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TRANSLATES OF M-SEQUENCES

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OTTAWA

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ABSTRACT

A program, written in machine language for the IBM 1620 Model 1 Computer with indirect addressing, 40 K memory, and a type-1443 on-line printer, is described. The program allows the analysis of two broad classes of linear sequence generators for possible translated outputs: (i) maximal-period generators, and (ii) generators whose periods are not maximal but whose state diagrams comprise a number of isolated loops, each loop containing the same number of states. The maximum register length of the generator is restricted to 35. Experimental check points for generators up to 20 stages are presented.

The program provides a number of options, of which the most useful are:

a) the advance of a translated sequence obtained by not more than two half-adders;

b) the advance of a translated sequence obtained by not more than three half-adders;

c) the location and number of half-adders required for a translated sequence of any specified advance;

d) the solution of difference equations related to different configurations of the generator.

Typical results of the various program options are presented.

CONTENTS

FIGURES

TRANSLATES OF M-SEQUENCES

- M.A. Gullen* -

INTRODUCTION

Linear, binary, maximal-period sequence generators are a class of autonomous sequential networks. The generator, in a standard configuration, comprises a clocked shift register of N memory elements, with feedback taken from one end of the register and taps at the outputs of various elements, combined in sum-modulo-2 adders, and fed to the other end of the register. Many other configurations are possible.

These generators find application in radar and communication systems, automatic error-correction circuits, and digital computer control circuitry. The work described in this report arose from an application to the measurement of errors in a telemetry system using the pseudo-random properties of the generated sequence [1].

The properties of linear, autonomous, sequential networks, and the sequences they provide, have been described [2, 3]. The property of immediate interest relates to the generation of delayed (or advanced) replicas of maximal-length, linear, binary sequences or m-sequences. The work described in this report has, as background, a paper presented by Tsao [4], with subsequent correspondence [5], and two notes prepared by Davis [6] and Nuspl [7]. Reference is also made to a paper by Scholefield [8] and a note by Fitch [9].

This report contains three sections. First, systematic computation of the register taps required to produce advanced replicas, or translates, of the sequence obtained from a simple generator, is performed for illustration and to clarify the symbology used. Illustrative calculations are performed, also, for a generator which does not provide a period of maximal length. A second, short section describes experimental work conducted on generators containing 15, 16, 17, and 20 memory elements in which the advance of the translate obtained from various tap settings was measured. The technique is simple, is easily applied, and can be extended readily to generators of any register length. Measurements made provide check points for the digital computer program described in the third section.

The program described under "Program" (p.15) is written in machine language for the IBM 1620 Data Processing System, Model 1, and demands 40,000 positions of core storage; a Card Read-Punch, Model 1622; Indirect Addressing, and an On-line Printer, Model 1443. The program provides several options and may be applied to the analysis of any generator characterized by an irreducible polynomial of degree less than or equal to 35. Program running time is a function of register length (or degree of the polynomial), the number of taps in the generator

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feedback network, and the option chosen. A complete analysis of the translates obtainable with 3, or less than 3, additional sum-modulo-2 adders (Option 2) from a generator of register length 20 with one adder in the feedback network takes just under 2 hours. Running time for the same option, register length 19, three adders in the feedback network, is $1\frac{1}{4}$ hours.

One option provided computes the advance of the translate obtained by summing from a specified set of taps. This option also simplifies the solution of difference equations arising in the analysis of a generator which is in other than the standard shift-register configuration.

Options and card input formats are described in detail. Sample output sheets are contained in Appendix C. The structure of the program and the function and addresses of various subroutines are outlined. Flow charts are not presented.

COMPUTATIONS

Consider the circuit shown in Fig. 1. Memory elements, such as flip-flops, are represented by numbered squares. These elements have the property that the signal at the input side appears at the output side after a clock pulse. Outputs are constant between clock pulses. Arrows indicate the direction of flow. Circles with a cross represent sum-modulo-2 adders, termed "half-adders", or simply "adders". The terms "half-adder" and "exclusive-or gate" are synonymous. It is assumed that no delay arises in adders.

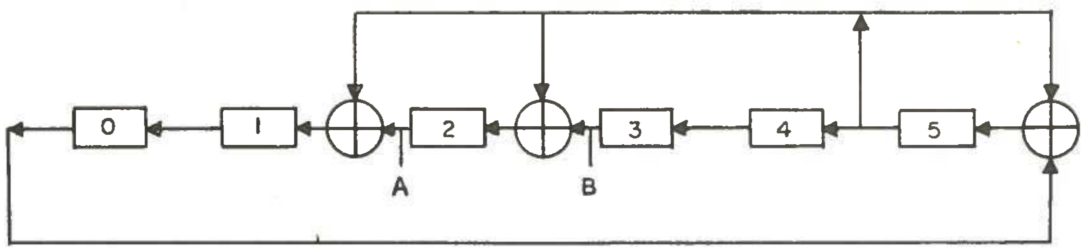


Fig. 1 Circuit A

The output of an element j at time t + k is denoted by y, appropriately superscripted and subscripted: y^j_{t+k}. A plus sign appearing in a subscript indicates the operation of normal arithmetic addition. A plus sign appearing between variables, y, indicates sum-modulo-2 addition. The output of element 3, above, at time t + 1 is equal to the output of element 4 at time t. Equations describing the operation of circuit A, then, are obtained by inspection:

$$y^0_{t+1} = y^1_t \qquad y^1_{t+1} = y^2_t + y^5_t \qquad y^2_{t+1} = y^3_t + y^5_t$$

$$y^3_{t+1} = y^4_t \qquad y^4_{t+1} = y^5_t \qquad y^5_{t+1} = y^0_t + y^5_t$$

In general, $y^j_{t+1} = a_0 y_t^{\ 0} + a_1 y_t^{\ 1} + a_2 y_t^{\ 2} \ldots a_n y_t^N$, $0 \leq j \leq N$, for N memory elements where the coefficients, a, are 0 or 1. The relations, written in matrix form for circuit A, give

$$\begin{bmatrix} y^0_{t+1} \\ y^1_{t+1} \\ y^2_{t+1} \\ y^3_{t+1} \\ y^4_{t+1} \\ y^5_{t+1} \end{bmatrix} = \begin{bmatrix} y^0_t \, y^1_t \, y^2_t \, y^3_t \, y^4_t \, y^5_t \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Manipulating the equations

$$y^0_{t+2} = y^1_{t+1} = y^2_t + y^5_t ,$$

$$y^0_{t+3} = y^2_{t+1} + y^5_{t+1} = y^0_t + y^3_t ,$$

since

$$y^0_R + y^0_R = 0 \text{ and } 0 + y^0_p = y^0_p$$

$$y^0_{t+4} = y^0_{t+1} + y^3_{t+1} , \text{ etc.,}$$

$$y^0_{t+6} = y^0_{t+5} + y^0_{t+3} + y^0_{t+2} + y^0_{t+0} .$$

We may, conveniently, take the time origin at t = 0 and write

$$y^0_6 = y^0_5 + y^0_3 + y^0_2 + y^0_0 .$$

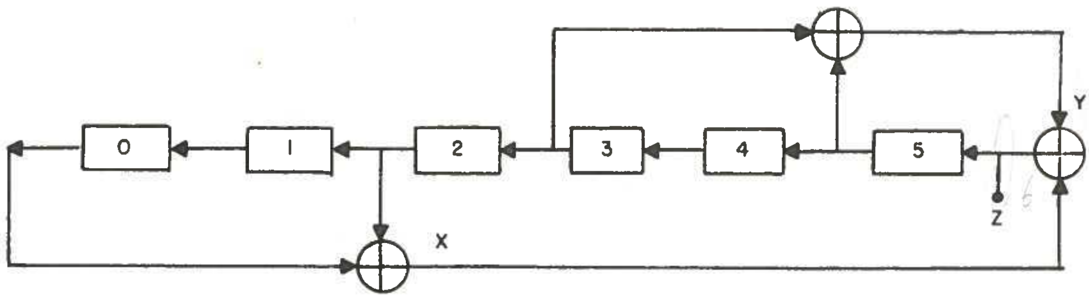


Fig. 2 Circuit B

This relation also describes circuit B (Fig. 2) which shows a generator in the standard shift-register configuration. In circuit B:

$$y^0_5 = y^5_0, \; y^0_4 = y^4_0, \text{ etc.}$$

Note that with N memory elements in the register, there are N + 1 points at which connections can be made. In circuit B a connection, or tap, can be made at point Z. If, in circuit B, the output from element 0 is taken as a reference, the output from element 1 will be advanced by one clock period, from element 2 by two clock periods, etc. The output at point Z will be advanced by six clock periods. Describing the signal at Z as y^6_t,

$$\text{then } y^0_6 = y^6_0 = y^0_5 + y^0_3 + y^0_2 + y^0_0$$

which is the generating function of a sequence in difference equation form. It follows that

$$y^0_7 = y^0_6 + y^0_4 + y^0_3 + y^0_1,$$

$$= y^0_5 + y^0_3 + y^0_2 + y^0_0 + y^0_4 + y^0_3 + y^0_1;$$

then

$$y^0_7 = y^0_5 + y^0_4 + y^0_2 + y^0_1 + y^0_0,$$

and an output, advanced by seven clock periods, can be obtained by tapping the output of elements 5, 4, 2, 1 and 0 and summing modulo-2.

$$y^0_8 = y^0_6 + y^0_5 + y^0_3 + y^0_2 + y^0_1,$$

$$= y^0_0 + y^0_1.$$

The computational process for higher translates is most readily effected by arranging columns corresponding to the various subscripts (or taps). An entry 1 in any column indicates that the term is present, an entry zero indicates that the term is not present. By convention the subscript 0, corresponding to a tap on the output of element 0, is on the left. When y^0_6 appears it is replaced by $y^0_5 + y^0_3 + y^0_2 + y^0_0$.

Advance	0	1	2	3	4	5	6
6	1	0	1	1	0	1	0
7	0	1	0	1	1	0	1
	1		1	1		1	
7	1	1	1	0	1	1	0
8	0	1	1	1	0	1	1
	1		1	1		1	
8	1	1	0	0	0	0	0
9	0	1	1	0	0	0	0
10	0	0	1	1	0	0	0
11	0	0	0	1	1	0	0
12	0	0	0	0	1	1	0
13	0	0	0	0	0	1	1
	1		1	1		1	
13	1	0	1	1	0	0	0

This process of shifting and adding modulo-2 is described, for example, in Nuspl's work [7]. The string of 1's and 0's, indicating connections to the register, is the connection vector. Note that if a translate of advance 13 is required from circuit B, it may be obtained using one adder in addition to those in the feedback loop, or it may be obtained without additional adders by regrouping the adders in Fig. 2.

The table of connections for translates in circuit B is continued below. Entries are made only after each shift and add operation.

	0	1	2	3	4	5	6
16	1	0	1	0	0	0	0
20	1	0	1	1	1	1	0
21	1	1	1	0	1	0	0
23	1	0	0	0	1	1	0
24	1	1	1	1	0	0	0
27	1	0	1	0	1	0	0
29	1	0	0	1	1	1	0
30	1	1	1	1	1	0	0
32	1	0	0	0	1	0	0
34	1	0	0	1	0	1	0
35	1	1	1	1	1	1	0
36	1	1	0	0	1	0	0
38	1	0	0	0	0	1	0
39	1	1	1	1	0	1	0
40	1	1	0	0	1	1	0
41	1	1	0	1	0	0	0
44	1	0	1	0	1	1	0
45	1	1	1	0	0	0	0
49	1	0	1	1	1	0	0
51	1	0	0	1	1	0	0
53	1	0	0	1	0	0	0
56	1	0	1	0	0	1	0
57	1	1	1	0	0	1	0
58	1	1	0	0	0	1	0
59	1	1	0	1	0	1	0
60	1	1	0	1	1	1	0
61	1	1	0	1	1	0	0
63	1	0	0	0	0	0	0

$$y^0_{63} = y^0_0 .$$

The sequence has a period of 63. In circuit B, from the table:

		0	1	2	3	4	5	6
19	Y	0	0	0	1	0	1	0
16	X	1	0	1	0	0	0	0

Translates advanced by 19 and 16 may be obtained at Y and X, respectively. Alternatively, by regrouping the adders, translates advanced by different numbers of clock periods may be obtained without additional circuitry.

	0	1	2	3	4	5	6
53	1	0	0	1	0	0	0
55	0	0	1	0	0	1	0
38	1	0	0	0	0	1	0
10	0	0	1	1	0	0	0

The register of 6 elements, circuit B, may contain $2^6 = 64$ different six-bit words. The state 000000 will not lead to any other state. The state diagram of circuit B comprises a loop containing 63 states and a trivial loop containing one state, the zero state. The sequence generated by circuit B is a maximal period or m-sequence. Circuit A generates the same sequence. The two networks have isomorphic behaviour.

We may enquire into the translates available at points A and B in circuit A. This reduces to solving two equations:

$$y^2_0 = y^0_x \quad \text{and} \quad y^3_0 = y^0_z \quad \text{for } x \text{ and } z .$$

By manipulating the circuit equations we find that

$$y^2_0 + y^2_3 + y^2_4 = y^0_0 + y^0_2 + y^0_5 .$$

Since the sequence is an m-sequence, translates arise at all points and, using the table

$$y^2_0 + y^2_3 + y^2_4 = y^2_{51} = y^0_0 + y^0_2 + y^0_5 = y^0_{56} .$$

$$y_0^2 = y_5^0 .$$

A translate advanced by five clock periods is obtained at A. Similarly

$$y_2^3 = y_{55}^0 \text{ or } y_0^3 = y_{53}^0 ,$$

and a translate, advanced by 53, is obtained at B.

Consider circuit C (Fig. 3).

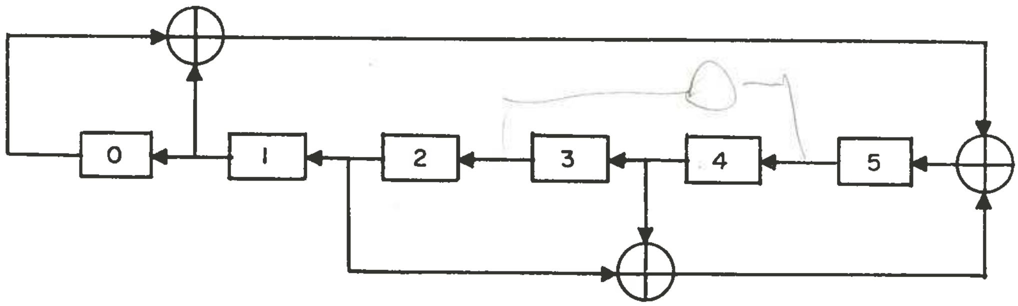


Fig. 3 Circuit C

The table for circuit C follows:

	0	1	2	3	4	5	6
6	1	1	1	0	1	0	0
8	1	1	0	1	0	0	0
11	1	1	1	1	0	0	0
14	1	1	1	1	0	1	0
15	1	0	0	1	0	0	0
18	1	1	1	1	1	0	0
20	1	1	0	1	0	1	0
21	1	0	0	0	0	0	0

$$y_{21}^0 = y_0^0 .$$

The circuit generates a sequence of period 21, which is not an m-sequence. Summing at the taps indicated in the table, or at shifts of them, will yield translates of the sequence. Connection vectors

$$\begin{matrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{matrix} \quad \text{etc.}$$

are not contained in the table and will not yield translates.

Characteristic Polynomial

The characteristic polynomial [2] associated with a square matrix, M, is defined by

$$C(x) = |M - xI|.$$

The characteristic polynomial associated with the matrix describing circuit A is

$$C(x) = \begin{vmatrix} X & 0 & 0 & 0 & 0 & 1 \\ 1 & X & 0 & 0 & 0 & 0 \\ 0 & 1 & X & 0 & 0 & 0 \\ 0 & 0 & 1 & X & 0 & 0 \\ 0 & 0 & 0 & 1 & X & 0 \\ 0 & 1 & 1 & 0 & 1 & 1+X \end{vmatrix}$$

$$= x^6 + x^5 + x^3 + x^2 + 1.$$

Note that the operation of subtraction modulo-2 is the same as the operation of addition. Note, also, that the indices of the powers of the variable x in the characteristic polynomial are the same as the subscripts in the generating function of the sequence provided by the circuit described by the matrix. If the characteristic polynomial of such a matrix is irreducible and primitive, the sequence provided by the corresponding circuit has a maximal period. If the characteristic polynomial is irreducible but not primitive, the corresponding circuit has a state diagram comprising a trivial loop containing the zero state, and a number of disjoint loops, each loop containing the same number of states; the period of the sequence is, then, a divisor of

$$2^N - 1,$$

and the particular sequence provided depends on the starting condition of the register [2].

The standard shift register configuration of the generator corresponds to the transpose of the companion matrix.

Tables of irreducible polynomials are given in Appendix C of Peterson's book [3]. The coefficients of the various powers of the variable are listed in octal notation with the highest power to the left. Thus, the entry 1743

	11	10	9	8	7	6	5	4	3	2	1	0
1743	0	0	1	1	1	1	1	0	0	0	1	1

represents the polynomial

$$x^9 + x^8 + x^7 + x^6 + x^5 + x + 1,$$

and a circuit, corresponding to a matrix which has this expression as its characteristic polynomial, will provide a sequence whose generating function is

$$y_9^0 = y_0^0 + y_1^0 + y_5^0 + y_6^0 + y_7^0 + y_8^0 .$$

Since the polynomial is irreducible and primitive, the sequence provided by the circuit is an m-sequence.

Duplication Property

If interest is centered on translates which have specified advances, the labour of working through many shift and add operations on the connection vector can be reduced by recognizing what Fitch [9] has called the duplication property.

If

$$y_a = y_x + y_y + y_z \;\ldots\ldots + y_w, \quad a > w > \ldots > z > y > x$$

then

$$y_{a+a} = y_{a+x} + y_{a+y} + y_{a+z} \;\ldots\ldots + y_{a+w} .$$

But

$$y_{a+x} = y_{2x} + y_{x+y} + y_{x+z} \;\ldots\ldots + y_{x+w} .$$

Similarly

$$y_{a+y},\ y_{a+z},\ \ldots\ldots\ y_{a+w}.$$

Substituting

$$y_{2a} = y_{2x} + y_{2y} + y_{2z}\ \ldots\ldots\ y_{2w}.$$

To illustrate, assume that it is required to obtain a translate of advance 283 from a generator corresponding to the polynomial 205201.

17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
0	1	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	1

$$y_{16} = y_0 + y_7 + y_9 + y_{11}.$$

The superscript, 0, denoting the reference point is omitted but is understood. In general, for a register of N elements, we may prepare a table containing entries up to an advance of 2N - 1.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
16	1	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0
17	0	1	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0
18	0	0	1	0	0	0	0	0	0	1	0	1	0	1	0	0	0
19	0	0	0	1	0	0	0	0	0	0	1	0	1	0	1	0	0
20	0	0	0	0	1	0	0	0	0	0	0	1	0	1	0	1	0
21	1	0	0	0	0	1	0	1	0	1	0	1	1	0	1	0	0
22	0	1	0	0	0	0	1	0	1	0	1	0	1	1	0	1	0
23	1	0	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0
24	0	1	0	1	0	0	0	0	0	0	0	0	0	0	1	1	0
25	1	0	1	0	1	0	0	1	0	1	0	1	0	0	0	1	0
26	1	1	0	1	0	1	0	1	1	1	1	1	1	0	0	0	0
27	0	1	1	0	1	0	1	0	1	1	1	1	1	1	0	0	0
28	0	0	1	1	0	1	0	1	0	1	1	1	1	1	1	0	0

29	0	0	0	1	1	0	1	0	1	0	1	1	1	1	1	1	0
30	1	0	0	0	1	1	0	0	0	0	0	0	1	1	1	1	0
31	1	1	0	0	0	1	1	1	0	1	0	1	0	1	1	1	0

Now

$$y_{32} = y_{22} + y_{18} + y_{14} + y_0,$$

$$y_{64} = \text{etc.},$$

where the expression on the right can be reduced to terms with a subscript less than 16, at each multiplication by 2, by applying to the table. We find

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
32	1	1	1	0	0	0	1	0	1	1	1	1	1	0	1	1	0
64	1	0	1	0	1	0	1	0	1	1	0	0	0	1	1	0	0
128	1	1	0	0	1	0	0	1	0	0	0	0	1	0	1	0	0
256	1	1	0	0	0	1	0	1	1	1	1	1	1	1	1	1	0

Further doubling will carry beyond the point required. We can, however, add to the subscripts by an amount equal to one register length and again apply to the table.

272	1	1	0	0	0	1	1	0	0	1	1	0	1	0	1	1	0

Further addition of one register length will carry beyond the point required. Adding 11 and applying to the table, we obtain

283	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0

The process involved here is similar to that described by Davis [6].

<u>Reciprocal Polynomial</u>

The reciprocal polynomial of $f(x)$, which is of degree m, is

$$f^*(x) = x^m f(1/x).$$

It can be shown [3] that $f^*(x)$ is irreducible, if, and only if $f(x)$ is irreducible; and that if $f(x)$ is irreducible then $f^*(x)$ is primitive, if, and only if $f(x)$ is primitive. In Peterson's tables reciprocals are not listed. The reciprocal of 205201, for example, is 201241.

If the sequence provided by a generator corresponding to a matrix whose characteristic polynomial is irreducible and primitive is

$$a_0\ a_1\ a_2\ a_3\ a_4 \cdots\cdots\ a_{p-1}\ a_p\ a_0\ a_1\ a_2\ \cdots,$$

the sequence provided by a generator whose matrix has the reciprocal polynomial is

$$a_2\ a_1\ a_0\ a_p\ a_{p-1} \cdots\cdots\ a_4\ a_3\ a_2\ a_1\ a_0\ \cdots,$$

where p is the period. The matrices corresponding to circuits A and B have 155 as their characteristic polynomial. Circuits A and B provide the sequence

...1 1 1 1 1 1 0 1 0 0 0 0 0 1 1 1 0 0 0 0 1 .... 1 0 1 0 1 0 0 1 1 1 1 1 1 ...

The reciprocal polynomial is 133, and circuits corresponding to matrices with 133 as a characteristic polynomial provide the same sequence reversed in time.

...1 1 1 1 1 1 0 0 1 0 1 0 1 .... 1 0 0 0 0 1 1 1 0 0 0 0 0 1 0 1 1 1 1 1 1 ...

Davies [5] has noted one simple relationship existing between the advance of the translate obtained from two taps in the standard configuration of a generator and the same two taps in the standard configuration of the reciprocal generator. If summing at taps 0 and a in an m-sequence generator provides a translate advanced by E, and summing at the same taps in the reciprocal generator provides a translate advanced by F, then

$$E + F = p + a,$$

where p is the period.

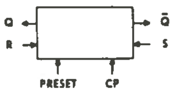
MEASUREMENTS

Measurement of the advances associated with translates obtained from the sum of the reference output and each remaining tapping point in turn, in various registers, was made, primarily to provide check points in the output of the computer program. Series 51 semiconductor networks (Texas Instruments) were used to construct the shift registers and ancillary logic circuits (Fig. 4). Supply voltage was +3 volts. In this NAND system, binary 0 corresponds to a voltage of some 2.5 volts, binary 1 to ground, or close to ground. The clock signal is 2 volts, positive-going from ground, duration 2 μsec, PRF 10 kc/s.

Consider the polynomial 100003, or $x^{15} + x + 1$, which is irreducible and primitive. The sequence generated is

SN 510 R-S Flip-Flop/Counter Network

Power Dissipation
2 mw @ V_{cc} = 3 v (Fan-out = 4)
8 mw @ V_{cc} = 6 v (Fan-out = 4)

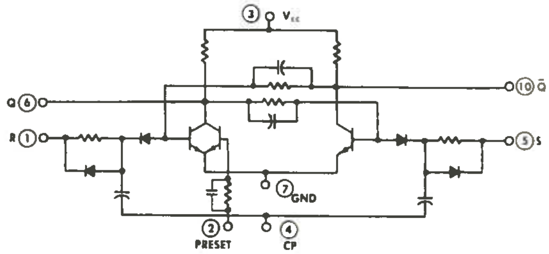


Fan-out
4 max

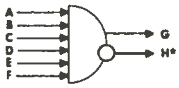
Truth Table

t_n		t_{n+1}
R	S	Q
0	0	Q_n
0	1	1
1	0	0
1	1	Indeterminate

Clock pulse is internally capacitive-coupled, allowing a single phase clock source



SN 513 NOR/NAND Network

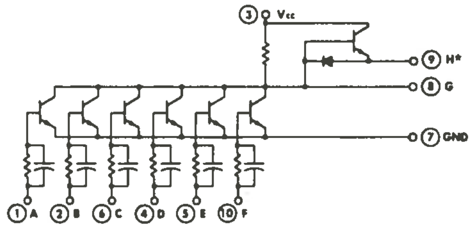


Logic

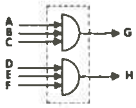
	NAND	NOR
G	$\overline{ABCDEF}$	$\overline{A+B+C+D+E+F}$
H	$\overline{ABCDEF}$	$\overline{A+B+C+D+E+F}$

Fan-out
Terminal G – 5 max
Terminal H – 25 max

Power Dissipation
3 mw @ V_{cc} = 3 v (Fan-out = 10)
13 mw @ V_{cc} = 6 v (Fan-out = 10)



SN 514 Two NOR/NAND Networks



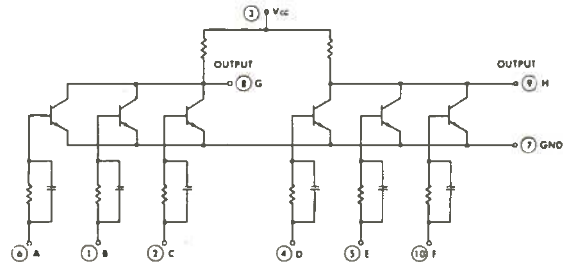
Logic

	NAND	NOR
G	$\overline{ABC}$	$\overline{A+B+C}$
H	$\overline{DEF}$	$\overline{D+E+F}$

Fan-out
Terminal G – 5 max
Terminal H – 5 max

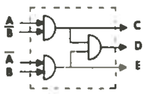
Power Dissipation (per logic stage)
2 mw @ V_{cc} = 3 v (Fan-out = 5)
8 mw @ V_{cc} = 6 v (Fan-out = 5)

Note:
The SN 514 contains two isolated logic functions. However, a common B+ terminal is used.

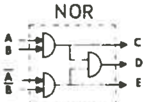


SN 515 Exclusive OR Network

NAND



NOR



Logic

	NAND	NOR
C	$\overline{A}+B$	$\overline{A}B$
D	$A\overline{B}+B\overline{A}$	$A\overline{B}+B\overline{A}$
E	$A+\overline{B}$	AB

Fan-out
Terminal C and E – 4 max
Terminal D – 5 max

Power Dissipation
4 mw @ V_{cc} = 3 v
16 mw @ V_{cc} = 6 v

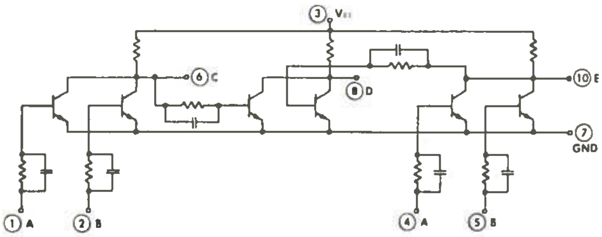


Fig. 4 Series 51 semiconductor networks

$$y_{15} = y_0 + y_1,$$

which has the form $y_n = y_0 + y_m$. If p is the period

$$y_0 = y_p \text{ and } y_a = y_{p+a};$$

then

$$y_n = y_0 + y_m, \quad p > n > m \tag{1}$$

$$y_{n-m} = y_0 + y_{p-m}, \tag{2}$$

$$y_{p+m-n} = y_0 + y_{p-n}. \tag{3}$$

These three equations, by application of the duplication property, will yield up to 3N relations describing the sequence. N is the register length. Applying equation (3) to the generating function, we find, since p is 32767,

$$y_{32760} = y_0 + y_{16376},$$

$$y_{16383} = y_0 + y_7,$$

or $$y_{16390} = y_7 + y_{14}.$$

Taps 7 and 14, in the experimental circuit, happened to be most readily accessible. The circuit constructed to measure the advance of the translate obtained by summing at taps 7 and 14 is shown in Fig. 5. The shift register itself is not included in the figure.

Either the translate or the reference sequence will appear at output 1, depending on the state of the 510 flip-flop at A. This flip-flop is preset by closing and opening S2; the translate appears at output 1. The logic network, below output 1, provides the functions

$$\text{Preset} = C + S = \overline{(\overline{C}.\overline{S})},$$
$$\text{Count} = C + \overline{S} = \overline{(\overline{C}.S)},$$

where C represents the clock signal and S the sequence bit. If the sequence bit is 1, a count signal is supplied to the ripple-through counter. If the sequence bit is zero, a preset signal is supplied to the counter. The 513 multiple input NAND gate is connected to the counter. The output of the gate is at ground until, in this

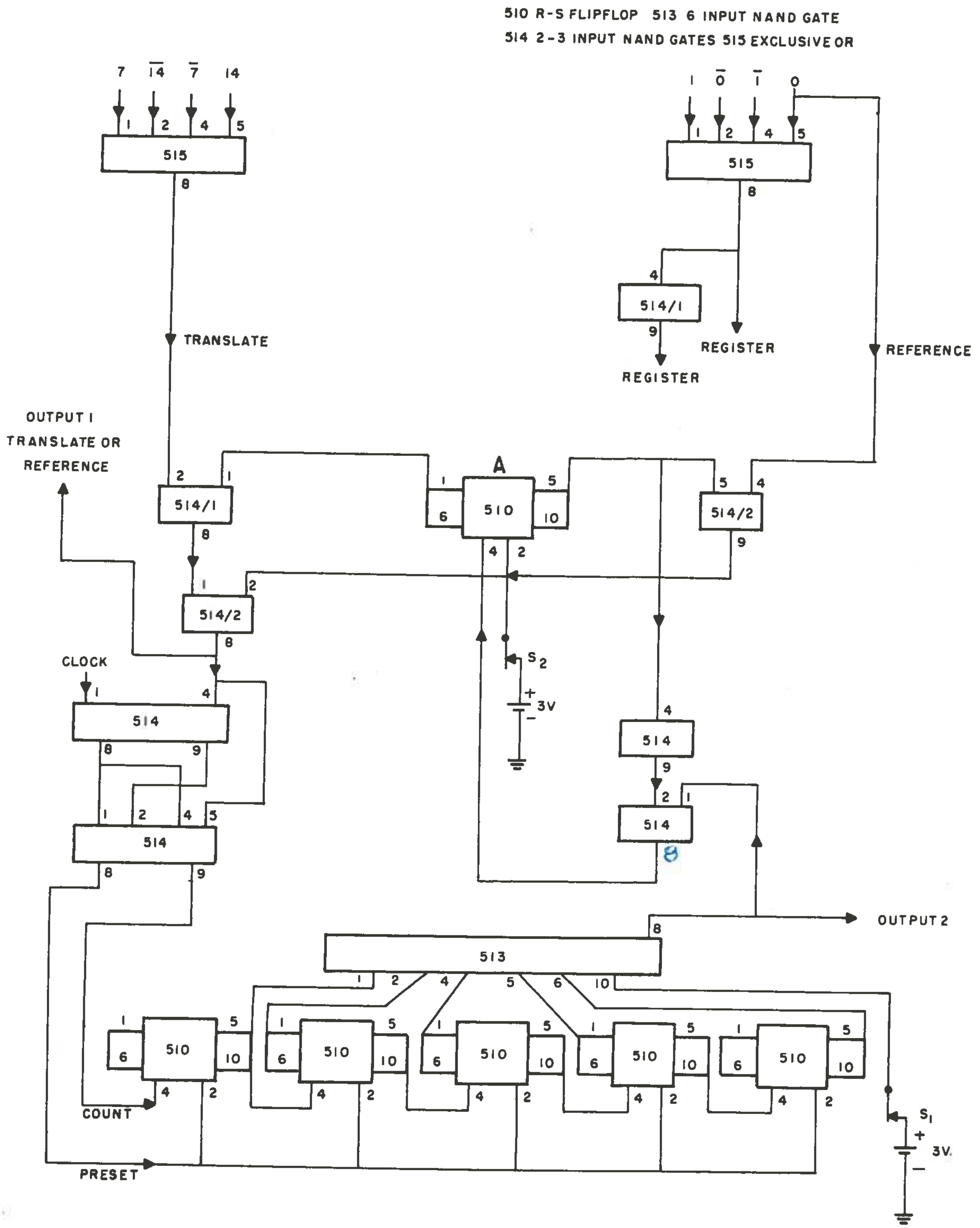


Fig. 5 Measurement circuit — 100003

case, the sequence bit has been 1 for 15 consecutive clock periods. After 15 consecutive 1's the gate output, output 2, rises to some 3 volts for one clock period, providing S1 is open. An output pulse at output 2 switches flip-flop A, and the reference sequence is routed to the counter. Once switched, the flip-flop will remain in this state until preset by S2.

When 15 consecutive 1's are detected in the reference, a positive 3-volt pulse again appears at output 2. Output 2 pulses are supplied to the START and STOP inputs of a suitable digital counter, such as the General Radio type-1130A. Clock pulses, suitably amplified and, if necessary, inverted, are supplied to the external timing signal terminal. The advance of the translate is read directly from the counter.

Summarizing the procedure:

1) Preset the flip-flop at A by closing and opening S2, with S1 closed.

2) Start the m-sequence generator.

3) Set the display time on the counter, type-1130A, to infinity and reset.

4) Open S1.

5) Read the advance of the translate from the counter and reset the counter.

6) Read the period of the sequence from the counter.

Knowledge of the advance of one translate, for example 7 and 14 in 100003 above, with step 6 in the operation summary, provides convenient checks of proper functioning of the measurement assembly.

The measurement assembly can be altered to accommodate generators of register length up to 31 by a simple change of interconnections between the ripple-through counter and the 513 gate. The network, with an appropriate generator, suggests a very flexible method of obtaining extremely long but highly precise delays.

Measurements made with the assembly on various generators are shown in Appendix A.

PROGRAM

The services offered by the program, with its various options, are described first, along with input card formats. Some details of program construction are presented later in Appendix B. An exhaustive description of the program, complete with flow charts, has not been attempted here, but a block diagram is shown in Fig. 6. This report is directed towards the engineering user.

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PROGRAM

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Some 15,000 positions of core storage in the 40,000-position memory unit required are not used by the program. A user who wishes to add options is invited to correspond with the writer at the Faculty of Engineering, Carleton University, Ottawa, Ontario, Canada.

Three precautionary statements are in order: (a) memory must be cleared with zeros before loading the program; (b) the program must be run with the OVERFLOW switch on PROGRAM; and (c) the program card entered at address 01600 to 01659 contains numerical blanks for printer format. If the program deck is copied by the 1620, this card must be duplicated separately.

The LOAD card contains a standard loader and must be the first card of the program deck to be read. The LAST card contains a branch instruction and must be the last program card read. The intervening program cards may be in random order. The last four cards of the deck contain alphamerics for headings, error messages, etc. These cards are labelled Alpha A, B, C, and D and must follow the LAST card in that order. Data input cards should be loaded immediately behind the Alpha cards.

The program is written in 1620 Machine Language and uses Indirect Addressing. An m-sequence generator of register length 35 has a period of 34,359,738,367. The connection vector contains 36 bits. Attention is restricted to connection vectors which are left-registered; i.e., the left-most bit must be a 1 and the right-most bit a 0. In a complete translate analysis, the machine considers some 10 to the power 10, 36-digit fields. Shift, add, and examine operations are performed in a loop of instructions. The execution time per instruction is 200 microseconds minimum. Machine language programming offers the greatest possibility of reducing program running time to a minimum.

Some problems involving high register lengths and a large number of terms in the sequence generating function are within the scope of the program but outside the speed capabilities of the 1620. Nevertheless, a little preliminary work with paper and pencil and judicious use of the program options can make the number of such problems surprisingly small.

The block diagram of the program is shown in Fig. 6. Note that the program is an exitless loop and will, if allowed to run indefinitely, terminate on READER NO FEED. An interrupt halt may be obtained immediately after printing by putting Program Switch 2 "on".

Routine Analysis

The generator is described by the characteristic polynomial in octal notation. This number will be termed the "characteristic". The input card has the characteristic punched, beginning in column 1 with a record mark (028) immediately

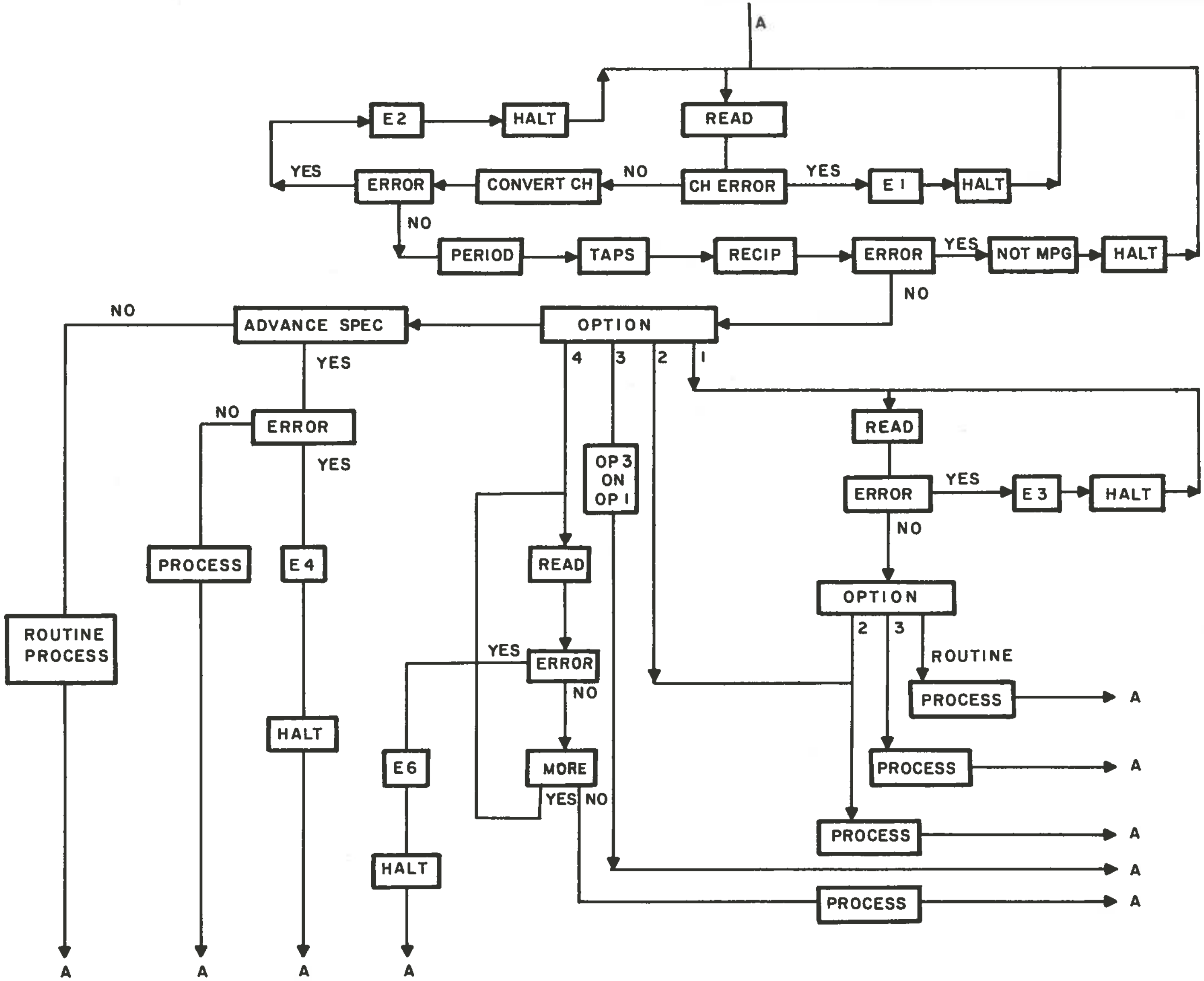

Fig. 6 Block diagram of program

following the last digit. In a routine analysis the computer will complete a series of shift and add operations on the connection vector until the vector

100000 0

is detected, indicating that one period has been searched. The advance associated with this vector and the characteristic are printed, and the machine branches to read the next input card. If the advance printed is not equal to the period but is a divisor of it, the characteristic represents a polynomial which is irreducible but not primitive.

Print-out occurs whenever the connection vector contains two or three 1's indicating a translate obtainable with one or two adders in the standard configuration of the generator. The program does not take into account those adders involved in the feedback loop, but, since these adders are clearly shown in the heading, it is a simple matter to determine, for example, translates available using only the adders of the feedback loop in pairs.

The connection vector is printed left-registered, the left-most bit corresponding to the tap on the output of memory element 0, the next the tap on the output of element 1, etc. The advance associated with the vector is printed to the left.

For a routine analysis only the characteristic followed by a record mark is punched in the input card. Error messages E1, E2, and NOT MPG are intended to cover gross punching errors. The machine halts.

E1	No record mark follows the characteristic.
E2	An 8 or a 9 appears in the characteristic.
NOT MPG	The total number of taps represented in the generating function is even.

After these three error messages, depression of the console START key will cause a new card to be read.

Option 1, Continuation

If a routine analysis, or an OP2 analysis (below) is interrupted, it may be continued from the last entry printed using OP1. Alternatively, it may be desirable to continue analysis with an advance and connection vector computed by hand as the start point.

To use OP1, the characteristic and its record mark are punched on one card with a 1 punched in column 50. This is followed by a second card with a left-registered connection vector beginning in column 1 and with a record mark following the last bit. The advance associated with this vector is punched, beginning in column 38 with a record mark following the last digit. A record mark must be punched in column 80 to guarantee that the card is a continuation card. If column 50 of the continuation card is left blank, a routine analysis to the end of one period will be obtained. If column 50 contains a 2, the analysis will continue OP2. If column 50 contains a 3, the analysis will continue OP3 (below).

If a record mark is omitted from column 80 of a continuation card or if the number of bits in the connection vector differs from the number of bits computed from the characteristic, error message E3 will be obtained. On depressing the START key, the program will branch to read a new continuation card. The start point is printed as a heading in the output format.

Option 2, 3 Adders or Less per Translate

The card format is the same as for a routine analysis, but a 2 is punched in column 50 of the characteristic card. Analysis in greater detail is provided. Connection vectors containing up to four 1's are printed in a search covering one period. Since the print time is still small compared with program running time, the greater detail is provided at little extra cost.

Continuation, OP2, is covered under OP1 above.

Option 3, Check Points

A need may arise to accept an entry from a routine or OP2 analysis as a starting point and continue the analysis in fine detail by hand. Option 3 provides check points for such an operation. Option 3 is only available on a continuation card. If column 50 of a characteristic card contains a 3, the error message OP3 on OP1 will be received and the program will branch to read a new card without halting.

Card format is the same as for OP1 but a 3 is punched in column 50 of the continuation card. The option takes the starting point specified, performs shift and add operations, and prints out the connection vector after every 10th or every 6th operation, depending on whether the number of taps involved in the generating function is equal to, or less than 7, or greater than 7.

Eighty print-outs are made before the option terminates and a new card is read, unless the connection vector

1000 0

is detected before this point.

Only a blank (or zero), a 2 or a 3 are permissible in column 50 of a continuation card. If a digit other than these is read, the error message E5 will be received, the machine will halt and branch to read a new card on depression of the START key.

Option 4, Solution of Difference Equations

Option 4 allows a connection vector to be specified and the advance of the corresponding translate is computed. The characteristic card has a 4 punched in column 50 and may be followed by up to 20 continuation cards, each containing a left-registered connection vector beginning in column 1 with a record mark immediately after the last bit. Each continuation card must have a record mark in column 80. A record mark punched in column 50 indicates that other continuation cards are to follow for simultaneous processing under this option. The last continuation card, of course, will have no record mark in column 50. The program will not read a 21st card whether a record mark is punched in column 50 of the 20th card or not.

If a record mark in column 80 is omitted, or if the number of bits in any connection vector differ from the number computed from the characteristic, if the first and last bits in a connection vector are not 1 and 0, respectively, error message E6 will be received, the machine will halt, and, on depression of the START key, will branch expecting to read a new characteristic card.

Option 5, Specified Advance

The connection vector corresponding to a specified advance is required. The characteristic is punched from column 1 with a record mark following the last digit . The advance required is punched from column 14, followed by a record mark. The tolerance on the advance specified, up to 3 digits, is punched from column 26 followed by a record mark. Column 50 is left blank.

If the record mark is omitted after the advance specified, or if a tolerance is specified but is not followed by a record mark, or if the advance specified is greater than the period or less than the degree of the characteristic polynomial, error message E4 will be received, the machine will halt, and on depression of the START key, will branch to read a new card.

The program performs the doubling process with table look-up until further doubling will carry beyond the advance specified. The operation of adding shifts of one register length with table look-up continues until addition of one more register length will carry beyond the advance specified. The connection vector at this point is printed out and may contain a zero at the left-most bit. Thereafter, a shift and add operation proceeds, with print-out following each operation, until the advance exceeds the sum of the advance specified and the tolerance.

Running Times - Model I

1) Running time, Routine and OP2, is a function of register length and the number of terms in the generating function. Typical times are shown below:

Register Length	Characteristic	Running Time
12	10407	< 1 minute
13	20033	< 1 minute
14	76055	3 minutes
15	100003	4 minutes
15	102043	5 minutes
16	201241	10 minutes
17	440001	14 minutes
18	1000201	28 minutes
19	2000047	76 minutes
20	4000011	113 minutes

2) Running time, OP3, is always less than 1 minute.

3) Running time, OP4, depends on the register length, the number of terms in the generating function, the number of continuation cards and how close the last solution comes to one period. Typical times are shown below:

Characteristic	Cards	Period	Last Solution	Time
266663	7	65535	20393	10 minutes
201241	8	65535	63945	25 minutes
10000005	5	2097151	28922	15 minutes

4) Specified Advance — The doubling operation is completed quickly for any advance and any characteristic. The time required for completion after the addition of register lengths terminates is less than one minute. Most time is spent in the second stage. With the characteristic 400000107 and a specified advance of 143710, the 1431 operations of adding one register length occupied 12 to 13 minutes. With the same characteristic and a specified advance of 383678, the 6565 operations of adding one register length took 75 minutes,

an average of 685 milliseconds per operation. If the register length is high and the required advance high and just below a doubling level, machine time can be saved by: (a) specifying an advance at, or just above the doubling level; (b) specifying an advance somewhat less than the difference between the doubling level and the advance required with an adequate tolerance; and (c) completing the calculation by hand.

Sample output sheets for all options are shown in Appendix C.

Running Times - Model II

By courtesy of the Northern Electric Research and Development Laboratories, Ottawa, running times on the IBM 1620 Model II Console have been obtained.

Option 2

Length	Characteristic	Running Time
16	201241	4.25 minutes
17	440001	6.50 minutes
18	1000201	11.75 minutes

Routine

19	2000047	24.00 minutes

Option 4

Characteristic	Cards	Period	Last Solution	Time
266663	7	65535	20393	2.75 minutes
201241	8	65535	63945	7.25 minutes

CONCLUSION

It is believed that the program and the information it provides will be helpful in the design of m-sequence generators to provide translates of known advance. The ability to solve difference equations, provided by Option 4, is believed to be the most valuable contribution here described.

Reference is made to Scholefield's paper [8]. It would appear that if a period and an advance are specified there is a solution, or set of solutions, involving particular characteristic polynomials and generator configurations which will yield a minimum number of adders. This topic is suggested for further work.

The work described was undertaken during a summer period with the National Research Council. The writer expresses his thanks to the Council for this opportunity, to Mr. T.H. Shepertycki of the Council who suggested the problem and who contributed to many stimulating discussions, and to Mr. Wayne A. Davis of the Defence Research Board who brought the work of the Communications Laboratory of the Board to the writer's attention.

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APPENDIX A

EXPERIMENTAL RESULTS

A. 100003 and the reciprocal 140001
Period 32767

100003		140001*
15	0 + 1	32753
30	0 + 2	32739
255	0 + 3	32515
60	0 + 4	32711
7790	0 + 5	24982
510	0 + 6	32263
16383	0 + 7	16391
120	0 + 8	32655
10449	0 + 9	22327
15580	0 + 10	17197
3836	0 + 11	28942
1020	0 + 12	31759
238	0 + 13	32542
32766	0 + 14	15
1	0 + 15	14

*Reciprocal computed.

B. 201241 and the reciprocal 205201
Period 65535

201241		205201
47300	0 + 1	18236
29065	0 + 2	36472
65255	0 + 3	283
58130	0 + 4	7409
13660	0 + 5	51880
64975	0 + 6	566
56931	0 + 7	8611
50725	0 + 8	14818
8609	0 + 9	56935
27320	0 + 10	38225
22257	0 + 11	43289
64415	0 + 12	1132
30354	0 + 13	35194
48327	0 + 14	17222
53211	0 + 15	12339
35915	0 + 16	29636

C. 400011 and the reciprocal 440001
Period 131071

400011		440001
9300	0 + 1	121772
18600	0 + 2	112473
17	0 + 3	131057
37200	0 + 4	93875
4585	0 + 5	126491
34	0 + 6	131043
65534	0 + 7	65544
74400	0 + 8	56679
18357	0 + 9	112723
9170	0 + 10	121911
18334	0 + 11	112748
68	0 + 12	131015
6482	0 + 13	124602
131068	0 + 14	17
38502	0 + 15	92584
17729	0 + 16	113358
3	0 + 17	14

D. 4000011 and the reciprocal 4400001
Period 1048575

4000011		4400001
212012	0 + 1	836564
424024	0 + 2	624553
20	0 + 3	1048558
848048	0 + 4	200531
786432	0 + 5	262148
40	0 + 6	1048541
524987	0 + 7	523595
647521	0 + 8	401062
1425	0 + 9	1047159
524289	0 + 10	524296
491520	0 + 11	557066
80	0 + 12	1048507
392521	0 + 13	656067
1399	0 + 14	1047190
352	0 + 15	1048238
246467	0 + 16	802124
1048572	0 + 17	20
2850	0 + 18	1045743
310122	0 + 19	738472
3	0 + 20	17

APPENDIX B

PROGRAM DETAILS

The program begins at 00520, continues to 10000, branches to 18740, and terminates at 19820. The interval 10000 to 18620 contains 16 subroutines. A branch to 00688 will cause a new card to be read, a new page to position on the printer, and the program to recycle.

The addresses and functions of various stores and the subroutines are given below:

39901 - 39999	Program read-in and distribution
39800 - 39899	Print-out area
39700 - 39799	A stored field of 100 zeros for clear operations
39500 - 39699	Alphameric store
39401 - 39499	Read-in data area
39200 - 39399	The work area for operations on the connection vector
38680 - 38799	120 numerical blanks stored for printer format
38650 - 38665	Characteristic stored as a record for pagination
00400 - 00519	Data area store, taps, period, etc.
03280	Routine option begins
08440	Specified advance option begins
04360	Option 2 begins
06340	Option 4 begins
05140	Option 3 begins

SUBROUTINES

SR1	17100020000$\bar{0}$	Clear print-out area for alphameric
SR2	27100640052$\bar{6}$	Print numeric as alphameric
SR3	27103820052$\bar{6}$	Conversion of the characteristic from octal
SR4	17111020000$\bar{0}$	Tap finding
SR5	27113081123$\bar{3}$	Period calculation and number of digits in the period
SR6	271167601107	Calculation of the reciprocal
SR7	17121080000$\bar{0}$	Taps print-out
SR8,1	17127100000$\bar{0}$	Numeric blanks, print format
8,2	17127460000$\bar{0}$	Same
8,3	17127820000$\bar{0}$	Same
SR9	27130001109$\bar{8}$	Prepare the work area for translate computation
SR10	17133720000$\bar{0}$	Continuation cards
SR11	17139240000$\bar{0}$	Specified advance tables
SR12	1715780$\bar{3}$9211	Adjust the work area
SR13	17162040000$\bar{0}$	Translate and connection vector print-out format
SR14	17170000000$\bar{0}$	OP4 continuation cards

A storage area associated with SR11 is located at 24998 to 27080; another associated with SR14 is located between 37000 and 38000.

If Program Switch 1 is on, various interim check points will be typed on the console typewriter. These were included, originally, to aid program debugging and have not been erased.

A copy of the program deck may be obtained by writing on letterhead to the Space Electronics Section, Radio and Electrical Engineering Division, National Research Council of Canada, Ottawa, Ontario.

APPENDIX C

SAMPLE OUTPUT SHEETS FOR ALL OPTIONS

ROUTINE ANALYSIS

100003 RECIPROCAL 140001 PERIOD, 32767
TAPS 00 01 15 03

29	1100000000000010	30	10 1000000000000
43	110000000000100	59	100 10000000000 10
60	1000100000000000	71	11000000000 10000
119	1000000 100000010	120	10000000 10000000
127	1100000 100000000	237	100000000000 10 10
238	100000000000000100	240	111000000000000
254	101000000000010	255,	100 1000000000000
267	110000000000 1000	476	10000000000 11000
480	101010000000000000	509	10000 100000000 10
510	100000 1000000000	519	11000000 1000000
631	1000000 10 1000000	960	1000 1000 10000000
1019	10000000000 100 10	10 20	100000000000 1000
1023	110 1000000000000	10 51	100 1000000000 100
1263	1000 11000000000000	1274	10 100000000 10000
1918	100000 1000000 100	20 40	100000000 1100000
2046	101000 1000000000	2526	10000000 10 100000
3835	1000000000 1000 10	3836	10000000000 10000
3840	1100 10000000000000	3895	100 1000000 100000
4092	1000 10000000 1000	5224	1000 100 100000000
7426	100000000 100 1000	7672	1000000 110000000
7680	10 100000 10000000	7771	10000000000 10 100
7789	1000 100000000000 10	7790	10000 1000000000000
7800	1100000000 100000	8191	100 1000 100000000
9029	100000 10 10000000	9120	100 10 10000000000
9540	100000000 1000 100	10 135	1000000 1000 10000
10194	100 100 1000000000	10 448	10000000 100000 10
10 449	100000000 1000000	10 455	110000 1000000000
10958	10000 100 10000000	15342	100000000000 1100
15345	101 1000000000000	15358	10 10000000000 100
15387	100 100000000 1000	15579	100000000 10000 10
15580	1000000000 100000	15585	11000 10000000000
15641	10000 100000 10000	16382	100000 10000000 10
16383	1000000 100000000	16391	11000000 10000000
16502	100000 110000000	16511	10 1000000 1000000
18056	1000000000 100 100	18240	100000 1000 100000
18301	10000 10 100000000	19123	10000 1000 1000000
20271	10000000 11000000	20 278	10 10000 100000000
20388	100000 100000 1000	20898	100 1 1000000000000
20910	10 1000000000 1000	20 941	10000 10000000 100
21071	1000 100000 100000	21132	100000000 10 10000
21608	10000 1000000 1000	21914	10000000 10000 100
22604	100000 100 1000000	23113	1000000 100 10000
23369	1000 10000 1000000	240 54	1000000 100000 100
24575	1000 1000000 10000	24634	1000000 100 100000
24978	10000 10000 100000	26410	100 100000 1000000
26914	1000000000 110000	26919	10 100 10000000000
28393	1000 100000000 100	28613	10000000 1000 1000
29553	1000000000 101000	29584	100 10000000 10000
29840	1000000 10000 1000	300 29	100000 10000 10000
30581	100 10000 10000000	30690	1000 10 1000000000
31160	10000 11000000000	31170	10 10000000 100000
32765	10000000000000 110	32766	1000000000000 10

32767 100003

140001 RECIPROCAL 100003 PERIOD 32767

TAPS 00 14 15 03

16	11000000000000 10	1607	10000000 10 100000
1613	110000 1000000000	2083	10 1000 1000000000
2194	10000 100 10000000	2749	10000 100000 10000
2939	10000 1000000 1000	3194	10000000 100 10000
3226	10 1000000000 1000	4166	1000 10000000 1000
4387	100000000 1000 100	5853	100 10 10000000000
5864	11000000000 10000	6366	100000 100 1000000
7799	10000 10000 100000	8143	100 1000000 100000
8203	1000000 1000 10000	8726	100000 1000000 100
9407	10000 1000 1000000	9665	100 10000000 10000
10172	100 100000 1000000	10866	10000 10000000 100
11171	1000000 10000 1000	11646	10 100000000 10000
11706	100000 1000 100000	11839	10000000 10000 100
11869	1000000000 10 1000	11873	1100 100000000000
12391	100000 100000 1000	12496	10000 10 100000000
12505	110000000 1000000	13653	1000 10000 1000000
14473	10 10000 100000000	14537	1000 100000 100000
14724	100 1000000000 100	16265	1000000 10 1000000
16272	1100000 100000000	16384	1000000 110000000
16391	1000000 100000000	16399	10000000 100000 10
17137	100000 10000 10000	17187	1000 110000000000
17197	1000000000 100000	17202	10000 100000000 10
17392	100000000 100 1000	17422	10000000000 10 100
17425	110 1000000000000	17438	1100000000000 100
21817	100 10000 10000000	22318	10000 11000000000
22327	100000000 1000000	22333	100000 10000000 10
22579	100 100 1000000000	22643	1000 1000000 10000
23240	1000 100000000 100	23652	10 100 10000000000
23746	10 100000 10000000	24583	1000 100 100000000
24977	100000000 1100000	24982	10000 10000000000
24992	1000000000 1000 10	25009	10 10000000000 100
25095	100000 10 10000000	25103	11000000 10000000
25353	100 100000000 1000	27550	100 1000 100000000
28687	10000000 1000 1000	28882	1000000 100 100000
28931	100 1100000000000	28942	10000000000 10000
28946	1000 1000000000 10	30251	10 10000000 100000
30727	1000 10 1000000000	30737	1100000000 100000
30862	1000000 100000 100	31504	100000000 10 10000
31509	11000 10000000000	31729	1000000000 100 100
31747	10 11000000000000	31759	100000000000 1000
31762	100 10000000000 10	31815	1000 1000 10000000
32145	10 1000000 1000000	32257	10000000 11000000
32263	100000 1000000000	32272	100000000 10000 10
32291	10 10 100000000000	32303	110000000000 1000
32512	10000000000 11000	32515	100 1000000000000
32527	100000000000 10 10	32529	1110000000000000
32542	1000000000000 100	32544	10 100000000000 10
32647	100000 1100000000	32655	10000000 10000000
32662	1000000 1000000 10	32707	1000000000 110000
32711	1000 100000000000	32722	10000000000 100 10
32737	100000000000 1100	32739	10 10000000000000
32752	1000000000000 110	32753	1100000000000000

32767 140001

205201 RECIPROCAL 201241 PERIOD 65535
TAPS 00 07 09 11 16 05

73	10000010000000100	285	10010000000000000
568	10000010000000000	1132	10000000000010000
2361	11000000100000000	3087	10010000000000100
3219	10001100000000000	3344	10010000000000010
3941	10001010000000000	4715	10000010000100000
6117	11000000010000000	6438	10000000101000000
7192	10000001000000010	7409	10001000000000000
7882	10000000100010000	8589	10010100000000000
8602	10100001000000000	8611	10000001000000000
8620	10000001000100000	8844	10000100000000010
9901	10100000000000010	10044	10000000000010010
10518	10000000011000000	12227	10000010010000000
12339	10000000000000010	14215	10010000000010000
14727	10000001001000000	14818	10000000100000000
15347	11000000000000100	15981	10000000000010100
16016	10000100000010000	16785	11000100000000000
17178	10000010001000000	17204	10001000000000100
17222	10000000000000100	18236	11000000000000000
18402	11001000000000000	18927	10010000010000000
19119	11000001000000000	19153	10001000000010000
19506	10100100000000000	20604	10000000000100010
20832	10000101000000000	20935	10000000001010000
21172	11010000000000000	22826	10000001000000100
23739	10000100000001000	24201	10000000110000000
25545	10011000000000000	25994	10000100001000000
26231	10000000010000010	26922	10000000000011000
27535	10000001100000000	27870	10000100000000100
28237	11000000000010000	28618	10000000100100000
28753	10000010000000010	29629	10101000000000000
30985	10001000000000010	31459	10000000000001100
32152	10000100010000000	32804	10010001000000000
33570	10100000001000000	34682	10000000010001000
34738	10111000000000000	34831	10001000000100000
35194	10000000000001000	36459	11000000001000000
36472	10100000000000000	36715	11000000000000010
36804	10100000100000000	37592	11000000000100000
38032	10000000100000010	38225	10000000010000000
38238	10100000000000100	39012	10001000001000000
40758	10000011000000000	41664	10000000010001000
42344	10100010000000000	43235	10000110000000000
43282	10100000010000000	43289	10000000000100000
43429	10100000000010000	44548	10000000001100000
45070	10000001000010000	45081	10000000010000100
45090	10000000000100100	45770	10000000001000010
46461	10100000000100000	47322	11000000000001000
47582	11100000000000000	47769	10000010000010000
48004	10000000000001010	48400	10000100000100000
51090	10000010100000000	51766	10000000001001000
51880	10000100000000000	51891	10000001010000000
52475	10000100100000000	53681	10010000001000000
54482	11000010000000000	54661	10010000100000000
55072	10000001000001000	55455	10000000100000100
56652	10010010000000000	56928	10001000010000000

205201

56935	100000000 10000000	56942	100000000 10 100000
56949	10 10000000000 1000	57311	10000000000 110000
57397	100 1000000000 1000	58069	10000000000 10 1000
59258	1000 1000 100000000	59296	100000000 100 10000
60495	1000 100 1000000000	62307	1000000000000 110
63382	100 10000000 100000	63706	100000 1000000 1000
64116	1000 100000000 1000	65060	10000000 10000 1000

65535 ?05201

400011 RECIPROCAL 440001 PERIOD 131071
TAPS 00 03 17 03

31	100 100000000000 1000	34	100000 1000000000 00
45	100 10000000 1000000	65	100000000 10000 10 00
68	100 000000000 100000	73	100 10 1000000000000 00
136	1000000 100 10000000	146	100000 1000 1000000 00
266	1000000 100 1000000	275	100000 100 1000000 00
286	100000 10000 1000000	317	10000000 11000000 00
1144	1000000 100000000 10	2291	1100000 10000000000 00
3241	10000000 1000000 100	4582	10 1000000000000 10 00
4585	10000 100000000000000	4597	100 100000000 100000
4650	10 1000000 100000000	6479	1000000000 1000 10 00
6482	1000000000000 10000	6486	100 1 1000000000000 00
8447	10001 100000000000000	9167	1000000 1000000 10 00
9170	1000000000 10000000	9177	100 1000 100000000 00
9299	10 1000000000000000 10	9300	1 10000000000000000 00
9316	100 10000000000000 10	12964	10000000 100 1000 00
12972	100000 10 100000000	13009	1000000000 1000 100
13218	1000000 10000 10000	15785	1000 1000000000000 10
16758	1100000000 1000000	16894	10000000 10 100000 00
17726	100000000000 11000	17729	10000000000000000 10
17730	110 10000000000000000	18331	10000000 100000 10 00
18334	10000000000 1000000	18340	100 100 10000000000 00
18354	100000 10000000 1000	18357	10000000 100000000
18365	100 10000 1000000000	18598	1 1000000000000 100
18600	10 100000000000000000	18615	100 10000000000000 100
18671	10000 1000000000 100	19251	10000000 1000000 10
22945	10000000 1000 100000	23902	110000000 1000000 00
25944	100000000000 1000 10	26307	10000000 1 1000000 00
27028	10 1000000000000 100	27899	1 10000000000000000 10
29221	1 1000 1000000000000	31287	1 10000 10000000000 00
32767	10000 1000000 100000	33531	10000000 100000 100
33782	100000000 11000000	33917	10000 10000 100000 00
35460	10 1000 100000000000	36668	10000 100 10000000 00
36680	100 000 100000 100000	36711	10000000000000 1100
36714	1 100 1000000000000000	36730	100000 1000000000 10
37197	1 1000000000000 1000	37200	1000 100000000000 00
37213	100 10000000000 10000	37349	1000000 10 1000000 00
38473	10000 100000 1000000	38499	1000000000 10 10 00

38502	1000000000000000 100	38504	10 1100000000000000 00
38743	100010000 100000000	42259	1000 100000 10000000
43331	10000001 1000000000	43679	1100000000 10000000
45625	1 1000000000 100000	46373	1000000 100000 100 00
52608	1000000000 1100000	54437	100000000 10 1000 00
56807	10000000 10000000 10	57100	1 100000000000 100 00
58442	1010000000 10000000	62574	10 1000000000 1000 00
65531	10001000000000 1000	65534	1000000 1000000000 00
65544	100100000000 10000000	65667	1000000 1000 1000000
66403	100 00 100000000000 10	67558	1000000000 10000 10
67828	10001000000 1000000	68775	10000 10 10000000000
70185	1 1000000 1000000000	70920	1000 10000000 1000 00
73040	10000 10000000 10000	73336	100000000 100000 10
73428	10 100000 1000000000	73465	10000 1000 100000 00
74397	10000 100000000 1000	74400	1000000 10000000 00
74409	100 100000 100000000	74437	100000000 10 1000 00
74701	1000 100 10000000000	74714	100000 1000000 100 00
77004	1000000000000 100 10	77008	1000 10 100000000000
78053	100000000000 110000	79462	1000000000 10 100 00
81177	1000000 10000000 100	84512	10 100000000 100000
84778	1000000000 100 10000	84785	100000 11000000000
86662	1000000000000 10 10	86665	10 100 1000000000000
86680	100000 100000000 100	87356	1000 100000000000 100
92749	1000000000000 100 100	92754	10000 110000000000
93939	1000 1000 1000000000	98301	10 10000 1000000000
107923	1000000 10000 100000	110106	100000000 10000 100
112505	10 10 10000000000000	116880	10 1000000000 100 00
121788	1 1100000000000000	125161	100000000 1000 1000
126354	1000000000000000 110	127791	1000 100000000 100 00
129231	1000000000000 10 100	131065	1000000000 100 1000
131068	10000000000000 1000		

131071 400011

440001 RECIPROCAL 400011 PERIOD 131071
TAPS 00 14 17 03

20	100 1000000000000 1000	1855	10 1000000000000 100
3293	100000000 1000 10000	4733	1100000000000000 10
5923	1000 100000000 10000	9285	1 11000000000000000
14204	10000000000 10 10000	18570	10 10 100000000000000
20980	10000 1000000000 100	23160	10000 1000000 100000
32777	10000 10 10000000000	37140	1000 1000 1000000000
38323	110000 100000000000	38337	100 100000000000 100
43730	10000000000 1000 100	44406	100000000 100000 100
44411	100 10 10000000000000	44425	10 10000000000000 10
46293	1100000 10000000000	46306	100 1000000000 10000
46570	100000000 10 1000000	49909	10000000 1000000 100
51622	10 1000000000000 10000	53031	1 1000000000000 10000
54069	10 1000 100000000000	54083	100 100000000000 10
56370	1000000 100000 10000	56377	100 1000 1000000000

56645	10 100000000 1000000	56671	100000 100 100000000 00
56679	10000000 1000000000	56688	100000000 10000 1000
57615	1000 10000 100000000	57651	100000 10 1000000000 00
57751	100000 1000000000 10	58044	10000000 10000 100 00
60163	10000000 1000 100000	60894	1000000 1 1000000000 00
62303	10 10000 1000000000000	63254	1000000 1000 1000000 00
63529	10000 1000000000000 10	64684	10000000000 10000 10
65415	1000 1000000 1000000	65537	1000000 100 1000000 00
65544	1000000 10000000000	65554	1000000000 1000 10 00
68509	1000000000 10 100000	72639	10000000 10 1000000 00
73984	1000000000000 110(00	74280	10000000 10000000 10
76646	10 1000000000000 100000	78475	110000000000 100000 00
84711	100000 1000000 10000	85458	10000000000 1 1000 00
87402	100000000 110000000	87748	11000000 100000000 00
88822	100000 1000 10000000	92337	10000 1000 10000000 00
92570	110 1000000000000000	92584	100000000000000 100
92586	10 100000000000 1000	92609	100000 10000 100000 00
93731	10 1000000 100000000	93871	1000000000 100 100 00
93875	1000 10000000000000	93888	1000000000000 110 00
94357	1000000000 1000000 10	94361	100 11000000000000 00
94375	11000000000000 100	94403	100000 100000 100000
94411	100 10000 1000000000	95617	1000 10 100000000000 00
97164	10000 10000 10000000	97300	11000000000 100000 00
97555	100000 100000000 100	98316	1000000 10000 1000 00
99790	10000 1100000000000	101855	1000 1 100000000000 00
103188	100000000000000 110	104058	1000000000000 10 100
104774	1100000000 10000000	105143	1000 100000 0000000 10
107178	10000000 1100000000	108138	1000 1000000 00 1000 00
111836	1000000 100000000 10	112415	1000000000 10000 100
112471	1000000000000 100 100	112473	10 100000000000000 00
112488	100000000000000 1100	112714	10000 100 1000000000 00
112723	100000000 100000000	112731	10000000 100000 1000
112737	100 100 10000000000	112748	1000000000 100000 00
112754	100000 10000000 1000	113344	10 1 1000000000000000 00
113358	10000000000000 10	113359	110000000000000 1000
114187	10 10000000 10000000	114324	1000000000 1100000 00
115302	10000000000 1000 10	117866	10000 10000000 100 00
118077	1000 1000000000 100	118107	10 100000 1000000000 00
118119	100 100000000 100000	121771	1000000000000 100 10
121772	110 0C0000000000000000	121788	100C000000000 10 10
121901	1000100 10000000000	121911	1000000000 1000000 00
121918	1000000 1000000 1000	122629	11000 1000000000000 00
124589	1100 1000000000000000	124602	1000000000000 100 00
124606	1000 100000000000 1000	126430	1000000 10 10000000 00
126486	100000000 100 100000	126491	10000 100000000000 00
126503	1000000000 10 1000	127845	1000000 10000000 100
128787	100000 1100000000000	129943	100000000 10000000 10
130763	110000000 100000000	130796	10000 100000 100000 00
130805	100 100000 100000000	130816	100 10000000 100000 00
130935	1000 100000 10000000	130945	100 1000000 1000000 00
131003	10100 100000000000000	131015	10000000000 10000 00
131020	10000 100000000 1000	131037	10000000 100 100000 00
131043	100000 10000000000	131054	10000000000 100 10 00
131057	100 10000000000000000		
131071	440001		

1000201 RECIPROCAL 1004001 PERIOD 262143
TAPS 00 07 18 03

29	1000000 1000 10000000	36	10000000000000 10000
40	1000 100 100000000000	72	100000000 10000 0010
80	1000000 100000 10000	9627	11000000000 1000000
9633	100000 1000000000000	9645	1000000 10000 1000000
10826	10000 10000 100000000	16313	10 1000 100000000000
19259	10000 100000 10000000	19266	100000000000 1000000
19272	100000 1100000000000	25039	100 1100000000000000
26326	10000000 1000000 100	26335	100000000 10000 10000
27258	1000000 10000 100000	28893	110000 100000000000
32626	1000 10000000 1000000	33966	100 10000000 10000000
33973	100000000 100000000	33981	1000000 110000000000
38525	100000000000 100000 10	38532	100000 1000000 100000
38544	100000000000 10 10000	38844	1000000000000 100 100
40444	100000000000 10000 10	43300	100 10000000000000 100
45985	10000 11000000000000	47357	10 100 10000000000000
50078	100000 10 1000000000	52627	100 1000000000000 10000
52631	1000 100000000000000	52645	1000000 1000000 10000
52656	10000000000 100 10000	52663	100 1000000 100000000
52678	10000000000000 11000	55631	100000 10000000 10000
56349	100000000000000 1100	57786	10 100000000 1000000
59664	1100 100000000000000	61339	10 10000000000000 1000
65351	1000 110000000000000	65541	10000000 110000000000
67939	1000000000 10 100000	67946	10 1000000 100000000
67962	1000000000000 10 100	74030	11000000 1000000000
77667	10000 10000000000 1000	79442	1000 100000000 100000
84400	1000000000000 10 1000	86423	110000000000000 100
88479	1000 10 10000000000000	91970	100000000 10 1000000
94714	1000 100000 100000000	100156	100000000000 1000 100
101434	11000 1000000000000	104886	1000 10000 100000000
105255	110000000000 10000000	105262	10000000 1000000000
105272	1000000 100 100000000	105766	10000 100000000 10000
109279	10000 10000000 100000	112677	10000000 100000 1000
114389	10 10 100000000000000	116709	100 10000000 1000000
119328	10 100000 10000000000	120219	100000 1000 100000000
120903	100000 100000000 1000	125532	1000000000 100 100000
130047	110000000000000000 10	130702	10000000 10 100000000
131068	10 100000000 10000000	131075	100000000 1000000000
131084	1000000 10 1000000000	135885	1000 10000000000 1000
135888	100 1000000000000000	135903	1000000 10000000 1000
139228	1101000000000000000	144238	100000000 100000 0010
148053	10 100000000000 100000	148058	10000 100000000000000
148071	1000000 100000 100000	157385	10000 1000000000 0100
157387	10 10000000000000000	157403	1000000 10000000 100
157698	100000000000 1100000	158877	10000000 100 10000000
158884	100000000000000 1000	158887	100 1000 10000000000
158909	1000 100000000000 10000	159153	100000 10000000000 100
160923	10 100000000000 10000	164668	110000000000 100000
171345	10000000000000000 10 10	173282	1000000000000000 110
174293	100000000 100000 100	175311	10 110000000000000000
176958	10000000 1000 1000000	188266	1110000000000000000
188516	11000000000000 1000	191181	100 10 10000000000000
194050	100000000000 1000 10	195413	10000000 10000000 100
195551	10 10000000000000000 10	196609	110000000 100000000

1000201

199019	1000010001000000000	200291	1001000000000001000
202868	1010000000100000000	204693	1000000010000001000
204703	1000000000100010000	204740	1100000000000010000
205104	1000001001000000000	209764	1000001000000000010
209765	1100000000000000000	209782	1000000100000000010
210517	1000000001010000000	210524	1000000000000000100
210526	1010000100000000000	210648	1001000010000000000
211525	1000000000110000000	211532	1000000000000000010
211533	1100000100000000000	218028	1001000000000100000
218218	1000010010000000000	219397	1000010000000000010
221469	1010000000000000100	225340	1000000000010000100
225347	1000010000001000000	225360	1000000000000110000
228778	1000100010000000000	228806	1001000000000000010
231144	1000000000001001000	233411	1000001000010000000
233418	1000000000000100000	233423	1000010100000000000
235957	1001000001000000000	236841	1000000001001000000
238656	1000100000000000100	241157	1000000000100001000
241799	1000000000011000000	241806	1100000010000000000
241823	1000000000000010010	243745	1000000100000000010
247784	1000000001100000000	251978	1000000001000100000
252517	1000001000001000000	255749	1000100000000000010
257330	1001001000000000000	262122	1000000000010001000
262129	1000100000010000000	262136	1000000000010000000
262143	1000201		

2000047 RECIPROCAL 3440001 PERIOD 524287
TAPS 00 01 02 05 19 05

70	10000000000001001000	1185	10000000000011000000
1276	10000100001000000000	1766	10000000010100000000
2650	10100000001000000000	4287	10000000010010000000
4907	10000100100000000000	5300	10001000000000000010
5301	10100000000000000000	5375	10000000100000000100
7390	10000000010001000000	9814	10000000001000001000
10602	10001000000000000000	13298	10000000000100000000
21204	10000000100000000000	22127	10000000000000000010
22128	10100100000000000000	25511	11000000000001000000
26797	10100001000000000000	29863	10000000000001100000
29869	10010100000000000000	37129	10010000100000000000
42408	10000000000000001000	43876	11000000000100000000
44256	10001000001000000000	48294	10000000000000010100
53594	10001000000000100000	56250	10001000000100000000
56845	10000100000000000010	59738	10000010001000000000
62097	10010000000010000000	62402	10000010000100000000
68958	10000100000000000100	70396	10000000000000100100
74258	10000010000000001000	75862	10000000000100100000
77167	10000001000000000100	81169	10001000000000000100
83211	11000010000000000000	91532	10000001100000000000
96403	10000000000000010010	96964	10000000011000000000
100259	10000000100000001000	101824	10001000010000000000

103907	10000000000110000000
106845	10000000000000001010
108391	10100000001000000000
121178	10000100010000000000
124856	10000100000001000000
132619	10000000001010000000
144880	10000001001000000000
150558	10000001000001000000
158784	11000000000000001000
166422	10100000000010000000
173100	10000010000000010000
183064	10000000000000101000
188279	10000100000000001000
190402	10000000000100010000
191408	10000010000001000000
197113	10000000100000100000
203648	10000000100000000010
209135	10000000000000100000
209884	10000001010000000000
214398	10001010000000000000
219991	10010000000001000000
231262	10000000000001010000
240788	10000100000001000000
253892	10000001010000000000
256744	10000011000000000000
258559	10000000000010000010
264792	10010000000000000100
267584	10000000000001000000
270092	10100000000000001000
275394	10100000000000000100
278582	10000001000010000000
280218	10001000000001000000
300666	10000000000001000010
315566	10000000110000000000
316339	11000100000000000000
318670	10000000010000100000
322875	10000000000010000000
323560	10010010000000000000
328453	10000110000000000000
340664	10000100000000000000
345964	10000010000000000010
360700	10001001000000000000
364453	10000001000000000010
367460	10000000000100000010
369734	11000000000000000100
375032	10100000000000100000
384864	10000000010000000010
391795	10010000000100000000
397191	10010000001000000000
407299	10100000000100000000
408410	10000000000001000100
418280	10101000000000000000
419768	10000000000010001000
422962	10100000100000000000
423873	10000000010000000100
450191	10000000000000000110
461852	10010001000000000000

104942	10011000000000000000
107199	10110000000000000000
113607	10000001000000010000
122833	10000010000010000000
125499	10000000000100000100
135046	11000000100000000000
147341	10000001000000100000
157041	10000000001000000000
162497	11000000000000100000
172982	10010000010000000000
178635	10000000010000010000
187516	11000001000000000000
188647	11000000000100000000
190696	10010000000000010000
192921	10000000100001000000
198348	10000101000000000000
203858	10001100000000000000
209140	11100000000000000000
211481	11001000000000000000
219738	10100000000000010000
222081	10000010000000000100
239617	11000000001000000000
242356	10000000001000000010
254552	11000000000010000000
256798	10000000100100000000
263725	10000100000000010000
264794	11000000000000000000
268836	10000000000001000100
273207	10000000010000000000
276082	10000001000100000000
279053	10010000000000100000
286919	10010000000000001000
312273	10001000100000000000
316337	10000000000000000100
316415	10000000000101000000
321637	10001000000000001000
323555	10000000000000110000
328450	10000000000000001100
340663	11000000000000000010
341940	10000000000000010000
351505	10000000100000010000
362642	10000001000000001000
366711	10000001000000000000
368211	10000000010000001000
371989	11000000000000010000
382589	10100000000001000000
391423	10000010010000000000
396696	10000000001000100000
399417	10000010000000100000
407716	10000000101000000000
411976	10001000000000010000
419015	10000000001000010000
420617	10000000000000011000
423581	10000010000000000000
428796	10000000100010000000
455889	10000000000100001000
464457	10000000001001000000

465478	10000100000000100000	473934	10010000000000000000
476274	10010000000000000010	479234	10100000000000000010
486029	10000000001000000100	496542	10000000000010010000
503159	10001000000010000000	507784	10000000000000100010
512825	10000100000100000000	513488	10000000000010100000
513723	10100010000000000000	519005	11010000000000000000
521197	10000000001100000000		
524287	2000047		

4000011 RECIPROCAL 4400001 PERIOD 1048575

TAPS 00 03 20 03

37	100 10000000000000 1000	40	100000 1000000000 00000
54	100 1000000000 1000000	77	100000000 1000000 01000
80	100 000000000 100000000	88	100 10000 10000000 00000
160	1000100 1000000000000	176	100000 1000000000 10000
320	100 00000 100000 1000000	349	10000000000 10000 1000
352	1000000000000000 100000	357	100 10 100000000000 00000
704	100 0000000 100 10000000	714	100000 1000 10000 00000
1396	1000000000 100000 1000	1399	100000000000000 1000000
1405	100 100 10000000000000	1422	100000 100000000 01000
1425	10000000 10000000000	1436	100 1000000 10000 00000
2798	1000000 100 1000000000	2810	100000 100000 1000 00000
2847	10000000000000 101000	2850	10000000000000000 00100
2852	101 10000000000000000	5700	10000000000000000 10010
5704	100010 10000000000000	11408	1000000 1000 1000 00000
21130	100 00000000 1000 100000	28390	100000000000 10000010
45937	11000000000000 10000000	55360	10000 100000 1000 00000
55403	1000000 110000000000	57551	10000000000 1100 00000
58165	1000000 10000000 100000	98131	1000 110000000000 00000
106007	110000000 10000000000	110806	100000000000000 10100
113662	10000100 1000000000000	113677	100000 10000000 100000
114008	100000000 1000000000 10	122880	10 10000 10000000 00000
136280	1000000 10000000000 100	154366	1000000 100000 100 00000
155061	1000000 1000000000 100	191091	1000000000 1000 00010
191917	10001000000 100000000	191997	1000000 100000000 10000
196262	1000000 10 10000000000	200607	1000 1000 10000000 00000
212011	101000000000000000 10	212012	11000000000000000 00000
212031	100 10000000000000 10	220014	10 10000000 1000 00000
221437	1100000 1000000000000	227324	100000000 100000 10000
228013	1000000000000000 1100	228016	1100 1000000000 00000
228035	100000 1000000000000 10	232651	100000000000000 100010
242713	1000000 110000000000	245760	1000 1000000000 1000000
246464	1000000000000 1000 1000	246467	100000000000000 10000
246471	100 1100000000000000000	246621	1000000000 1000000 100
262495	10000 10000 100000000000	269242	10000 1000000000 1000000
273440	1100000 100000000000	290560	10000 10 100000000 00000
300907	1000000000 1000000 100	310119	100000000000000 11000
310122	100 00000000000000 10	310123	1 10 10000000000 00000
357281	101000000 10000000000	384303	1000 1000000 10000 00000
385026	1000000000 10000 100000	392518	1000000000 100000 01000
392521	1000000000000 10000000	392528	100 1000 100000000 00000
393216	1100000000 1000000000	401214	1000000 10000000 10000
417878	1000000 10000000000 10	421717	1000000000 100000 00010
424022	110000000000000000 100	42402 4	10 10000000000000 00000

424042	100 1000000000000000 100	424379	10000 10000000000000 0100
426872	110 00000000000000 10000	442874	10 10000000000 10 00000
450047	1000 1000000000000 100000	454645	100000000 100 100000000
454656	100 000 10000 1000000000	454699	10000000000000 1100000
456032	10 100000 1000000000000	458402	1000000 10000 10000 0000
458482	1000 100000000000000 10	485426	10000000000000 10 10000
491517	100 00000 100000000 1000	491520	1000000000 100000 0000
491529	100 100000 10000000000	491563	100000000 10000 10 00000
492934	100 0000000000 100 100000	492942	100000 10 1000000000 0000
493239	100 0000000000 100 10000	493246	100000 11000000000 00000
502547	10000000000000 110000	521509	10000000000 10000 0100
522133	10 100000000000000 100	524286	1000000 100000000 01000
524289	100000000 10000000000	524299	100 1000000 10000 000000
524465	100000000 11000000000	524984	1000 10000000000 001000
524987	1000000 1000000000000	525000	100 1000000000 10 000000
534828	1100000000000 1000000	538132	1000000000000 10 100000
546880	10 10000000000000 10000	551985	100000000 1000000 10000
552132	10000000000 10 10000000	562879	10000000000 100000 10000
574418	10000000000000000 110	577265	1000000000000 110 00000
581120	100000000 1000 1000000	591953	1000000000000 10000100
601818	1000 10000 1000000000 00	620246	10 1000 1000000000 00000
624591	10 10 100000000000000 00	630295	10 10000000 10000000000
636035	110000000000000000 10	638297	100000000 10 100000000
638887	10000 100000000000000 10	647518	10000 10000000000 01000
647521	10000000 100000000000	647533	100 100000000 100 000000
647598	10000 1000 10000000000	648949	10000000000 1100000000
658926	100000000 1000 10000000	679350	100000000 1100000 0000
702927	10000000000 10000000 10	711534	10000 10000000 10 000000
714562	1000 100000000000000 100	720548	10000 100000000 100000
748983	100000000 1000000 100000	751604	1000000 1000000 10 00000
751766	10 100000000000 10000000	760828	10 1000000000000 100000
763403	1000 100000000 10000000	768624	100000000 10000000 0100
770049	1000000 100 10000000000	770062	100000 100000 10 00000
785033	10000000000 100 1000000	785042	100000 100 10000000 0000
785056	100000 10000000 1000000	785093	100000000 10 1000 000000
785354	11000000 10000000000000	786429	10 10000000000000 01000
786432	10000 100000000000000	786447	100 10000000000 100000
786515	10000000 1000000 100000	786781	10 1000000000 100 00000
800272	10000 1000000 100000000	836583	111000000000000 00000
839435	11000 10000000000000000	843431	10000000000000 100100
843436	10000 110000000000000	844454	10000000000000 100010
848045	1100000000000000 1000	848048	1000 1000000000000 00000
848064	100 1000000000000 10000	884537	1000000000000 1000100
898942	1000000 100000000000 10	912064	1000 10000000000 10000
917505	1000 100000 10000000000	917678	110000 10000000000 0000
923472	1000000 1000 1000000000	946154	11000000000000 100000
962159	10000 100000000000 10000	966556	1000000 10 100000 00000
983037	10000000000000000 10 10	983040	10 100 100000000000 00000
983058	100000 100000000000 100	985884	10000000000 100 10000
986492	10000000000 10 1000000	987863	11000000000 100000000
997289	10000000 10000 10000000	1048569	1000000000000 1001000
1048572	10000000000000000 1000		
1048575	400011		

TRANSLATES OF MAXIMAL PERIOD SEQUENCES

LINEAR GENERATOR MAXIMUM REGISTER LENGTH 35

4400001 RECIPROCAL 4000011 PERIOD 1048575

TAPS 00 17 20 03

23	100 1000000000000 1000	51299	10000 1000000 100 00000
60724	1000000000 1100000000	62097	10 100000000000 10 00000
62707	100 0100000000000 10000	65535	10000000000 1000 00100
65540	100 10 100000000000000	65557	10 1000000000000 00010
82028	101000000 10000000000	86432	1000000000 10000 10000
102436	1000000000000 1100000	125114	1000 100000 10000 00000
130903	10000 110000000000000	131080	100000 1000 100000 00000
136527	10000000000 1000 10000	149652	10000000000 1000 00010
164056	1000 1000000000000 100	200527	100000000000 100 10000
200531	10001000000000000000	200547	10000000000000 11000
204140	1000 10000000000000 10	205145	11000 1000000000 00000
205162	100 1000000000000 100	209145	1000 1100000000 00000
211994	11100000000000000000	248315	1000000 10000 100 00000
261806	100000000 10100000000	262075	1000000 1000000 100000
262143	10000000000 100 100000	262148	10000 1000000000 00000
262163	100000000000000 101000	263230	1000000 11000000 00000
263493	1010000000 1000000000	263533	1000000 100000 10 00000
263542	100 10000 100000000000	263556	100 1000000000 10 00000
278526	1000000 100000 1000000	278536	100 100000 100000 00000
279969	10000000 100000000 100	285185	1000000 1000 100 00000
287762	100000000000 10 100000	296822	100000000 10 100 00000
296985	1000000 100000 1000000	299607	100000 100000000 100000
328042	100000000 10000 100000	334031	1000000000000 10 00100
337054	1000000 10000 1000000	345667	1000000 100000000 00010
369236	1100000000 1000000000	389662	1000 1000000 100 00000
399638	110000000 100000000	400986	1000 10000 100000 00000
401054	10000000 100 100000000	401062	1000000 10000000 00000
401074	1000000000 10000 1000	409707	1000000000000 10 00010
410290	1010000000 1000000000	412559	100000000000000 00110
418290	1000000 10 10000000000	423988	10 10 1000000000 00000
428335	10001 010000000000000	446766	10000 1000 100000 00000
456640	10000 100000000000 100	467469	1000 100000000 10 00000
471324	110000000000 1000000	474176	110000000000000 00010
485712	10000 10000000000 10000	496456	10 1000000000 100 00000
496606	1000000 1000000 10000	501711	1000000000000 10 10000
510458	10100000000000 100000	513761	1000000000000 110 00000
523588	100000000 100 10000000	523595	1000000 10000000 00000
523608	10000000000000 1000 1000	524120	110000000 100000 00000
524286	1000000 100 10000000000	524296	100000000 100000 00000
524306	1000000000 1000000 1000	526460	100000000000000 10100
527084	100000 100000000000 100	546044	110000000000000 10000
555336	1100000 1000000000000	555352	100 10000000000 10000
555641	1010000 1000000000000	555656	100 1000000000 100000
557026	10000 100000000 1000000	557055	100000 100 100000 00000
557066	1000000000 1000000000	557075	10000000 100000 01000
563165	101000000000000 10000	590112	10000000000000 100010
590185	10000 1000000 10000000	592551	100000 10 10000000 00000

4400001

593891	110000000000000100000	593930	100001000001000000000
593942	100100000000100000000	598543	100000000001000100000
605715	100000000000101000000	621719	100000000000000110000
624214	100000000000010000100	624551	100000000000000100100
624553	101000000000000000000	624571	100000000000000001100
626877	100000000100000000010	630716	100000000001000000010
647377	100000001000000010000	655370	100000000011000000000
656054	100010010000000000000	656067	100000000000010000000
656074	100000010000000001000	663564	100001000000000100000
664283	100000010001000000000	691303	100000010100000000000
738455	101100000000000000000	738472	100000000000000000010
738473	110000000000000001000	747686	100000010000000000100
758022	101000010000000000000	775143	100000011000000000000
779347	100000000100001000000	786090	100001000010000000000
801972	100000001000000000100	802108	110010000000000000000
802124	100000000000000010000	802128	100010000000000001000
802829	100000000010001000000	805870	110000001000000000000
815943	100001000000000000010	820559	100000000000010000010
820563	100110000000000000000	820580	110000000000000000100
821267	100000100000000010000	827145	100000110000000000000
828572	100000000101000000000	836563	100000000000000010010
836564	110000000000000000000	836583	100000000000000001010
847976	100010001000000000000	852323	101000000010000000000
856594	100000000100000010000	856670	100000001000100000000
857503	100000001000000000010	893532	100000000010000000100
894222	100000100000010000000	912313	100000000001000000100
925702	100001010000000000000	934586	100000000010000000010
934913	100000000100000100000	934921	100100001000000000000
937787	101000000000000000100	942578	100000000110000000000
950449	110001000000000000000	990425	100000001000000100000
991037	110000000000001000000	993181	110000000100000000000
993226	100000100001000000000	1002651	100000000000110000000
1020204	100000100000000000010	1027460	100010000000000100000
1037179	100010000000100000000	1042877	101000100000000000000
1042894	100100000000000000010	1045726	110100000000000000000
1045743	100000000000000000100	1045745	101000000000000001000
1045777	100000100000100000000	1045788	100100000000100000000
1047150	100000001001000000000	1047159	100000000100000000000
1047170	100000000001000001000	1047176	100100100000000000000
1047190	100000000000001000000	1047196	100000100000000001000
1047871	100010000010000000000	1047884	100100000000010000000
1048223	101001000000000000000	1048238	100000000000000100000
1048243	100001000000000001000	1048269	100000100000001000000
1048415	100000000010000010000	1048422	100100010000000000000
1048495	100001001000000000000	1048507	100000000000100000000
1048515	100000001000000001000	1048535	100000000000100100000
1048541	100000100000000000000	1048555	100000000000001001000
1048558	100100000000000000000		
1048575	4400001		

OPTION 2 — THREE ADDERS OR LESS PER TRANSLATE

1000201 RECIPROCAL 1004001 PERIOD 262143
TAPS 00 07 18 03

OP2

29	1000000100010000000	36	1000000000000010000
40	1000100100000000000	51	1000000100010001000
58	1000100000010010000	65	1001000000110000000
72	1000000000100000010	73	1100000100010000000
80	1000000010000010000	84	1000100100001000000
160	1000000000100000110	253	1100110000000000000
300	1100000000000010100	506	1010000010100000000
1722	1000000001100001000	3430	1000001000010000100
3437	1000010000001100000	3971	1100000000000001010
4274	1001010000000000010	4820	1001000001001000000
5000	1001000000000001100	5414	1100001001000000000
5653	1000100010000000010	6679	1100000000001000010
6685	1000011000001000000	7286	1010110000000000000
8819	1000000000001101000	9626	1000001000010000010
9627	1100000000001000000	9633	1000001000000000000
9645	1000000100001000000	9651	1000001100000100000
9807	1100001000000001000	10821	1010010000000100000
10826	1000010000100000000	10834	1000000110000100000
11299	1100000000010000100	11306	1000010010001000000
11323	1000100000000010010	12530	1000000001001000100
12543	1000100000000110000	12973	1100101000000000000
13156	1001000000011000000	13163	1100000010100000000
13629	1001100000001000000	14572	1000100010100000000
16311	1000110000000000100	16313	1010001000000000000
16325	1000000100001010000	16990	1000010001000010000
18177	1101001000000000000	19042	1000111000000000000
19254	1010001000000100000	19259	1000010000010000000
19266	1000000000001000000	19272	1000001100000000000
19283	1000000100010000010	19290	1000001000000110000
19422	1001000010001000000	19612	1000010000000000110
19614	1110000010000000000	20093	1001000000000000110
20222	1000011000000010000	20466	1000010000000100010
21645	1001000000010100000	21652	1010000001100000000
21803	1001010010000000000	22507	1010000000001001000
23906	1000100000000100010	25036	1100100000000001000
25039	1001100000000000000	25053	1000000100000010010
25086	1000000000100001010	25594	1001000001000000010
25946	1010000010001000000	26305	1000001000010001000
26312	1000100000010100000	26319	1010000001010000000
26326	1000000001000000100	26328	1010000100010000000
26335	1000000001000010000	26339	1000100100000100000
26887	1000010010000000010	27251	1100001000010000000
27258	1000000010000100000	27263	1000010100000100000
27554	1011000000000001000	28892	1000011000000000010
28893	1100001000000000000	28905	1000000100001100000
29835	1001010001000000000	30680	1000000010000000110
31132	1011010000000000000	32322	1001000000000001010
32622	1001000010000010000	32626	1000100000001000000
32632	1000001100100000000	32686	1000000001000100100
33963	1000100010000001000	33966	1001000000010000000
33973	1000000000100000000	33981	1000000110000000000

1000201

33991	10000001001000000010	33999	10000000100000011000
34045	10100000000000000110	34472	11100100000000000000
36354	10100010000001000000	38084	10000000101010000000
38511	10010000000010001000	38518	10001000001100000000
38525	10000000000010000010	38526	11000001000010000000
38532	10000010000000100000	38537	10000101000100000000
38544	10000000000001010000	38548	10001001000000000100
38837	10000010010100000000	38844	10000000000000100100
38846	10100001000000001000	39228	10101000000000000100
39721	10110000000001000000	40437	10000100001100000000
40444	10000000000001000010	40445	11000001000001000000
42466	10001000010000000010	42551	10000000011001000000
43297	10001000000000101000	43300	10010000000000000100
43302	10100101000000000000	43606	10000010001000000100
44076	11000000000000100100	45980	11100000000000100000
45985	10000110000000000000	45997	10000001000010000010
47129	10001000000001100000	47355	10010100000000000100
47357	10100100000000000000	47370	10000001000000101000
47814	10100000000000010010	49056	10100110000000000000
50072	11100000000001000000	50078	10000010100000000000
50088	10000001001000000100	50189	11000000000000010010
50727	10000000011000000100	51135	10010000100000001000
51145	10000000001001100000	51182	11000000001000000010
51190	10000000110000100000	52331	11000000000000100010
52443	11100000001000000000	52603	10000000000110001000
52617	10000000100100001000	52624	10001000000010001000
52627	10010000000000010000	52631	10001000000000000000
52645	10000001000000010000	52649	10001001000100000000
52656	10000000000100100000	52660	10001001000000001000
52663	10010000000100000000	52671	10000001100100000000
52678	10000000000000011000	52681	10010001000000000010
52877	10100000000011000000	52884	11000000110000000000
53767	10010000000100000100	53774	10000100001010000000
54128	10100000000000001010	54516	10000000100000001100
54643	10000010011000000000	55604	10000000010100001000
55611	10001000000100000100	55618	10000100000110000000
55625	11000000100010000000	55631	10000010000000010000
55635	10001001001000000000	55675	10000000101000000010
55683	10000000100000010100	56144	10100000000100000010
56151	10000010010001000000	56167	10001000000000010100
56342	10000000110100000000	56349	10000000000000001100
56351	10100001000000000010	57378	11100000000000001000
57784	10000100001000000100	57786	10100000000001000000
57792	10000011100000000000	58181	10000000000100100100
59663	10010010000000000010	59664	11001000000000000000
59678	10000001000000011000	59703	10001000001000000010
59711	10000000100010100000	60049	10011000100000000000
60210	10100100000000000010	61337	10000100000000100100
61339	10100000000000001000	61342	10010101000000000000
62264	10001010001000000000	62766	10010100000001000000
62779	10000000000000110100	63544	10000000100000100100
63946	10000000010001000010	64737	10000010001000001000
64926	10000110100000000000	65027	10001000010010000000
65166	10000100010000000010	65245	11000000000100000010
65252	10000010100001000000	65347	11010000000000010000
65351	10001100000000000000	65364	10000001000000100010
65534	11100000000100000000	65541	10000000110000000000
65550	10000001010000000010	67193	10100000010000001000

1000201

67203	1000000000101010000
67932	1000101000010000000
67944	1000010100000000100
67955	1000000101010000000
67964	1010000100000000100
69781	1000010000000001010
71897	1000100000001000100
73709	1100000000100010000
74030	1100000001000000000
76180	1001000000001000010
77089	1100000000010010000
77518	1101000000000001000
77667	1000010000000001000
78697	1100000001100000000
79438	1001000001000010000
79447	1000010101000000000
80564	1000000000000011010
83133	1000000000001010010
84393	1000001010010000000
84403	1001000100000000100
84934	1000000001100010000
85095	1100000010000100000
85405	1000000001001000010
85683	1100000010000000100
86423	1100000000000000100
86604	1000100000100010000
86850	1011100000000000000
88475	1011000000000010000
88491	1000000100001000100
91766	1000000001010000010
91963	1001010000010000000
91976	1000001100000000100
94251	1000000010010000010
94710	1001001000000010000
94722	1000000110001000000
95680	1100011000000000000
96454	1000000000000111000
97029	1000100001000000100
97785	1000001000000001100
99405	1010000010000000100
100149	1000010001010000000
100158	1010000100000010000
100456	1001000000000101000
101434	1100010000000000000
102366	1010000000010010000
102552	1101000000100000000
103051	1000001000001000100
104133	1000000010100100000
104882	1001010000000010000
104895	1000000101000100000
105254	1000001000100000010
105262	1000000010000000000
105280	1000000110000001000
105320	1100000000001010000
105740	1000000000110001000
105754	1000001000010100000
105766	1000010000000010000
106506	1100000000000001100
67926	1100000000001000100
67939	1000000000010100000
67946	1010000000100000000
67962	1000000000000010100
68944	1010100000100000000
70555	1001110000000000000
72471	1110000000000000100
74029	1000001010000000010
74039	1000000101100000000
77064	1000000010001001000
77096	1001000010100000000
77662	1010000000100100000
77670	1001000110000000000
78849	1001001000001000000
79442	1000100000000100000
80549	1011000000100000000
83121	1000011000000100000
83510	1101100000000000000
84400	1000000000000101000
84925	1100000000100000100
85088	1000001000011000000
85257	1000000010001000100
85676	1000000000101100000
86422	1000001000000001010
86425	1011000100000000000
86641	1000010010000010000
87434	1001100000000000100
88479	1000101000000000000
90387	1010001000000001000
91773	1000001000000100100
91970	1000000000101000000
94136	1001100001000000000
94258	1000001000000101000
94714	1000100000100000000
95594	1000011001000000000
96441	1010010000001000000
96563	1001010000000100000
97038	1000000001000110000
98112	1000100000101000000
99513	1000001011000000000
100156	1000000000001000100
100169	1000000000010110000
101433	1000101000000000010
101447	1000000100000110000
102373	1001000001100000000
102567	1000000000000011100
103597	1010000000100001000
104347	1000101000000000100
104886	1000100001000000000
105009	1111000000000000000
105255	1100000000010000000
105272	1000000100100000000
105290	1000000000100010010
105326	1010001001000000000
105747	1000100000010000010
105761	1010000001000100000
105770	1000100101000000000
107163	1001000000000100100

1000201

108010	1000000000000001110
109110	1100010001000000000
109279	1000010000000100000
109473	1000010010100000000
110586	1001000001000001000
111201	1000010000001001000
111343	1000000000010100100
111889	1010010010000000000
112295	1100010010000000000
112677	1000000001000001000
113603	1011000000000000010
114389	1010100000000000000
114826	1100000001000001000
115115	1010000000001000010
115584	1000000000001010100
116706	1000100001000001000
116715	1000001101000000000
119326	1000011000000000100
119338	1000000100101000000
120098	1000001010000000100
120219	1000001000100000000
120897	1100000001001000000
120906	1001000101000000000
122933	1110100000000000000
123892	1100010000100000000
125532	1000000000100100000
125995	1000001001000000100
126907	1100000000100100000
128667	1010010001000000000
129006	1100000001000010000
129852	1000000000101000100
130047	1100000000000000010
130325	1001000001000000100
130695	1101000000010000000
130710	1000000110000000100
131068	1010000000010000000
131084	1000000101000000000
131102	1000000001000010100
133426	1000010000000101000
135881	1001000000010010000
135888	1001000000000000000
135906	1001000100100000000
135928	1001100000100000000
136590	1001000010000000100
137341	1100000000100000100
138445	1000001000001010000
139228	1101000000000000000
139698	1000010000000010010
140708	1000001001000001000
143589	1001000000000010100
144231	1010000000110000000
144239	1100000100100000000
146484	1000000000101001000
147345	1000000010100010000
147426	1000000010000010010
148053	1010000000000100000
148071	1000000100000100000
152362	1010000000000011000
108790	1000000010001000010
109274	1010000010000100000
109284	1000010100100000000
109702	1000001001001000000
111194	1000000010010000100
111262	1000000000101000010
111635	1000010001000100000
112290	1010000000000110000
112670	1010000010010000000
112680	1001000100001000000
114387	1010010000000000100
114403	1000000100000010100
114895	1000001010000010000
115572	1000101000000100000
116342	1100001000000000100
116709	1001000000001000000
117982	1000010011000000000
119328	1010000010000000000
119508	1010010000000001000
120213	1100100000001000000
120227	1000000110000010000
120903	1000001000000001000
122684	1000001000100010000
123551	1000000010100000100
125525	1001001000010000000
125537	1000010100000001000
126004	1000000001000011000
128662	1010000000000101000
128961	1010001000000010000
129467	1000000000011100000
130046	1000001000000000110
130048	1110000100000000000
130334	1000000001001010000
130702	1000000010100000000
131066	1000010001000000100
131075	1000000001000000000
131093	1000000101000000100
132073	1000000010000100010
134414	1001000010000000010
135885	1000100000000001000
135903	1000000100000001000
135924	1001000000000010010
136138	1010000000000001100
136731	1001000000001000100
137653	1000011000000001000
139227	1010001000000000010
139243	1000000100000001100
140390	1000011000100000000
141110	1000001010100000000
143593	1010100001000000000
144238	1000000001000000010
145521	1001001001000000000
147337	1010000000100000010
147418	1000000001100000010
148051	1000010000010000100
148058	1000010000000000000
148076	1000010100001000000
152931	1010010000100000000

1000201

154065	1100100001000000000
154767	1000000000000010110
155340	1000001000000010100
156641	1010000001000001000
157385	1000010000000000100
157403	1000000100000000100
157423	1010000000000010100
157691	1000011000010000000
157703	1000010100000000010
157918	1100100000000010000
158870	1100100000010000000
158884	1000000000000001000
158898	1000000100010010000
158909	1000100000000010000
158941	1001001000000100000
159147	1100000000101000000
159155	1010000110000000000
160900	1000000000010101000
160921	1000010000001000100
160927	1000101100000000000
161483	1101000000000100000
162458	1000010001000001000
163842	1100100000100000000
164266	1000110000000000010
164668	1100000000000100000
165215	1010000000000100100
167020	1010001010000000000
167324	1000010000001000010
168800	1100000000001001000
169841	1000000000011000100
169861	1001000000100100000
171338	1000000010110000000
171346	1100000100000000100
172850	1000101000000010000
173275	1000000000111000000
173283	1100000100000000010
173913	1000001010000001000
174286	1001000001010000000
174295	1010000100001000000
175311	1011000000000000000
176653	1010000000001010000
176958	1000000010001000000
176975	1000000000010010010
179285	1000000000111000000
180770	1001000000000011000
180799	1000100010000010000
181310	1000000000001000110
182031	1000010000100001000
182351	1000100001000010000
183245	1011000010000000000
183706	1000100001000100000
183945	1000010000010010000
184383	1010001000000000100
187219	1000000010000001010
188265	1100001000000000010
188282	1000000100000000110
188515	1000001000000010010
188519	1001100100000000000
154751	1110000000100000000
155154	1001000000001001000
156634	1000000010010100000
157380	1010000000010100000
157387	1010000000000000000
157405	1010000101000000000
157517	1000000011000100000
157698	1000000000001100000
157893	1000100010001000000
158146	1100000000000000110
158877	1000000010010000000
158887	1001000100000000000
158905	1001000000100010000
158913	1000100110000000000
158953	1000000000001011000
159153	1000001000000000100
159246	1000100010000100000
160914	1000000001010000100
160923	1010000000000010000
161058	1001000000000100010
161741	1100100000000100000
163748	1101000001000000000
164170	1000010000100100000
164667	1000001000001000010
164673	1000011100000000000
165220	1001010000100000000
167317	1000000000110000100
168090	1000100011000000000
168806	1001001000100000000
169854	1000001000010010000
170803	1000000001000000110
171345	1000000000000001010
172633	1100000001000100000
172982	1100000000000101000
173282	1000000000000000110
173700	1000101010000000000
173923	1000000000100010100
174293	1000000000100000100
175309	1100010000000000100
175326	1000000100000001010
176951	1100010000010000000
176964	1000001100000010000
177452	1000000000100101000
179292	1100000010000000010
180774	1100100010000000000
181040	1100000000000110000
181792	1000101001000000000
182257	1000000001000001010
182597	1001001010000000000
183525	1000000001000001100
183933	1000000000010100010
183952	1001000000101000000
186375	1000000000001001100
187872	1000000000000101010
188266	1110000000000000000
188495	1000000000010001100
188516	1100000000000001000
188544	1001000000010000010

1000201

188551	10000010001001000000	189421	11000000000101000000
189428	10100000110000000000	191178	10101000000000001000
191181	10010100000000000000	191194	10000001000000100100
191360	10100000001010000000	191523	10011000000000100000
192441	10000010000000011000	192847	10001100100000000000
194043	10000010001100000000	194050	10000000000000100010
194051	11000001000000010000	194062	10000000000110100000
195406	11000000010100000000	195413	10000000100000000100
195415	10100001001000000000	195471	10100000100000010000
195549	10000100000000001100	195551	10100000000000000010
195552	11010001000000000000	195931	10100000000001000100
196608	10000010010000000010	196609	11000000001000000000
196617	10000001110000000000	197795	10000000000000100110
197897	10001000000000000110	198890	11010000000000000100
199014	10101000000000100000	199019	10000100010000000000
199028	10000001010000010000	199752	10000000001000001100
200288	10001000000001001000	200291	10010000000000001000
200294	10010011000000000000	200348	10110000000000000100
200689	10001100010000000000	201832	10000000000001001010
201907	10000010100010000000	202866	10000100100000000100
202868	10100000001000000000	202876	10000001101000000000
203194	10000000110010000000	204672	10000100000010001000
204679	10001000000110000000	204686	11000000100100000000
204693	10000000100000001000	204696	10010001000100000000
204703	10000000001000100000	204707	10001001000000010000
204718	10000000000010011000	204732	10000000001100000010
204739	10000010000000100010	204740	11000000000000010000
204744	10001101000000000000	205098	11010000000001000000
205104	10000010010000000000	205113	10000001010000001000
207465	10100010001000000000	208450	10010000010001000000
209392	10001000001001000000	209758	11000000000110000000
209764	10000010000000000010	209765	11000000000000000000
209782	10000001000000000010	209783	11000001100000000000
209801	11000000000000011000	210018	10101010000000000000
210510	10101000000010000000	210517	10000000010100000000
210524	10000000000000000100	210526	10100001000000000000
210537	10000001000101000000	210544	10100000001000010000
210645	10001100000000001000	210648	10010000100000000000
210658	10000001001001000000	211290	11100000000000000010
211518	10011000000010000000	211525	10000000001100000000
211532	10000000000000000010	211533	11000001000000000000
211544	10000001000110000000	211551	11000000100000010000
211877	10000000000001100010	212311	10000000000000101100
213409	10001000011000000000	213685	10000010010000010000
214737	10000000001001001000	215161	10000000001001000010
215173	10001000000001010000	215482	10001000000000100100
215999	10000000001000101000	217156	10010100000000001000
217573	10000000000010000110	218025	10001000001000001000
218028	10010000000000100000	218033	10000101100000000000
218213	10110000000000100000	218218	10000100100000000000
218228	10000001001000001000	218558	10000000101000001000
218726	10001000000000001010	218729	10110000010000000000
219392	10100000000001100000	219397	10000100000000000010
219398	11000011000000000000	220721	10000000010011000000
221166	11000010000000100000	221178	10000000000001110000
221448	10000000000010001010	221467	10000100000000010100
221469	10100000000000000100	221471	10101001000000000000
222246	11000000010000000010	222665	11000100000000001000

1000201

223260	10001000000000001100
223778	10001000000100000100
224554	10000000000000110010
224681	10000100000000110000
225333	10001000010100000000
225342	10100001000000100000
225353	10000001100010000000
225364	10001001000000000010
227289	11000000000100001000
228477	10010000100001000000
228778	10001000100000000000
228803	10001000000000011000
228807	11001001000000000000
229031	11000000000001100000
229379	10010000110000000000
229650	10000100001000000010
230589	10000000110000000100
230791	10001100001000000000
231144	10000000000001001000
231172	10011000000000000010
233148	11100000000000010000
233411	10000010000100000000
233423	10000101000000000000
233441	10000100000001010000
234974	11000100000000000010
235957	10010000001000000000
236337	10000000110000000010
236841	10000000010010000000
237906	10000000000110100000
238652	10010000000001010000
238658	10100011000000000000
240089	10000100011000000000
240438	10100000010010000000
241157	10000000001000001000
241787	10100000000000100010
241799	10000000000011000000
241806	11000000100000000000
241823	10000000000000010010
241864	10000100000000011000
242891	10000010000001100000
243201	10101000000001000000
243738	11000000000110000000
243746	11000000101000000000
244614	11000001000000010000
245478	10000100010010000000
245866	10101000100000000000
247701	10001010000001000000
247784	10000000011000000000
249357	10100000000100010000
250028	10000100001000010000
251650	10001100000000100000
251971	10100010000100000000
251983	10000101000000010000
252511	11000001000001000000
252523	10000001100001000000
252672	11100010000000000000
253320	10000100100001000000
253814	10000000110000001000

223263	11010000100000000000
224541	10001100000001000000
224590	10100000000100000100
225326	10100000000010001000
225340	10000000000010000100
225347	10000100000001000000
225360	10000000000000110000
226331	10000001000101000000
228100	10000000001101000000
228774	10011000000000010000
228788	10000001001000100000
228806	10010000000000000010
228847	11000100000000010000
229037	11000001010000000000
229589	10001000100000000100
229658	10000000100001100000
230598	10000000010000010010
231137	10000100100100000000
231147	10010001000000001000
232025	10011010000000000000
233405	11000100000001000000
233418	10000000000000100000
233434	10000001000100000100
234804	11010100000000000000
235954	10001010000000001000
235966	10000001010010000000
236834	10100100000010000000
236847	10000011000000001000
237913	10100000001000000010
238656	10001000000000000100
239660	10000000000001100100
240431	10000100000010100000
241150	10010000100100000000
241160	10010001000001000000
241792	10001100000100000000
241805	10000011000000000010
241816	10000001001100000000
241824	11000001000000001000
242884	10000100000100000010
243044	10000000000011000010
243342	10100000100000000010
243745	10000000100000000010
244237	10010110000000000000
245471	10100000000010000100
245590	10010010000000001000
247298	11000100000000100000
247777	10110000000010000000
247792	10000001100000000010
249492	11000010001000000000
251433	10000000100011000000
251879	11001000000000000010
251978	10000000010001000000
251994	10000000000010010100
252517	10000010000001000000
252597	10000010000001001000
252944	10001100000000010000
253616	11010000000000000010
254140	10010010000000000100

1000201

254922	1011001000000000000	255745	1001000000000110000
255749	1000100000000000010	255750	1100010100000000000
257064	1000000011100000000	257327	1001100000000001000
257330	1001001000000000000	257342	1000000100001001000
257531	1000000010000101000	258010	1000010010000001000
258020	1000000000100011000	258198	1000000000101100000
258946	1010010000000010000	259506	1010100000000000010
259684	1100100000000000100	259837	1001000000001100000
260096	1010100000000010000	260637	1000000010011000000
260644	1100000010000001000	260654	1000000000110010000
260661	1001000000100000010	260669	1000000010010010000
260676	1001000000100001000	261321	1000100000001000010
261397	1000000001010100000	261404	1010000000100000010
261413	1000000001010010000	261420	1001000000100000100
262108	1100000000010001000	262115	1000100010010000000
262122	1000000000010001000	262125	1001000100000010000
262129	1000100000010000000	262136	1000000000010000000

262143 1000201

OPTION 3 — CHECK POINTS

440001 RECIPROCAL 400011 PERIOD 131071
TAPS 00 14 17 03

OP1 113358 100000000000000010 3

113378	111110010010011100	113390	111110111110110000
113410	110110001111001010	113427	101011101100010000
113450	100101101000010110	113474	100101100010001110
113492	100010000110110110	113513	101100010011010010
113542	100000101001001100	113561	111010010011000100
113575	111101111100100000	113596	110110101011101110
113613	101011101000010100	113631	110100001111010110
113650	110001100100110100	113668	110101101011000110
113684	111010001011100000	113702	101110011011111010
113718	100111011101011110	113738	111000100011001010
113759	111001110010001110	113772	100111011111110100
113790	110100111010100110	113807	101011111000011100
113823	100110001111110010	113846	111001000100011100
113861	101001101111111100	113883	100111010000010000
113905	100110111110001100	113927	100001100101001110
113940	111100101111111110	113954	111011110001110000
113972	110101000111110100	113993	110011000010111000
114013	100000110101100110	114030	101001001111011100
114056	100000011101001000	114072	111111111000011100
114086	111000111111100000	114107	101101110010000100
114134	101100001000011110	114153	101010001001010000
114185	100000001000100100	114207	111111010010011110
114220	101011111011110100	114242	110100001011101110
114262	100011101001001010	114282	101000100111011000
114314	110010010000000000	114341	111011011011001110
114355	100111100111110110	114374	110001000111111100
114392	101110110010000100	114413	100101111000010010
114433	111110010111001110	114453	100011100010001110
114469	100100011011110110	114492	111011000100001100
114511	111010100010010100	114531	111101010000101000
114548	101010111101100010	114564	111101000111101000
114587	110100000111011110	114606	110001100100100110
114623	101011010110011100	114641	110100001100110010
114672	100001000000101110	114688	100100110110110110
114719	100000000110111000	114741	100011010010011010
114760	111100100010111100	114781	100011000110101100
114800	100001001111111010	114818	111001001101011010
114838	110011100101101100	114859	101110101100011010
114879	110011110000100100	114903	100100101110110000
114925	101011010011011010	114946	101111001100000010

80 440001

TRANSLATES OF MAXIMAL PERIOD SEQUENCES
LINEAR GENERATOR MAXIMUM REGISTER LENGTH 35

10000005 RECIPROCAL 12000001 PERIOD 2097151
TAPS 00 02 21 03

OP1 6801 1000 100000000 10000 1000 3

6839	1000 10000 1000 1100 11100	6864	11100 100 10 100 10 1011110
6883	10110 11000 110000 100000	6909	1000 1100 1101110 1111000
6925	1011110 10 10 100 111000 10	6943	110 1000000 1110 11011110
6961	10 10000 110 10 110 1000000	6989	10110 10 1000 100 11100010
7009	110 10000 10 10 1110 110 110	7027	10 100 10000 10 10 11010000
7053	10 1100000 110 100 10000 10	7075	11000 11000 11100 1101000
7096	11110 11110 110 111100 10	7110	1110 110 1000 11110 110110
7126	1000 10 1100 10 110 100 10 10	7146	10 1100 111100 1100 110010
7164	10 11100 111111111111110	7174	1100000000 11110011110
7199	1111000000000000 1100 110	7224	1100 111111000000000110
7247	111100 110000 1100000000	7275	111 1000 1100 1111100110
7289	11111110000 111100 110000	7308	1100 1100 1100 1111110110
7321	111111 0000 10 10000 11000	7342	110000 1100 1000 10011110
7367	1111000000 111110 10 10 10	7386	11110000 11000 100000100
7415	1000 10 10 1110 1100 111100	7430	100 10 10 1111100 10000100
7453	10 100 10000 1000 1110 1000	7478	10 111100 110 100 10101100
7496	110 111100 110000 1100 10	7514	11000 111111100 111111000
7528	1100 11110000 1111000 110	7545	1100 1100 1100 110 1100000
7566	1111111111111110 111000	7579	1100000 10 10 111111111110
7589	1100000000000000000 10 100	7651	10 10 10 1000000000100010
7677	10000 100 10000 100000000	7714	100 1100 11000 1000101000
7740	10 10 1111111111111 10 10 100	7753	11000000 1000000 1011110
7787	1000 100 10 110 1000000000	7822	10 1111110 10000 10101010
7840	11000 100 100 10000000 1110	7871	1000 100 1110000 1011010
7891	10 110 11100 1100 100 110 10	7909	100 10 11111110 111101110
7921	110 10 100 10 100 10 10 11110	7939	1000 11000 110000 1000000
7970	11111000 10 10 10 11110 110	7987	10 10 1000000 100 10100110
8010	1110 1100 10000 10 11000 10	8032	111000 11110 100 10011100
8051	10 10 1100 100 110 1110 10 10	8068	111110 11110 10 100011100
8085	100 100 10000 10 110 10 1110	8105	1000 110 100 100 110001000
8129	100 1110 11100 110 111100	8143	110 111110 1000 100011100
8161	11000 100 10 10 10 110 10 100	8187	10 1000 10 10 10 1100000010
8209	1100 110 1000000 11100000	8234	11111111111100 100001100
8251	1100000 1110 100 1110 1110	8265	110 100 1110 10 1000000000
8298	10 100 10 10000 11100 11100	8320	10 110 11000 100 110111100
8338	100 1110 10 11110 100 100 10	8355	111000 100 100 110 1111010
8374	10 10 10 110 11110 100 10 100	8391	100 110 100 100 1100000100
8419	11110 10 100 11100 110 1110	8435	1000 1110 11111 10 10001100
8451	1110 1000 100 10 1110000 10	8473	1110000 10 10 1100 1011000
8494	1101100 10000 1111001110	8512	111110 100 1100 111100000

80 10000005

OPTION 4 — SOLUTION OF DIFFERENCE EQUATIONS

```
201241          RECIPROCAL  205201          PERIOD  65535
TAPS    00   05   07   09   16   05



OP4

     488  10000100000001000                 24784  1100000 1000000000
   28835  1000000000000110                  44612  101000000000 10000
   47137  1001100000000000                  58175  1001000001 1000000
   63155  1100001001000000                  63945  10101000000000010



266663          RECIPROCAL  315555          PERIOD  65535
TAPS    00   01   04   05   07   08   10   11   13   14   16   11



OP4

     273  1001100000000000                    768  1111000000000000
    1536  1010101000000000                   8653  10000001110000000
    9890  1110000001000000                  10249  1110000000100000
   20393  1110000000000100


10000005        RECIPROCAL  12000001        PERIOD  2097151
TAPS    00   02   21   03



OP4

     693  111000000 1000000000000            3806  110000000000 100000001 0
    7589  11000000000000000 10100            7757  110000000000 100000 0100
   28922  11000 1000000000000 1000



400000107       RECIPROCAL  704000001       PERIOD  67108863
TAPS    00   01   02   06   26   05



OP4

              52        10 10 10000000 100000000000000
             106        1100000000 1000000000000000
             218        11100000 1000000000000000000
             436        10 10 100000000000 10000000000
            3488        1000000000000000 100000 11000
```

OPTION 5 — SPECIFIED ADVANCE

10000005 RECIPROCAL 12000001 PERIOD 2097151
TAPS 00 02 21 03

ADVANCE 13570 + 40

13566	000000 1000 110 10 10 10 100	13568	10 100000 1000 110 1010100
13570	1000 100000 1000 110 10 100	13572	100000 100000 1000 110100
13574	10000000 100000 1000 1100	13576	1000000000 100000 100010
13577	11100000000 100000 10000	13581	10 10 111000000000 1000000
13587	101000 10 10 111000000000	13596	10 1000000 10 1000 1010110
13597	1111000000 10 1000 10 10 10	13598	110 1100000 10 100 010100
13600	100 10 11000000 10 1000 100	13602	10000 10 11000000 1010000
13606	10 10 10000 10 11000000 100	13608	1000 10 10000 10 110 000000
13615	10 10000 1000 10 10000 10 10		

251132516577 RECIPROCAL 376562532225 PERIOD 17179869183
TAPS 00 01 02 03 04 05 06 08 10 11 12 15 17 19 21 22
24 27 30 32 34 21

ADVANCE 610 + 100

578	0 1 10 1 1 10 1 1 10 10 100000000 100 1 100 1 1000
581	1 1 1 100 1 10 1 100 100000 10 1 10 10 1 10 100 1 10
582	10000 1 1 10000 10 1 10 10 1 1 10 1 1 100 1000 1 10
583	10 1 1 1 10 100 1 1 1 100 1 1 1 1 10000 1 1 10 1 10 1 10
584	10 10000000 100 1 1 100 10 10 10 10 10 100 1 1 10
585	10 10 1 1 10 10 10 10 10 1 10000 1 1 1 1000 1 100 10
586	10 10 100 1 1 1 10 1 10000 1 10 1 1 10 1 1 1000 1 100
588	1 10 10 100 1 10000 100 10 1 10 1 10 100 1 1 10 1 10
589	100 10 100 1 10 1 10000 1 1 1 10 1 100 1 10 10 1 1 10
590	10 1 10 100 1 10 10 10 10 1 10 10 1 10000 10000 10
591	10 100 100 1 10 100 1 1 1 1 1000 1 1000 10 1 10 100
593	1 10 10 1 1 1 1000 1 10 1 10 10 1 1 100 10 10 1 1 1000
596	1 1 100 1000 100 1000 1 1 1000 1 10 10 1 10000 10
597	1000 1 100 100 1 1 10 100 100 1 1 100 1 1 1 1 10 100
599	1 10 1 1 10 1 100 1 1 1 10000 1 1 1 1 10 10 1 1 10 1000
602	1 1 100 10 10000 10 10 100 10 10 10 1 1 1 100 1000
605	1 1 1000 10000 1 100000000 10000 1 1 1 10 1 100
607	1 1000 1 1000 1 1 1 1 1 10 10 10 1 1 1 100 1 1 10 1 1 10
608	100 1 1 10 1 10 100 1 10 1 1 1 1 1 10 10 10 1 1 1000 10
609	10 1 100000 1 10 10 1000 10 100000 1 1 1 100 100
611	1 10 100 10 10 1000 1 1 1 10 1 1 100 100 1 1 10 1 100
613	1 100 10 10000 1000 1 10 10000 1 10 1 10 10 1 1 10
614	100 1 10 1 1 10 1 1000 1 10000 1 100 100 10000 10
615	10 1 100 1 10 1 10000 1 100 10 10 1 10 1 10 1 10 100
617	1 10 100 100 1 10000 100 1 100 1 1 1 1 1 1 1 1 1 1 1000
620	1 1 100 100 1 1 1 10 10 10 1 1 10000 1 1 10 1 10 10 10
621	1000 1 100 1 10000 1 1 1 1 10 1 1 10 1 1 100 100000

626	11111 0 10 1 10 11111 0 100 100 111 100 10 1 100
628	11000000000 0 1110 10000 100 1110 10 11110
629	100 11110 10 11111 0000 10 100 11100 1110 10
630	10 11000 111100 1100 10 11100 1110000 1000
633	1110 1000 10000 10 1100 1110 10000 1110 100
635	11000 100 100 1100000 11000 1110 1000 1000
638	11100 11000 10 10 100 10 10000 10 10 1000 100
640	11000 11100 1100 11110000 10 10 111000 100
642	1100 11110 1110 10 110 100 11000 111100 100
644	1100 110 10 1100 10000 11111 1000 1110 1100
646	1100 110 1111000000 10 1100 10 10 10 10 1110
647	100 110000 100 100 10 11110 1000 1110000 10
648	10 1100 10 100 1110 11110 10 111000 1110 100
650	110 100 10000 111000 10 11000 111000 1000
653	11100 100 11111 0 10 100 100 11000 11100 100
655	11000 11110000 11111 1100 100 10 10 10 1100
657	1100 11110 10 11000 10 10 10 1000000 111110
658	100 1100 1000 10 10 1000000 11100 1000 10 10
659	10 1100 1000 1100 11110 10 1110 10 110 10000
663	11110 10 1100 110 100 110 10 111100 111000
666	11100000000 0 10 10000 110 11111 0 11100 10
667	1000 1110 10 1111000 10 110 110 1100 10 1100
669	110 1110 1000 10 1100 10000000 100 10 11110
670	100 1000000 1100 100 1110 110 10 110 1110 10
671	10 110 110 10 1000000 110 110 11100 100 1000
674	1110 10000 110 110 10 10 110 1100 10 10 11100
676	11000 100 10 1000 10000000000 10 110000 10
677	100 11100 1110 10000 10 10 110 10 11111 0 100
679	110 1100 1100000 110 10000 1100 11110 1000
682	11100 10 11000 100 100 11111 0 1111 0 10 1000
685	11 1000 100000 10000 111000 10 100 1100000
690	1111 100 110 10 100 1000 10 10 1000 11000 110
691	100000 100 110 110 1110 11100000 11110 110
692	10 1111111000 11111 0 111000 100 1110 1110
693	10 10000 10 111111 0 1000 10 10 110 111000 10
694	10 10 111000000 110000 100 11111111 100 100
696	110 10 10 100 111000 110 100 100 110 110 1100
698	1100 10 111111 0 1110 11000 100000 100 1110
699	100 110 110 10000 10 11100 111100 10 1100 10
700	10 1100 11000 1100000 100 10 10 10 1100 1100
702	110 100 100 11111111 0 10 11111111000 100 110
703	100 10 11110000 110 11111 00 10 1110000 110
704	10 110 10 10 11110 1000 10 10 1000 10 10 10 110
705	10 100 10000000 1000 10000 1110000 111110
706	10 10 1100 10 1110 110 1110 1110 10 1000 10 10
707	10 10 1000 11100 100 1110 110 100 1110 10000
711	11110 10000 110 111000 110000 100000 1000