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CORONA PULSE CHARACTERISTICS AND THEIR RELATIONSHIP TO RADIO NOISE

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OTTAWA

OCTOBER 1964

ABSTRACT

Analyses of the responses of radio noise meter quasi-peak and field intensity circuits to the following types of input pulses are given: periodic rectangular pulses, random rectangular pulses, random double exponential pulses (e.g., positive corona pulses). An experimental investigation of corona pulses originating on points and conductors is described. The probability density functions for the time intervals between pulses, and the shapes and amplitudes of these pulses were determined.

It is shown that for random corona pulses, with characteristics disclosed in tests on conductors, the quasi-peak reading of a noise meter is proportional to the average charge content of the pulse and to the square root of the average repetition rate.

CONTENTS

FIGURES

CORONA PULSE CHARACTERISTICS AND THEIR RELATIONSHIP TO RADIO NOISE

- A. Reed* -

INTRODUCTION

This report presents a summary of an investigation of corona pulse characteristics and their relation to readings on a radio noise meter. Mathematical treatments of the effects of various types of pulse inputs to the meter are related to results of experimental work done in the high-voltage laboratory and at the high-voltage building at the Metcalfe Road Field Station of the National Research Council.

The mathematical treatment consists firstly of an analysis of the effect of periodic, rectangular pulses fed into the input of an idealized radio noise meter, and the resultant effect on the Quasi Peak and Field Intensity circuits. Secondly, a spectral density analysis of the effect of random rectangular and double exponential pulses is presented. The noise meter readings are related to pulse characteristics such as amplitude, duration, and average repetition rate.

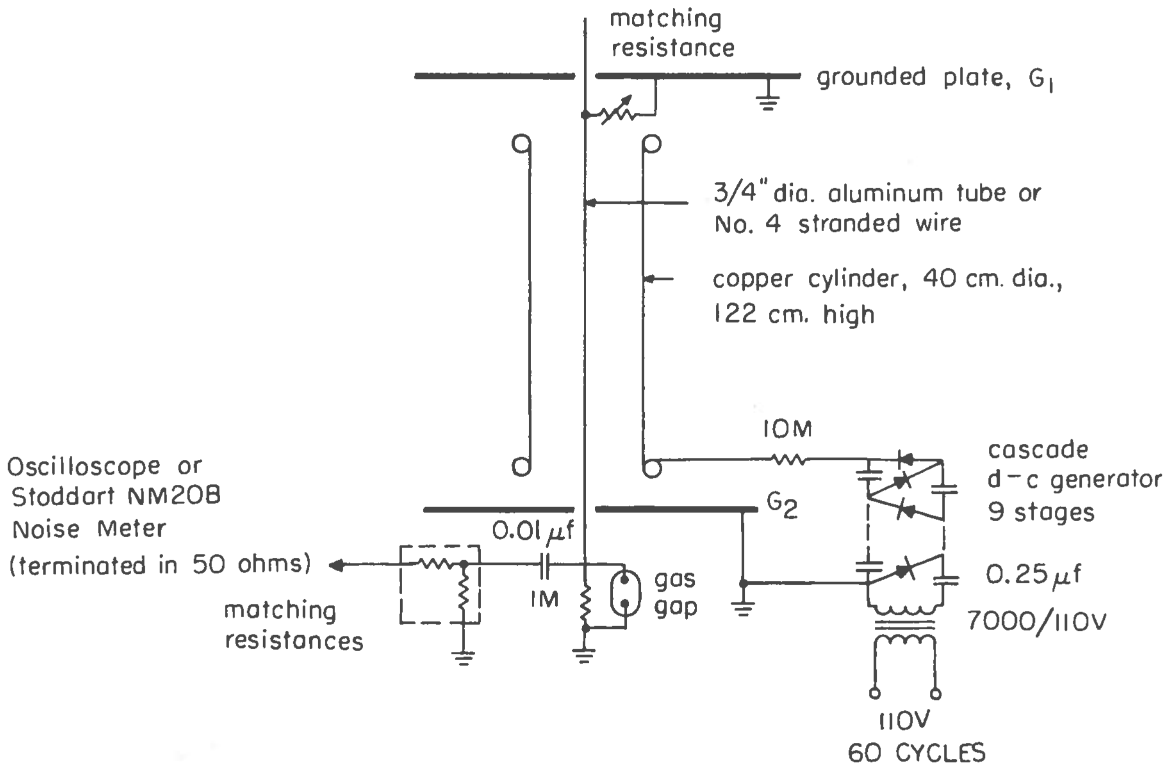

Fig. 1 Laboratory test setup

* NRC Summer Student, 1964

The initial experimental work was done with a vertical coaxial arrangement (Fig. 1), with the corona produced on metal points which were screwed into the central conductor. Other experiments were performed using No. 4 stranded wire in the coaxial setup, and 370-foot Lark and No. 2 conductors at the Metcalfe Road Field Station.

THEORY

The idealized noise meter was assumed to consist of the following sections (Fig. 2):

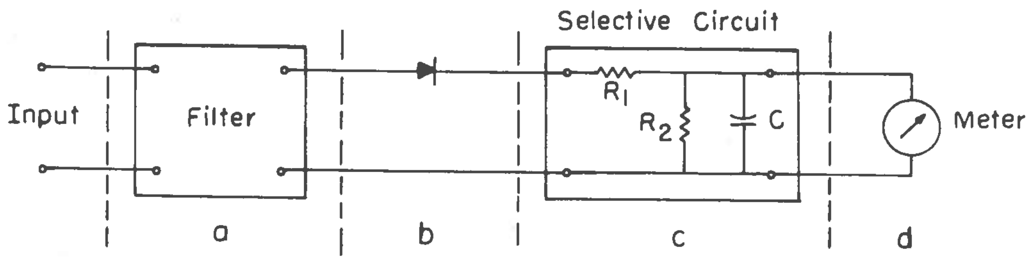

Fig. 2 Ideal noise meter

a) An ideal filter with center frequency f_c and bandwidth Δf

b) A rectifier

c) A selective circuit (QP or FI)

d) A meter to indicate the response

Periodic Pulses

A Fourier analysis of the noise meter filter response to a periodic rectangular pulse train is made (Appendix I), and using this result, a transform analysis of the QP and FI response is made. From this analysis, four main points are evident:

a) If the pulse repetition rate is greater than the bandwidth of the filter, the meter reponse is proportional to the repetition rate.

b) If the pulse repetition rate is less than the bandwidth of the filter, the meter response is proportional to

$$\frac{\Delta f}{2} + f_o,$$

where Δf is the bandwidth of the filter and f_o is the pulse repetition rate.

c) Because of the above two points, an analysis of this type is not truly applicable to the case of corona pulses originating from more than one point. Although corona pulses occurring at a single point are very nearly periodic, the combined effect of many points results in a random pulse train. Hence, the noise meter sees pulses at repetition rates which are sometimes greater and sometimes less than its bandwidth. Because of the differences of the effects of these two conditions, a periodic pulse train with the same average repetition rate cannot be used to approximate the random train.

d) The QP and FI circuits are of the same basic configuration. When the input to the noise meter is a periodic pulse train, the QP reading should be very nearly twice the FI reading.

Random Pulses

A spectral density analysis (Appendix II) of the response to a random input of rectangular pulses gives the result that

$$(\text{QP reading})^2 = \frac{2A^2\,(1 - \cos\omega\tau)}{\omega\,\overline{T}}\left[1 + 2\,\mathrm{Re}\left(\frac{P}{1 - P}\right)\right] \tag{1}$$

$$(\text{QP reading}) = K\sqrt{\bar{f}_o}\cdot Q \quad . \tag{2}$$

For a random input of double exponential corona pulses of the form

$$A\,(e^{-at} - e^{-bt})\ ,$$

we obtain

$$(\text{QP reading}) = K\cdot\sqrt{\bar{f}_o}\;\; Q\ ,$$

where $\bar{f}_o$ is the average repetition rate

$Q = A\,\frac{b-a}{ab}$ = the area of a single pulse, proportional to the charge content of the pulse.

EXPERIMENTAL PROCEDURE

Periodic Pulses

Items (a) and (b) of the theory indicate that the meter response should decrease suddenly as the pulse repetition rate is increased through the bandwidth of the filter.

Because the actual filter in the noise meter is not an ideal filter as was assumed for the derivation of this result, one would not necessarily expect an abrupt decrease in meter response, but rather a gradual one at repetition rates near the bandwidth of the filter.

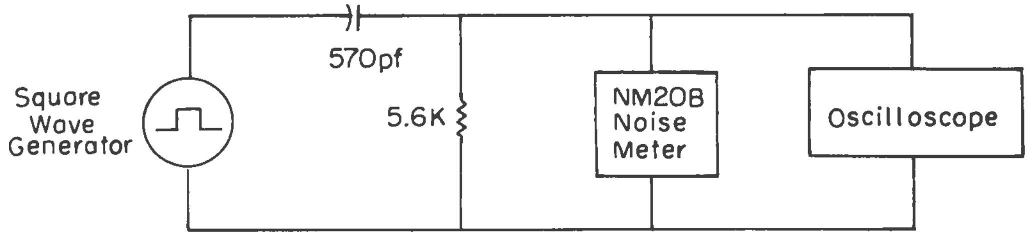

Fig. 3 Periodic pulse generator

An experiment was performed using generated pulses as the input to the noise meter. These pulses were obtained by differentiating the output from a variable-frequency square-wave generator (Fig. 3). The plot of noise meter response vs pulse repetition rate (Fig. 4) shows the expected dip at a repetition rate near the bandwidth of the filter.

<u>PERIODIC PULSES</u> - QP reading at 1 Mc vs. Pulse Repetition Rate at Repetition Rates near Bandwidth of Filter

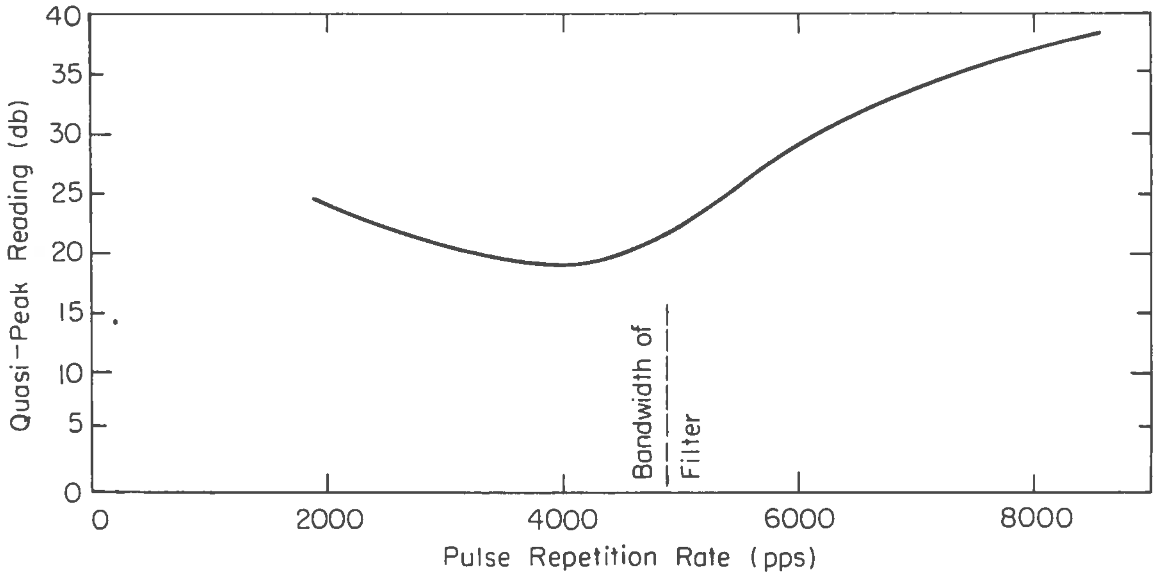

Fig. 4 Periodic pulses — graph of QP reading at 1 mc/s versus pulse repetition rate

Corona Pulses

a) Description of Laboratory Apparatus

The laboratory test setup (Fig. 1) was essentially the same as that used by Denholm [1] and Rakoshdas [2]. It consisted of a vertical copper cylinder with a coaxial center conductor. The inside conductor was either No. 4 stranded wire or a $\frac{3}{4}$-inch-diameter aluminum tube with threaded holes into which metal points could be screwed, as done by Akazaki [3]. The power supply was connected to the vertical cylinder and all measurements were made on the inner conductor which was essentially at ground potential. The oscilloscope was equipped with a camera and fast-rise preamplifier.

b) Method of Obtaining Data

(i) Laboratory Setup

Various numbers of metal points were attached to the inner conductor and the voltage was increased so as to cause the points to be in corona. Noise meter readings and photographs were taken within minutes of each other so as to minimize experimental error due to aging of the points. Photographs of a cathode-ray oscilloscope display were taken so as to determine the probability density function for the time intervals between successive pulses and also to determine the shapes and amplitudes of the pulses. All noise meter readings were taken at 1 mc/s.

Data were taken with 2, 4, 5, and 10 points in positive corona and with 4 and 10 points in negative corona. Measurements were also made using polished No. 4 stranded wire as the central conductor. The results of these experiments are shown in Figs. 9 to 13.

(ii) Field Station Setup

Two 370-foot test lines at the Metcalfe Road Field Station were used and the noise meter was coupled to the line with the tuned circuit, as shown in Fig. 5. Because of the strong signal from a local radio station it was necessary to include a portion AB of the circuit which was tuned so as to minimize the background noise. Photographs were taken of the pulse-wave shape and of the pulse repetition rate, along with noise meter readings, at two different voltages on each of the two lines. One of the lines was a Lark conductor and the other a No. 2 stranded wire.

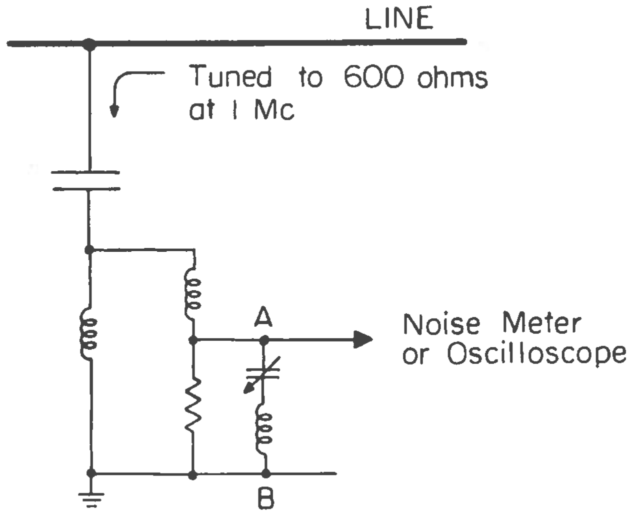

Fig. 5 Noise meter coupling to the 370-foot outdoor test line

DISCUSSION OF RESULTS

Coaxial Setup

It was found that pulses originating from a single point were very nearly periodic and of the same amplitudes. When two different types of points were used so that each gave a distinctive type of pulse, it was noted that each point gave periodic pulses independently of the other point so long as the separation was 6 inches or more. The result of several similar points in corona was a random pulse train which was due to the superposition of several periodic pulse trains.

When more than two metal points were in corona, an exponential form of frequency distribution for the pulse repetition rate was indicated; i.e.,

$$p(x) = \frac{1}{\bar{T}} e^{-x/\bar{T}} , \qquad (3)$$

where $\bar{T}$ is the mean interval.

For only two points in corona, the distribution was rectangular because of the periodicity of the two superimposed periodic pulse trains. The distribution of negative corona pulses from the polished No. 4 wire was also of the exponential form, but for positive pulses the distribution was nearly Normal or Gaussian.

TABLE I — SUMMARY OF RESULTS

Corona formed on:	Actual QP reading (μ V)	Average repetition rate (pps)	Average measured amplitude (volts)	$Q = A^*\left(\frac{b-a}{ab}\right)$ for double-exponential pulses	$K = \frac{\text{QP rdg.}}{Q\sqrt{f}}$	Distribution of repetition rates	Predicted QP reading using $QP = 33.5\ Q\sqrt{f}$	Deviation (%)
Group I			(50-ohm termination)					
Laboratory Setup								
2 pts (+) 54.4 kv	425	4,115	1.125	0.181 ($\times 10^{-6}$)	36.6	Rectangular	390	8.2
4 pts (+) 54.4 kv	500	7,407	1.08	0.173	33.6	Exponential	498	0.4
5 pts (+) 54.4 kv	510	9,911	0.968	0.156	32.8	"	520	2.0
10 pts (+) 54.4 kv	700	19,011	0.934	0.150	33.8	"	692	1.1
No. 4 wire + 58.7 kv	2000	1,688	1.15	0.185	26.3	Normal	2550	27.5
No. 4 wire + 60.9 kv	3000	2,825	1.15	0.185	30.4	Normal	3300	10.0
4 pts (-) 54.4 kv	35	195,000	0.009	0.00216	36.7	Exponential	32.2	4.9
10 pts (-) 54.4 kv	60	449,000	0.01	0.00241	37.2	"	54	10.0
No. 4 wire at -56.6 kv	80	9,340	0.1	0.0241	34.4 Avg=33.5	"	78	2.5
Group II			(500-ohm termination)					
Field Stn Setup								
Lark + 300 kv	9000	1,436	2.43	—	—	Exponential		
Lark + 275 kv	8500	1,272	2.34	—	—	"		
No. 2 + 150 kv (Fair)	2000	6,882	1.24	—	—	"		
No. 2 + 150 kv (Rain)	1800	10,121	0.93	—	—	"		
No. 2 + 200 kv	4000	16,447	1.17	—	—	"		

However, the extra term in the expression for the spectral density, which involves the standard deviation, is negligible compared with the rest of the expression, so that the same general relationships hold in this case. A possible explanation for this change in distribution function is that the positive corona on the polished wire does not occur at fixed points, but rather travels back and forth on the line. If this is the case, it seems feasible that the exponential distribution will go over to a Normal distribution. This implies that negative corona occurs at fixed points. The difference may be due to the different shielding effects of positive and negative ions.

According to the theory, the QP meter reading obeys relationship (2). K is a constant at a particular measuring frequency which depends on the bandwidth, gain of the noise meter, and other fixed parameters. Equation (2) was inverted to solve for K and an average value was used to give the "predicted" QP reading. The predicted and actual readings generally differed by less than 10% for a wide range of repetition rates and meter readings (Table I). There was one case where the reading differed by 27%.

Rakoshdas [2] found that corona pulses approximate the shape of a double exponential:

$$A^* \left(e^{-at} - e^{-bt}\right) .$$

This was verified. For a 50/150 nanosecond positive pulse, $a = 10.5 \times 10^6$ and $b = 34.6 \times 10^6$. Negative pulses have a 20 ns rise time and a 50 ns decay time. For negative pulses, $a = 38.3 \times 10^6$ and $b = 83 \times 10^6$. The above values were used to find Q and A* in Table I. Q is proportional to the charge content of the pulse.

The average pulse amplitude for positive pulses decreases slightly with an increase in the number of points in corona, whereas for negative corona there is no significant change. This indicates that the shielding effect of the negative ions is smaller than that of the positive ions.

The corona pulses formed on the stranded wire had a much lower repetition rate and much higher amplitude than the pulses formed at the metal points owing to the much higher gradient at the metal points. The higher gradient causes the ions to move faster, and thus their shielding effect is more short-lived. This effect was reported by Akazaki [3] as a result of his work with waterdrops.

Field Station Setup (370-foot Line)

The input to the noise meter when connected to the long line was not a double-exponential pulse, but rather a decaying oscillatory waveform caused by the response of the tuned coupling circuit and reflections from the open end of the line. For this reason, the relationships derived in Appendix I cannot be directly applied to the results. Only positive corona was examined at the field setup because the background noise due to radio stations and airplane beacons was too great to examine the effect of the smaller negative corona pulses successfully.

It was found that the repetition rate of the pulses increased and the amplitude decreased during rain, thus verifying the results of Akazaki [3]. The result was a decrease in radio noise during rain. It was also found that the distribution of the time intervals between pulses was of the exponential type, indicating that the corona was formed at specific points on the line. This fact was stated in a closure by Morris and Rakoshdas [4].

CONCLUSIONS

1) Corona pulses originating from a single point are very nearly periodic, of the same amplitude, and have a double-exponential shape.

2) For random corona pulses, the quasi-peak reading of a noise meter is proportional to the average charge content of a pulse and to the square root of the average repetition rate.

3) The time intervals between successive pulses originating from a finite number of points have an exponential distribution.

4) The time intervals between successive positive pulses originating from a polished wire have a Normal distribution.

5) The time intervals between successive positive pulses originating from a 370-foot outdoor line have an exponential distribution.

6) Radio noise is primarily due to positive corona because, although the repetition rate is much greater, the charge content of negative pulses is much less than that of positive corona pulses. The radio interference is proportional to the first power of the charge content, but only to the square root of the repetition rate.

7) The average pulse repetition rate increases during rain, but the charge content of pulses decreases, giving a net reduction in radio noise.

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APPENDIX I

RESPONSE OF FILTER TO A PERIODIC RECTANGULAR PULSE TRAIN

Consider a string of rectangular pulses with height A and duration τ (Fig. 6).

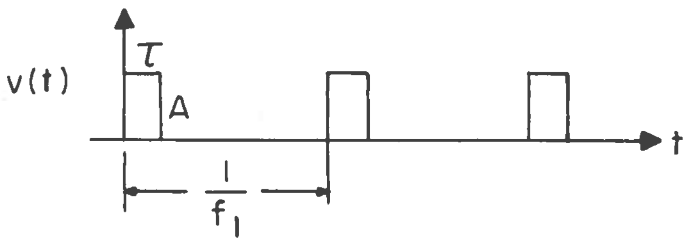

Fig. 6 Series of periodic rectangular pulses

The Fourier series for this waveform is:

$$(1) \qquad v(t) = (\text{dc term}) + \sum_{n=1}^{\infty} \frac{2A}{\pi n} (\sin \pi n f_1 \tau)(\cos \pi n f_1 [\tau - 2t])$$

If we pass this signal through an ideal filter with center frequency f_c, and bandwidth Δf, we obtain output, $v_f(t)$ in the form:

$$(2) \qquad v_f(t) = \frac{2Af_1}{\pi f_c} \sin \pi f_c \tau \sum_{n=n_1}^{n=n_2} \cos 2\pi n f_1 \left(t - \frac{\tau_1}{2}\right),$$

where $\quad n_1 f_1 = f_c - \frac{\Delta f}{2}$, the lower band edge

$\qquad\quad n_2 f_2 = f_c + \frac{\Delta f}{2}$, the upper band edge.

By the use of trigonometric identity* , $v_f(t)$ can be written as:

$$(3) \qquad v_f(t) = \frac{2f_1 A}{\pi f_c} \sin \pi f_c \tau \left[\frac{\cos 2\pi f_c t \cdot \sin 2\pi \left(\frac{\Delta f + f_1}{2}\right) t}{\sin 2\pi \left(\frac{f_1}{2}\right) t} \right].$$

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* H.B. Dwight, Tables of Integrals and Other Mathematical Data No. 420A,(Macmillan and Co., New York, 1961)

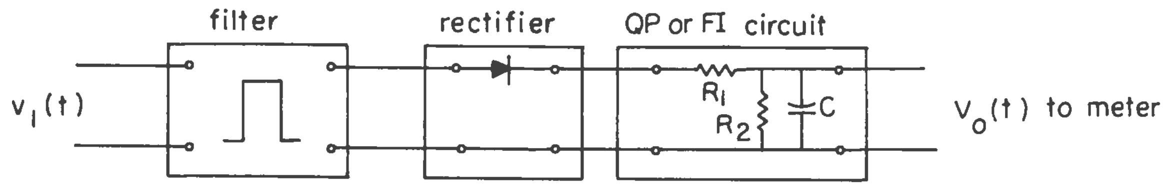

Fig. 7 General circuit for analysis

By writing the filter output in the form of a sum of cosine terms, we can consider the input to the Quasi-Peak circuit as the sum of rectified cosine terms. The Laplace transform F(s) of a repetitive rectified cosine wave is:

$$(4) \qquad F(s) = \left[\frac{2\pi f e^{-sT/4}}{1 - e^{-sT/2}} + s\right] \cdot \frac{1}{s^2 + 4\pi^2 f^2} \,,$$

where the cosine wave is of the form: $\cos 2\pi f t$. The transfer function for the QP or FI circuit is

$$(5) \qquad H(s) = \frac{\xi}{s + a\xi} \,, \quad \text{where} \quad \begin{cases} \xi = 1/CR_1 \\ a = \left(1 + \frac{R_1}{R_2}\right) \end{cases}$$

Hence the input to the meter for a single cosine term is:

$$(6) \qquad V_o(s) = \left[2\pi f \frac{e^{-sT/4}}{1 - e^{-sT/2}} + s\right] \cdot \frac{1}{s^2 + 2\pi^2 f^2} \cdot \frac{\xi}{s + a\xi} \,.$$

In the time domain, this is:

$$(7) \qquad v_o(t) = \frac{\xi}{(a\xi)^2 + (2\pi f)^2} \begin{cases} \left[-a\xi e^{-a\xi t} + a\xi \cos 2\pi f t + 2\pi f \sin 2\pi f t\right] \cdot u(t) \\ + \left[2\pi f e^{-a\xi\,(t - T/4)} - a\xi \cos 2\pi f t - 2\pi f \sin 2\pi f t\right] \end{cases} \Bigg\} \times \left[u(t) + u\left(t - \frac{T}{2}\right) + \ldots\right]$$

where $u(t)$ is a unit step at time $t = 0$.

Now this represents the input to the meter from a single input term to the rectifier of the form

$$v_{f_i} = \cos 2\pi f_i t \,.$$

Our actual output from the filter is

(8) $v_f(t) = \sum_i v_{f_i}(t) = \cos \alpha t + \cos(\alpha + \delta)t + \ldots . + \cos(\alpha + [n-1]\delta)t$,

where

$\delta = 4\pi f_1$	f_1 = repetition rate
$\alpha = 2\pi\left(f_c - \frac{\Delta f}{2}\right)$	f_c = center frequency
$n = \frac{\Delta f}{2 f_1} + 1$	Δf = bandwidth of filter

At a measuring frequency of about 1 mc/s, the steady-state input to the meter becomes approximately:

(9) $$v_o(t) \propto \frac{8 n f_1 A f_c \sin \pi f_c \tau}{a\xi} \cdot \frac{\xi^2}{(a\xi)^2 + (2\pi f_c)^2} ,$$

because $f_c >> \Delta f$ and $\frac{a\xi}{2 f_1} << 1$.

To find the ratio between the QP reading and the FI reading for a periodic input we note that

$$a_{QP} = 1 \qquad\qquad a_{F.I.} = 2$$

and

$$\xi_{QP} = 1000 \qquad\qquad \xi_{F.I.} = 1.67 \ .$$

Using the above values in equation (7) we find that $\frac{A_{QP}}{A_{FI}} = 2.0$ to within the accuracy of the above approximations.

APPENDIX II

SPECTRAL DENSITY ANALYSIS OF NOISE METER RESPONSE

a) Derivation of the Spectral Density of a Random Train of Rectangular Pulses

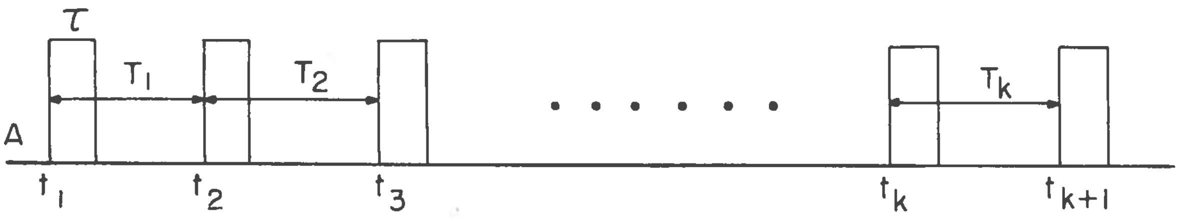

Fig. 8 Series of random rectangular pulses

Consider a series of pulses g(t) which occur at times t_1, t_2 .... t_k. Let these $g(t_i)$ be rectangular with amplitude A and duration τ, and let them be considered as the input, $v_1(t)$, to a radio noise meter.

$$v_i(t) = \sum_{k=1}^{\infty} g(t_k) \ . \tag{1}$$

This series may be truncated at t = -T and t = +T, where T is arbitrarily large, in order to find the Fourier transform of the input. For convenience, let t_1 occur at -T and t_{n+1} at +T, then

$$V_T(\omega) \triangleq \sum_{k=1}^{N} \int_{-T}^{T} g(t_k)\, e^{-j\omega t}\, dt \ , \tag{2}$$

which may be written as

$$V_T(\omega) = \frac{A}{j\omega} \cdot (1 - e^{-j\omega\tau}) \cdot \sum_{k=1}^{N} e^{-j\omega t_k} \ . \tag{3}$$

Now, the Spectral Density is defined as:

$$\Phi_{VV}(\omega) = \lim_{T\to\infty} \frac{1}{2T} \mid V_T(\omega) \mid^2 . \tag{4}$$

When we let $\overline{T} = \dfrac{T_1 + T_2 + \ldots + T_N}{N}$, Equation (4) becomes

$$\Phi_{VV}(\omega) = \frac{2A^2(1 - \cos\omega\tau)}{\overline{T}\,\omega^2} \cdot \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \sum_{j=1}^{N} e^{-j\omega(t_k - t_j)} . \quad (5)$$

If p(x) is the probability density function for the time intervals between consecutive pulses, it follows that the assembly average of terms of the form $e^{-j\omega x}$ is $P(\omega)$ where:

$$P(\omega) = \overline{e^{-j\omega x}} = \int_{-\infty}^{\infty} p(x)\, e^{-j\omega x}\, dx . \quad (6)$$

To evaluate the sum in Equation (5), consider first those terms for which $k > j$. Because

$$e^{-j\omega(t_k - t_j)} = e^{-j\omega T_j} \cdot e^{-j\omega T_{j+1}} \ldots . e^{-j\omega T_{k-1}} ,$$

it follows that

$$\overline{e^{-j\omega(t_k - t_j)}} = \left[P(\omega) \right]^m , \quad (7)$$

and thus for $k > j$:

$$\lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \sum_{j=1}^{N} e^{-j\omega(t_k - t_j)} = \sum_{m=1}^{\infty} P^m = \frac{P}{1-P} . \quad (8)$$

For $k < j$ we obtain the complex conjugate of (8), and for $k = j$ the limit is unity. Thus our input spectral density for a rectangular pulse train is:

$$\Phi_{VV}(\omega) = \frac{2A^2(1 - \cos\omega\tau)}{\omega^2\,\overline{T}} \left[1 + 2\,\mathrm{Re}\left(\frac{P}{1-P} \right) \right] . \quad (9)$$

This signal is fed through a filter (assumed ideal) and is then rectified and put through the selective circuit (QP or FI) before being fed into the meter. To find the meter response, we must consider four points:

i) If a signal with spectral density $\Phi_{VV}(\omega)$ is fed through a "black box" with transfer function $H(\omega)$, the relation between input and output spectral density is:

$$\Phi_{out}(\omega) = |H|^2\, \Phi_{in}(\omega) . \quad (10)$$

ii) If the transfer function of the selective circuit is $H(\omega)$, then

$$|H(\omega)|^2 = \frac{\xi^2}{(a\xi)^2 + \omega^2}, \quad \text{where } \xi = 1/CR_1, \quad a = 1 + \frac{R_1}{R_2} \quad \text{as in Fig. 6.}$$

iii) An ideal filter with center frequency ω_c and bandwidth $\Delta\omega$ will pass only frequencies ω, such that

$$\left(\omega_c - \frac{\Delta\omega}{2}\right) \leq \omega \leq \left(\omega_c + \frac{\Delta\omega}{2}\right).$$

iv) The meter will respond to only the d-c component of the signal fed to it because the meter response is much slower than the measuring frequencies. As a result the meter response will be proportional to $1/\pi$ times the integral of the spectral density over the frequency band of the filter. Because the bandwidth is very much less than the measuring frequency, the integral I of the spectral density over the bandwidth is very nearly:

$$I = \frac{\xi^2}{(a\xi)^2 + \omega^2} \cdot \frac{\Delta\omega}{\pi} \cdot \frac{2A^2(1 - \cos\omega\tau)}{\omega^2 \overline{T}} \left[1 + 2\,\mathrm{Re}\left(\frac{P}{1-P}\right)\right]. \quad (11)$$

b) Spectral Density Analysis of Noise Meter Response to a Random Train of Double-Exponential Pulses

From the study of high-voltage corona phenomena, it has been found that the noise-producing pulses have a double-exponential shape, the equation for which may be written in the form

$$g(t) = A(e^{-at} - e^{-bt}). \quad (12)$$

As before, consider a series of these pulses, repeated randomly in time, being fed into an RN meter. Call the input to the noise meter $v_i(t)$. Then

$$v_i(t) = \sum_{k=1}^{\infty} g(t_k). \quad (13)$$

By truncating the above series and taking the Fourier transform we obtain

$$V_T(\omega) = \sum_{k=1}^{N} A \int_{t_k}^{\infty} \left[e^{-a(t-t_k)} - e^{-b(t-t_k)}\right] e^{-j\omega t}\, dt. \quad (14)$$

The upper limit in this expression is taken to be infinity because the pulse separation is assumed to be much greater than the pulse duration. As before, we go through the limiting procedure to find the spectral density, $\Phi_{VV}(\omega)$. The energy to the meter in this case is then

$$I \propto \frac{\xi^2}{(a\xi)^2 + \omega^2} \cdot \frac{A^2 \Delta\omega}{\pi \overline{T}} \frac{(b-a)^2}{(a^2+\omega^2)(b^2+\omega^2)} \left[1 + 2\,\mathrm{Re}\left(\frac{P}{1-P}\right)\right]. \qquad (15)$$

Now, for a measuring frequency of 1 mc/s, because a and b for corona pulses are greater than 10^7 (B.R. Das) and because $a\xi << \omega$

$$I \simeq K^1 \frac{\xi^2 \Delta\omega}{\omega^2 \pi} \frac{A^2}{\overline{T}} \frac{(b-a)^2}{(ab)^2} \left[1 + 2\,\mathrm{Re}\left(\frac{P}{1-P}\right)\right]. \qquad (16)$$

The QP meter reading will be proportional to the square root of I, the integrated spectral density. Thus, at a particular measuring frequency, we can say

$$(\text{QP reading}) = \frac{KA(b-a)}{\sqrt{\overline{T}}\,ab} \left[1 + 2\,\mathrm{Re}\left(\frac{P}{1-P}\right)\right]. \qquad (17)$$

If the time intervals between consecutive pulses are exponentially distributed; i.e., if

$$p(x) = \frac{1}{\overline{T}} e^{-x/\overline{T}}, \qquad (18)$$

then

$$P(\omega) = \frac{f_o}{f_o + j\omega}, \qquad (19)$$

where $f_o = 1/\overline{T}$.

And as a result, the term

$$2\,\mathrm{Re}\left(\frac{P}{1-P}\right) \equiv 0 \qquad (20)$$

for exponentially distributed pulses.

If the time intervals between pulses are normally distributed; i.e., if

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\overline{T})^2}{2\sigma^2}}, \qquad (21)$$

where σ is the standard deviation

$\overline{T}$ is the mean value of x,

we find that

$$P(\omega) = e^{-j\omega\overline{T}} \, e^{-\frac{\omega^2 \sigma^2}{2}} , \tag{22}$$

and the term

$$2 \operatorname{Re}\left(\frac{P}{1-P}\right) = \frac{2 e^{-\frac{\omega^2 \sigma^2}{2}} \left(\cos \omega m - e^{-\frac{\omega^2 \sigma^2}{2}} \right)}{1 + e^{-\omega^2 \sigma^2} - 2 e^{-\frac{\omega^2 \sigma^2}{2}} \cdot \cos \omega m} .$$

When experimental values are substituted in this expression, it becomes negligible. Thus, for both exponentially and Normally distributed corona pulses, the Quasi Peak relationship is very nearly:

$$(\text{QP reading}) = \frac{KA\,(b - a)}{\sqrt{\overline{T}}\, ab} , \tag{23}$$

where the constant, K, depends on the parameters of the measuring instrument.

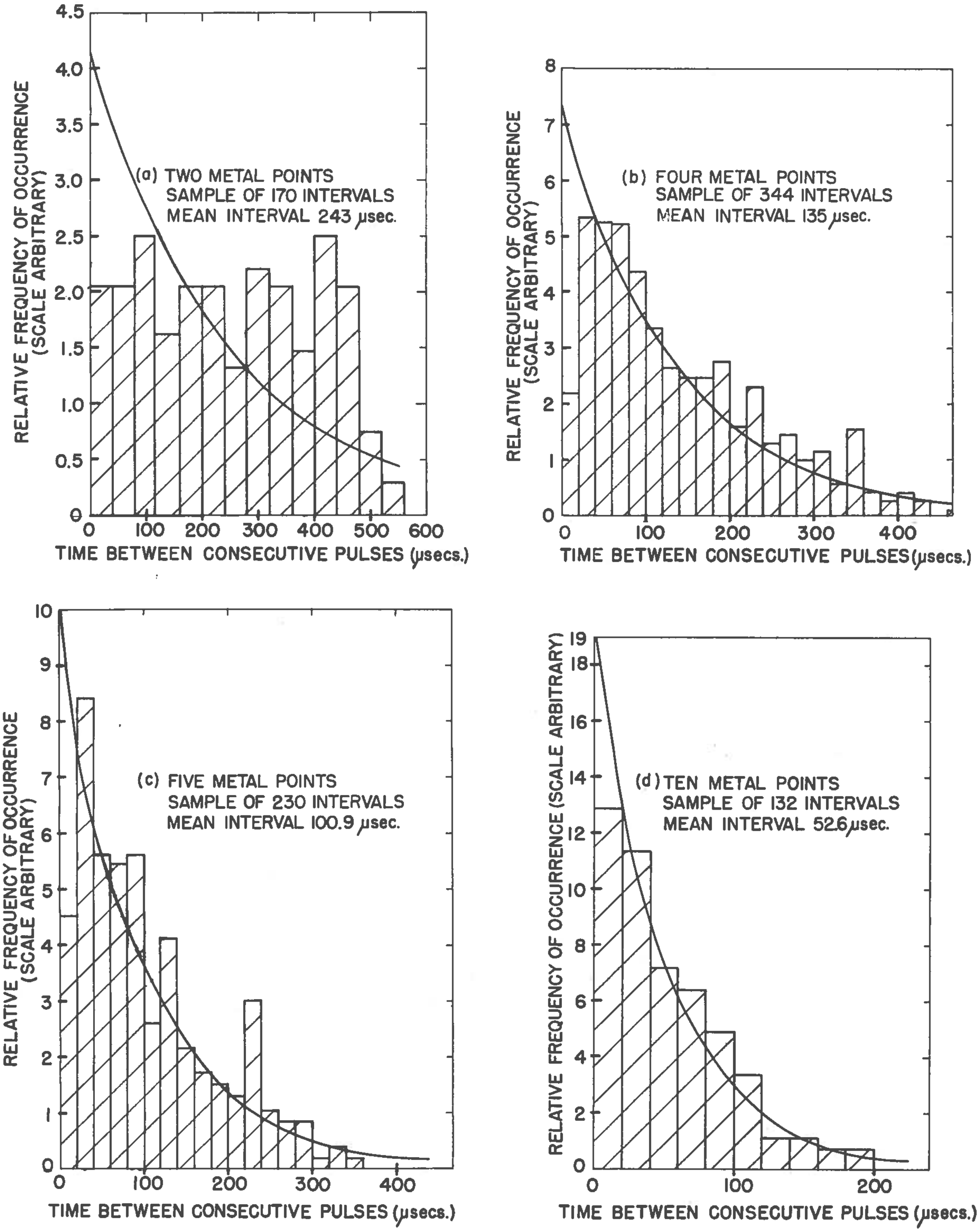

Fig. 9 Distribution of pulse repetition rates for positive corona pulses, 54.4 kv

(solid line: theoretical exponential distribution shaded histogram: experimental results)

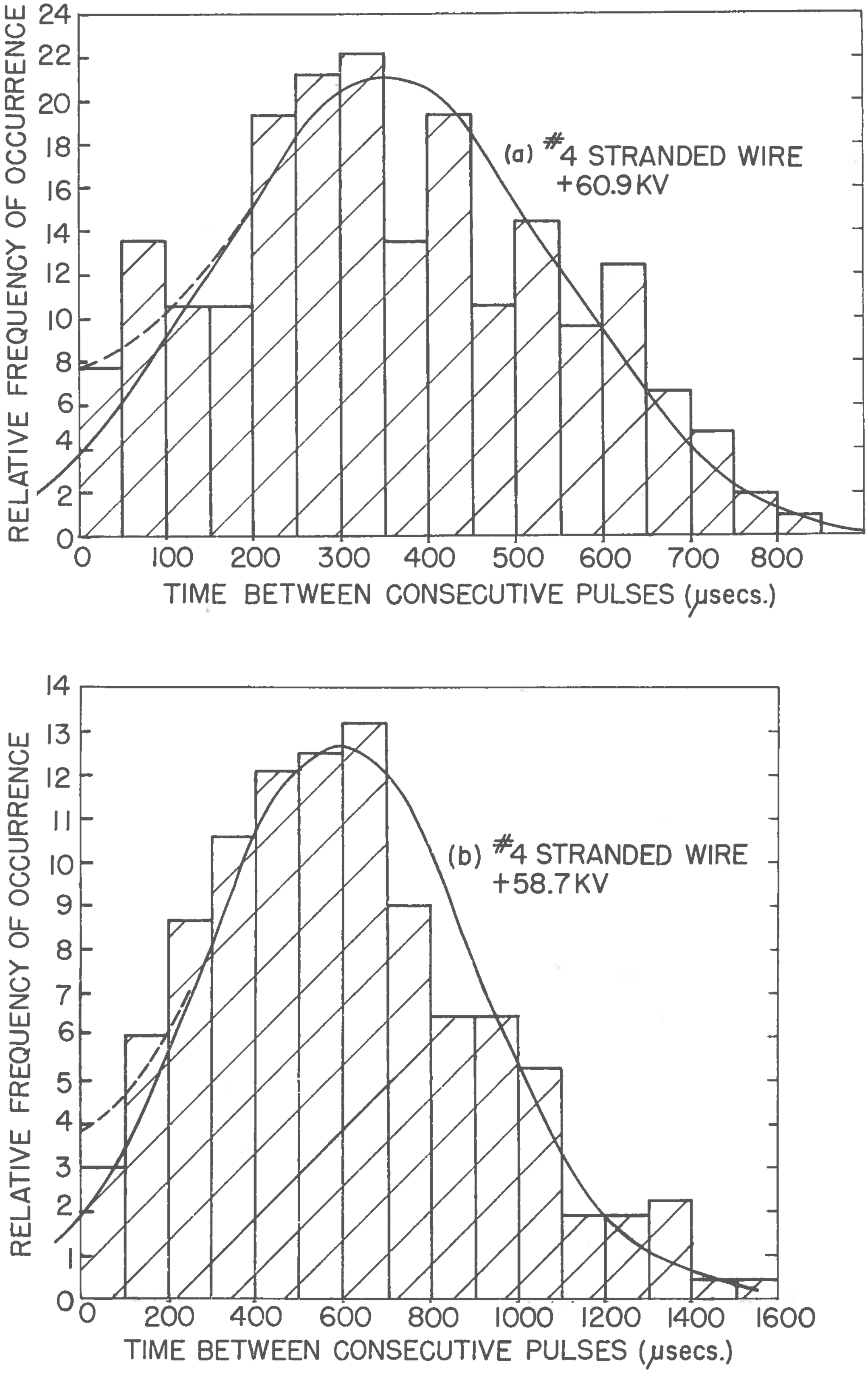

Fig. 10 Distribution of pulse repetition rates for positive corona pulses

(solid line: theoretical Gaussian distribution corresponding to mean and variance of shaded histogram)

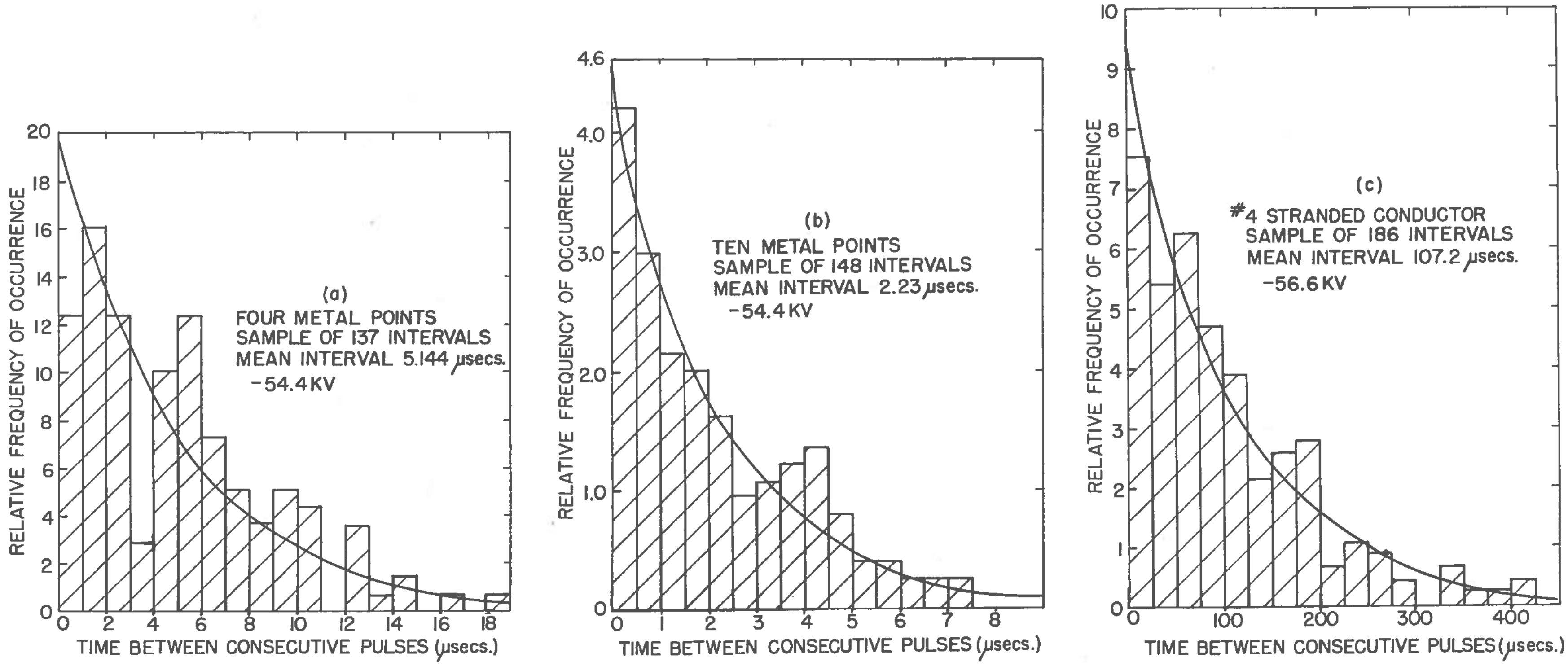

Fig. 11 Distribution of pulse repetition rates for negative corona pulses

(solid line: theoretical exponential distribution shaded histogram: experimental results)

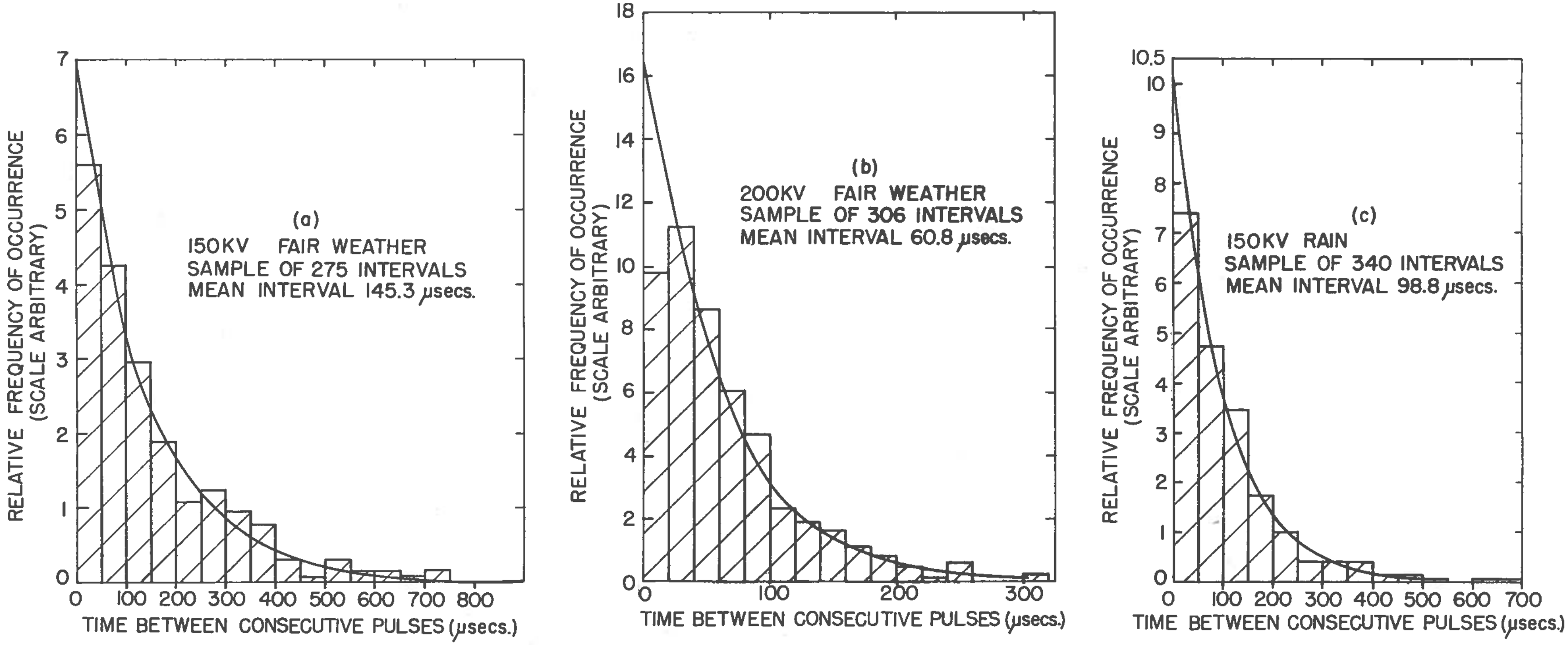

Fig. 12 Distribution of pulse repetition rates for positive corona pulses on 370 feet of no. 2 stranded conductor

(solid line: theoretical exponential distribution   shaded histogram: experimental results)

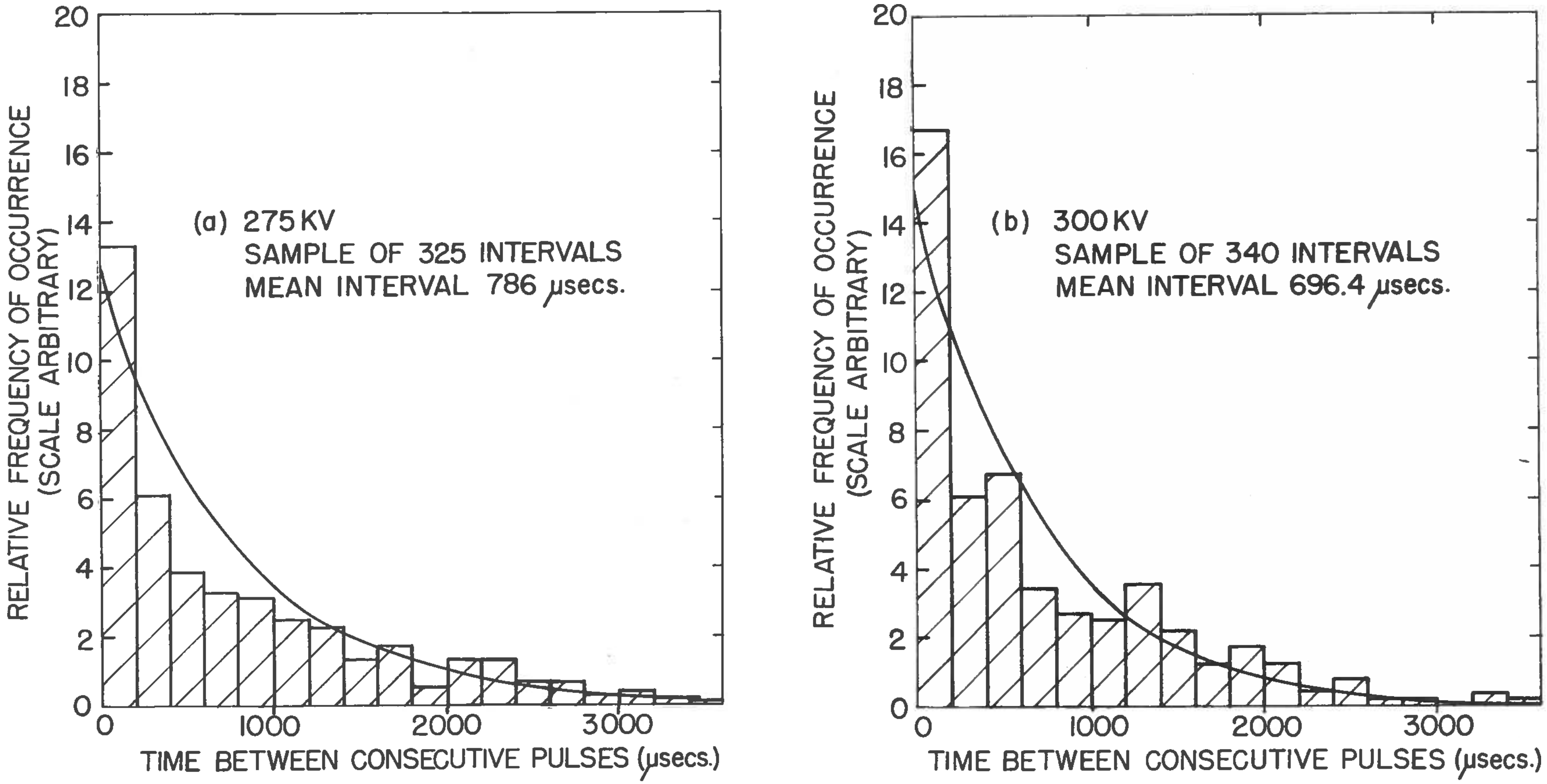

Fig. 13 Distribution of pulse repetition rates for positive corona pulses on 370 feet of Lark conductor

(solid line: theoretical exponential distribution shaded histogram: experimental results)