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SOME FUNDAMENTAL ERRORS OF A TRIANGULATION COMPUTER

E. H. STOCK

OTTAWA

JULY 1951

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National Research Council of Canada

Radio and Electrical Engineering Division

SOME FUNDAMENTAL ERRORS OF A TRIANGULATION COMPUTER

E. H. Stock

ABSTRACT

Some fundamental errors of aircraft position computers employing plane resolution of distances and operated from Shoran or similar radar systems are discussed. The effect of a bulk correction on these errors as applied to the NRC Straight-line Flight Computer is shown for some typical cases. Graphs showing the effect of ray and earth curvatures and heights of the radar stations upon sea-level map distance are included in an appendix.

CONTENTS

SOME FUNDAMENTAL ERRORS OF A TRIANGULATION COMPUTER

Introduction

Triangulation computers of various types have been constructed to determine the position of an aircraft from measurements of distance from two ground positions. Most of these ignore the following errors:

(a) errors inherent in the method of measurement,
(b) fixed instrumental errors,
(c) geometric errors.

With the development by the Radio and Electrical Engineering Division, National Research Council, of a Straight-line Flight Computer* for use with Shoran radar equipment to guide an aircraft along topographic survey flight lines, the significance of these errors was investigated. The errors inherent in this method of measurement arise from uncertainties in the velocity of, and the path followed by, electromagnetic waves in the atmosphere. The fixed instrumental errors, time delays, etc., in the radar equipment can be evaluated by calibration procedures. Geometric errors, such as those caused by differences of station heights and curvatures of the earth, are apparent when a plane, rather than a three-dimensional model, of triangulation computer is employed.

Using a plane triangle, the corners of which depict two fixed radar stations and one moving station, a mathematical analysis will be made whereby the errors introduced into the position of the moving point can be evaluated if the errors in the three sides of the triangle are known.

Error Introduced in a Perpendicular of any Plane Triangle by Errors in the Lengths of the Three Sides

In Fig.1 is shown a geometrical representation of a triangulation computer. The small triangles, at $(-^{Z}/_{2}, 0)$ and $(^{Z}/_{2}, 0)$, depict two fixed ground stations, and Z is the distance between them. The variable point (x, y) represents an aircraft at a distance X from one ground station and a distance Y from the other.

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* NRC Progress Report ERA-151, April, 1948.

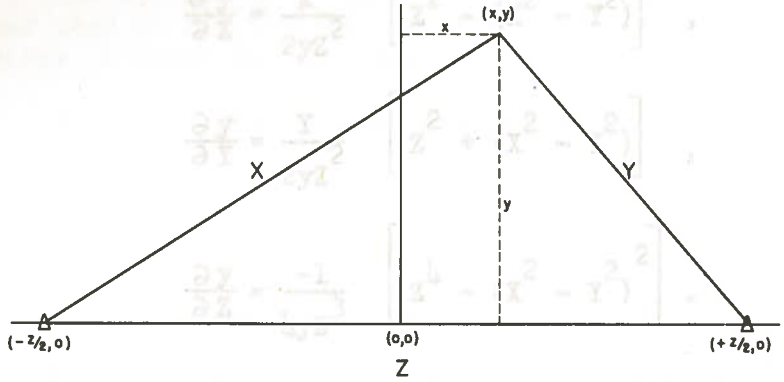


FIG. I

The error in the perpendicular distance, y, from the aircraft to the line joining the two ground stations, introduced by errors in the three distances X, Y and Z, is derived below.

From the triangle (Fig.1),

$$X^2 = y^2 + \left(\frac{Z}{2} + x\right)^2 , \qquad (1)$$

$$Y^2 = y^2 + \left(\frac{Z}{2} - x\right)^2 . \qquad (2)$$

Subtracting (2) from (1) we find

$$x = \frac{X^2 - Y^2}{2Z} . \qquad (3)$$

Adding (1) and (2) we find

$$y = \pm\sqrt{\frac{X^2 + Y^2 - Z^2/2}{2} - x^2}$$

$$= \pm\sqrt{\frac{X^2 + Y^2 - Z^2/2}{2} - \left(\frac{X^2 - Y^2}{2Z}\right)^2} . \qquad (4)$$

Taking partial derivatives of (4)

$$\frac{\partial y}{\partial X} = \frac{X}{2yZ^2}\left[Z^2 - (X^2 - Y^2)\right], \tag{5}$$

$$\frac{\partial y}{\partial Y} = \frac{Y}{2yZ^2}\left[Z^2 + (X^2 - Y^2)\right], \tag{6}$$

$$\frac{\partial y}{\partial Z} = \frac{-1}{4yZ^3}\left[Z^4 - (X^2 - Y^2)^2\right]. \tag{7}$$

The increment of a function of three variables, $y = f(X, Y, Z)$, may be written as

$$\Delta y = \frac{\partial y}{\partial X}\Delta X + \frac{\partial y}{\partial Y}\Delta Y + \frac{\partial y}{\partial Z}\Delta Z + \epsilon\Delta X + \epsilon'\Delta Y + \epsilon''\Delta Z, \tag{8}$$

in which ΔX, ΔY, and ΔZ are increments of the variables X, Y, and Z, respectively, and ϵ, ϵ', and ϵ'' are infinitesimals all of which approach zero with ΔX, ΔY, and ΔZ.

Since ΔX, ΔY, and ΔZ are all very small with respect to X, Y, and Z we may write (8) as

$$\Delta y = \frac{\partial y}{\partial X}\Delta X + \frac{\partial y}{\partial Y}\Delta Y + \frac{\partial y}{\partial Z}\Delta Z. \tag{9}$$

Substituting (5), (6), and (7), into (9) we find

$$\Delta y = \frac{1}{2yZ^2}\left\{X_o\Delta X\left[Z^2 - (X^2 - Y^2)\right] + Y_o\Delta Y\left[Z^2 + (X^2 - Y^2)\right] - \frac{\Delta Z}{2Z}\left[Z^2 - (X^2 - Y^2)\right]\left[Z^2 + (X^2 - Y^2)\right]\right\} \tag{10}$$

Similarly it can be shown that the error introduced in the horizontal distance, x, by errors in the lengths of the three sides of the plane triangle, can be represented by

$$\Delta x = \frac{1}{Z}\left[X_o\Delta X - Y_o\Delta Y - x_o\Delta Z\right] \tag{11}$$

Evaluation of the Quantities Δy and Δx

The evaluation of Δy and Δx for the NRC Straight-line Flight Computer used in conjunction with Shoran radar has been made for four cases in which the following symbols are used:

Z — distance between ground stations

ΔZ — error in Z

X — map distance

X_v — Veeder (Shoran) distance

X_a — equivalent computer arm distance

ΔX — error in X

$$\Delta X = X - X_a$$

C_X — computer X-arm setup value

$$C_X = X_v - X_a$$

$(-D_c)_X$ — delay correction for X in the Shoran radar equipment

$(-H_r)_X$ — height reduction term for X

$(\overline{-H_r})_X$ — a chosen value of $(-H_r)_X$.

Ignoring velocity corrections, it is known* that

$$X = X_v + (-D_c)_X + (-H_r)_X \text{ .}$$

The symbols for the Y-arm are similar to those above. The computer arm setup value, or "bulk correction" C_X or C_Y, is introduced by slipping the couplings between the Veeder counter, the arm lead screw, and the reference autosyn, thus causing a difference between the Veeder (Shoran) distance and the equivalent computer arm distance. The evaluation of the height reduction term $(-H_r)$ is shown in the Appendix.

* U.S.A.F. Report on Phase III, Caribbean Area Shoran Project.

Usually the Straight-line Flight Computer is set up so that the readings on the Veeder counters are the same as the equivalent computer arm distances ($C_X = C_Y = 0$). The instrument will receive the raw Shoran values, that is distances uncorrected for velocity, delays, wave curvature, slope, and reduction to sea level, and set these values into two plane arms which intersect at the third point of a plane triangle, the other two points being fixed by the ground station spacing.

In each of the four cases (Figs. 2, 3, 4, 5) the true plumb point position of the aircraft is shown by the rectangular grid coordinates (neglecting the small difference between a spheroidal earth and a plane surface). The values shown on the curved solid and dotted lines are the y and x differences, Δy and Δx, respectively, between the true plumb point position and a position given by the Shoran values. The Shoran values may be "uncorrected" as in Fig.2, or "bulk corrected" as in Figs. 3, 4, and 5. Neglecting the difference between map distance and Shoran distance, the values shown for the y and x differences may be used as corrective factors to determine a map position from a position found from the Shoran distances.

Fig.2 shows the values of corrective factors which can be applied to a position determined from uncorrected Shoran distance values in order to give the approximate map position. The corrective factors, or errors, as they may be called, are relatively large, and during a topographic operation they will cause the flight lines actually flown to be a few hundred yards in error.

"Bulk corrections", introduced into the computer arms and designated as C_X and C_Y, can be chosen such that the errors will be considerably reduced, and the resulting flight lines will be closer to those required for the operation. Fig.3 shows that a suitably chosen value for the bulk correction reduces the greatest error to approximately one tenth of the former value.

Fig.4 and Fig.5 show the variations caused by different ground-station spacings without changing the initial bulk correction.

Curve Evaluation Data

Values of $(-H_r)_X$ and $(-H_r)_Y$ have been calculated from the equation given in the Appendix for an aircraft altitude, H, of 20,000 feet and ground station elevations, K_X and K_Y, of 0 feet. These values are used in Cases 1, 2, 3, and 4.

In the choice of values for ΔX, ΔY, and ΔZ, a factor for the difference between a plane and spheroidal earth has not been considered, as it will be of second order for the distances under consideration. The errors inherent in the Shoran method of measurement of distances, as mentioned earlier, are considered to be small and outside the scope of this report.

Case 1 (Fig.2)

$$Z = 150 \text{ miles}$$
$$\Delta Z = 0 \text{ miles}$$
$$C_X = C_Y = 0 \text{ miles}$$
$$\Delta X = (-D_c)_X + (-H_r)_X$$
$$\Delta Y = (-D_c)_Y + (-H_r)_Y$$
$$(-D_c)_X = (-D_c)_Y = -0.180 \text{ miles}$$

Case 2 (Fig.3)

$$Z = 150 \text{ miles}$$
$$\Delta Z = 0 \text{ miles}$$
$$(-D_c)_X = (-D_c)_Y = -0.180 \text{ miles}$$
$$(-\bar{H}_r)_X = (-\bar{H}_r)_Y = -0.120 \text{ miles}$$
$$C_X = -\left[(-D_c)_X + (-\bar{H}_r)_X\right] = 0.300 \text{ miles}$$
$$C_Y = -\left[(-D_c)_Y + (-\bar{H}_r)_Y\right] = 0.300 \text{ miles}$$
$$\Delta X = (-H_r)_X - (-\bar{H}_r)_X$$
$$\Delta Y = (-H_r)_Y - (-\bar{H}_r)_Y$$

Case 3 (Fig.4) and Case 4 (Fig.5)

The same data as in Case 2 is used, except that Z = 100 miles in Case 3 and Z = 200 miles in Case 4.

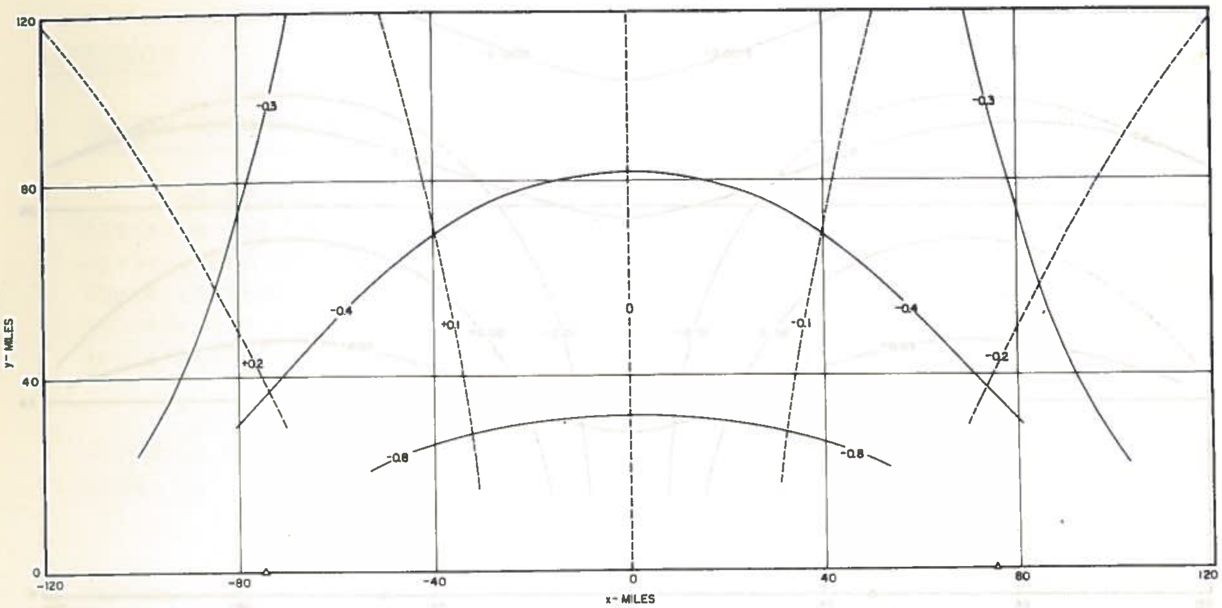

120
80
40
0
-120
-80
-40
0
40
80
120
x - MILES
y - MILES
-0.3
-0.3
-0.4
-0.4
0
+0.1
-0.1
+0.2
-0.2
-0.8
-0.8

FIG. 2

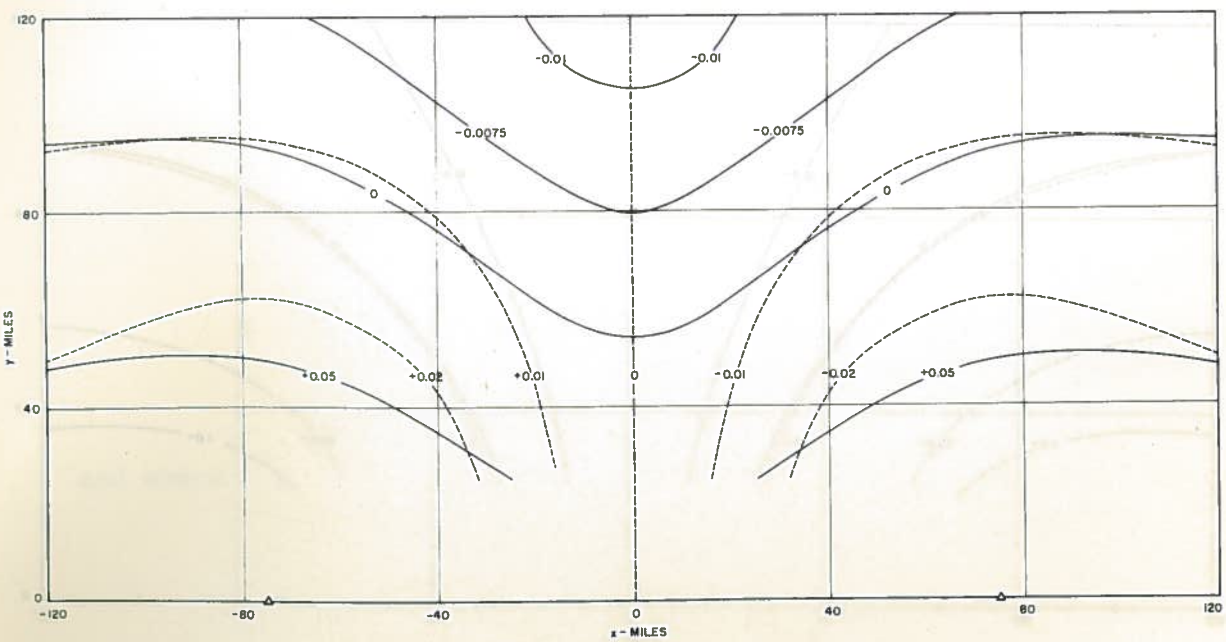
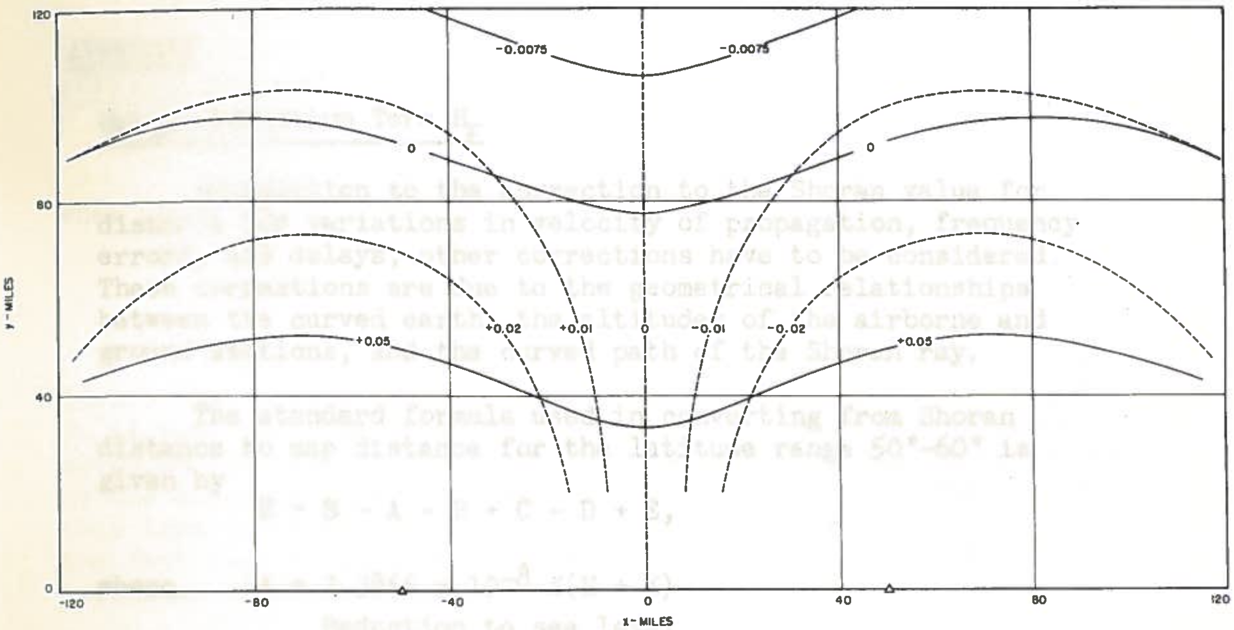

120
80
40
0
-120
-80
-40
0
40
80
120
x - MILES
y - MILES
-0.01
-0.01
-0.0075
-0.0075
0
0
+0.05
+0.02
+0.01
0
-0.01
-0.02
+0.05

FIG. 3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80
40
0
y-MILES
-0.0075
-0.0075
0
0
+0.02
+0.01
0
-0.01
-0.02
+0.05
+0.05
-120
-80
-40
0
40
80
120
x-MILES

FIG. 4

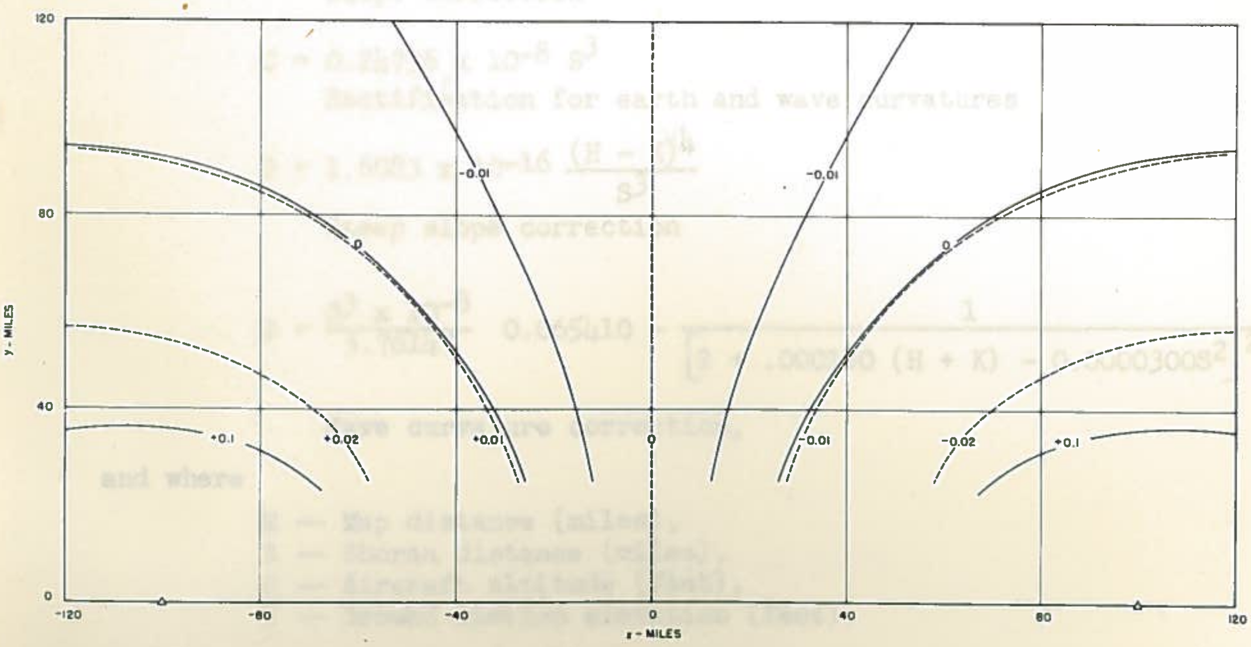
120
80
40
0
y-MILES
-0.01
-0.01
0
0
+0.1
+0.02
+0.01
0
-0.01
-0.02
+0.1
-120
-80
-40
0
40
80
120
x-MILES

FIG. 5

APPENDIX

Height Reduction Term H_r

In addition to the correction to the Shoran value for distance for variations in velocity of propagation, frequency errors, and delays, other corrections have to be considered. These corrections are due to the geometrical relationships between the curved earth, the altitudes of the airborne and ground stations, and the curved path of the Shoran ray.

The standard formula used in converting from Shoran distance to map distance for the latitude range 50°-60° is given by

$$M = S - A - B + C - D + E,$$

where $A = 2.3866 \times 10^{-8}\, S(H + K)$

Reduction to sea level

$$B = 1.7935 \times 10^{-8}\, \frac{(H - K)^2}{S}$$

Slope correction

$$C = 0.24736 \times 10^{-8}\, S^3$$

Rectification for earth and wave curvatures

$$D = 1.6083 \times 10^{-16}\, \frac{(H - K)^4}{S^3}$$

Steep slope correction

$$E = \frac{S^3 \times 10^{-8}}{3.76147} \quad 0.065410 - \frac{1}{\left[2 + .000180\,(H + K) - 0.0000300S^2\right]^2},$$

Wave curvature correction,

and where

M — Map distance (miles),
S — Shoran distance (miles),
H — Aircraft altitude (feet),
K — Ground station elevation (feet).

Apart from changes in the constants caused by a difference of latitude range, the foregoing has been taken from the Report on Phase III, Caribbean Area Shoran Project, U.S.A.F.

APPENDIX (Cont'd)

The corrective terms have been lumped together to give what is called a Height Reduction Term

$$H_r = - A - B + C - D + E \quad .$$

Plots of this term have been made for aircraft altitudes of 10,000 feet (Fig.6), 15,000 feet (Fig.7), and 20,000 feet (Fig.8). It is seen that the term is a function not only of the aircraft altitude but also of ground station elevation and the Shoran distance of the aircraft from the ground station in question. When using this term in Cases 1, 2, 3, and 4, special attention is drawn to the fact that at any position two values are required, one dependent on the X distance and the other on the Y distance.

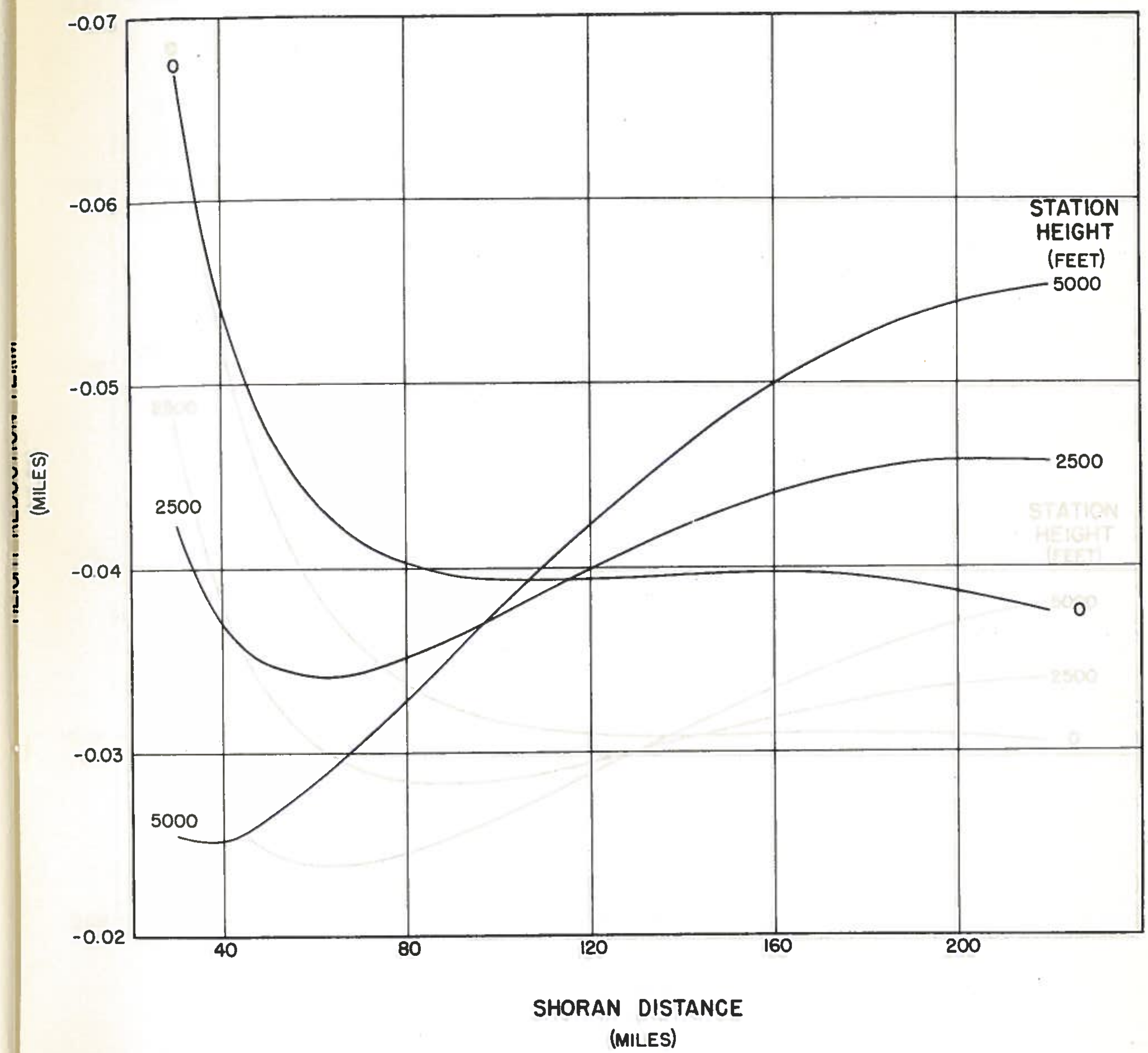

FIG. 6

HEIGHT REDUCTION TERM

FOR AIRCRAFT ALTITUDE

OF 10,000 FEET

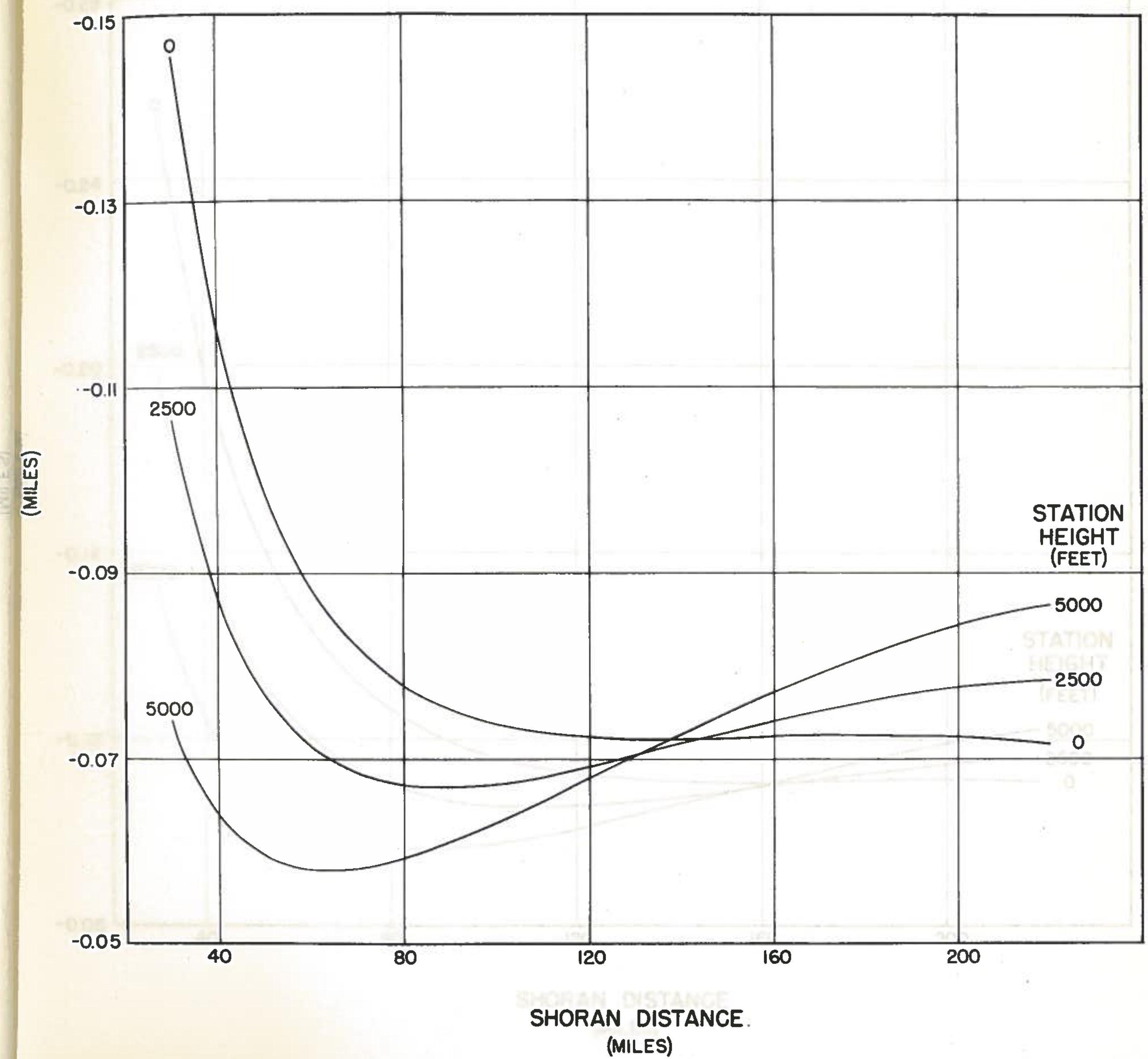

FIG. 7

HEIGHT REDUCTION TERM

FOR AIRCRAFT ALTITUDE

OF 15,000 FEET

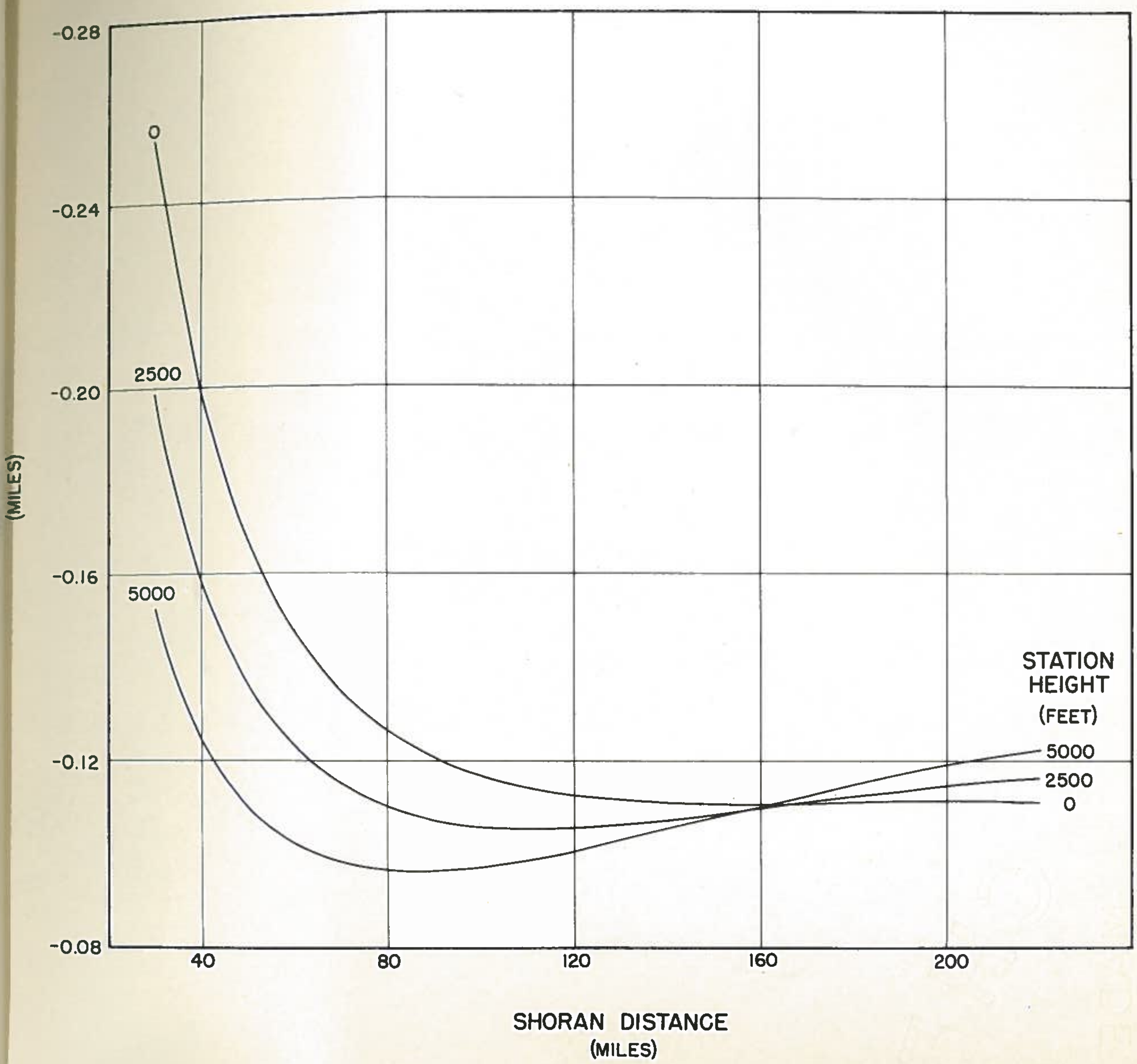

FIG. 8

HEIGHT REDUCTION TERM

FOR AIRCRAFT ALTITUDE

OF 20,000 FEET

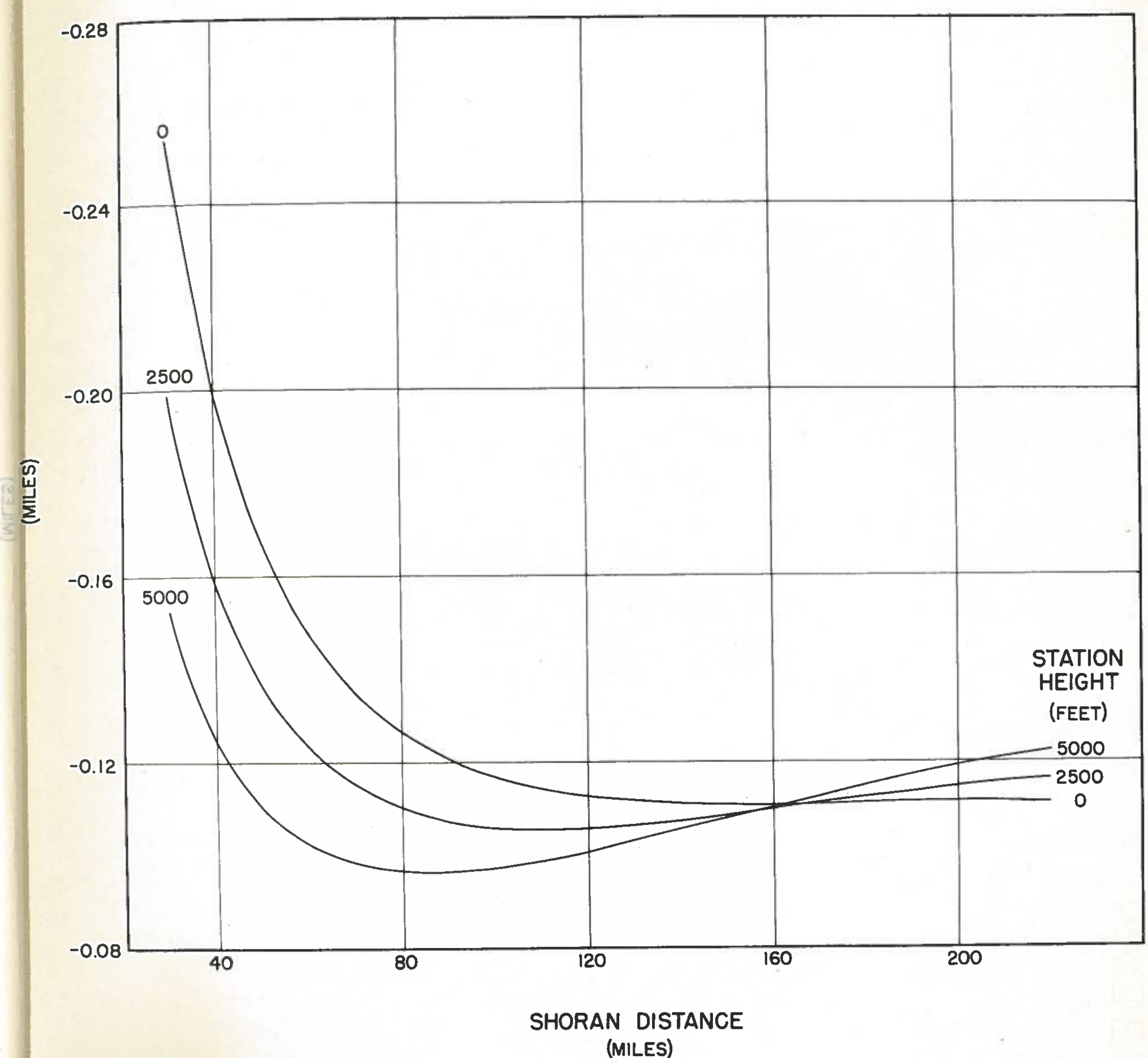

FIG. 8

HEIGHT REDUCTION TERM

FOR AIRCRAFT ALTITUDE

OF 20,000 FEET