Radio frequency high voltage power supplies

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RADIO FREQUENCY HIGH VOLTAGE POWER SUPPLIES

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RADIO FREQUENCY HIGH VOLTAGE POWER SUPPLIES

by

G.W.C. Mathers,

with revisions by S.J. Buchsbaum, November, 1952.

ABSTRACT

The theoretical analysis of the double-tuned, overcoupled air transformer, used in radio frequency high voltage supplies shows that the equivalent circuit will oscillate at two frequencies — one above, and one below the self-resonant frequency of the secondary circuit. These frequencies can be varied, within limits, around the self-resonant frequency of the secondary circuit, by changing the tuning of the primary circuit. The limits in the variation of the operating frequency are imposed by the effective Q of the primary circuit, which has a definite minimum value for good operation of the oscillator. The oscillator can be forced to operate at either the lower or higher resonant frequency by reversing the coupling between the secondary coil and the tickler coil.

Starting from the required d-c output power, a design procedure is outlined which will give all the circuit constants and operating conditions to obtain this output.

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I

INTRODUCTION

Recently, considerable work has been done on the development of high-voltage, low-power, direct-current supplies, using a radio frequency oscillator and a double-tuned, overcoupled air transformer for obtaining the high voltage. The usual output of these units is from 5,000 to 30,000 volts, at from 2 to 20 watts, and they have been used extensively as a source of high voltage for the accelerating anodes of cathode-ray tubes, where most of the power output is absorbed in the bleeder resistance.

The chief advantages of this method of developing high voltages are:-

1. At these frequencies (100 to 500 kc) a small air-core voltage step-up transformer and one or two small filtering condensers in the rectifying circuit are sufficient, resulting in small size and light weight.

2. Since the energy storage is low, and the voltage falls off rapidly with excessive loading, the units are comparatively safe to handle.

3. Insulation, and other related problems common to large iron-core transformers and large condensers, are avoided.

Several articles (7, 8, 9) on the construction and operation of these units have already been published. It is the purpose of this report to develop some of the theory and to outline a design procedure. A description of the construction and operation of the unit developed at the National Research Council will be included.

II

THEORY

Because the source of power is a Class C oscillator, the usual method of finding the output of a double-tuned transformer, using an equivalent generator and internal impedance, is not very satisfactory and a new approach was found necessary.

Essentially, the circuit is quite simple, as shown by the typical example in Fig. 1.

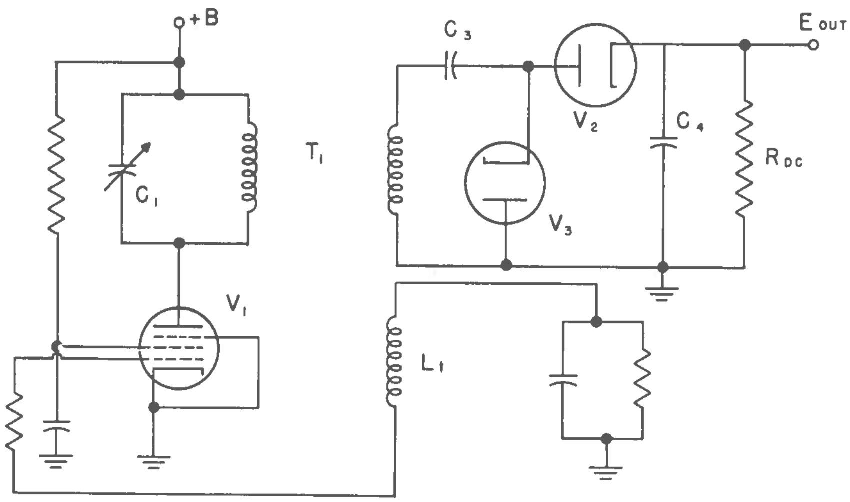


Fig. 1 Typical R.F.H.V. Supply Circuit

Here:-

V1	-	oscillator tube
V2, V3	-	rectifier tubes
T1	-	double-tuned, step-up transformer
L_t	-	tickler coil
C3, C4	-	filter condensers
R_{DC}	-	direct current load resistance

The equivalent alternating current circuit is shown in Fig. 2.

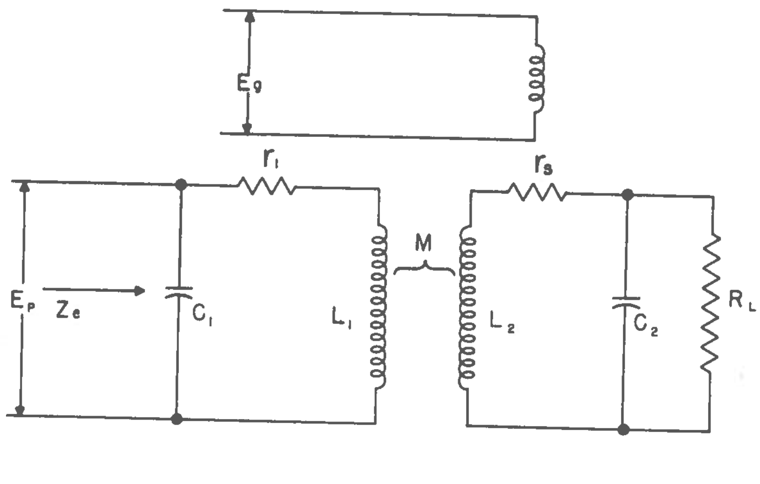
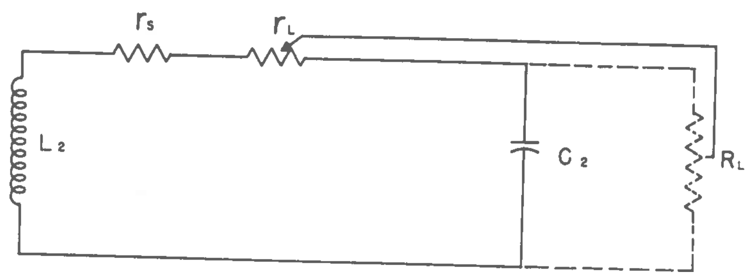


Fig. 2 Equivalent A.C. Circuit For Fig. I



Equivalent Secondary Circuit
Fig. 3

R_L^*, the equivalent alternating current load resistance, equals $\frac{R_{DC}}{2p^2}$, where p equals the number of stages of voltage multiplication.

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* See "List of Symbols", p. 50

Consider the parallel combination of C_2 and R_L:-

$$Z = \frac{R_L \cdot \frac{1}{j2\pi f C_2}}{R_L + \frac{1}{j2\pi f C_2}} = \frac{R_L - j2\pi f C_2 R_L^2}{1 + 4\pi^2 f^2 C_2^2 R_L^2} \qquad (1)$$

In practice, R_L is large, and $1 \ll 4\pi^2 f^2 C_2^2 R_L^2$

Therefore
$$Z = \frac{1}{4\pi^2 f^2 C_2^2 R_L} - \frac{j}{2\pi f C_2} \qquad (2)$$
$$= r_L - \frac{j}{2\pi f C_2}$$

The equivalent impedance of the network as seen from the plate of the oscillator tube is

$$Z_e = -j\,\frac{2\pi f_{01}^2 L_1}{f} + \frac{\dfrac{2\pi f_{01}^4 L_1}{f^3}}{\dfrac{r_1}{2\pi f L_1} + \dfrac{2\pi f k^2 (r_L + r_S) L_2}{(r_L + r_S)^2 + (2\pi f L_2 \theta_2)^2} + j\left(\theta_1 - \dfrac{(2\pi f k L_2)^2 \theta_2}{(r_L + r_S)^2 + (2\pi f L_2 \theta_2)^2}\right)}$$

If this equation is reduced into individual equivalent impedances, the resulting primary circuit is as shown in Fig. 4, where:-

$$X_C = -j\,\frac{2\pi f_{01}^2 L_1}{f}$$

$$X_{1e} = -j\,\frac{2\pi f_{01}^4 L_1}{f^3 \theta_1}$$

$$X_{2e} = j\left\{\frac{f_{01}^4 L_1 (r_L + r_S)^2}{2\pi f^5 k^2 L_2^2 \theta_2} + \frac{2\pi f_{01}^4 L_1 \theta_2}{f^3 k^2}\right\}$$

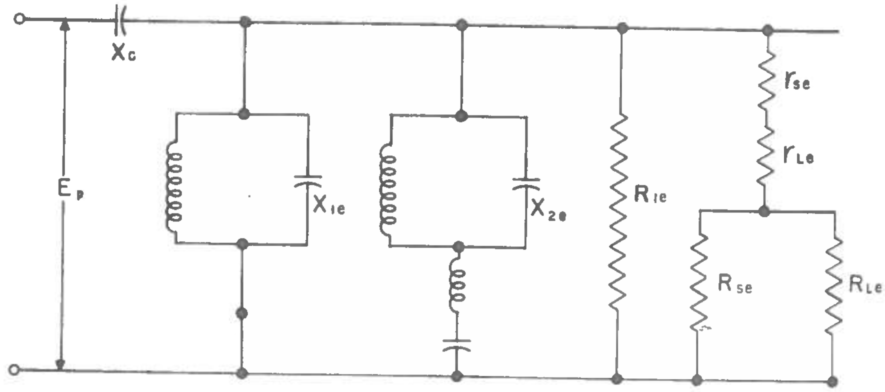
$$R_{1e} = \frac{4\pi^2 f_{01}^4 L_1^2}{f^2 r_1}$$

$$r_{Se} = \frac{1}{k^2}\,\frac{f_{01}^4}{f^4}\,\frac{L_1}{L_2} \cdot r_S$$

$$r_{Le} = \frac{1}{k^2} \; \frac{4\,\pi^2\, f_{01}^4\, f_{02}^4}{f^6} \; L_1\, L_2 \cdot \frac{1}{R_L}$$

$$R_{Se} = \frac{1}{k^2} \; \frac{4\,\pi^2\, f_{01}^4}{f^2} \; L_1\, L_2 \; \frac{1}{r_S} \; \theta_2^2$$

$$R_{Le} = \frac{1}{k^2} \; \frac{f_{01}^4}{f_{02}^4} \; \frac{L_1}{L_2} \; \theta_2^2 \; R_L$$



Equivalent Primary Circuit Impedances

Fig. 4

The variation of the equivalent impedances with frequency, is shown in Fig. 5 and Fig. 6, for fixed values of f_{01} and f_{02}. Instead of plotting reactance and resistance, the susceptances and conductances were plotted, since the susceptance goes to zero at resonance. The equivalent secondary resistances have been combined into one, and are represented by the conductance

G_{2e}. The graphs were plotted for the following circuit constants which were obtained from an actual circuit:

f_{02} = 200 kc.	L_1 = 0.15 mh.
f_{01} = 200 kc. and 210 kc.	r_1 = 2 ohms
$r_S + r_L$ = 700 ohms	k^2 = .05
L_2 = 43 mh.	

The capacitive reactance, X_c, (Fig. 4) was found to be negligible for all values of frequency plotted. It will later be shown that this is negligible in most practical circuits. To simplify computation, the resistances r_1, r_S, and r_L were assumed constant for all frequencies.

There are three frequencies at which the equivalent susceptance is zero -- f_1, f_2, and at or very near to f_{02}. The oscillator will operate at either f_1 or f_2 but not at the other apparent resonant frequency. This may be explained in the following manner.

Suppose it does start to operate at f_{02}. If the frequency increases slightly due to a small change in voltage or load, the equivalent susceptance will now be inductive. Thus, the frequency will increase further, since the susceptance of a parallel circuit is inductive below resonance. Stable operation will be reached finally at f_2. The reverse is true for a small decrease in frequency from f_{02}, in which case stable operation will be attained at f_1. At f_1 or f_2, a change in frequency will produce a change in susceptance which will tend to hold the frequency constant.

If the coupling between the primary and secondary is decreased, the two resonant frequencies (f_1 and f_2) approach f_{02}, until at critical coupling and for under-coupled circuits, they coincide. In this work, critical or under-coupled circuits, are not used because of the poor efficiency of energy transfer from the primary to secondary and the poor frequency stability.

As shown in Fig. 5, the resonant frequencies may be changed by changing f_{01}, other factors being constant.

Theoretically, it is possible to operate at practically any desired frequency since the oscillator works at the resonant frequency of the <u>equivalent</u> primary circuit.

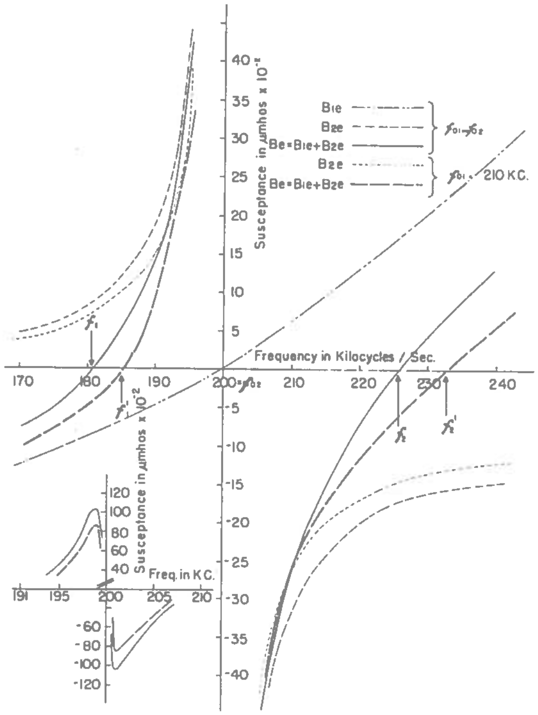


Fig.5 Equivalent Primary Susceptances Versus Frequency For Two Values Of f_{01}, The Primary Resonant Frequency.

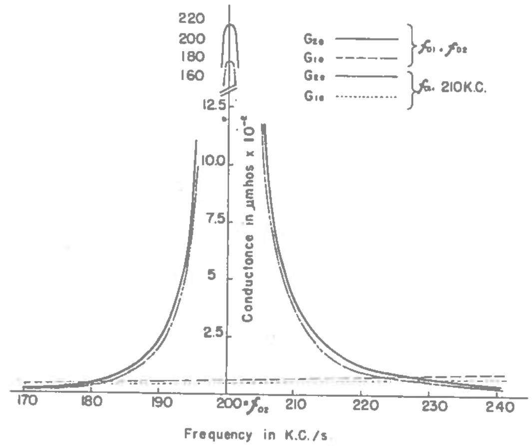


Fig. 6 Equivalent Primary Conductances versus Frequency for 2 Values of f_{01}, the Primary Resonant Frequency.

The resonant frequencies can be found by putting the imaginary term in the expression Z_e equal to zero, again neglecting the small effect of X_c. Thus we have:-

$$\theta_1 - \frac{4\pi^2 f^2 k^2 L_2^2 \theta_2}{R_2^2 + 4\pi^2 f^2 L_2^2 \theta_2^2} = 0 \qquad (4)$$

As has been shown already, a trivial solution results if

$$\theta_1 = \theta_2 = 0 \quad \text{i.e.} \quad f = f_{02}$$

The expression may be simplified as follows:-

$$\theta_1(R_2^2 + 4\pi^2 f^2 L_2^2 \theta_2^2) \quad = \quad 4\pi^2 f^2 k^2 L_2^2 \theta_2^2$$

dividing by $4\pi^2 f^2 L_2^2 \theta_2^2$

$$\frac{\theta_1}{\theta_2} \cdot \frac{1}{Q_2^2} + \theta_1 \, \theta_2 \quad = \quad k^2,$$

$$\text{where } Q_2 = \frac{2\pi f L_2}{R_2},$$

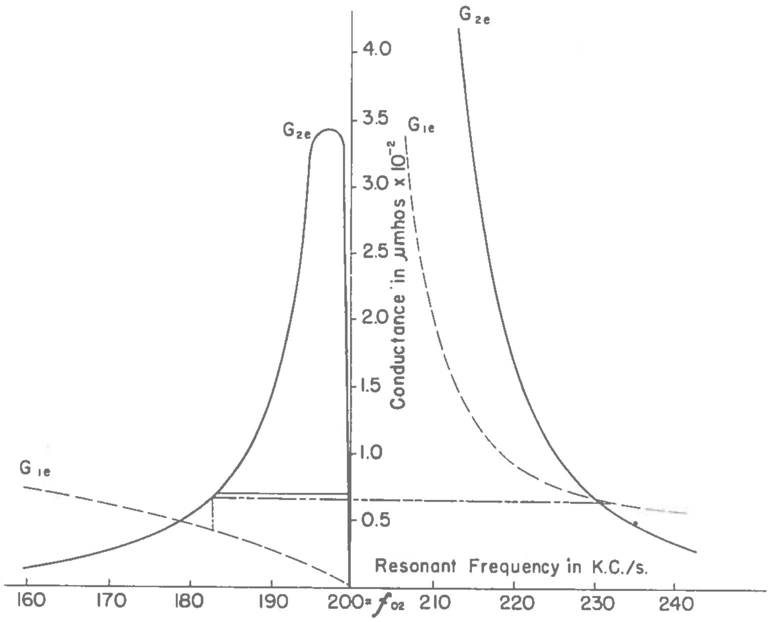
$$\text{or } \theta_1 \, \theta_2 \quad = \quad k^2 - \frac{\theta_1}{\theta_2} \cdot \frac{1}{Q_2^2} \qquad (5)$$

One interesting point shown by the equation is the fact that the resonant frequency can never be in between f_{01} and f_{02}. If the resonant frequency was in this region θ_1 and θ_2 would be of opposite sign. This would mean that the left hand side of the equation would be negative and the right hand side positive.

In practice, the term $\frac{\theta_1}{\theta_2} \cdot \frac{1}{Q_2^2}$ is negligible compared with k^2 providing θ_1 is not large, and θ_2 small. The operating frequencies are therefore given by

$$\theta_1 \, \theta_2 = k^2 \qquad (6)$$

Since the circuit will operate at the resonant frequencies, the equivalent reactance at the operating frequency is infinite, leaving only the conductances G_{1e} and G_{2e}. The effect of varying the resonant frequencies on the conductances G_{1e} and G_{2e}, by changing f_{01}, is shown in Fig. 7.



Note: Larger G_{1e} for the same G_{2e} at the higher resonant frequency.

Fig. 7 Variation Of Equivalent Conductances, G_{1e} & G_{2e}, With Resonant Frequency.

The equations for these variables, after substituting for f_{01} from equation 6, are:

$$G_{1e} = \frac{1}{R_{1e}} = \frac{r_1}{4\pi^2 f^2 L_1{}^2 (1 - \frac{k^2}{\theta_2})^2} \tag{7}$$

$$G_{2e} = \frac{1}{R_{2e}} = \frac{k^2 L_2}{(1 - \frac{k^2}{\theta_2})^2 L_1 (R_2 + \frac{4\pi^2 f^2 L_2^2 \theta_2^2}{R_2})} \tag{8}$$

Since the value of R_2 was again assumed constant, the actual value of G_{2e} should be greater on the low frequency side of f_{02}, and smaller on the high frequency side of f_{02}, than is shown in the graph.

The power delivered by the oscillator to the primary circuit is $E_p^2 G_{1e}$, and to the secondary circuit $E_p^2 G_{2e}$. The fraction of the secondary power delivered to the load resistance, R_L, depends on the ratio of the secondary circuit resistance r_S, and the effective series load resistance r_L. This ratio is established primarily by the secondary circuit design but is not affected appreciably by a change in operating frequency. Therefore, for maximum power output, the power delivered to G_{2e} should be a maximum. How much of this power is converted to output power is a matter of secondary circuit design which will be dealt with later.

For a constant value of E_p, the maximum power output will be obtained when G_{2e} is a maximum, with G_{1e} a minimum for good efficiency. Referring to Fig. 7; this condition will be realized if the circuit operates at a frequency just below f_{02}. On the high frequency side of f_{02}, a higher value of G_{2e} is obtainable but there is also a much higher value of G_{1e}.

The resonant frequency at which G_{2e} is a maximum can be determined as follows:

$$\frac{1}{G_{2e}} = \frac{1}{k^2}\left(1 - \frac{k^2}{\theta_2}\right)^2 \frac{L_1}{L_2}\left[R_2 + \frac{(2\pi f L_2 \theta_2)^2}{R_2}\right] \qquad (9)$$

Putting $f_{02} = f + \Delta$

$$f_{02}^2 = f^2 + 2\Delta f + \Delta^2$$

$$\theta_2 = 1 - \frac{f_{02}^2}{f^2} = -\frac{2\Delta}{f} \quad (\text{since } \Delta^2 << f^2)$$

$$\frac{1}{G_{2e}} = \frac{1}{k^2} \left(1 + \frac{k^2 f_{02}}{2\Delta} - \frac{k^2}{2}\right)^2 \frac{L_1}{L_2} \left(R_2 + \frac{16\pi^2 L_2^2 \Delta^2}{R_2}\right)$$

Differentiating with respect to Δ and equating the result to zero, the equation becomes

$$\Delta^3 \cdot \frac{16\pi^2 L_2^2}{R_2} (2 - k^2) = k^2 f_{02} R_2$$

from which

$$\Delta = \left[\frac{k^2 R_2^2 f_{02}}{16\pi^2 L_2^2 (2 - k^2)}\right]^{1/3}$$

Other factors being constant, the efficiency is better at the lower resonant frequency because:-

1. For corresponding values of G_{2e}, the value of G_{1e} is lower. This is shown in Fig. 7.
2. The effective series load resistance, r_L, varies inversely with frequency squared, since

   $$r_L = \frac{4\pi^2 f_{02}{}^4 L_2{}^2}{f^2 R_L}$$

Thus the ratio $\frac{r_L}{r_S}$ is higher and more power is delivered to the load. At this point, it may seem possible, with any reasonable value of circuit constants, to obtain any required power output simply by varying the primary tuning (f_{01}), (Assuming that E_p can be kept constant by driving the oscillator tube harder for larger power outputs). However, there are three factors which limit the range of the operating frequency and make good circuit design essential.

These are:-

1. Effective Q of the primary circuit.
2. Load resistance, $\frac{1}{G_{1e} + G_{2e}}$, must match the oscillator tube.
3. The phase of the grid voltage feedback.

These will be considered in detail, individually.

1. Effective Q of the primary circuit.

Consider the simple circuit as shown in Fig. 8.

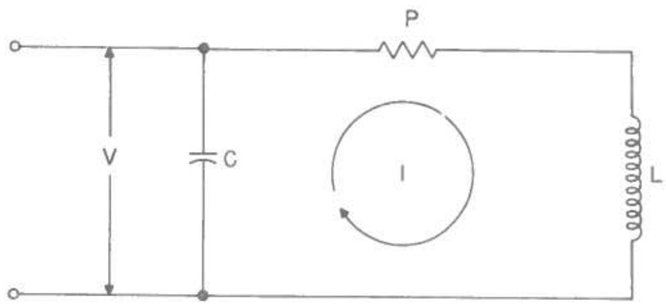


Fig. 8 Parallel-tuned Resonant Circuit.

P = the power dissipation

The energy stored in the condenser per cycle = CV^2

The energy dissipated per cycle = $P/_f$

The ratio of the energy stored to the energy dissipated per cycle =

$$\frac{CV^2}{P/_f} = \frac{2\pi fCV^2}{2\pi P} \tag{11}$$

The circulating current in the circuit = $2\pi fCV$

$$\text{Therefore, } \frac{\text{the energy stored/cycle}}{\text{the energy dissipated/cycle}} = \frac{V\ I}{2\ \pi\ P} \tag{12}$$

$$= \frac{\text{volt amp.}}{2\ \pi\ \text{x power output}}$$

For good oscillator operation the energy stored per cycle should equal twice the energy dissipated per cycle. If the energy stored is less than this, operation will be very unstable, and if the reverse is true, the efficiency will be low. For the relatively small powers involved in this particular application, the latter condition is preferable to the former; therefore, the energy stored per cycle should be equal to, or slightly greater than, twice the energy dissipated per cycle.

That is:- $$V\,I \geq 4\,\pi\,P \qquad (13)$$

but $$V\,I = \frac{V^2}{X_C}$$

and equation (13) becomes

$$X_C \leq \frac{V^2}{4\,\pi\,P} \qquad (14)$$

If R is the equivalent parallel resistance of the circuit,

$$R = Q\,X_L = Q\,X_C$$

$$P = \frac{V^2}{R} = \frac{V^2}{Q\,X_C}$$

Therefore : $$\frac{V^2}{X_C} \geq \frac{4\,\pi\,V^2}{Q\,X_C}$$

$$Q \geq 4\,\pi \qquad (15)$$

Referring to Fig. 2,

the circulating volt-amps in the primary circuit

$$= \frac{E_p^2}{X_{C1}}$$

and $$X_{C1} = \frac{2\,\pi\,f_{01}^2\,L_1}{f}$$

Thus from equation (14):-

$$X_{C1} \leq \frac{E_p^2}{4\,\pi\,P_T} \qquad (16)$$

where P_T is the a-c power output of the oscillator tube.

If the resonant frequency is approximately equal to f_{02}, (e.g. at maximum G_{2e}), f_{01}, and thus X_{C1} will be very large. As shown by the preceding discussion a limit is placed on the value of X_{C1} (equation (16)). Thus, depending on the plate voltage and power output required, the circuit will not always operate at a frequency which will give maximum G_{2e}.

In the graphs and theory used so far, the reactance of the series condenser X_c (see Fig. 4) has been neglected. The magnitude of this reactance is equal to X_{c1}, and is therefore limited to a low value with respect to the other primary impedances. Thus, in practical cases, it may be neglected with respect to the parallel load resistance.

2. Load Resistance

This has a definite value depending on the power output required, the voltage across the tank circuit, and the oscillator tube or tubes used, in order to obtain maximum power output and efficiency. In this case the total alternating current load resistance across the plate of the tube must equal $\frac{E_p^2}{P_T}$.

3. Grid Voltage Feedback

So far, it has been assumed that the oscillator operates at the resonant frequency of the equivalent primary circuit. Depending on the phase of the grid voltage, with respect to the plate current, the actual operating frequency will differ slightly from the resonant frequency. This difference may be fairly large if the equivalent primary Q is low, or if f is nearly equal to f_{02}. The efficiency will therefore be reduced and, if the difference is too large, the circuit will not operate. In practice, the operating frequency can be taken equal to the resonant frequency, as the difference is only of the order of 0.5 kc at maximum load, i.e. equivalent primary Q equal to 4 π. These factors limit the values of the circuit constants and operating frequency. Thus careful design is essential to obtain the required voltage and power output. To show how the theory developed above is used in the design of a high voltage supply, the design procedure will be outlined briefly. A more detailed analysis is contained in the design section of the report and the various factors which influence the selection of the circuit constants are more fully discussed.

Knowing the power and voltage output required and the d-c plate supply voltage for the oscillator tube, a suitable rectifier circuit and value of secondary inductance are selected. The self-resonant frequency of the secondary circuit is estimated and thus the approximate value of $Q_2 = \frac{2\pi f_{02} L_2}{R_2}$ can be found. Assuming a value of Q_1 of 100, the coefficient of coupling k can be computed. For good efficiency k should equal 20 k_c, where k_c equals $\frac{1}{\sqrt{Q_1 Q_2}}$. The secondary circuit power is therefore known and the primary circuit power loss may be estimated at about 25% of secondary circuit power. Thus, the required total alternating current power output of the tube is known. A rough calculation can be made to determine a suitable plate voltage swing, E_p, for good efficiency with the oscillator tube or tubes selected.

Now referring to Fig. 4:-

The resistances r_{Se} and r_{Le} in practice, are negligible compared with R_{Se} and R_{Le}, thus the required effective load resistance R_{Le} is equal to $\frac{E_p^2}{P_{DC}}$.

There are still three factors to be determined. These are:-

1. The operating frequency, f
2. The primary self resonant frequency, f_{01}
3. The primary inductance, L_1

These factors are related by the equations:-

$$\left(1 - \frac{f_{01}^2}{f^2}\right)\left(1 - \frac{f_{02}^2}{f^2}\right) = k^2 \qquad (17)$$

$$R_{Le} = \frac{E_p^2}{P_{DC}} = \frac{1}{k^2} \frac{f_{01}^4}{f_{02}^4} \frac{L_1}{L_2} \left(1 - \frac{f_{02}^2}{f^2}\right)^2 R_L \tag{18}$$

$$X_{C1} = \frac{2 \pi f_{01}^2 L_1}{f} \leqq \frac{E_p^2}{4 \pi P_T} \tag{19}$$

The solutions of these equations yield a cubic equation for f, or any of the other unknowns selected.

The best method of solving this set of equations is to use a series of approximations, taking for the first, $f = f_{02} - 4\Delta$, where Δ is given by equation (10). f_{01} and L_1 can therefore be obtained from equations (17) and (19), and equation (19) checked. If satisfactory, the design of the primary inductance, tuning capacity and the oscillator can be completed. In this way it is possible to obtain an optimum design for the circuit with resulting maximum efficiency.

Note: It was found from experience that the first approximation $f = f_{02} - 4\,\Delta$ worked very well in any circuits designed.

Now we return to the theoretical discussion. The method used to couple energy from the output, to the grid of the oscillator tube also determines the operation of the circuit. This is usually done by mutual coupling between the tickler and secondary or primary coils. Consider the equivalent circuit shown in Fig. 9. Since the equivalent plate resistance R_p of the tube is large, the circuit may be reduced to the one shown in Fig. 10 where:-

$$R_1 = r_1 + \frac{1}{4 \pi^2 f^2 C_1^2 R_p}$$

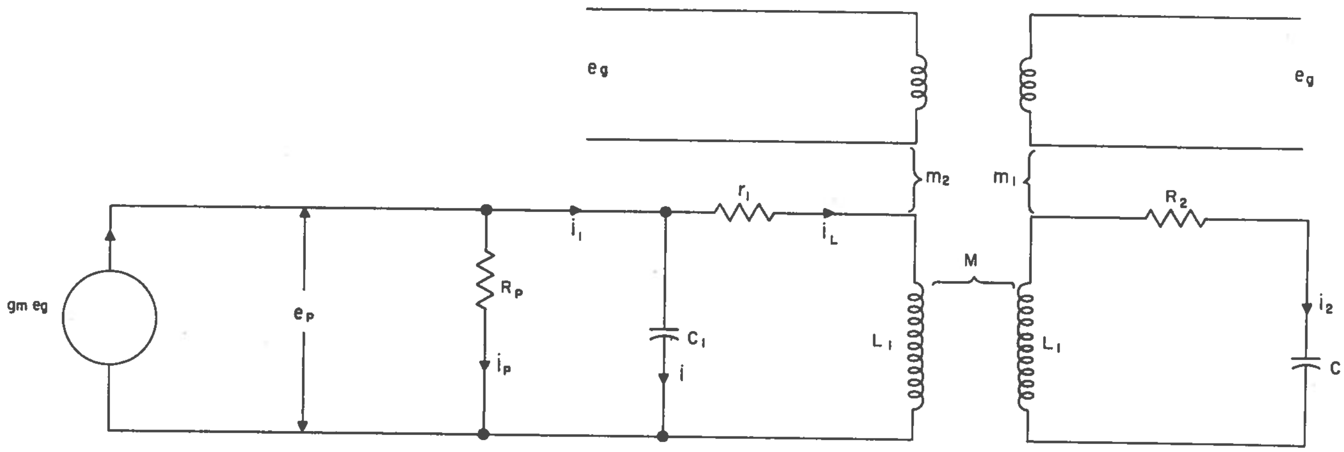


Fig. 9 Equivalent Oscillator Circuit

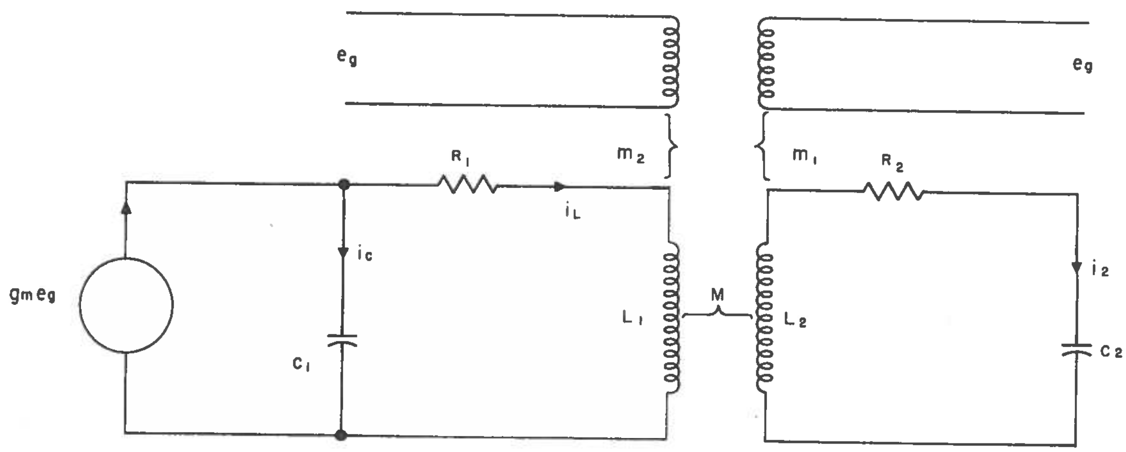


Fig. 10 Equivalent Oscillator Circuit With Rp Included
In Tank Circuit

The frequency at which e_g is in phase with $g_m e_g$ can be found as follows:-

Putting $\omega = 2\pi f$,

$$+ g_m e_g = i_C + i_L \tag{20}$$

$$\frac{-j\, i_C}{\omega C_1} = i_L (R_1 + j \omega L_1) \pm j \omega M i_2 \tag{21}$$

$$-j \frac{g_m e_g}{\omega C_1} = i_L \left(R_1 + j \omega L_1 - \frac{j}{\omega C_1}\right) \pm j \omega M i_2 \tag{22}$$

$$0 = i_2 \left(R_2 + j \omega L_2 - \frac{j}{\omega C_2}\right) \pm j \omega M i_L \tag{23}$$

1. Tickler coil coupled to the secondary coil:

Assuming negligible grid current flow:-

$$e_g = \pm j \omega m_1 i_2 \tag{24}$$

$$\text{also } j \omega L_1 - \frac{j}{\omega C_1} = j \omega L_1 \theta_1$$

$$\text{and } j \omega L_2 - \frac{j}{\omega C_2} = j \omega L_2 \theta_2$$

Equations (22), (23) and (24) can be combined and an expression found in terms of e_g alone.

$$- j \frac{g_m e_g}{\omega C_1} = \frac{e_g}{\omega^2 M m_1} \left[R_2 R_1 - \omega^2 L_2 L_1 \theta_1 \theta_2 + j(\omega L_1 \theta_1 R_2 + \omega L_2 \theta_2 R_1) \right] + \frac{e_g M}{m_1} \tag{25}$$

For e_g in phase with $g_m e_g$:-

$$\frac{R_2 R_1 - \omega^2 L_2 L_1 \theta_1 \theta_2}{\omega^2 M m_1} + \frac{M}{m_1} = 0 \tag{26}$$

Putting

$$M = k\sqrt{L_1 L_2}$$

$$\frac{R_2}{\omega L_2} = \frac{1}{Q_2}$$

$$\frac{R_1}{\omega L_1} = \frac{1}{Q_1}$$

equation (26) becomes

$$\theta_1 \theta_2 = k^2 + \frac{1}{Q_1 Q_2} \qquad (27)$$

The circuit will oscillate at the frequency given by this equation, which is very close to the resonant frequency given by $\theta_1 \theta_2 = k^2$ (6)

Let the frequency defined by equation (27) = f_p

and let $f_{02} - f_p = \Delta_p$

and $f_{01} - f_p = \alpha_p$

Let the resonant frequency defined by equation (6) = f_r and let $f_{02} - f_r = \Delta_r$ and $f_{01} - f_r = \alpha_r$

Assuming Δ_p^2, Δ_r^2, α_p^2, α_r^2, $<< f^2$ and also letting $f_p = f_r = f$ since the difference is very small, equations (27) and (6) become:-

$$\frac{4\Delta_p \alpha_p}{f^2} = k^2 + \frac{1}{Q_1 Q_2} \qquad (28)$$

$$\frac{4\Delta_r \alpha_r}{f^2} = k^2 \qquad (29)$$

thus $$\Delta_p \alpha_p = \Delta_r \alpha_r + \frac{1}{Q_1 Q_2} \cdot \frac{f^2}{4} \qquad (30)$$

Therefore $\Delta_p \alpha_p > \Delta_r \alpha_r$

Since f_{01} and f_{02} are the same for f_p and f_r we can write

$$\Delta_p > \Delta_r$$

$$\alpha_p > \alpha_r \qquad (31)$$

Therefore, on the low frequency side of f_{02}, the actual operating frequency is slightly lower than the resonant frequency and on the high frequency side it is higher. In both cases, this frequency shift tends to lessen the load on the oscillator, thus reducing the power output.

The vector diagram for the circuit at resonance is shown in Fig. 11, for the resonant frequency both greater and less than f_{02}. Consider the case in which the mutual inductance between the primary and secondary is positive. For $f < f_{02}$, the secondary current vector is shown as 10 $i_2'(+M)$. For the grid voltage to have the right phase with respect to $g_m e_g$, the tickler-secondary coupling must be positive i.e. $e_g' = + j\omega m_1 i_2'$.

For $f > f_{02}$, a similar line of reasoning shows that the tickler-secondary coil coupling must be negative, i.e. $e_g'' = - j\omega m_1 i_2''$.

It is, therefore, possible to <u>force</u> the oscillator to work at either the lower or higher resonant frequency by changing the coupling of the tickler coil. This gives stable operation at one or the other selected frequency. As shown in the vector diagram, there is a greater phase difference between e_g and $g_m e_g$ for $f > f_{02}$. This results in still poorer efficiency at the higher resonant frequency.

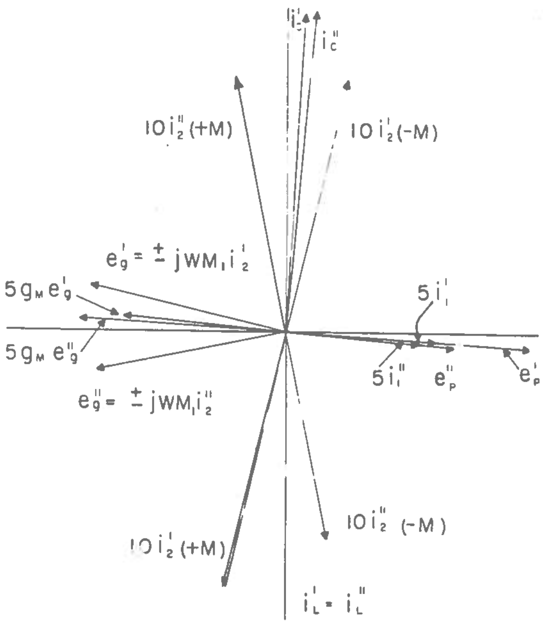


Fig. 11 Phase relations in the circuit of Fig. 9, at resonance, with the tickler coil coupled to the secondary coil. i_L' etc. for resonant frequency $< f_{o2}$ i_L'' etc. for resonant frequency $> f_{o2}$

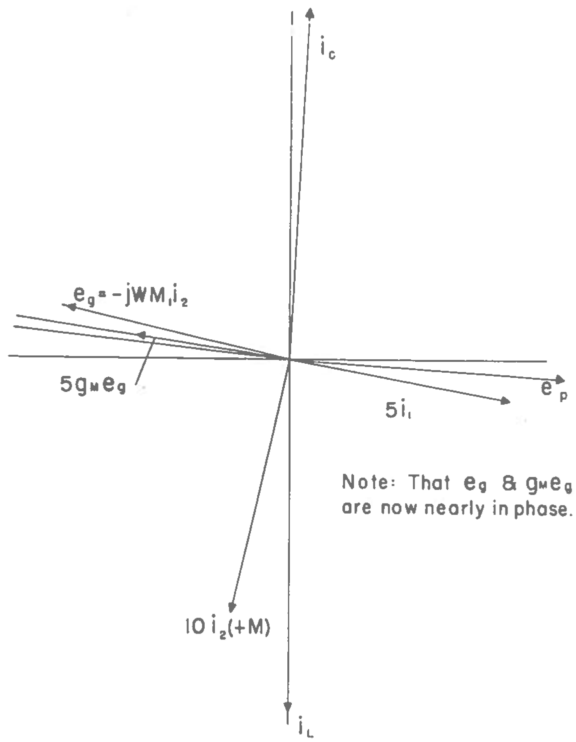


Fig. 12 Phase relations in the circuit of Fig. 9, after a decrease in frequency of 0.3 K.C/s from resonance ($f < f_{o2}$ and tickler coil coupled to the secondary coil.)

In Fig. 12 is shown the effect of a frequency decrease of 0.3 kc from the lower resonant frequency. This small change is sufficient to bring e_g and $g_m e_g$ nearly into phase.

2. Tickler coil coupled to the primary coil. $e_g = \pm j \omega m_2 i_L$

From equations (22) and (23):-

$$\frac{-\, g_m e_g m_2}{C_1} = e_g \left[R_1 + \frac{\omega^2 M^2 R_2}{R_2^2+\omega^2 L_2^2 \theta_2^2} + j \left(\omega L_1 \theta_1 - \frac{\omega^3 L_2 \theta_2 M^2}{R_2^2+\omega^2 L_2^2 \theta_2^2} \right) \right] \tag{32}$$

For e_g in phase with $g_m e_g$

$$\omega L_1 - \frac{\omega^3 L_2 \theta_2 M^2}{R_2^2+\omega^2 L_2^2 \theta_2^2} = 0 \tag{33}$$

This reduces to $\theta_1 \, \theta_2 = k^2 - \frac{\theta_1}{\theta_2} \cdot \frac{1}{Q_2^2}$ (34)

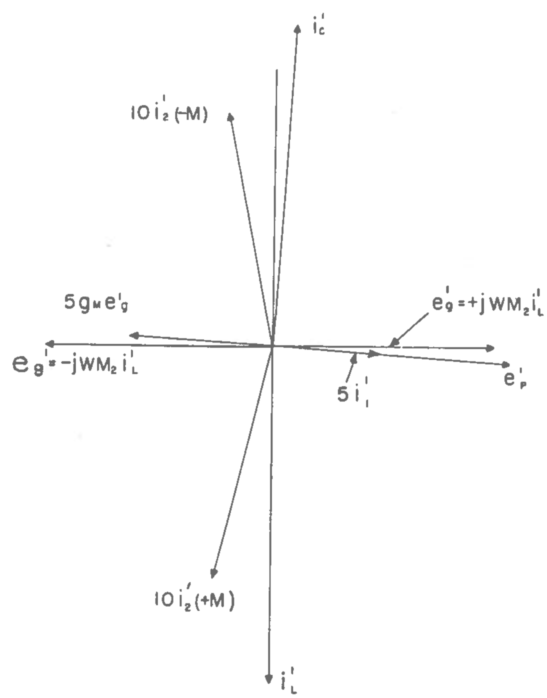


Fig. 13 Phase relations in the circuit of Fig. 9 ; with the tickler coil coupled to the primary coil.

The vector diagram for the primary coupled tickler coil is shown in Fig. 13. Notice that circuit will operate only if the tickler-primary coupling is positive and also that it does not matter at which resonant frequency the circuit is operating, the phase angle between $g_m e_g$ and e_g is the same.

The circuit will start oscillating at the frequency which presents the least load to the tube, and if the circuit is tuned to obtain maximum output, when the load reaches some value at or near maximum, the frequency will suddenly change over to the other resonant point which, for this f_{01}, will load the tube very lightly. Sudden small changes in load or voltage will also cause the frequency to jump if the circuit is loaded. As a result of this instability with primary-tickler coupling, the secondary-tickler coupling is used, unless it is specifically desired to operate with the former, in which case a phase-shifting network is required in the grid circuit.

III

DESIGN PROCEDURE

The principles developed in the theoretical discussion will now be applied to the design of a high-frequency power supply. For convenience, the equivalent radio frequency circuits used in the theory will be reproduced here with a few minor modifications. Fig. 14 shows the equivalent secondary circuit.

r_L = the effective secondary series load resistance

$$= \frac{4\pi^2 f_{02}^4 L_2^2}{f^2 R_L}$$

r_2 = the a-c resistance of the secondary coil

r_{HW} = the resistance coupled into the secondary circuit from rectifier heaters.

r_E = resistance coupled into the secondary circuit from grid excitation loss and the power loss in shielding.

r_S = $r_2 + r_{HW} + r_E$.

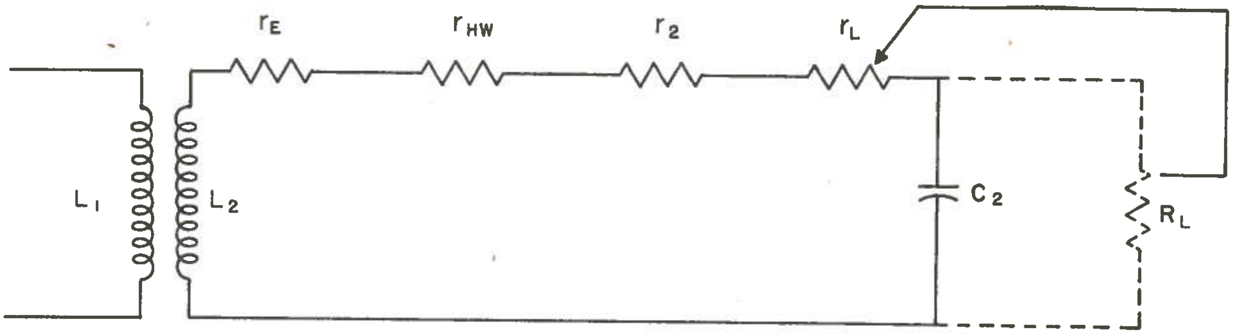


Fig. 14 Equivalent Secondary Circuit

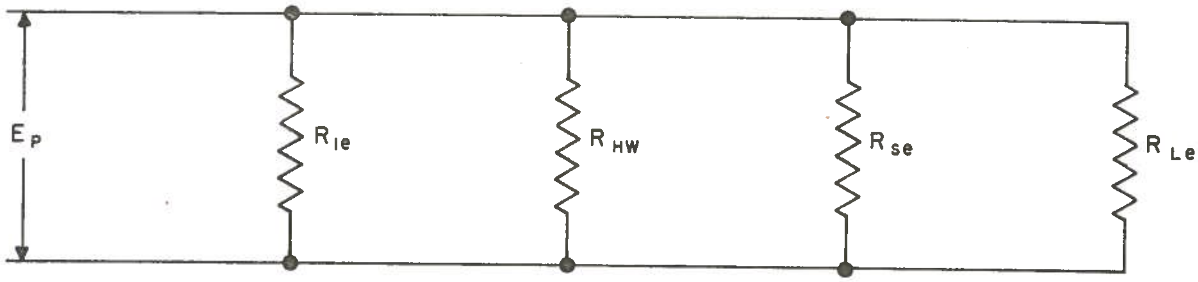


Fig. 15 Equivalent Primary Circuit At Resonance

Fig. 15 shows the equivalent primary circuit, with the reactances omitted, since at resonance the equivalent reactance is infinite. The resistances r_{se} and r_{Le} shown in Fig. 4 are neglected since they are negligible compared with R_{se} and R_{Le}, and

R_{HW} = resistance introduced into the primary circuit by the rectifier filaments coupled to the primary coil.

This circuit enables us to represent the various power losses in the following manner.

$$\text{Power loss in the primary coil} \quad \frac{E_p^{\,2}}{R_{le}}$$

Power loss in the rectifier heaters coupled to primary coil $\frac{E_p^2}{R_{HW}}$

Power loss in secondary circuit $\frac{E_p^2}{R_{Se}}$

Power output of the circuit $\frac{E_p^2}{R_{Le}} = P_{DC}$

The steps in the design are as follows:-

1. The required output voltage E_{DC}, power P_{DC}, and resistance R_{DC} are known.
2. Selection of the rectifier circuit:

The rectifier circuit may be a half-wave, or a voltage-multiplying circuit. In practice, the voltage drop in the rectifier tubes is neglected since the currents drawn are very small. Some of the more important considerations in the selection of a rectifier circuit are listed below:

2:1 A half-wave circuit, requiring only one tube and one or two condensers, will occupy much less space than a voltage-multiplying circuit.

2:2 Since the half-wave circuit uses only one tube, there is only one filament to heat.

The low filament-power tubes (0.5 to 1 watt) usually used can be readily heated by coupling with a few turns of wire to the primary or secondary coil, thus eliminating large insulated filament transformers. With a voltage-multiplying circuit more filament power is required. With a half-wave circuit, it is possible to use a filament transformer, if necessary, for higher-power filaments, but with a voltage-multiplying circuit the insulated filament transformer, if connected to the filaments which are at r-f potentials, will act as a short circuit across the secondary coil, because of its large distributed capacity to ground.

2:3 A voltage-multiplying circuit will increase the efficiency considerably because of the lower power loss in the secondary coil. For a half-wave circuit, the secondary coil power loss will be approximately four times that for a voltage-doubling circuit, with the same output. This may be a very important consideration in a high-voltage circuit, for in this case the power loss in the secondary coil may equal the power output with a half-wave rectifying circuit. In some cases the maximum safe power dissipation of the secondary coil may be exceeded.

2:4 For a high-voltage output and a correspondingly high peak r-f voltage across the secondary coil, there will be severe corona loss in the secondary circuit unless extreme care is taken to guard against this by using large diameter wire where possible, and smooth, rounded connections. With a voltage-multiplying circuit the corona loss is considerably reduced, since only a fraction of the r-f voltage is required.

2:5 Using a half-wave rectifier circuit, a large ratio of transformation from the secondary to the primary coils is required. Some difficulty may be experienced in obtaining practical values for secondary and primary circuit constants. With a voltage-multiplying circuit, the effective load resistance is decreased considerably and the ratio of the secondary to primary inductance can be smaller.

2:6 In some cases, where two outputs are required from a single secondary coil and oscillator, it may be necessary to use a voltage-multiplying circuit for one, or both, rectifying circuits.

A full-wave rectifier circuit has not been considered because at these frequencies the ripple voltage is very low, adequate filtering being provided by one or two condensers.

With these points in mind, and also the particular requirements with respect to size and efficiency, a suitable rectifying circuit can be selected. The effective a-c load resistance, R_L (Fig. 2) can be computed from $R_L = \dfrac{R_{DC}}{2p^2}$

where p = the number of stages of voltage multiplication

= 1, for half-wave

= 2, for doubler circuit etc.

3. D-C plate voltage.

The d-c plate voltage for the oscillator tube is next selected, since this is usually limited. Other factors being constant, the larger the d-c supply voltage, the smaller the secondary inductance required with a resulting increase in efficiency, because of the higher secondary resonant frequency.

4. Secondary Inductance.

A suitable trial value of secondary inductance is selected from past experience and results, after comparing the following factors - d-c plate voltage, power output, effective a-c load resistance, and the type of oscillator tube or tubes.

Some sample values of L_2, with various other associated constants, are shown in Table 1.

D.C. OUTPUT	RECTIFIER	R_L	PLATE VOLTAGE	L2	TUBE	EFFICIENCY
10 KV 10 WATTS	DOUBLER	1.25 MEG.	300	45 MH	6L6 6Y6	40% 45%
10 KV 10 WATTS	HALF-WAVE	5.00 MEG.	300	45 MH	6L6 6Y6	25% 30%
30 KV	TRIPLER	—	280	45 MH	2-6Y6'S IN PARALLEL	—
5 KV 5 WATTS	HALF-WAVE	2.5 MEG.	300	30 MH		
(10KV (2WATTS (2KV (4WATTS	DOUBLER DOUBLER	6.25 MEG. .125 MEG.	300	45 MH	6L6 6Y6	30%

TYPICAL R.F. H.V. SUPPLY CONSTANTS

TABLE I

All the data was not available, hence the table is not complete. It does show the variety of conditions over which the 45 mh. coil will operate satisfactorily. It is not intended to imply that this size coil is <u>best</u> for all the above conditions.

The secondary inductance should vary approximately as follows:-

4:1 For constant power output.

(a) $L_2 \propto R_L$

(b) $L_2 \propto \frac{1}{E_b^2}$

4:2 For a constant load resistance R_L,

$$L_2 \propto P_{DC}$$

The author does not desire to specify any rigid rules with regard to the secondary inductance. Rather, the above statements are to aid in the selection of a secondary inductance which will give the greatest efficiency and power output under specified conditions.

5. Secondary circuit distributed capacity.

This is the most difficult factor to find, since it depends on the actual construction of the secondary circuit and coil. It will usually vary between 10 and 30 µµfarads, depending on the following factors.

5:1 Construction of the secondary coil: the larger the individual pies (see next section) and the fewer the number of pies, the greater will be the distributed capacity.

5:2 A voltage-multiplying circuit, because of the additional circuit elements and heater windings, will increase the distributed capacity. The heater windings tend to act as short-circuited turns around the coil.

5:3 Shielding of the coil and circuit will also increase the effective capacity across the coil.

The distributed capacity of the 45 mh, 7-pie, secondary coil used in the 10 kv supply is 9µµf. with a half-wave rectifying circuit (no shield), and 15µµf. with a voltage-doubling circuit (shield and two heater windings). The distributed capacity can be estimated only with due consideration to the preceding discussion. There is one consolation in the fact, that the distributed capacity, and therefore the secondary resonant frequency, do not affect the output appreciably, if within ±15% of the actual value.

Having estimated C_2, it is possible to obtain some idea of the efficiency of the secondary circuit, and thus check the selected value of secondary inductance. The secondary coil resistance varies approximately as $\sqrt{L_2}$ and is 270 ohms for a 45 mh. coil at 200 kc.

Referring to Fig. 14,

$$r_L = \frac{4\,\pi^2 f_{02}^4 L_2^2}{f^2\,R_L} \div \frac{4\,\pi^2 f_{02}^2 L_2^2}{R_L} = \frac{L_2}{C_2 R_L} \qquad (36)$$

The power output is known, thus the power loss in the secondary coil can be computed. This is approximately equal to $\frac{P_{DC}}{r_L} \times r_2$. An approximate idea of the efficiency is therefore obtainable.

6. Design of Secondary Coil.

6:1 The main considerations in selecting and designing the type of coil to be used are:-

(a) Allowable voltage between turns on the coil.

(b) Power dissipation.

(c) Corona loss.

(d) Q value.

The best type of coil consists of a series of equally spaced pies, wound on a hollow, insulated, coil form. A Q of from 100 to 300 is readily obtainable and a power dissipation of from 10 to 20 watts (15 watts for the coil used in the 10 kv - 2 kv supply built at NRC).

The voltage between pies should not exceed one-half the needle gap breakdown voltage for air. Several attempts may have to be made before a satisfactory design is achieved, but since this is one of the most important elements in the circuit, it is worth the effort.

6:2 Consider the pie-wound coil with the dimensions shown in Fig. 16.

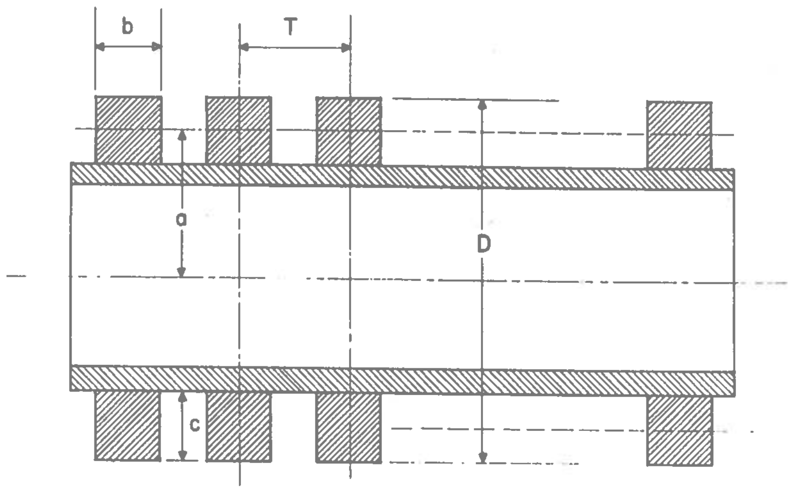


Fig. 16 Construction Of a Secondary Coil With B pies.

Let the number of pies = B

The condition for maximum efficiency is never greatly departed from if the relation $5_c + 3_b = D$ is satisfied for the individual pies. The actual values of c, b and D are determined by the ease of winding, mechanical strength, desirable physical size of the coil, etc. In general, the value of b should be kept small, consistent with winding space, rigidity and ease of construction. Having selected values of b, c and D, the number of pies and the spacing between pies must be determined. The inductance of a pie-wound coil is given very nearly by

$$L_2 = \frac{0.8\, a^2\, B^2\, n_2^2}{6a + 9(B-1)T + 10c}\ \mu h \qquad (37)$$

with a, c, and T in inches. This formula is obtained from the formula for the inductance of a single multilayer pie, by considering the whole coil as a single pie.

Values of B and T can be substituted in the formula and the value of n_2 found which will give the required inductance.

6:3 Second method.

The above formula is sufficiently accurate for most purposes. Another method is included here, which is more accurate, particularly for small and very large coils (below 20 and above 200 mh.).

From the formula given by Terman (1), the mutual inductance between multilayer pie-wound coils can be computed.

For equal sized coils (Fig. 16) spaced a distance T, center to center, and where b is less than T, the mutual inductance will be given by

$$M = 2.54 \text{ a } N n_2^2 \ \mu h \qquad (38)$$

with 'a' in inches. N is a constant which depends on T and a, and is listed by Terman (2).

The mutual inductance between coils spaced T, 2T ---- (B-1)T inches apart can therefore be found. If we let:-

M_1 = the mutual inductance between coils spaced T inches apart.
M_2 = the mutual inductance between coils spaced 2T inches apart etc.
L_S = self inductance of one pie

$$= \frac{0.8 \ a^2 \ n_2^2}{6a + 9b + 10C} \ \mu h$$

then, the total inductance of the secondary coil will be given by:-

$$L_2 = B \ L_S + 2 \left[(B-1)M_1 + (B-2)M_2 + \text{---} M_{B-1} \right] \ \mu h \qquad (39)$$

If the individual pies are wide compared with the pie spacing, the more complex formulae given by Terman (3) must be used for the mutual inductance.

6:4 Best wire size.

Litz wire will give a better Q value at these frequencies than solid wire. The exact number of strands and wire size, however, may vary between 3 and 10 strands, of from No. 36 to No. 41 B and S gauge wire, depending on the size of the coil. The best wire diameter, d, is found as follows:-

(a) For $\frac{f}{P^2} < 10^4$, $d^3 = \frac{7,600}{f \ P}$ millimetres^3 (40)

(b) For $\frac{f}{P^2} < 10^8$, $d^3 = 0.165$ millimetres^3 (41)

(c) For $10^4 < \frac{f}{p^2} < 10^8$; d is obtained from Fig. 17.

$$p^2 = \sigma + \frac{(n')^2 S^2 L}{D^3} \qquad (42)$$

n' = the number of strands in the cable

σ = given in Table 2 for various n'

S = a constant, depending on the coil shape, given in Fig. 18.

$L = \frac{L_2}{B}$ = effective inductance of one pie in µH.

D = external diameter of the coil in cm.

This will not usually give the best wire diameter, but the a-c resistance of the coil can be computed for one or two gauges on either side of the value obtained using the above method, and thus the lowest possible theoretical resistance can be found. In the examples worked to date, one or two gauges smaller than that given by the above method were found satisfactory. The winding space, b x c, should be checked using a winding table, and, if not satisfactory, a small change in b may be made without affecting the final results appreciably.

TABLE II

No. of Strands n'	1	3	9	27	large
	0	0.9	3.3	10.4	0.4 n'

TABLE III

No. of Strands n'	3	4	7	12	19	37	61	91
	2.97	3.95	6.94	11.85	18.8	36.6	60.3	90

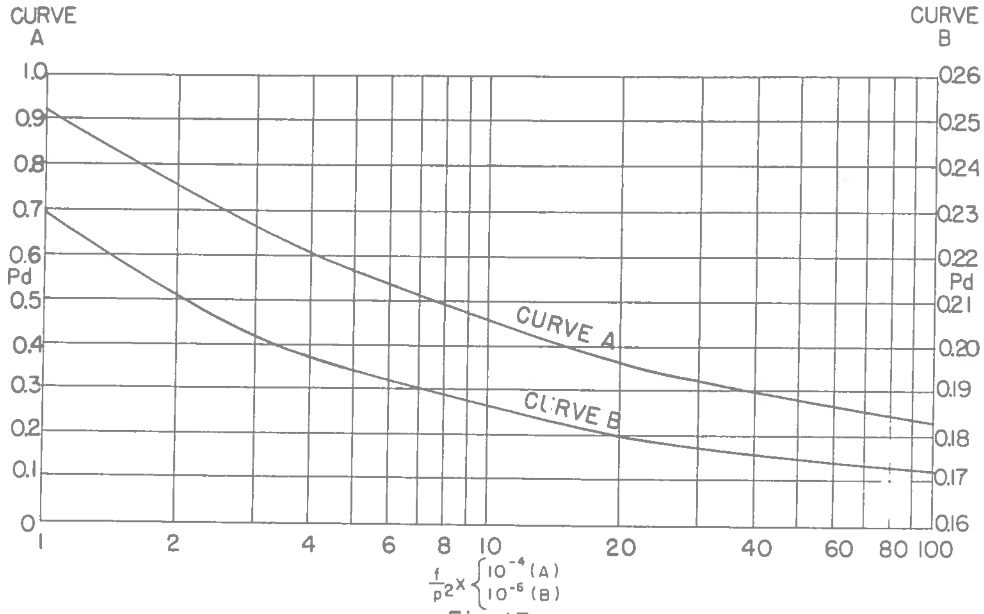


Fig. 17

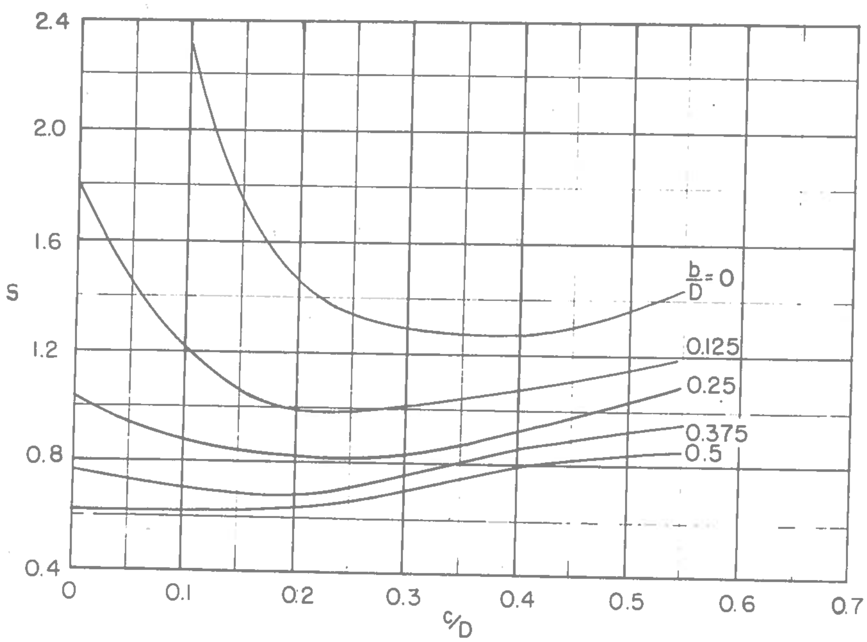


Figure 18

6:5 A-C and D-C resistance.

$$r_{dc} = \frac{2\pi \; a \; B \; n_2 \rho}{\alpha} \tag{43}$$

where ρ = the resistance in ohms/inch of a single strand.

α is a constant given in Table 3, which corrects the single-strand resistance for the number of strands in parallel and the increased length due to stranding.

The a-c resistance is computed from the formula and tables given by Terman (4).

7. Coefficient of coupling k.

7:1 Approximate Q_2

The total secondary series resistance

$$R_2 = r_2 + r_L + r_{HW} + r_E \qquad \text{(see Fig. 14)} \tag{44}$$

Take $r_L \doteqdot \dfrac{4\pi^2 f_{02}^2 L_2^2}{R_L}$ (45)

$r_{HW} = \dfrac{P_{HW}}{P_{DC}} \times r_L$, where P_{HW} is the power used by the rectifier filaments.

r_2 = a-c resistance of the secondary coil.

$r_E = \dfrac{P_E}{P_{DC}} \times r_L$, where P_E is the estimated power loss in the shield and grid circuit. This latter is usually of the order of .25 watts or less.

Then $Q_2 \doteqdot \dfrac{2\pi \; f_{02} L_2}{R_2}$ (46)

7:2 For good stability and efficiency the coefficient of coupling should be greater than 20 k_c, where k_c equals $\dfrac{1}{\sqrt{Q_1 \; Q_2}}$. Since there is considerable freedom in the design of the primary inductance, it is safe to assume that a value of 100 will be obtainable for Q_1. The exact value of k is not critical.

8. Oscillator Plate Voltage Swing.

The oscillator plate voltage swing will depend on the d-c plate voltage, power output required, and the type of tube used. It may be necessary to use two or more tubes in parallel to obtain the power required.

Since the a-c plate voltage determines the operating frequency, primary inductance and equivalent load resistance, it is advisable to make an approximate computation to determine a suitable value for the tube or tubes to be used before computing the above.

The power used in the secondary circuit

$$= \frac{P_{DC}}{r_L} \times R_2 \text{ watts} \tag{47}$$

Estimated power loss in the primary circuit = .25 x Power used in the secondary circuit. (48)

The total a-c power output of the oscillator tube is therefore known.

Let E_{pp} = the peak value of the a-c plate voltage.

I_{pp} = the peak value of the fundamental frequency component of plate current

Then the required $I_{pp} = \dfrac{2 P_T}{E_{pp}}$ (49)

Suitable values for E_{pp}, E_{gmax} (the maximum positive grid voltage) and the d-c screen voltage, can be selected. For pentode tubes, the minimum plate voltage $(E_b - E_{pp})$ should not be less than 15 to 20% of the d-c plate voltage. From the static tube characteristics the maximum plate current can be found and from the graph given by Terman (5) the required angle of plate current flow θ_p. The values of E_{pp}, E_{gmax} and θ_p can then be adjusted until suitable operating conditions for the tube are attained. This will establish a good value for E_p. After finding L_1, f_1 and f_{01}, the exact power output required can be found and the oscillator tube design completed.

9. Operating frequency, primary inductance, and primary tuning capacity.

The load resistance required to obtain the d-c output is

$$R_{Le} = \frac{E_p^2}{P_{DC}} \tag{50}$$

Also

$$R_{Le} = \frac{1}{k^2} \frac{f_{01}^4}{f_{02}^4} \frac{L_1}{L_2} \left(1 - \frac{f_{02}^2}{f^2}\right)^2 R_L \qquad (51$$

For good oscillator operation:

$$X_{C1} = \frac{2\pi f_{01}^2 L_1}{f} \leq \frac{E_p^2}{4\pi P_T} \tag{52}$$

f and f_{01} are related by the equation

$$\left(1 - \frac{f_{01}^2}{f^2}\right)\left(1 - \frac{f_{02}^2}{f^2}\right) = k^2 \tag{53}$$

The solution of these equations for f, f_{01}, and L_1 involves a cubic expression, therefore, it is easier to try a successive approximation method to find f, solving for f_{01} and L_1 from equations (53) and (51), then checking equation (52). It has been found from experience that a good first approximation is to to take $f = f_{02} - 4\,\Delta$, where

$$\Delta = \left[\frac{k^2 R_2^{\,2} f_{02}}{16\pi^2 L_2^{\,2} (2-k^2)}\right]^{1/3}$$

10. Design of the primary inductance.

The primary coil may be a single pie, parallel to the secondary coil on the same form, or a solenoid wound on an insulating former, and placed over the secondary coil. Litz wire is preferable, the common sizes being from 10 to 60 strands of No. 36 to No. 41 wire.

The design of the coil is very similar to that of a single pie of the secondary coil, the best coil size being determined by the relation 5c + 3b = D. The number of turns n_1, required to give the value L_1, as computed in section 9, is found from

$$L_1 = \frac{0.8\, a^2 n_1^2}{6a + 9b + 10c}\ \mu h.$$

The best wire diameter, and a-c resistance r_1, are computed by the same formulae and methods as used for the secondary coil.

11. Total a-c power output P_T

Referring to the equivalent primary circuit shown in Fig. 15 and the equivalent secondary circuit shown in Fig. 14.

$$R_{Le} = \frac{1}{k^2} \frac{f_{01}^4}{f_{02}^4} \frac{L_1}{L_2} \left(1 - \frac{f_{02}^2}{f^2}\right)^2 R_L \tag{54}$$

$$R_{Se} = \frac{4\pi^2}{k^2} \frac{f_{01}^4}{f^2} L_1 L_2 \left(1 - \frac{f_{02}^2}{f^2}\right)^2 \cdot \frac{1}{r_S} \qquad (55)$$

$$R_{1e} = \frac{4\pi^2 f_{01}^4 L_1^2}{f^2 r_1} \qquad (56)$$

$$R_{HW} = \frac{E_p^2}{P_{HW}} \qquad (57)$$

where P_{HW} is the power used by the rectifier filaments coupled to the primary coil. It is now possible to find the exact value for r_s, since the correct value of r_L can be found.

$$r_L = \frac{4\pi^2 f_{02}^4 L_2^2}{f^2 R_L} \qquad (58)$$

The total effective load resistance R_T is now known.

$$\frac{1}{R_T} = \frac{1}{R_{Le}} + \frac{1}{R_{Se}} + \frac{1}{R_{1e}} + \frac{1}{R_{HW}} \qquad (59)$$

and the resulting total a-c power output

$$P_T = \frac{E_p^2}{R_T} \qquad (60)$$

12. Class C oscillator design.

The Class C oscillator design is standard but for the fact that the load resistance is known. It is therefore necessary to adjust the grid voltage or angle of plate current flow to give the required plate current. This is done using the method as outlined in section 8. Having obtained θ_p, the design can be completed using either the graphical or approximate method for a class C oscillator.

13. Tickler Coil Design.

From the previous section the required alternating grid voltage is known. The grid excitation is obtained by mutual coupling to the secondary coil.

Let:- I_2 = the rms value of the secondary current
M_t^2 = mutual inductance between the tickler coil and secondary coil.
E_g = the rms value of the alternating grid voltage.

Assuming negligible current drawn by the grid circuit, then

$$E_g = 2 \pi f M_t I_2 = 2 \pi f M_t \sqrt{\frac{P_{DC}}{r_L}} \qquad (61)$$

The tickler coil is usually a flat pie, wound parallel to the secondary coil on the same form, with the spacing sufficient to prevent arcing between the two coils.

Assuming a reasonable value for the mean radius of the tickler coil (it is convenient to use the same radius as for the secondary), the mutual inductance between the tickler and secondary coil can be found by the method used in computing the mutual inductance of the secondary coil (see section 6:3).

Let:- a_t = the mean radius of the tickler coil.

a_2 = the mean radius of the secondary coil.

n_2 = the number of turns per pie on the secondary.

n_t = the number of turns on the tickler coil.

$m_o = 2.54 \, N \sqrt{a_t a_2}$ µh.

where N is the same constant as is used in section 6:3.

$m_{01} n_2 n_t$ = the mutual inductance between the tickler and the first pie of the secondary coil.

$m_{02} n_2 n_t$ = the mutual inductance between the tickler and second pie of the secondary.

$m_{0b} n_2 n_t$ = the mutual inductance between the tickler and the B'th pie of the secondary.

Then:- $M_t = n_2 n_t (m_{01} + m_{02} + m_{03} + \text{----} m_{0B})$ (62)

The required value of n_t can be found from equation (62).

Since the current drawn by the grid of the oscillator tube is small, the design of the tickler coil is not as critical with regard to efficiency as the primary and secondary coils. From the results obtained for the latter, the best coil size and wire size can be estimated.

To permit for adjustment and also to make allowance for the neglected grid current, the number of turns used should be 15 to 20% more than computed.

14. Primary - secondary spacing.

Let:- n_1 = the number of turns on the primary coil.

n_2 = the number of turns per pie on the secondary coil

The required mutual inductance $M = k\sqrt{L_1 L_2}$. As in section 13,

$$M = n_1 n_2 (M_{01} + M_{02} + \text{---} M_{0B})$$
$$= n_1 n_2 M_{0T}$$

Also $(m_{01}+m_{02}+m_{03}+\text{-----}m_{0B})$ will be approximately equal to B x m_{0x}, where x will equal 1, 2, 3 --- or B.

For an approximate solution we can assume that $M_{OT} = B\ M_{0(B+1-x)}$.

The required value for $M_{0(B+1-x)} = \dfrac{k\sqrt{L_1\ L_2}}{B\ n_1\ n_2}$

From the formulae and tables, as used in section 6:3, the approximate spacing between the primary and secondary coils can be found. It is advisable to leave the primary coil adjustable and to measure the actual coefficient of coupling afterwards.

15. Conclusion

The design values as computed by the above method should be accurate to within ± 10% of the actual operating conditions. The following factors may be easily adjusted to obtain the correct output after the circuit is built:

15:1 Screen voltage or grid bias.
15:2 Number of turns on the primary coil which may be reduced.
15:3 Spacing of the primary and secondary coils.
15:4 Primary tuning capacity C.

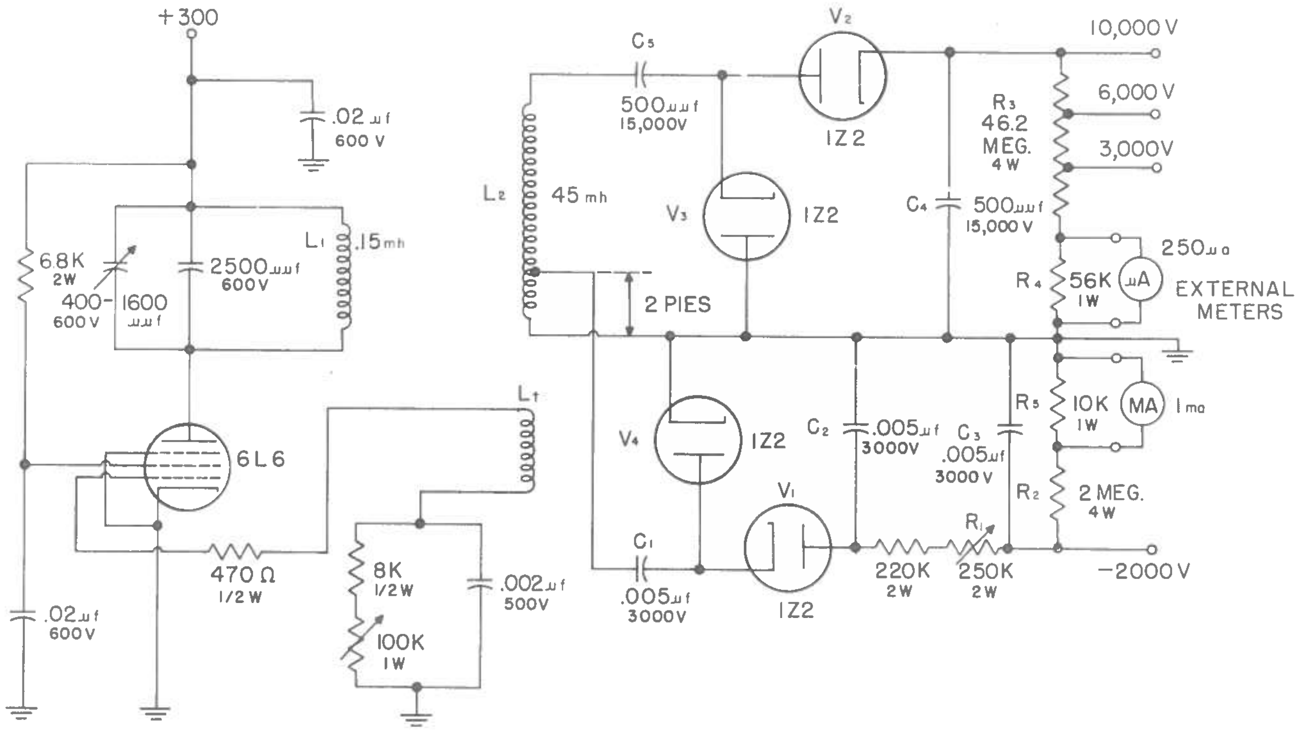
IV

10 kv - 2 kv Supply

The circuit diagram for the unit built at the National Research Council is shown in Fig. 19. The 10 kv-output is used for the post-accelerating anodes of a cathode-ray tube, and the negative 2 kv-output for the main accelerating anode, focus and brilliancy control.

Both outputs can be varied simultaneously by the variable grid-bias resistor, while the 2 kv output has a separate control in the 250,000-ohm series potentiometer. The output power at full load is 6 watts (exclusive of rectifier filaments) for an input to the 6L6 oscillator tube of 21 watts.

Several photographs of the apparatus are shown in Figs. 20,21,22,23,24,25. The main chassis is constructed from 3/8 inch Plexiglass, mounted in a sheet metal chassis. There is a copper screen shield over the complete apparatus. The Plexiglass chassis was used to avoid high voltage insulation difficulties. For connection to the secondary coil, rectifier tubes, condensers and bleeder resistance, No. 14 B and S gauge, enamelled copper wire was used, thus greatly reducing the corona loss.



Circuit Diagram for 10 K.V. and - 2 K.V. R.F.H.V. Supply

Fig. 19

The leads from the secondary coil, which is wound with No. 3-41 Litz wire, were kept as short as possible. The filaments of the rectifier tubes V_1, V_2, and V_3 are heated with r-f power, by coupling to the primary or secondary coil with one or two turns of No. 14 copper wire. Since the cathode of the other rectifier tube, V_4, is at ground potential, it is heated from the 6.3-volt, 60-cycle transformer winding which supplies the 6L6 filament. The small associated supply delivers 300 volts of regulated d-c for the oscillator tube and also 6.3 volts for the 6L6 filament.

The primary, secondary and tickler coils are wound on a common form of slotted Bakelite and coated with insulating varnish. The coils are constructed as follows:-

1. Secondary Coil.

   7 pies - 200 turns/pie of No. 3-41 Litz wire.
   c = 1/4" D = 1 3/4"
   b = 1/16" T = 1/4" } see Fig. 16
   L_1 = 45 mh.
   a-c resistance at 200 kc = 270 ohms

2. Primary Coil.

   1 pie - 50 turns of No.50-38 Litz wire.
   c = 7/16" D = 2 1/8"
   b = 3/16"
   L_1 = .15 mh.
   a-c resistance at 200 kc = 1.44 ohms

Spacing between primary and secondary coils = 7/16", with a coefficient of coupling k = .242.

3. Tickler Coil.

   1 pie - 100 turns of No.3-41 Litz wire.
   c = 3/16" D = 1 7/8"
   b = 1/16"

Spacing between tickler and secondary coils = 1".

There are several oscillator tubes which may be used to replace the 6L6.

1. 2 - 2E30's in parallel

   Supply voltage - 250 volts.
   Supply current - 84 ma.
   Screen resistance - 1000 ohms.
   Grid bias resistance - 36,000 ohms.

2. 6V6 or 6AQ5.

Plate supply - 300 volts.
Supply current - 70 ma.
Screen resistance - 6000 ohms.
Grid bias resistance - 25,000 ohms.

The circuit is adjusted for maximum output, by tuning the primary coil. The tuning will vary slightly if the screen voltage or grid bias is changed because of the resulting change in effective plate resistance. This partially substantiates the theoretical discussion regarding the maximum output.

The supply operates very satisfactorily, giving a stable output voltage. The output, however, is very sensitive to changes in supply voltage; thus a regulated 300-volt supply is used.

V

ILLUSTRATIVE PROBLEM

Design of 18 kv, 250 microampere Supply

1.* $P_{DC} = 4.5$ watts
$R_{DC} = 72 \times 10^6$ ohms

2. Use voltage doubler circuit:

$$R_L = \frac{72 \times 10^6}{2 \times 4} = 9 \times 10^6 \text{ ohms}$$

3. D-C supply voltage = 300 volts.

4. Secondary inductance

4:1 45 mh for 10,000 volts, with $R_L = 1.25 \times 10^6$ ohms
Power = 10 watts

4:2 $L_2 = \frac{45 \times 9}{1.25} \times \frac{4.5}{10} = 145$ mh (section 4:1 and 4:2)
Take $L_2 = 140$ mh

5. Estimate the distributed capacity = 20μμf

$f_{02} = 100$ kc
$r_L = 860$ ohms (Equation (36))
$r_2 = 485$ ohms
These values are satisfactory

---

* Numerical designations correspond to those used in Section III.

6. Secondary inductance design.

Required L_2 = 140 mh

Select b = 1/8", C = 5/16", therefore, D = 5C + 3b ÷ 2"

a = .843"

$$L_2 = \frac{0.8 \times .712\, B^2 n_2^2}{5.06 + 9(B-1)\, T + 3.12} \qquad \text{(Equation (37))}$$

Try:- B = 8

T = 5/16"

Therefore n_2^2 = 107,200

n_2 = 327 turns

Secondary inductance by computation of mutual inductances:

$$y = \sqrt{\frac{\frac{T^2}{a^2}}{4 + \frac{T^2}{a^2}}}$$

$M = 2.54 \times .843\ N\ n_2^2$

$= 2.14\ N\ n_2^2$

a = .843 $\qquad a^2 = .712$

$\underline{M_1}$

T = 5/16"

$T^2 = .0977 \qquad y = .181 \qquad M_1 = .0308\ n_2^2$

$\frac{T^2}{a^2} = .137 \qquad N_1 = .01443$

$\underline{M_2}$

$T = \frac{10"}{16}$

$T^2 = .391 \qquad y = .347 \qquad M_2 = .0153\ n_2^2$

$\frac{T^2}{a^2} = .549 \qquad N_2 = .007277$

$\underline{M_3}$

$T = \frac{15"}{16}$

$T^2 = .878 \qquad y = .485 \qquad M_3 = .00905\ n_2^2$

$\frac{T^2}{a^2} = 1.23 \qquad N_3 = .004229$

$\underline{M_4}$

$T = \frac{20''}{16}$ $\quad y = .594 \quad M_4 = .00556\ n_2^2$

$T^2 = 1.56$

$\frac{T^2}{a^2} = 2.19 \quad N_4 = .00260$

$\underline{M_5}$

$T = \frac{25''}{16}$

$T^2 = 2.44 \quad y = .679 \quad M_5 = .00358\ n_2^2$

$\frac{T^2}{a^2} = 3.43 \quad N_5 = .001673$

$\underline{M_6}$

$T = \frac{30''}{16}$

$T^2 = 3.52 \quad y = .743 \quad M_6 = .00242\ n_2^2$

$\frac{T^2}{a^2} = 4.94 \quad N_6 = .001130$

$\underline{M_7}$

$T = \frac{35''}{16}$

$y = .794$

$T^2 = 4.79 \quad M_7 = .00165\ n_2^2$

$\frac{T^2}{a^2} = 6.73 \quad N_7 = .000773$

$\underline{L_S}$

$$L_S = \frac{0.8 \times 0.712\ n_2^2}{5.06 + 1.13 + 3.12}$$

$$= 0.0612\ n_2^2$$

From equation (39)

$L_2 = 1.275\ n_2^2\ \mu h$

From which $n_2 = 332$

To summarize, $L_2 = 140$ mh

$n_2 = 330$ turns/pie

6:4 Best wire size:

Try 3-strand Litz wire.

Applying equation (40) and (42)

$L = \frac{140}{8} \times 1000 = 17500\ \mu h$

$\sigma = 0.9$ (Table 2) $D^3 = 131\ cm^3$

$C/D = .155$ $P^2 = 2040$

$b/D = .0625$ $\frac{f_{02}}{P^2} = 49.0$

$S = 1.3$ (Fig. 18) $P = 45.1$

$d^3 = 1.69 \times 10^{-3}\ mm^3$ (Equation (40))

$d = 4.69$ mils -- No. 37 wire

Try No. 38 wire:

$d = 3.965$ mils

$d_o = 9.9$ mils, corresponding to No. 30 wire.

From winding table, for random wound coil with 75% space factor, the required winding space for 330 turns of No. 30 wire = .0423 sq. in.

The available winding space = .0390 sq. in.

Try No. 39 wire:

$d = 3.531$

$d_o = 8.83$ mils -- corresponding to No. 31 wire

Required winding space = .0337 sq. in.

6:5 $r_{dc} = 390$ ohms (Equation (43))

$r_2 = 480$ ohms

No. of layers = 25

No. of turns/layer = 13

Q_2 (no load) = 183

This is satisfactory.

7. Coefficient of coupling k

$r_L = 860$ ohms (Equation (45))

$r_2 = 480$ ohms

Use a 1Z2 rectifier tube and supply one filament by coupling to the secondary coil.

Power required for filament = 0.5 watts

Estimated grid excitation and shielding loss = .25 watts

$r_{HW} + r_E = \frac{.75 \times 860}{4.5} = 140$ ohms

R_2 = 1480 ohms

Q_2 = 59

Assume Q_1 = 100

Therefore $k_c = \frac{1}{\sqrt{Q_1 Q_2}} = .013$

$k = 20\ k_c = .26$

$k^2 = .0678$

Take k^2 = .068

8. Oscillator Plate Voltage

Power used in the secondary circuit $= \frac{P_{DC} \times R_2}{r_L}$

= 7.75 watts.

Estimated power loss in the primary coil = 2 watts.

Power required for rectifier filament coupled to the primary coil = .5 watts.

Total a-c power output = 10.25 watts

$\longrightarrow$ 10 watts.

A 6V6 or 6AQ5 should be sufficient to supply this power as a Class C oscillator.

Select:- E_{pp} = 240 volts

Required I_{pp} = 83 ma.

Take Eg_{max} = + 15 volts

E_{sg} = 250 volts.

Then Ip_{max} = 205 ma. (from tube characteristics)

$$\frac{I_{pp}}{I_{pmn}} = .404$$

Assuming $\alpha = 3/2$ (see Terman p. 447) the required θ_p = 145 degrees.

This is satisfactory for Class C operation.

9. Operating frequency, primary inductance and primary circuit self-resonant frequency

For a first approximation, select

$$f = f_{02} - 4\left[\frac{-k^2 R_2^{\,2} f_{02}}{16\pi^2 L_2^{\,2}(2-k^2)}\right]^{1/3}$$

$= 94.7$ kc.

Take $f = 94$ kc.

$f_{01}^2 = 13400$ (Equation (53))

$f_{01} = 116$ kc.

Required $R_{Le} = 6400$ ohms (Equation (50))

Therefore, required $L_1 = .216$ mh. (Equation (51))

For good operation, $X_{C1} \leq \frac{E_p^2}{4\pi P_T} = 229$ ohms

Actual value of $X_{C1} = 193$ ohms.

10. Design of the primary inductance.

Try $b = 3/16"$
$c = 3/8"$
$s = 1.25"$

Therefore, $D = 2\ 3/8"$

Inside diameter of the coil = 1 5/8"
Mean radius $a = 1"$

$\therefore n_1^2 = 3090$,

$n_1 = 55$ turns

Try 27 strand Litz wire

Applying equations (40) and (42),

$\sigma = 10.4$	$P^2 = 1130$
$C/D = .158$	$P = 33.6$
$b/D = .079$	$d = 5.17$ mils (Equation (40))
$D^3 = 220\ cm^3$	

This value d corresponds to No. 36 wire.

Try No. 37 wire.

Required winding area = .0692 sq. in.
Available winding area = .0705 sq. in.
No. of layers = 11
No. of turns/layer = 5
D-C resistance = .566 ohms
A-C resistance $r_1 = 1.04$ ohms

Q_1 (no load) = 130

11. Equivalent primary resistance.

Correct value of $r_L = 975$ ohms (Equation (58))

Correct value of $r_{HW} + r_E = 160$ ohms

Correct value of $r_S = 640$ ohms

R_{Le} = 6400 ohms (Equation (54))
R_{Se} = 9750 ohms (Equation (55))
R_{le} = 36,200 ohms (Equation (56))
R_{HW} = 57,600 ohms (Equation (57))
R_T = 3300 ohms
P_T = 8.7 watts

12. Class C oscillator design (Approximate method)

The results of the design, using the approximate method as outlined by Terman (6),

E_{pp} = 240 volts
P_T = 8.7 watts
Eg_{max} = + 15 volts
E_{sg} = 250 volts
E_b = 300 volts
θ_p = 120°
E_c = -70 volts
D-C plate current = 40 ma
D-C screen current = 3.8 ma
Total power input = 13.2 watts
Plate dissipation = 3.3 watts
Screen dissipation = 1 watt
Screen resistance = 13,000 ohms
Grid bias resistance = 44,000 ohms
Power to grid circuit = 0.14 watts
Efficiency = 66%

13. Tickler Coil design

E_g = 60 volts rms = 85v peak
M_t = 1.49 mh. (Equation (61))
Choose the radius at the tickler coil = .843"
Choose the spacing between the tickler coil and the nearest pie of the secondary = 3/4" = T_1

The mutual inductance between the tickler coil and each successive pie of the secondary coil works out to:

m_{01} = .0125
m_{02} = .00758
m_{03} = .00458
m_{04} = .00302
m_{05} = .00214
m_{06} = .00144
m_{07} = .00108
m_{08} = .000775

Therefore, $M_t = .0331\ n_2 n_t\ \mu h$

or $n_t = \dfrac{1490}{.0331 \times 330} = 136$ turns

Take the number of turns on the tickler coil = 150
Use No. 3 39 Litz wire
with a = .843"
c = 1/4"   D = 1 7/8"
b = 1/16"

14. Approximate primary - secondary spacing.

Required mutual inductance M = .453 mh

$= n_1 n_2 M_{0T}$

$$M_{0T} \doteq 8\ M_0\ (B + 1 - x)$$
$$= 8\ M_{05}$$

Therefore $M_{05} = \dfrac{M}{8\ n_1 n_2} = .00312\ \mu h$

This gives $T_5 = 1.91$ inches.

The spacing between the primary coil and the 5th coil of the secondary (from the tickler coil end) - 1.9 inches.

The spacing between the primary coil and the nearest coil of the secondary = .97 inches (center to center).

15. Summary of design values.

Voltage output = 18,000 volts
Power output = 4.5 watts
Oscillator tube plate supply = 300 volts
Type of oscillator tube = 6AQ5
Screen grid voltage = 250 volts
Maximum grid voltage = +15 volts
Grid bias = -70 volts
Grid bias resistor = 44,000 ohms
Screen resistor = 13,000 ohms
Power loss in secondary circuit = 2.95 watts
Power loss in primary coil = .8 watts
Power used to heat rectifiers = 1 watt
Plate dissipation = 3.3 watts
Screen dissipation = 1 watt
Grid excitation = .14 watts
Power input = 13.2 watts
Efficiency = 30%
Secondary resonant frequency = 100 kc
Primary resonant frequency = 116 kc
C_1 = 870 $\mu\mu f$
Operating frequency = 94 kc

Coefficient of coupling = .26
Secondary inductance L_2 = 140 mh
Q_2 = 183
Primary inductance L_1 = .216 mh
Q_1 = 130

LIST OF SYMBOLS

R_{DC} = direct current load resistance

R_L = equivalent alternating current load resistance across the secondary coil $= \frac{R_{DC}}{2p^2}$

p = the total number of stages of voltage multiplication

r_L = equivalent secondary series load resistance $= \frac{4\pi^2 f_{02}^{\;4} L_2^{\;2}}{f^2 R_L}$

r_2 = the alternating current resistance of the secondary coil

r_{HW} = the resistance introduced into the secondary circuit from rectifier filaments coupled to the secondary coil

r_E = the resistance coupled into the secondary circuit from the grid circuit of the oscillator tube and shield

r_S = $r_2 + r_{HW} + r_E$

R_2 = $r_S + r_L$

r_{Se} and R_{Se} = equivalent primary resistance produced by r_S. See Fig. 4.

r_{Le} and R_{Le} = the equivalent primary resistance produced by R_L. See Fig. 4.

R_{2e} = the total equivalent secondary resistance in the primary circuit

r_1 = the alternating-current resistance of the primary coil

R_{HW} = the resistance introduced into the primary circuit by rectifier filaments coupled to the primary coil

R_{1e} = the equivalent parallel primary resistance produced by r_1

G_{1e} = $1/R_{1e}$

G_{2e} = $1/R_{2e}$

R_T = the equivalent primary resistance

Z_e = the equivalent primary impedance

E_p = the rms value of the alternating voltage across the tank circuit

E_{pp} = the peak value of the alternating voltage across the tank circuit

E_{DC} = the d-c output voltage of the supply

E_b = the d-c plate voltage of the oscillator tube

E_g = the rms value of the alternating grid voltage

E_{gmax} = the maximum positive value of the grid voltage

E_c = the grid bias voltage

E_{sg} = the d-c screen-grid voltage

I_{pp} = the peak value of the fundamental frequency component of the plate current

I_p = the rms value of the alternating plate current

θ_p = the total angle of plate current flow through the oscillator tube

P_{DC} = the d-c power output of the supply

P_T = the total alternating current power output of the oscillator tube

P_{HW} = the power loss in the rectifier tube filaments

P_E = the estimated power loss due to grid excitation and shielding

P_{Se} = the total power delivered to the secondary circuit

L_2 = the self-inductance of the secondary coil

L_1 = the self-inductance of the primary coil

$$Q_1 = \frac{2 \pi f L_1}{R_1}$$

$$Q_2 = \frac{2 \pi f L_2}{R_2}$$

M = the mutual inductance between the primary and secondary coils

k = the coefficient of coupling between the primary and secondary coils $= \frac{M}{\sqrt{L_1 L_2}}$

k_c = the critical coefficient of coupling between the primary and secondary coils $= \frac{1}{\sqrt{Q_1 Q_2}}$

M_t = the mutual inductance between the secondary and tickler coils

C_1 = the primary tuning capacity

C_2 = the distributed capacity in the secondary circuit

f = the operating frequency

f_{02} = the self-resonant frequency of the secondary coil $= \frac{1}{2\pi\sqrt{L_2 C_2}}$

f_{01} = the self-resonant frequency of the primary circuit $= \frac{1}{2\pi\sqrt{L_1 C_1}}$

$\theta_2 = 1 - \frac{f_{02}^2}{f^2}$

$\theta_1 = 1 - \frac{f_{01}^2}{f^2}$

n_1 = the number of turns of wire on the primary coil

n_2 = the number of turns of wire per pie on the secondary coil

n_t = the number of turns of wire on the tickler coil

B = the total number of pies composing the secondary coil

T = the center-to-center spacing of the pies of the secondary coil

d = diameter of one strand of Litz wire

d_o = total diameter of the Litz wire

ACKNOWLEDGMENT

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BIBLIOGRAPHY

1. Terman's Radio Engineers' Handbook p. 72
2. Terman's Radio Engineers' Handbook p. 69 - 70
3. Terman's Radio Engineers' Handbook p. 73
4. Terman's Radio Engineers' Handbook p. 77 - 82
5. Terman's Radio Engineers' Handbook p. 447
6. Terman's Radio Engineers' Handbook p. 444 - 452
7. O.H. Schade Proc. of I.R.E. 31:158 April 1943
8. H.C. Baumann Communications 26:26-70 March 1946
9. O.H. Schade R.C.A. Review 8:7 March 1947

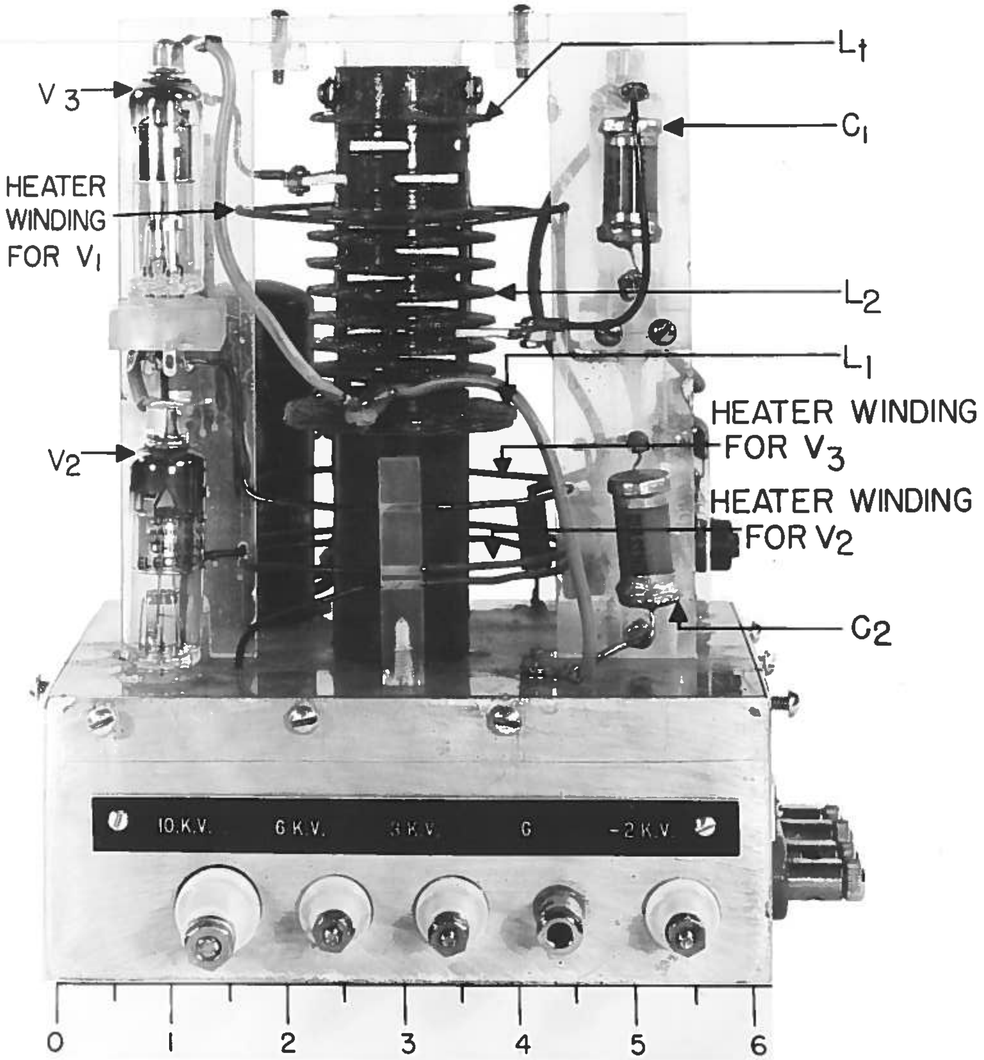

FIG. 20
SIDE VIEW OF SUPPLY
SHOWING OUTPUT TERMINALS

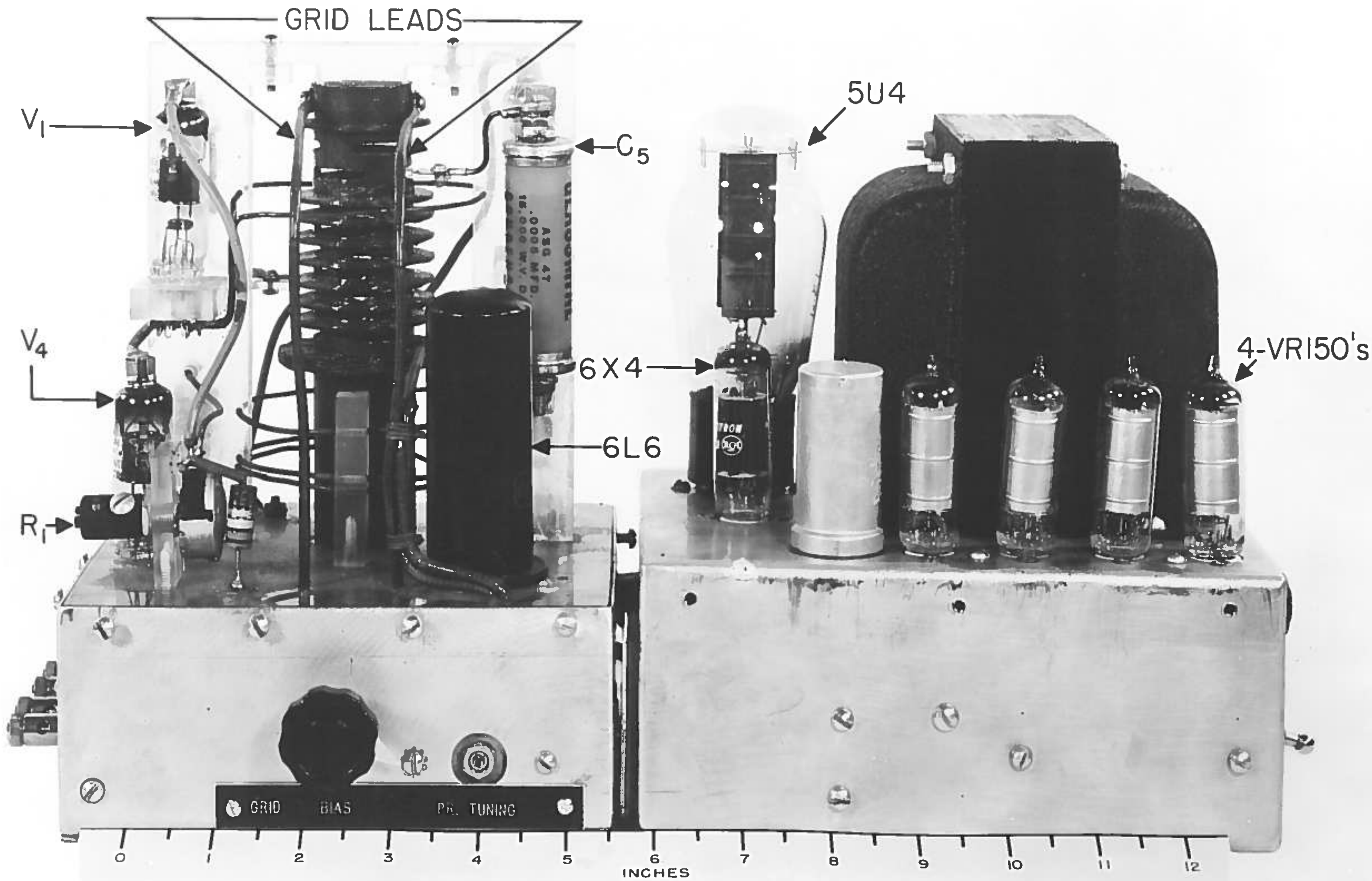

FIG. 21

RADIO-FREQUENCY HIGH-VOLTAGE SUPPLY

WITH ASSOCIATED 300 VOLT REGULATED SUPPLY ON RIGHT

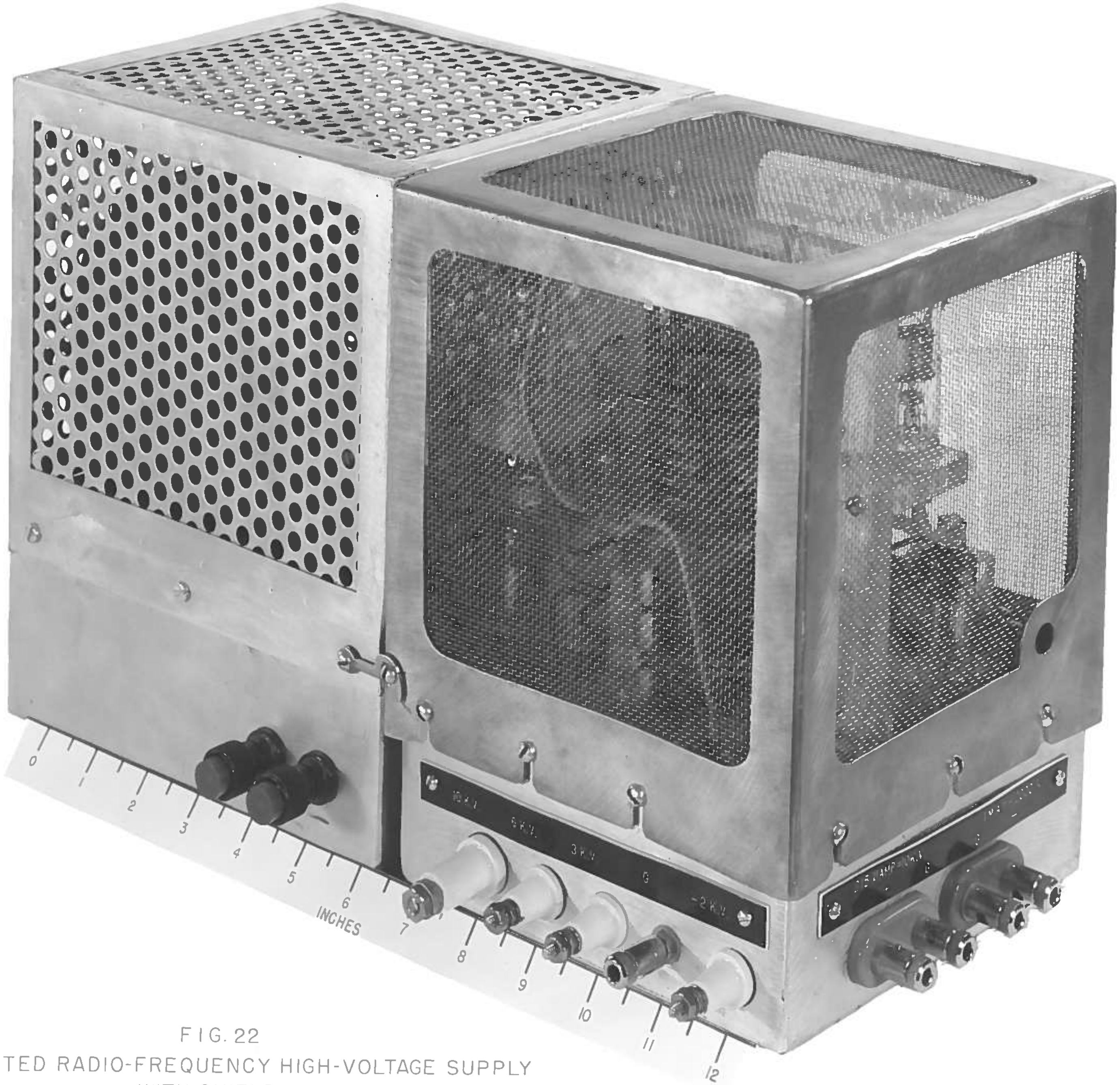

FIG. 22
COMPLETED RADIO-FREQUENCY HIGH-VOLTAGE SUPPLY
WITH SHIELD

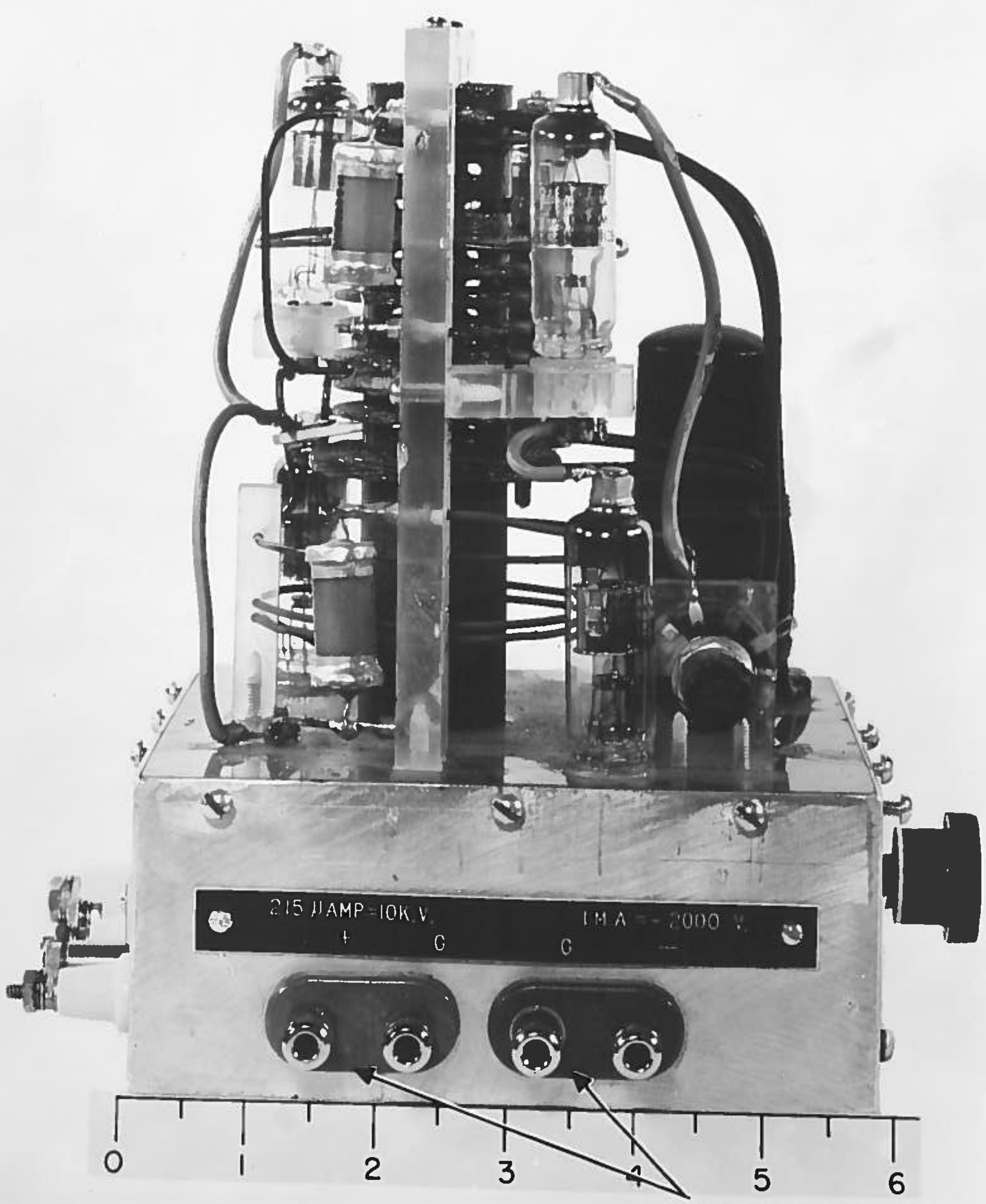

FIG. 23
END VIEW OF SUPPLY

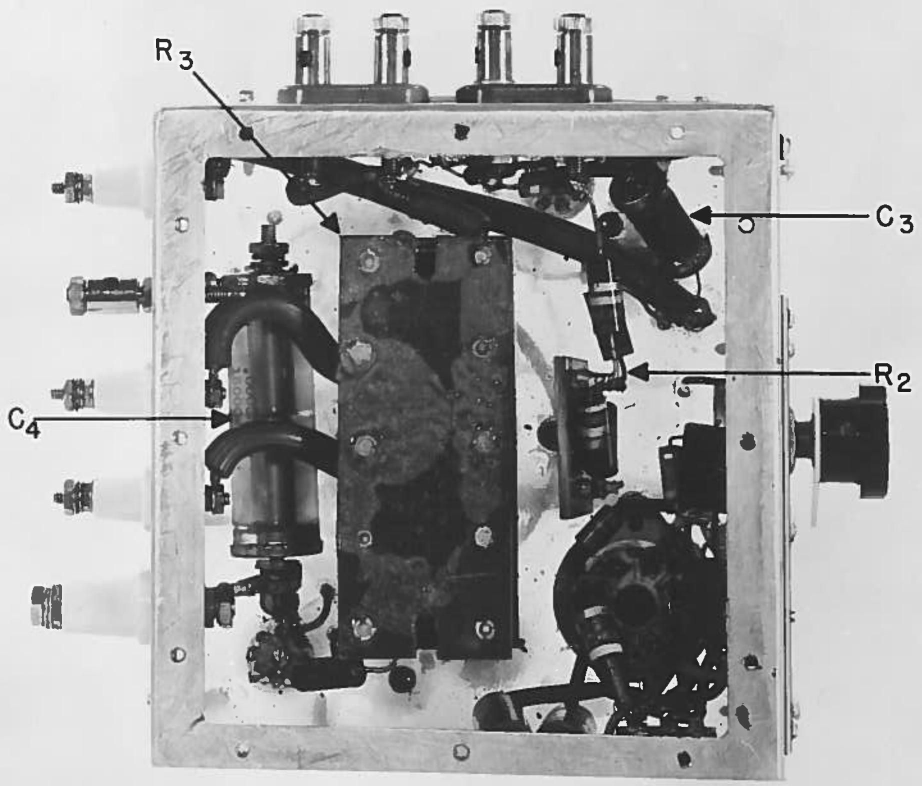

FIG. 24

BOTTOM VIEW OF SUPPLY

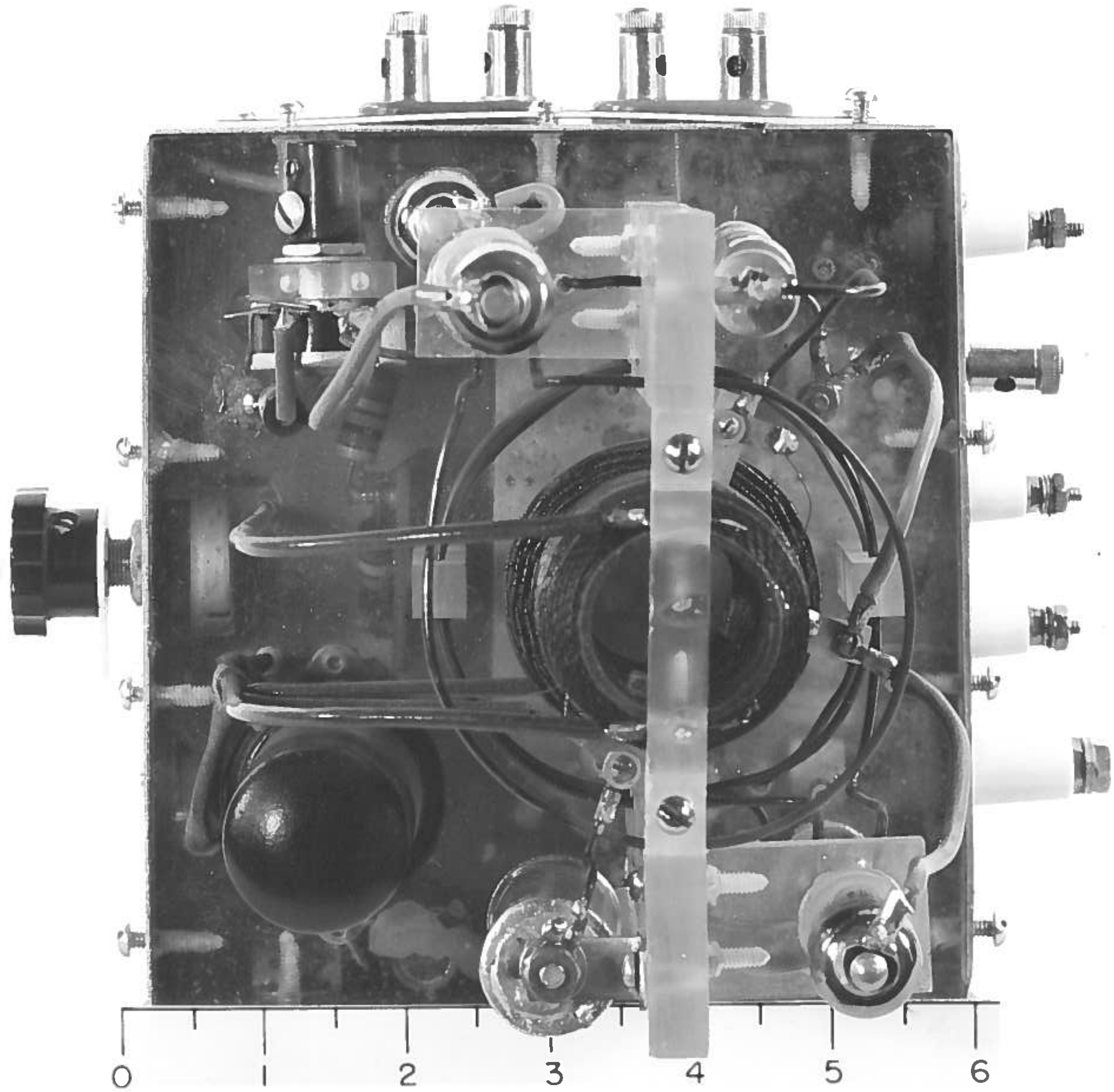

FIG. 25
TOP VIEW OF SUPPLY