation.

The value of the foundation displacement during the second stage can be obtained from equation (3) in which one should put $d\sigma = 0$,

$$S_2 = \frac{u\bar{\tau}_{\Pi} - (\gamma_M - \gamma_T)(F - f)}{E(F - f)}\, b\,\bar{\omega}_0 (1 - \mu^2)(h_M - h_{MH}) \qquad (11)$$

when

$$h_M > h_{MH}.$$

Here $\bar{\tau}_{\Pi}$ and $\bar{\omega}_0$ are averaged values corresponding to the value for the period of soil freezing from the depth h_{MH} to the depth h_M.

For practical calculations the value of tangential heaving forces converted to the unit of adfreezing surface can be considered constant with a sufficient degree of accuracy (equal to the mean value) during the entire period of increase in heaving forces T.

The full value of foundation displacement corresponding to the termination of soil freezing is determined by the formula

$$S_{\text{п}} \leqslant \Delta, \tag{12}$$

where S_k is the foundation displacement during the second stage calculated from formula (11) with $h_M = h_{M\Pi}$, and $h_{M\Pi}$ is the full (maximum) depth of freezing.

The value of S_{Π} at the moment of maximum generation of heaving forces T_{max} must be limited, i.e. the condition must be fulfilled

$$S_{\text{п}} = S_{\text{н}} + S_k, \tag{13}$$

where Δ is the permissible displacement of the foundation.

The question of permissible foundation displacement resulting from heaving forces has not yet been developed. In solving this problem one must consider the performance of the structure as a whole and the performance of the foundation soil. Obviously the latter will be the limiting factor. For the time being one can confidently say that displacement of the foundation equal to the swelling of the foundation soil because of its elastic counter-action to compacting, does not pose any danger because of this small value.

With fulfillment of equation (13) it may occur that the additional pressure on the upper surface of the anchor footing, defined by formula (5), is inadmissibly large and a region of plastic deformation will occur in the soil. Since the zone of plastic deformation cannot make an exit to the surface owing to the rigid layer of frozen soil, the region of plastic deformation in the unfrozen soil can be generated only in the direction of streamlining around the anchor footing, making its exit under the foot of the foundation. Calculation shows that for the formation of such regions of plastic deformation, the additional pressure p must be very large.

For regions with deep winter freezing, when the depth of the foot of the foundation is usually not much greater than the depth of the freezing, the real problem is the generation of a region of plastic deformation in the layer of frozen soil (Fig. 2). Here it is assumed that the layer of unfrozen soil lying above the anchor footing is in fact a continuation of the footing up to the lower boundary of the frozen soil.

We shall neglect the tangential stresses in the vertical planes AA' and BB' passing through the foundation. Moreover, in calculations of the strength of frozen soils one can consider only bonding[3], for the sake of simplicity with some approximation. Then keeping in mind the mentioned assumptions and also the fact that along the remaining part of the lower boundary of the frozen soil there is the pressure $\gamma_M h_M$, the limiting pressure is defined by the Prandtl solution, well known in plasticity theory

$$p_{п} = \gamma_{м} h_{м} + 5{,}14 C_{дл}, \qquad (14)$$

where $C_{дл}$ is the long term shear strength of frozen soil.

Formula (14) was obtained for a two-dimensional (perimeter foundation). With a certain safety factor it can be applied also to the three-dimensional problem. For example, for a square foundation (in plan view) as shown by investigations of the theory of limiting equilibrium[4], in formula (13), instead of coefficient 5.14 one should use the coefficient 5.35.

These considerations are valid if the region of plastic deformation does not extend to the ground surface which is determined by the relation $h_{пл} < h_M$. The value $h_{пл}$ can be obtained from the relation $h_{пл} b' \cdot$ sine 45^o, where b' is the greatest span of the anchor footing.

A properly anchored foundation, except for condition (12), should also satisfy the condition

$$p < \frac{p_{п}}{k}, \qquad (15)$$

where k is the safety factor which should be not less than 1.5, p is the additional pressure determined by formula (5) with S = S .

As an explanation to the suggested method of design a numerical example is given below.

It is required to check the foundation for heaving forces constructed in the form of a vertical member with a cross-section of 40 x 40 cm with an anchor footing having the dimensions of 100 x 100 cm in plan view, and the upper surface of the footing is at a depth of 2.2 m from the ground surface. The depth of freezing $h_{M\Pi} = 2$ metres. The initial pressure along the foot of the foundation $\sigma_o = 2$ kg/cm^2.

Foundation data: $F = 10^4$ cm^2, $f = 1600$, $F - f = 8400$ cm^2, $u = 160$ cm.

Soil characteristics: $\mu = 0.3$, $E - 80$ kg/cm^3, $E_H = 300$ kg/cm^2; $\bar{\tau}_\Pi = 1$ kg/cm^2, $C_{\text{ДЛ}} = 3$ kg/cm^2, $\gamma_T = 1.6$ t/m^3, $\gamma_M = 1.8$ t/m^3, $\omega = 0.95$.

The first component in expression (12), according to formula (9), is $S_H = 0.58$ cm.

The depth of freezing corresponding to the termination of foundation soil swelling is determined by formula (10). However, in this formula ω_o itself depends on h_{MH}, the value of which is as yet unknown, the calculation is carried out by series approximation. We assumed initially that $h_M \approx 0$, here $\bar{\omega}_o = \bar{\omega}_o = 0.64$. Then $h_{MH} = 112$ cm.

With this value of h_{MH} the value of $\omega_o = 0.41$ and the corresponding $\bar{\omega}_o = 0.5\ (0.64 + 0.41) = 0.52$.

Carrying out a second calculation using formula (10) with $\bar{\omega}_o = 0.52$, we obtain a second approximation $h_{MH} = 119$ cm, to which corresponds $\omega_o = 0.39$, and $\bar{\omega}_o = 0.51$. Hence, it is evident that the second approximation is completely satisfactory for practical calculations.

With soil freezing up to 2 m depth $\omega_o = 0.1$ and $\bar{\omega}_o = 0.5\ (0.39 + 0.1) = 0.24$.

The second component in equation (12) is determined by formula (11) $S_K = 0.30$ cm. Then the full value of upward foundation displacement at the end of freezing will be equal to $S_\Pi = 0.88$ cm.

It is assumed that $0.88 \text{ cm} < \Delta$.

The additional pressure on the surface of the anchor footing, according to equation (15) with $\omega_o = 0.5\ (0.64 + 0.1) = 0.37$ corresponding to the entire period of soil freezing from 0 to 2 metres, will be $p = 2.1$ kg/cm^2.

The limiting pressure from the condition of the formation of the region of plastic deformation, according to expression (14), is $p_{\Pi} = 16.4$ kg/cm^2. The safety factor $k = 7.8 > 1.5$.

References

1. Porkhaev, G.V. Sily zaankerivaniya fundamentov v talom grunte (Anchoring forces for foundations in unfrozen soil), Trudy Instituta Merzlotovedeniya AN SSSR, T. XIV, 1958.

2. Tsytovich, N.A. Mekhanika gruntov (Soil mechanics). Gosstroiizdat, 1951.

3. Osnovy geokriologii (Merzldovedeniya) (Principles of geocryology (Permafrost studies)), Part II. Izd. AN SSSR, 1959 (Translated by N.R.C.).

4. Prager, V. and Hodge, F.G. Theory of ideally plastic bodies. (Teoriya ideal'no plasticheskikh tel. Izd-vo Inostrannoi Literatury, 1956. Russian translation).

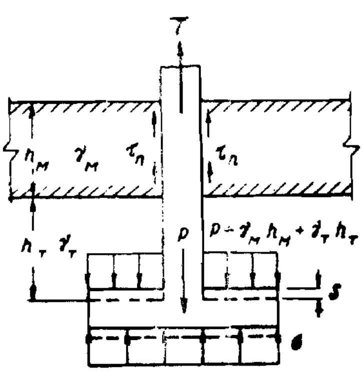

Fig. 1

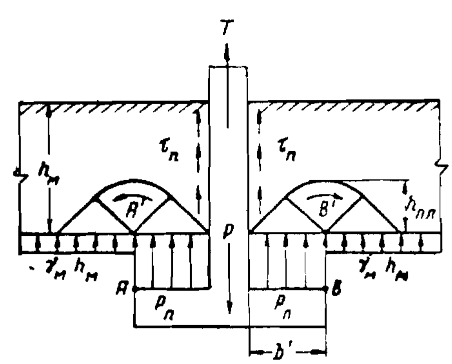

Fig. 2