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ANALYSIS OF THE PERFORMANCE OF THE BURIED PIPE GRID OF A HEAT PUMP

by

W.G. Brown

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PREFACE

Interest in the possible uses of equipment operating on the reversed refrigeration cycle, more commonly known as heat pumps, developed rapidly following the end of the war in 1945. It was believed by many that such equipment would find extensive use in space heating. Several Divisions of the National Research Council had an interest in these possibilities and it was finally agreed that the Division of Building Research should investigate them. Compressor equipment already available for this work was turned over to the Division and planning of the project was begun. The ground coil was selected as the heat source for study.

Before experimental work was begun the Division invited representatives from other Divisions, several interested university departments, refrigeration manufacturers and the Hydro-Electric Power Commission of Ontario, to meet as a Heat Pump Committee and to give the benefit of their thinking and experience to the project. This Committee met first in 1949 and again in 1952.

Progress on the project was slow for several reasons, including equipment difficulties, and the need for changes in the experimental procedures and immediate objectives which were indicated as experience was gained in the work. As a result, no extensive test program was carried out before it became necessary to reconsider complete replacement of the compressor equipment and reconstruction of the ground coil at a new site. When, at this stage, the project was reconsidered in relation to other important commitments of the Division, the decision was made, with regret, to discontinue experimental work until some future date.

The operating results obtained from numerous experimental runs made over two years have now been analysed. This analysis, together with the conclusions drawn from the experimental data are now presented. Specific portions of the work and the equipment used have been described in undergraduate theses prepared by two students who were employed in summer work on the project. The records of the two meetings of the Committee are contained in the Proceedings which have been issued.

The experience gained in this project, some of which still remains to be reported, indicates not so much any lack of technical feasibility of the heat pump, as it does the difficulty in dealing on a proper experimental basis with thermal problems involving the ground.

Ottawa
September 1957

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INTRODUCTION

In recent years considerable interest has arisen in Canada over the possibility of heating and cooling buildings by means of the heat pump. This interest has been due mainly to successful applications in the United States, particularly in the South.

The heat pump is a refrigeration system in which heat is extracted at low temperature from an external source and discharged at high temperature. For cooling purposes the cycle can be reversed. The advantage of the heat pump is that, generally, the heat supplied is two or more times as much as the electrical heat equivalent of the compressor motor power. The magnitude of this advantage, however, decreases as the difference between the heat discharge and extraction temperatures increases.

In southern regions two factors have combined to favour the heat pump: the availability of heat sources at relatively high temperatures results in correspondingly low operating costs for winter heating; and summer cooling loads, being large, require equipment of capacity similar to that for the heating load. Summer air-conditioning is being regarded more and more as a necessity in northern parts of the United States, and where it is required the high initial costs of the heat pump can be justified. In Canada, climatic conditions are considerably different from those in the South; winter heating requirements are greater while summer cooling loads are generally lower. Under these conditions a heat pump would not be as effective as in warmer climates but still may be feasible when both heating and cooling are required.

Natural heat sources in Canada, (air, water or ground) are available only at comparatively low temperatures when winter heating requirements are greatest. Since the feasibility of a heat pump installation may depend strongly on the quality of the source it is important to be able to predict the source characteristics accurately. Although air is the most accessible source it has two serious disadvantages in cold climates: its temperature is the lowest of the natural sources and; there is danger of evaporator coil frosting at times of greatest heat demand. Water is not usually available, hence for most potential heat pump installations the ground would serve as the heat source. As a step in investigating the possibilities of the heat pump in Canada, a project to study the ground heat source was instituted in 1949 by the Division of Building Research of the National Research Council.

The performance theory for pipes buried in the ground (1) has been available for many years. In general, however, this theory has been developed for idealized conditions and has not had extensive experimental verification, particularly for long-term operation. Among the factors usually neglected in theoretical calculations are the effects of soil freezing and temperature-induced moisture migration, variable soil thermal properties due to non-uniform moisture distribution, and non-uniform heat extraction rates due to fluctuating heating demands. In addition, design calculations usually assume long-term or steady-state operation at the maximum heat extraction rate, whereas in reality the performance depends on the previous history of heat extraction. Under these conditions a ground grid designed theoretically may be oversized. This report deals with these factors and their importance by comparing observed and calculated data during a full season of heating operation and by comparison of prior history-dependent operation with steady-state operation.

EQUIPMENT AND PROCEDURE

The Heat Pump

The heat pump used in conjunction with the investigation of the buried pipe grid consisted of 3 Brown-Boveri compressors of 2½ hp each which could be connected in parallel. The refrigerant, (Freon 12) was circulated through 3 heat exchangers (evaporators), where heat was extracted from the secondary refrigerant of the ground pipes. The condenser consisted of a forced air Freon heat exchanger.

The Ground Grid

The ground pipe grid (Fig. 1) consisted of 5 parallel loops of 1-in. copper pipe (1.125 in. O.D.) approximately 189 ft long, on 5-ft centres and buried 6 ft deep. The loops were connected to common supply and return headers and, by means of shut-off valves, could be used in any parallel combination. All pipes connecting the grid and the headers were insulated to reduce heat transfer between supply and return refrigerant. The refrigerant circulated in the pipes was calcium chloride brine having specific gravity 1.21 and specific heat 0.71.

The original soil at the grid site was mainly sand, thus sand was used to back-fill the 54- by 100-ft excavation to give the best possible soil uniformity. The completed excavation with the pipes in place prior to back-filling is shown in Fig. 2.

About 200 thermocouples, (20 gauge copper-constantan with neoprene insulation and a waterproof cover), were installed to measure pipe surface, brine, and surrounding ground temperatures. Pipe surface thermocouples were soldered in place at the middle of each supply and return run and also at 15-ft intervals along the middle loop, (loop C). Brine thermocouples were placed in wells at the beginning and end of each loop. Ground thermocouples, supported by wooden sticks, were installed vertically and horizontally, $\frac{1}{4}$, $\frac{1}{2}$, 1 and 2 ft from the centre-line at the middle of each pipe run. Additional ground thermocouples were installed at the same location at depths of 1, 3 and about 9 ft below the ground surface, (bed-rock was encountered about 9 ft deep). Thermocouples were also installed at horizontal distances of 4 and 6 ft beyond the outermost pipes. Installation details are shown in Fig. 3.

Measurements

Ground and pipe surface temperatures were measured on a self-balancing potentiometer indicator (sensitivity 0.1°F) and brine temperatures were recorded on a multipoint recorder, (accuracy approx. 1°F). Brine flow rates, (6 to 20 gal/min) were measured with a calibrated volumetric displacement meter. Heat extraction rates were calculated from the temperature increases along the pipe and the brine flow rates. Differential pressures across 5/8-in. orifices in each pipe loop were equalized to ensure identical brine flow rates. In conjunction with operation of the pipe grid, several samples of soil were analysed for density, moisture content and grain size distribution.

Scope of Tests and Analysis of Data

The data for the experimental investigation were obtained in three stages:

(1) Preliminary tests

Two short-term tests of 2- and 3-days' duration were made in July 1950 using pipe loop C. In the analysis these data were used to determine the thermal conductivity and diffusivity of the ground.

(2) Operation during the heating season

From October 1950 to May 1951 the ground grid pipe loops were used in various combinations and heat extraction rates and pipe surface temperatures were recorded. The method of analysis of the data for this period consisted of comparing observed pipe surface temperatures at various times with those calculated using the ground properties determined from the preliminary tests.

(3) Normal ground temperatures

Soil temperatures were recorded for an additional two years after completion of the heat extraction tests to determine normal temperature changes due to the annual weather cycle. These data were not used in the analysis but were of interest in illustrating how ground temperatures vary with depth and time during the year.

RESULTS AND DISCUSSION

(1) PRELIMINARY TESTS - DETERMINATION OF SOIL THERMAL PROPERTIES

Heat Extraction Rates

In the preliminary tests on pipe loop C, (July 18 to 21 and July 27 to 28, 1950) pipe surface and soil temperatures were recorded periodically. The heat transfer rates in B.t.u. per hr per ft of pipe were calculated using the formula,

$$Q^1 = w\ Cp\ \frac{\Delta T}{L}, \qquad (1)$$

where: w = brine flow rate in lb per hr (5240 for both tests),

Cp = specific heat of brine (0.71 B.t.u./hr/°F),

ΔT = temperature rise along the pipe (°F),

L = pipe length (189 ft).

During both preliminary tests, the multi-point temperature recorder was found to be insufficiently accurate for determination of the small rise in brine temperature along the pipe (0° to 4°F). Calculation showed, however, that at the brine flow rates used in the tests the temperature difference between the brine and pipe surface would be less than 0.05°F. Thus, pipe surface temperatures measured with the more accurate temperature indicator were used to calculate heat extraction rates. Since this instrument was read only to the closest $\frac{1}{2}$°F, however, heat extraction rates could not be accurately determined. Although the amounts of heat extracted by the brine during the July 18 to 21 test (Fig. 4) were uncertain, the extraction rates appeared to increase during the first several hours warm-up period, then decreased gradually for the remainder of the test. The gradually decreasing rates after the initial warm-up period were due to the method of operation of the primary refrigeration system, (constant superheat, in which heat extraction decreases as the evaporator or heat exchanger temperature decreases). For the first few minutes of operation the heat extraction rates were presumably negative since the brine in the heat exchangers linking the primary and secondary refrigeration systems was initially at ambient temperature, (about 70°F), whereas the ground temperature at the pipe was 59°F.

Pipe Surface Temperatures

Pipe surface temperatures, (Fig. 5) decreased continuously during the July 18 to 21 test from 59°F initially to 25°F after 5 days' time. Results were similar for the July 27 to 28 test. In this second test, however, all temperatures were slightly lower than in the first since the ground and pipe surface temperatures had not completely recovered in the intervening time between tests.

Determination of the Thermal Properties of the Soil

Available theory for buried pipes (1) shows that for short-term tests the temperature "decrease" below normal temperature at any radial distance from a pipe including the pipe surface* is given by:

$$\Delta T = \frac{Q^1}{2\pi k} \quad I \left(\frac{r}{2\sqrt{\alpha t}} \right), \qquad (2)$$

where: ΔT = the "decrease" in temperature below the normal temperature at the point in question,

Q^1 = the heat extraction rate per unit length of pipe,

k = the thermal conductivity of the soil,

I = the line source integral, (see reference (1) p. 297 for tabulation),

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* For very short times (less than about 1 hour) the theory does not hold.

r = the radius,

α = the thermal diffusivity,

t = the time from start of operation.

Equation 2 shows that if Q^1, k and α can be assumed constant as a first approximation then ΔT is a function only of $\frac{r}{\sqrt{t}}$ or t/r^2. Consequently, a graph of ΔT versus $\frac{t}{r^2}$ should give a single line from which the diffusivity α could be determined using the equation. (The thermal conductivity would remain unknown since Q^1 was not accurately known.)

In this method of comparing temperature "decrease" data for the pipe and various points in the surrounding soil, the normal soil temperatures at the points in question (Figs. 6 and 7) were assumed to be the same as those measured at the beginning of the test. This assumption was valid since measurements at unused pipes A and E showed that the normal temperatures remained essentially unchanged for several days.

Results for pipe surface, 0.25, 0.5 and 1.0 foot radii, (Figs. 8 and 9) showed the assumptions involved in using equation 2 were valid as a first approximation. Using the equation, the thermal diffusivity was found to be about 0.04 ft^2/hr (see Appendix I for all calculations of this section).

Although the thermal conductivity could not be determined directly from the test data it was determined indirectly from the moisture content and density data obtained in conjunction with the tests (Table I). This was possible since thermal conductivity, density, diffusivity and specific heat are related by the formula

$$k = \alpha \rho Cp, \tag{3}$$

where ρ is the soil density, and Cp the soil specific heat. Although the specific heat in turn was not measured, published information (3) shows it can be calculated from the equation:

$$Cp = \frac{100C + M}{100 + M},$$

where M is the percentage moisture content and C is the specific heat of dry soil (0.16 to 0.17 B.t.u./lb/°F). The thermal conductivity obtained by use of these equations was 1.2 B.t.u./hr/°F/ft.

With the conductivity established, equation 2 was then reused to determine the average heat extraction rate. The value obtained, (61 B.t.u./hr/ft) agreed moderately well with the apparent mean of the measured heat extraction rates (Fig. 4).

Although the data of Figs. 8 and 9 for the 0.25-, 0.5- and 1.0-ft radii appeared to correlate, the data for the pipe surface failed to do so at large values of t/r^2. As mentioned previously, however, the heat-extraction rates were known to vary with time, but the form of the relationship was unknown due to measurement inaccuracies. Thus, in order to determine more accurately the ground properties and the form of the heat extraction-time relationship, further analysis was carried out. The analysis method consisted of assuming constant heat-extraction rates for short periods of time and using equation 2 to sum the temperature "decreases" at the pipe surface caused by these rates. By equating this sum to the measured pipe surface temperature "decrease" the individual heat-extraction rates could be determined step by step. This procedure was used for several values of thermal conductivity and diffusivity as related by equation 3. The heat-extraction rates were determined; then the temperature "decreases" at the various radii in the soil were computed for comparison with observed values. The best agreement, (Fig. 10, calculations given in Appendix II) was obtained for a thermal conductivity of 0.9 B.t.u./hr/°F/ft and a diffusivity of 0.03 ft^2/hr. The values of heat-extraction rate determined from the calculations (Fig. 4) were in good agreement with the apparent measured rates.

There was some minor uncertainty about the values of heat flow, conductivity and diffusivity due to possible errors in thermocouple positioning (about $\frac{1}{4}$ in.). In addition, some temperature error was apparent, presumably due to switching errors or imperfect grounding of equipment (see 1.0-ft radius at beginning of test in Fig. 10). Misener (2) reports similar difficulties in temperature measurements and errors of the same magnitudes. The value of thermal conductivity of 0.9 B.t.u./hr/ft/°F, however, agrees fairly well with the accepted value of 1.0 B.t.u./hr/ft/°F for sandy soil (3). The assumption of constant diffusivity is supported by results of Penrod (4) who reports tests using a heat pump to determine the thermal diffusivity of a clay-type soil. His results for several days operation showed a constant diffusivity up to at least 1.5 ft from a buried pipe.

Effect of Soil Freezing and Moisture Migration

The predominance of large particles in the soil grain-size distribution (Fig. 11) indicates that almost all the moisture present would have been frozen at 32°F.

Published data (3) show that, at the moisture contents present in the tests (Table 1B), freezing would not change the soil thermal properties appreciably. However, the release of latent heat during the freezing process would cause pipe surface temperatures to be somewhat higher than with a non-freezing process. Complete theory, ((1) p. 265) is available for the freezing process for the condition of constant heat extraction rate. The equations governing are:

$$\left[\frac{Q^1}{2\pi} - k \frac{(T_o - T_f)}{I\left(\sqrt{\frac{b}{\alpha}}\right)} \right] e^{-\frac{b}{\alpha}} = 2Lb \qquad (4)$$

and

$$\Delta T = T_o - T_f + \frac{Q^1}{2\pi k} I\left(\sqrt{\frac{b}{\alpha}}\right) \left[\frac{I\left(\frac{r}{2\sqrt{\alpha t}}\right)}{I\left(\sqrt{\frac{b}{\alpha}}\right)} - 1 \right] \qquad (5)$$

$$\text{for } r \leq 2\sqrt{bt}$$

where: T_o = initial pipe surface temperature

T_f = freezing temperature

L = latent heat of fusion per unit volume of soil

b = a constant

Q^1 = heat extraction rate per foot of coil

k = thermal conductivity of soil

α = thermal diffusivity of soil

r = pipe radius

t = duration of operation

Using these equations and the values of the variables encountered in the July 18 to 21 test, the error in calculated pipe surface temperature due to neglect of freezing was found to be only about 0.2°F for any continuous test. (For calculations see Appendix III.) Calculations by Ingersoll, Zobel and Ingersoll ((1) p. 266) also indicate only small effects due to freezing. These authors point out that, on the basis of equations 4 and 5, freezing effects will contribute very little to the performance of a ground coil.

Moisture migration induced by temperature gradients may or may not have contributed significantly to the heat extraction. Since the mechanism is not yet fully understood (5), accurate estimates of its effect cannot be made. In general, however, with moisture migration present the heat extraction rates would be expected to be higher than without migration due to increased thermal conductivity of the soil near the pipe and possibly to the latent heat of condensation of the migrating moisture. It may be noted that the theoretical temperature "decreases" (Fig. 10) were too large particularly at the 0.5- and 1.0-ft radii. If the only effect of moisture migration was to cause a decreasing conductivity gradient from the pipe surface outward, then equation 2 shows that the theoretical temperature "decreases" at the 0.5 and 1.0 radii would depart even more from the observed values. If latent heat effects were present, however, due to moisture migration in the vapour phase or by alternate condensation and evaporation then only part of the total heat flow in the soil remote from the pipe would take place by conduction and the remainder would be transmitted by the moisture in the vapour phase. Near the pipe surface the vapour would condense and the total heat flow to the pipe would take place only by conduction in the resulting saturated soil. Under this condition the pure conduction heat flow would be greater near the pipe than in the remote soil, and the temperature "decreases" at the 0.5- and 1.0-ft radii would be smaller than those calculated and in better agreement with observed values. Unfortunately no definite conclusions can be drawn about the effects of the moisture migration due to the possibilities of experimental error in the measured data. The results of the data analysis, however, seem to indicate that moisture migration was not a significant contributor to the buried pipe performance.

(2) OPERATION DURING THE HEATING SEASON

Procedure and Measurements

From October 1950 to May 1951 the pipe loops were used in various parallel arrangements to study the effect of this type of operation on heat extraction rates and ground and pipe temperatures. For the first 2½ weeks in October, pipe C was used alone for 7 hours each day, (excepting week-ends). From November to April operation was continuous for about 100 hours each week. Beginning with loop C, progressively more pipe loops were added to the system during this period. During April and May short tests (3 to 30 hours) were made using the individual pipe loops. The number of compressors in use was also varied.

During the heating season all temperatures were measured twice daily with a precision of 0.1°F. With this precision, however, the heat extraction rates calculated from equation 1 still had considerable possible error. For pipe C, in which the temperature rise in 189 ft of pipe was measured, the possible errors would be 4.5 and 1.0 B.t.u./hr/ft of pipe at brine flow rates of 6000 and 1300 lb/hr respectively. The possible error for the other pipes of the grid would be double these values since temperature rises were measured along only one-half the pipe length (see Fig. 1). In addition, some of the thermocouples along pipe C periodically gave erroneous readings indicating that additional errors may have been present in the measurements on the other pipes which had only two surface thermocouples.

Data Analysis

Since the heat pump was operated arbitrarily the data would be of limited direct use for practical design purposes. The results of the preliminary tests, however, indicated that moisture migration effects were probably small and theoretical calculations might be expected to agree fairly well with observed data. If this could be established the design procedure for ground pipe grids could be given a more rigid basis. In this report, then, test data are discussed only in their relation to theory.

Average heat extraction rates and operating times for the full heating season are given in Table II. The theoretical pipe grid was assumed to operate with the same heat extraction rates and the same time periods as the actual pipe grid. The pipe surface temperatures at various times during the heating season could then be calculated using available theory and the ground thermal properties as determined from the preliminary tests. Since a great number of operating periods had to be considered the heat extraction rate during each period was assumed constant and equal to the average observed heat extraction rate during the same period. Neither moisture migration nor freezing effect theory is available for cyclic operation, thus no attempt was made to account for these phenomena in the calculations.

The method of calculating the pipe surface temperatures was essentially that of reference (1), p. 261, except that the contribution to any one pipe surface temperature due to all other pipes was included. The method involves summation of the line-source relationship (equation 2) to determine the temperature "decrease" at a pipe surface due to both heat extraction from the pipe itself and the other pipes at their respective distances from the pipe. Since the operation of the heat pump was intermittent, the method of summation was similar to that used for the preliminary tests, except that heat extraction rates during periods when the heat pump was inoperative were taken as the negative of those during the preceding operating period. The net mathematical effect of this, in summing the temperature "decreases", is the same as having no heat extraction during the inoperative period. Since the ground did not extend infinitely in all directions as called for by the line-source equation the method of negative images ((1) p. 260) was used. Mathematically, this consists of considering an imaginary pipe grid as far above the ground surface as the actual pipe grid is below (Fig. 12). The heat extraction rates in these imaginary pipes are taken as the negatives of those in the actual pipes; the net mathematical effect of both sets of pipes is equivalent to the assumption of unchanging ground surface and remote environment temperature. Since the object of these calculations was to compare actual and calculated pipe surface temperatures, the effect of the annual weather cycle also had to be considered. The calculated pipe surface temperatures were then determined by subtracting the calculated total temperature "decreases" from the normal ground temperatures at the pipe depth at the same time in the heating season. These normal temperatures were obtained by averaging the temperatures 6 ft beyond the outermost pipes of the grid.

The preliminary tests showed that when a constant heat extraction process was assumed, the best agreement among temperature data was obtained with a heat flow rate of 61 B.t.u./hr/ft. When account was taken of variable heat extraction, however, the apparent true average heat extraction rate for the test period was only 51 B.t.u./hr/ft. The assumption of constant heat extraction also yielded a thermal conductivity of 1.2 B.t.u./hr/ft/°F and a diffusivity of 0.04 ft^2/hr. For the heating season, actual average heat extraction rates were measured, but to simplify the calculations constant heat extraction rates were assumed. Since the duration of test periods in the heating season was about the same as for the preliminary tests, the same relationship was assumed between average heat extraction rates and the required constant rates. In this case, the constant rates would be in the ratio $\frac{61}{51} = 1.2$ to the average measured rates. Equation 2, the basic equation for all calculations, however, shows that multiplying the average heat extraction rates by 1.2 is equivalent to dividing the thermal conductivity value of 1.2 B.t.u./hr/ft/°F by the same factor. In all calculations therefore, the average heat extraction rates were used with a thermal conductivity of 1.0 B.t.u./hr/ft/°F. Details of the calculations are given in Appendix IV.

Summarized observed and calculated temperatures and temperature decreases for the various pipes of the pipe grid at the end of several operating periods during the heating season, (Table III) were in good agreement for essentially the whole heating season. In general, the calculated temperatures were slightly lower than observed values, indicating that a ground pipe grid designed on a theoretical basis would probably be somewhat oversized. The maximum difference noted between observed and calculated temperatures of 6°F, (pipe A, Mar. 22, 1951) was probably due in part to calculation errors caused by the uncertainty of the heat extraction terms.

The amount of overdesign which would result from theoretical calculations can be estimated directly from the observed and theoretical temperature decreases. Since equation 2 shows that heat flow is directly proportional to temperature decrease the results for pipe A on Mar.22, 1951 can be used to estimate the maximum overdesign likely to occur. The percentage overdesign in this instance would be $\frac{(20 - 14)}{14} \times 100 = 43$ per cent. The much better agreement between observed and calculated results for pipe C especially, would indicate a probable overdesign of only 10 to 20 per cent.

(3) NORMAL GROUND TEMPERATURE VARIATIONS

In the previous section the buried pipe or refrigerant temperature that resulted from heat extraction was equal to the normal ground temperature at the pipe depth less the temperature "decrease" due to the heat extraction. Since the ability of a heat pump to extract heat decreases with decreasing temperature, these normal ground temperatures are of primary concern in evaluating the feasibility of the heat pump method of heating buildings in different climates.

In the present investigation no means were derived for determining the normal ground temperatures during the heating season except at the pipe depth. However, measurements were obtained at all depths for an additional two years after the 1950-51 tests. Calculations, (Appendix V) showed the temperatures would be essentially recovered within at most 3 or 4 months after the tests (within $\frac{1}{2}$°F), hence data were representative of normal ground temperatures. Results (Fig. 13) showed that at all depths the temperatures decreased during the winter months until March then increased during the summer months. During the winter months the temperatures were greatest at the greatest depths and lowest near the ground surface. In the summer the opposite trend was apparent and temperatures near the surface were greatest. The mean annual ground temperature for all depths was about 48°F.

DESIGN METHOD FOR A HEAT PUMP GROUND PIPE GRID

The encouraging agreement between observed and calculated pipe surface temperatures during the heating season suggests that a theoretical design method for buried pipe grids can be evolved. Since test results were obtained for only one installation, however, the method must be considered tentative until further field observations are obtained.

The present analysis has shown that the essential information necessary for theoretical calculations consists of the thermal conductivity and diffusivity of the ground and a knowledge of the rates at which heat must be extracted to heat a proposed building. As pointed out in the introduction, a heat pump-ground pipe grid combination is unique among heating methods in that the performance depends on the prior history of heat extraction. Obviously, exact information of this kind cannot be obtained since the heating requirements of a building vary both daily and yearly. To avoid the design calculations resulting from an attempt to account for these variations the following simple equation (1) for steady-state heat flow is sometimes used for design purposes:

$$\triangle T = \frac{Q^1}{2\pi k} \quad \ln\left(\frac{2s}{r}\right), \qquad (6)$$

where:

$\triangle T$ is the temperature "decrease" after steady state is reached

Q^1 is the design or maximum expected heat transfer rate per foot of pipe

s the depth of pipe bury

r the pipe radius.

This equation, so far as is known, has not been verified for use when the heat extraction varies during the heating season. However, it is apparent that a buried pipe grid designed from this equation would always be somewhat oversized. In order to determine how much oversizing may result a more realistic heating cycle design can be considered.

The maximum demand on a heat pump is likely to occur when the normal ground temperatures at the pipe depth have their lowest value i.e., about mid-February in the Ottawa area. The duration of this demand probably would not exceed a week's time in practice, but for design purposes a two-week peak demand design load would probably contain a reasonable factor of safety. The heat load in the month prior to peak load requirements will have greater influence on the pipe surface temperature during the peak period than in the preceding months, and since this month may have been severely cold a reasonable value for the average heat extraction rate would be the highest rate for perhaps a 10-year period. The heat extraction, during previous months which have least effect on the pipe temperatures at the time of the peak load, could probably be taken equal to the average value from the beginning of the season. The values of average heat extraction rates and the 10-year maximum for the month preceding the peak load can be obtained with the aid of degree-day data. Having obtained design heat load data, the monthly heat extraction rates of the ground must be obtained by consideration of the average coefficient of performance of the proposed heat pump during each particular month.

Assuming the above design procedure for a house with a 90,000 B.t.u./hr heat load, calculations for ground having the thermal properties determined in the preliminary tests, degree-day data for the Ottawa, Canada region, and compressor performance data given by Smith et al (7) gave a temperature decrease in February of 25°F (Appendix VI). The simple equation 6 gave a temperature decrease of 27°F. Hence both design methods give very nearly the same result and the simpler method appears to be fully justified.

DISCUSSION

The analysis of the preliminary test data was sufficient to determine the thermal properties of the ground and to show how the heat extraction rates varied with time. Although heat extraction rates were not accurately measured, the agreement of the calculated rates and the apparent mean of the measured rates indicated that neither moisture migration nor freezing contributed appreciably to heat extraction. The general agreement of observed and calculated pipe surface temperatures during the heating season further indicated the minor roles played by moisture migration and freezing.

By comparing two buried pipe design methods it was found that the simpler method based on assumed steady-state operation would probably be sufficient for most purposes. Some caution should be used in design, however, because large vertical moisture content gradients may be present in some soils causing variations in the thermal properties (cf. 6). In addition, moisture contents may vary throughout the year (8). In the present tests these effects were apparently small (Table I).

CONCLUSION

Analysis of the performance of the buried pipe grid of a heat pump has indicated that theory, neglecting freezing and moisture migration effects, can be used with fair accuracy to predict pipe or refrigerant temperatures and heat extraction rates. For this purpose the following simple equation appears to be sufficient:

$$Q^{1} = \frac{2\pi k \Delta T}{\ln\left(\frac{2s}{r}\right)}$$

where, Q^{1} is the maximum or design heat extraction rate per unit length of pipe,

k is the ground thermal conductivity

ΔT is the temperature "decrease" below normal minimum ground temperature at the pipe depth,

s is the depth of pipe bury

r is the pipe radius.

To complete a heat pump design the normal minimum or undisturbed ground temperature must be known so that the actual pipe temperature can be determined and the heat pump selected. Generally, this normal temperature will not be available from measured data, but, methods are available (1) for estimating its value from a knowledge of the ground thermal properties and the annual ambient temperature variations.

Some caution should be exercised with regard to choice of depth of pipe bury and spacing between pipes of a grid. The simple equation as it stands indicates that a pipe buried close to the ground surface would have a lower temperature decrease than one buried at greater depth. The normal ground temperatures are higher at greater depth during the heating season, however, and the resulting pipe surface and refrigerant temperatures would generally be higher for a deep-buried pipe. In general the choice of depth of bury will depend on pipe size, normal ground temperatures and vertical variations in the ground thermal conductivity. For most purposes the most suitable depth would probably be between 4 and 6 ft. The spacing between pipes would not have much influence on the pipe surface temperatures provided the pipes are about 5 ft or more apart. For detailed calculations of this effect, however, the influence of one pipe on another can be determined using the simple buried pipe equation but substituting the distance d, between pipes for r, and the distance

$\sqrt{4s^2 + d^2}$ for 2s.

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REFERENCES

1. Ingersoll, L.R., O.J. Zobel and A.C. Ingersoll, Heat conduction. Revised Edition, University of Wisconsin Press, 1954.

2. Misener, A.D., Heat loss from the heated floor slab at Prince Charles School, London, Ont. Preliminary report to the National Research Council by the University of Western Ontario, March 1956.

3. Kersten, M.S., Thermal properties of soils, p. 161, Highway Research Board, special report No. 2, Frost action in soils. National Academy of Sciences, U.S. 1952.

4. Penrod, E.B., Thermal diffusivity of a soil by the use of a heat pump. Journal Applied Physics, Vol. 21, 1950, p. 425.

5. Kuzmak, J.M. and P.J. Sereda, The mechanism by which water moves through a porous material subjected to a temperature gradient. I. The introduction of a vapour gap into a saturated system. II. Salt tracer and streaming potential to detect flow in the liquid phase. Submitted to Soil Science.

6. Hooper, F.C., An experimental residential heat pump. Canadian Journal of Technology, Vol. 30, pp. 180-197, 1952.

7. Smith, G.S., H. Christensen, and A. Presson, The heat pump for residential heating. I Economics and operating characteristics. Technical Note No. 6, University of Washington Experiment Station, 1948.

8. Smith, G.S., Intermittent ground grids for heat pumps. Transactions A.S.H.A.E., 1956.

TABLE I A

SOIL DRY DENSITY TESTS - NOVEMBER 1949

Position (Fig.2)	Depth (ft)	Dry density (lbs/cu ft)	
a	1	103	Average 96
b	1	90	
a	6	111	Average 102
b	6	93	

TABLE I B

SOIL MOISTURE CONTENT TESTS (% OF DRY WEIGHT)

Date	Position (Fig.2)	Depth (ft below surface)					
		1	2	3	4	5	6
July 20, 1950	c		11.3				12.7
	c		10.4		10.9		11.9
						Average	12.4
Jan. 5, 1951	c	7.2	10.3	14.9	13.4	14.0	12.4
	d		12.2	12.4	13.0	13.3	15.0
	e	7.9	9.7	12.8	11.7	13.7	13.7
						Average	13.7

TABLE II

HEAT EXTRACTION FROM THE GROUND LOOPS

Date	Operating time		Pipe loops in use	Brine flow per loop lb /hr	Heat extraction rates (B.t.u./hr/ft of pipe)				
	Start	Hours of operation			Loop A	Loop B	Loop C	Loop D	Loop E
Oct.1950									
2	9:30 AM	7.2	C	7340			38		
3	9:00 AM	7.2	C	6890			43		
4	9:00 AM	7.2	C	6920			44		
5	8:55 AM	7.1	C	7030			44		
6	9:15 AM	7.1	C	6970			40		
10	9:05 AM	7.6	C	6950			42		
11	8:48 AM	7.5	C	6910			46		
12	8:52 AM	7.4	C	6980			42		
13	9:15 AM	7.3	C	6960			50		
16	9:14 AM	7.1	C, D	5900			40	38	
17	9:06 AM	7.2	C, D	5900			37	39	
18	9:10 AM	7.0	C, D	5900			56	56	
19-20	9:15 AM	31.5	C, D	5900			55	55	
23-26	9:30 AM	77.7	C, D	5890			48	44	
26-27	3:10 PM	25.5	C, D	2170			39	39	
30-3	8:57 AM	103.7	C, D	2590			39	39	
Nov.1950									
6-10	9:30 AM	102.8	C, D	2620			17	18	
13-17	2:40 PM	98.1	C, D	2550			20	21	
20-22	9:43 AM	52.2	C, D	2520			21	21	
22-24	1:55 PM	50.8	C, D	5250			24	26	
27-1	9:05 AM	103.5	C, D	5230			22	22	
Dec.1950									
4-8	9:32 AM	103.0	C, D	5250			21	21	
11-15	10:55 AM	101.7	C, D	5170			20	20	
18-22	10:00 AM	100.6	C, D	5350			21	22	
27-29	9:30 AM	54.8	C, D	5310			19	22	
Jan.1951									
2	9:35 AM	1.9	B, C, D	4910		24	18	18	
2-5	11:30 AM	77.2	B, C, D	2630		30	24	24	
8-12	9:05 AM	103.4	B, C, D	2270		31	27	26	
15-19	10:20 AM	102.2	B, C, D, E	2410		21	20	18	28
22-26	10:00 AM	102.8	B, C, D, E	2590		23	20	19	28
29-2	10:15 AM	100.0	B, C, D, E	2630		23	21	21	28
Feb.1951									
5-9	12:45 PM	99.8	B, C, D, E	2630		18	15	17	23
12-16	9:41 AM	102.8	B, C, D, E	1650		17	17	17	20
19-23	1:33 PM	99.0	A, B, C, D, E	1430	28	21	17	14	22
26-2	10:22 AM	102.4	A, B, C, D, E	1200	28	22	17	16	23
Mar.1951									
5-9	9:15 AM	96.5	A, B, C, D, E	1160	26	21	17	14	22
12-16	9:00 AM	103.0	A, B, C, D, E	1190	24	21	16	15	21
19-22	10:00 AM	78.5	A, B, C, D, E	1180	25	21	16	15	21
27-30	10:20 AM	78.2	A, B, C, D, E	1230	9	8	6	7	7
Apr.1951									
2-6	10:15 AM	102.3	A, B, C, D, E	1190	10	8	7	8	8
9-13	10:00 AM	102.5	A, B, C, D, E	1230	10	8	7	8	8
16-17	10:30 AM	23.0	A, B, C, D, E	1190	4	3	5	5	6
17	9:30 AM	7.4	A	5650	28				
17-18	4:53 PM	15.9	A, B, C, D, E	1380	6	7	5	6	7
18	8:45 AM	7.8	B	6110		24			
18-19	4:30 PM	16.3	A, B, C, D, E	1440	7	6	5	5	6
19	8:45 AM	7.8	C	6620			17		
19-20	4:30 PM	16.3	A, B, C, D, E	1370	6	5	3	5	5
20	8:45 AM	7.8	D	6190				20	
20-21	4:30 PM	16.0	A, B, C, D, E	1050	6	5	4	4	5
21	8:30 AM	9.5	E	5850					17
23-27	10:15 AM	102.3	A, B, C, D, E	1420	6	5	3	4	4
30	1:30 PM	3.0	A	1420					
May 1951									
1	12:30 PM	3.0	A	1420	22				
2	12:30 PM	3.0	A	2830	30				
3	12:30 PM	3.0	A	4120	30				
4	12:30 PM	3.0	A	5360	29				
10-11	9:15 AM	30.7	A, B, C, D, E	1460	18	22	16	17	19
14	8:30 AM	7.5	A	1460	31				
15	12:30 PM	3.3	A	1500	26				
16	12:45 PM	3.0	A	2920	44				
17	12:30 PM	3.3	A	4450	47				
18	12:30 PM	3.3	A	5750	46				

TABLE III - A

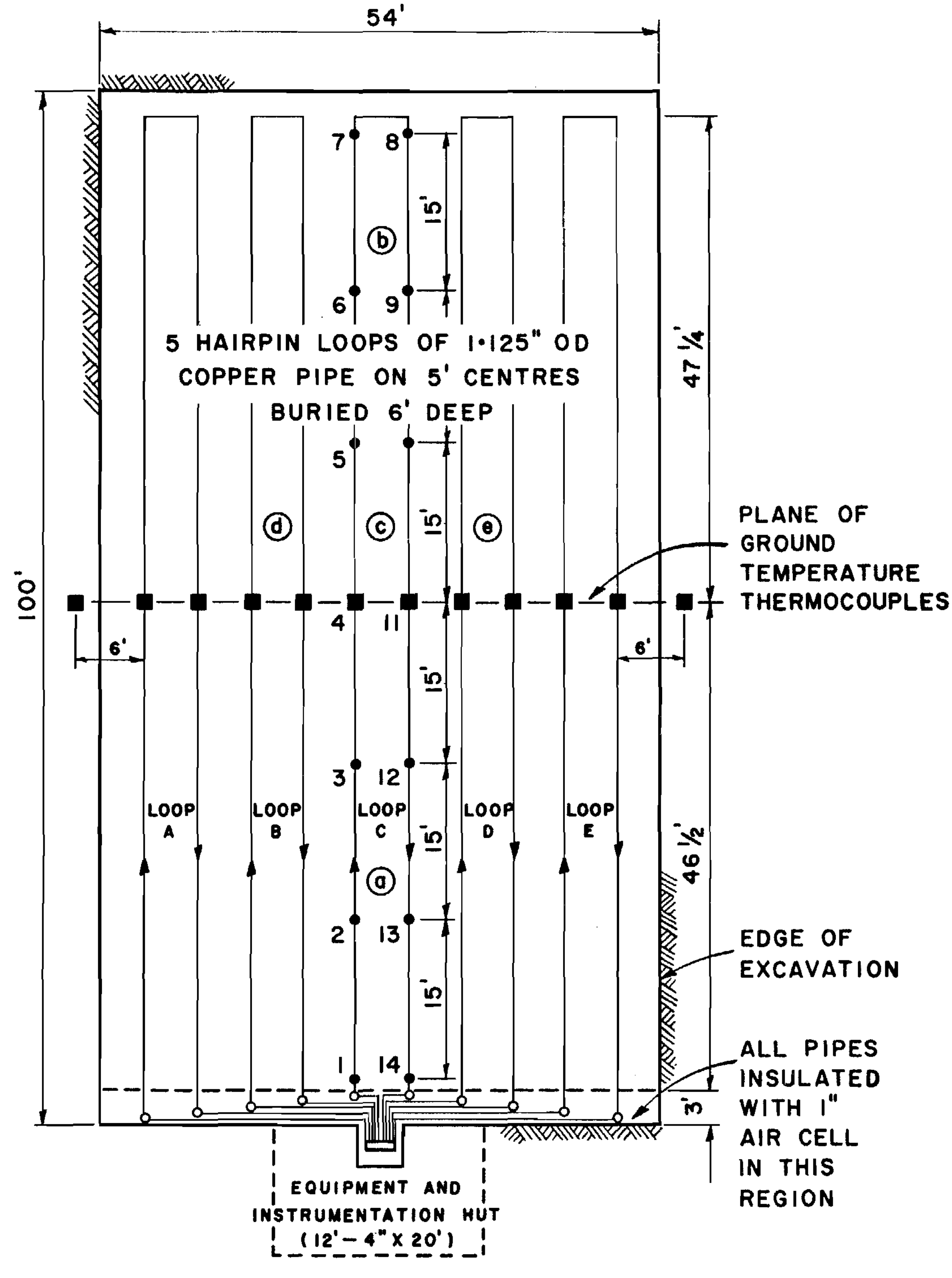
OBSERVED AND CALCULATED PIPE SURFACE TEMPERATURES

Date	Time	Temperature in degrees F									
		Supply pipe A		Supply pipe B		Supply pipe C		Supply pipe D		Return pipe E	
		O	C	O	C	O	C	O	C	O	C
Nov. 3,1950	4:40 PM					21	22	21	21		
Dec. 8,1950	4:32 PM					26	27	27	27		
Jan. 5,1951	4:40 PM					22	23				
Jan.19,1951	4:30 PM					24	23				
Feb. 2,1951	2:15 PM			24	21	24	21	24	20	25	19
Mar.22,1951	4:30 PM	25	19	24	20	24	23	25	24	26	22

TABLE III - B

OBSERVED AND CALCULATED PIPE SURFACE TEMPERATURE "DECREASES"

Date	Time	Temperature in degrees F									
		Supply pipe A		Supply pipe B		Supply pipe C		Supply pipe D		Return pipe E	
		O	C	O	C	O	C	O	C	O	C
Nov. 3,1950	4:40 PM					31	30	31	31		
Dec. 8,1950	4:32 PM					18	18	18	19		
Jan. 5,1951	4:40 PM					20	19				
Jan.19,1951	4:30 PM					17	18				
Feb. 2,1951	2:15 PM			16	20	16	20	16	20	16	21
Mar.22,1951	4:30 PM	14	20	15	20	15	17	15	16	14	18



■ Location of thermouple sticks

● Loop C pipe surface thermocouples

○ Thermocouple wells in pipe

ⓐ to ⓔ Moisture content and density determination stations

FIGURE 1

LAYOUT OF HEAT PUMP GROUND PIPE GRID

Fig. 2 Heat pump excavation - ground coils and thermocouple sticks

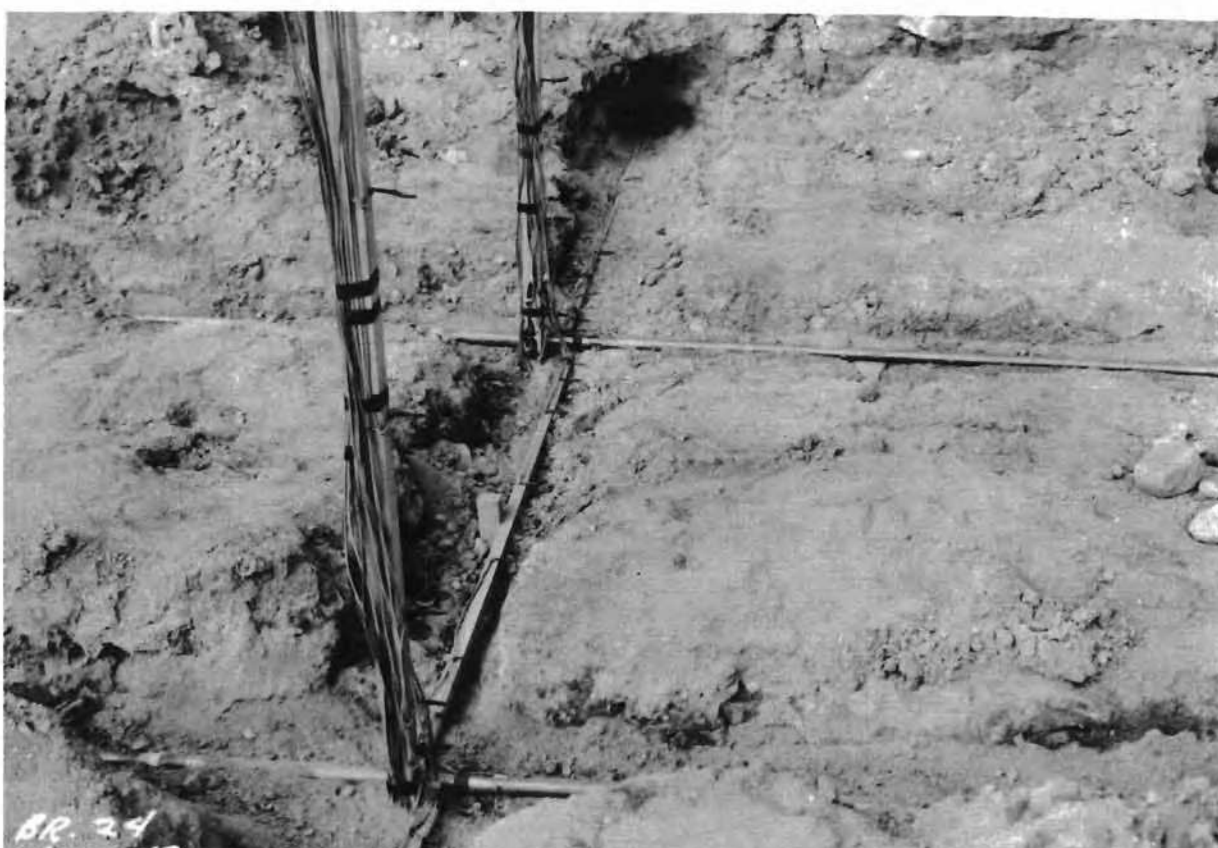


Fig. 3 Detail of ground and pipe surface thermocouple installation

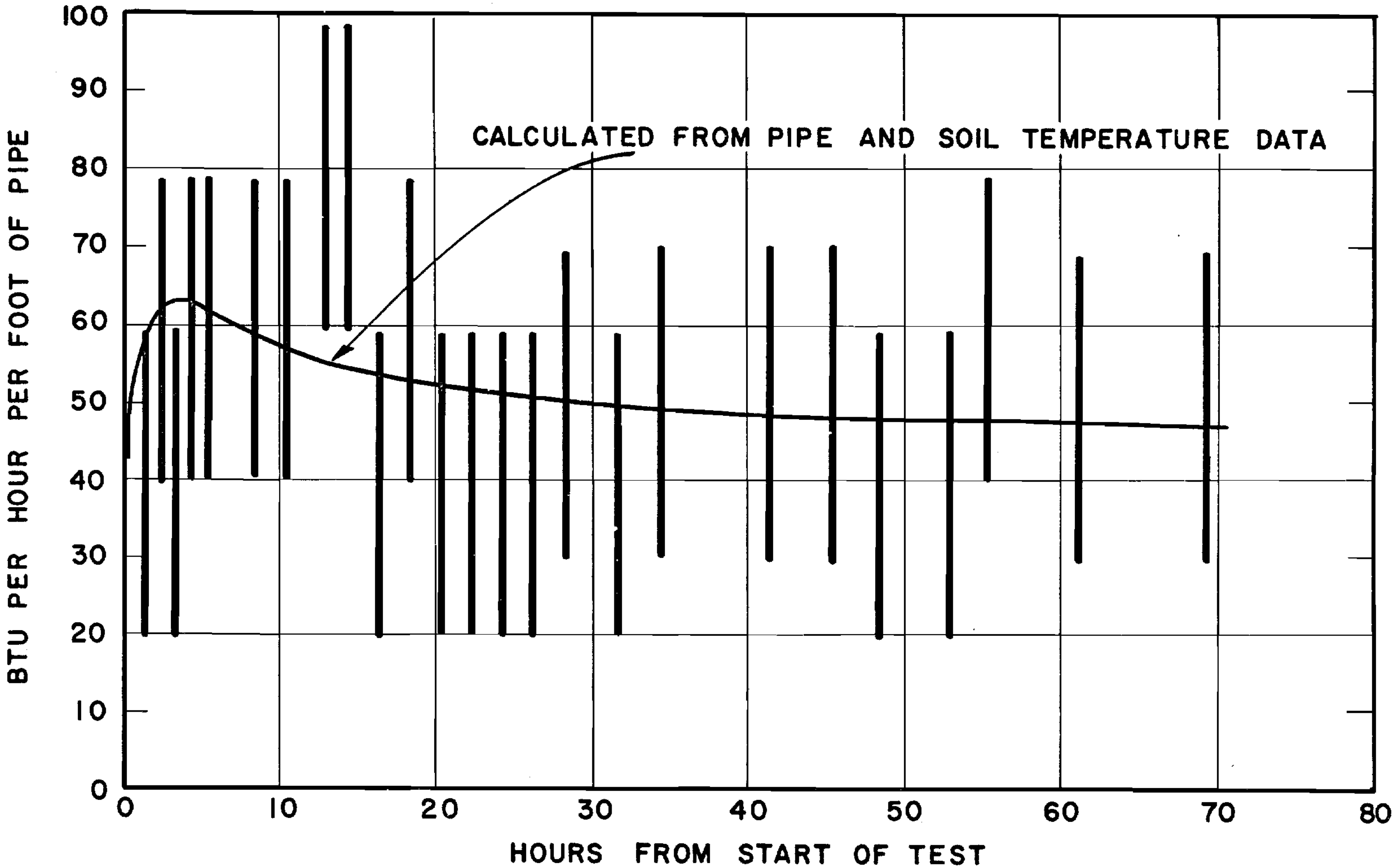


FIGURE 4

HEAT EXTRACTION RATES FROM PIPE C IN PRELIMINARY TEST, JULY 18 TO 21, 1950

(Vertical lines are the limits of the measured heat extraction rates)

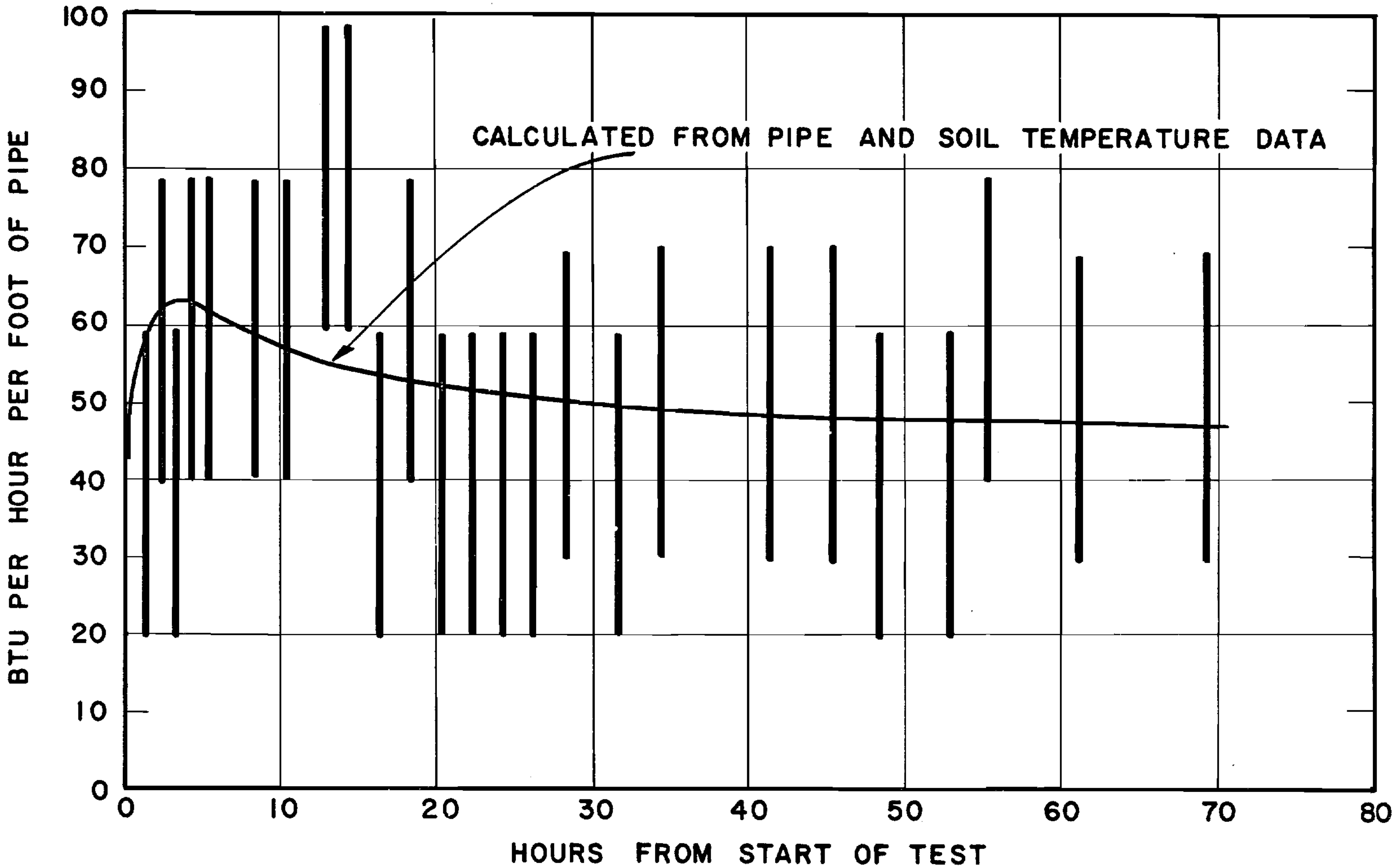


FIGURE 4

HEAT EXTRACTION RATES FROM PIPE C IN PRELIMINARY TEST, JULY 18 TO 21, 1950

(Vertical lines are the limits of the measured heat extraction rates)

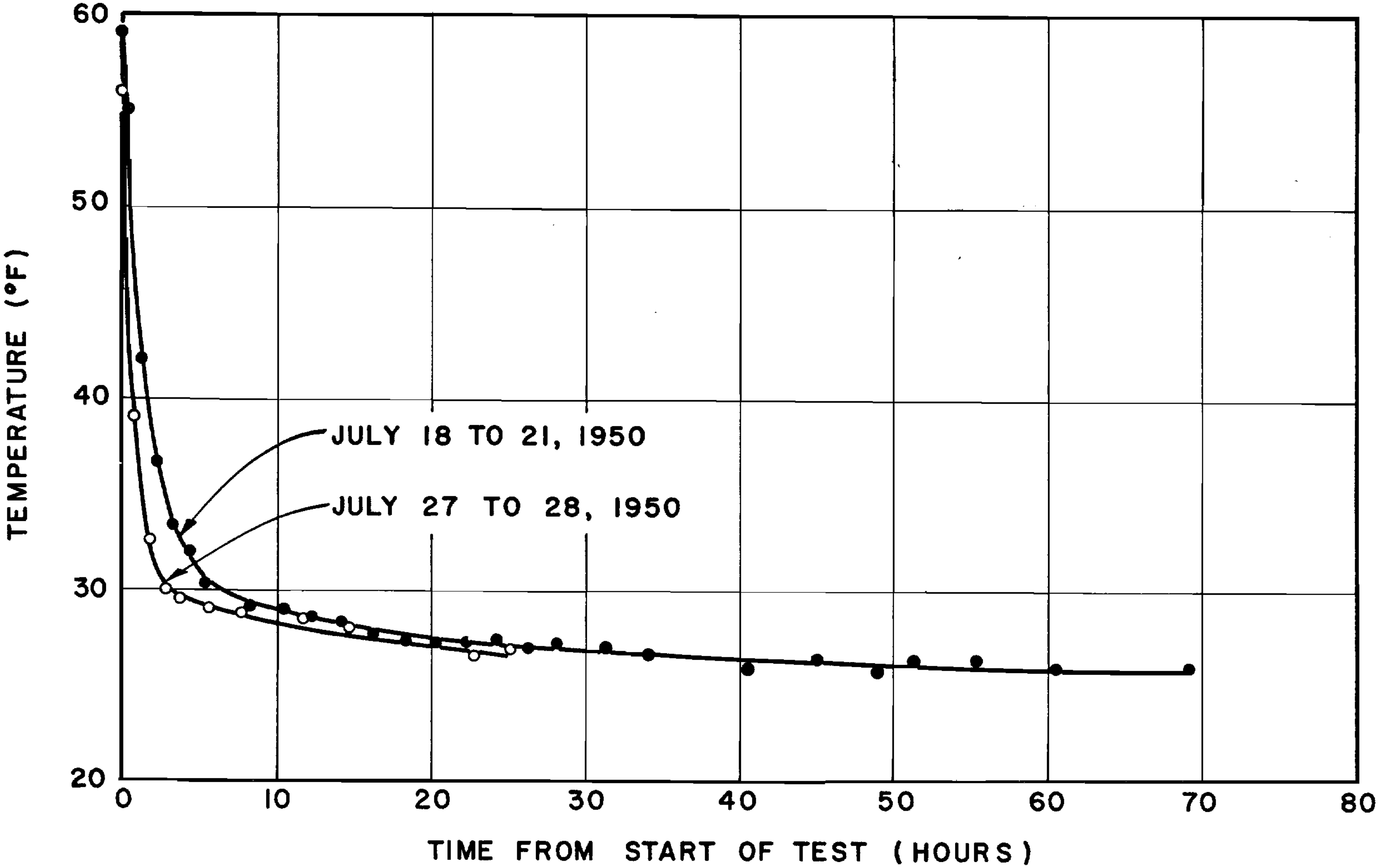
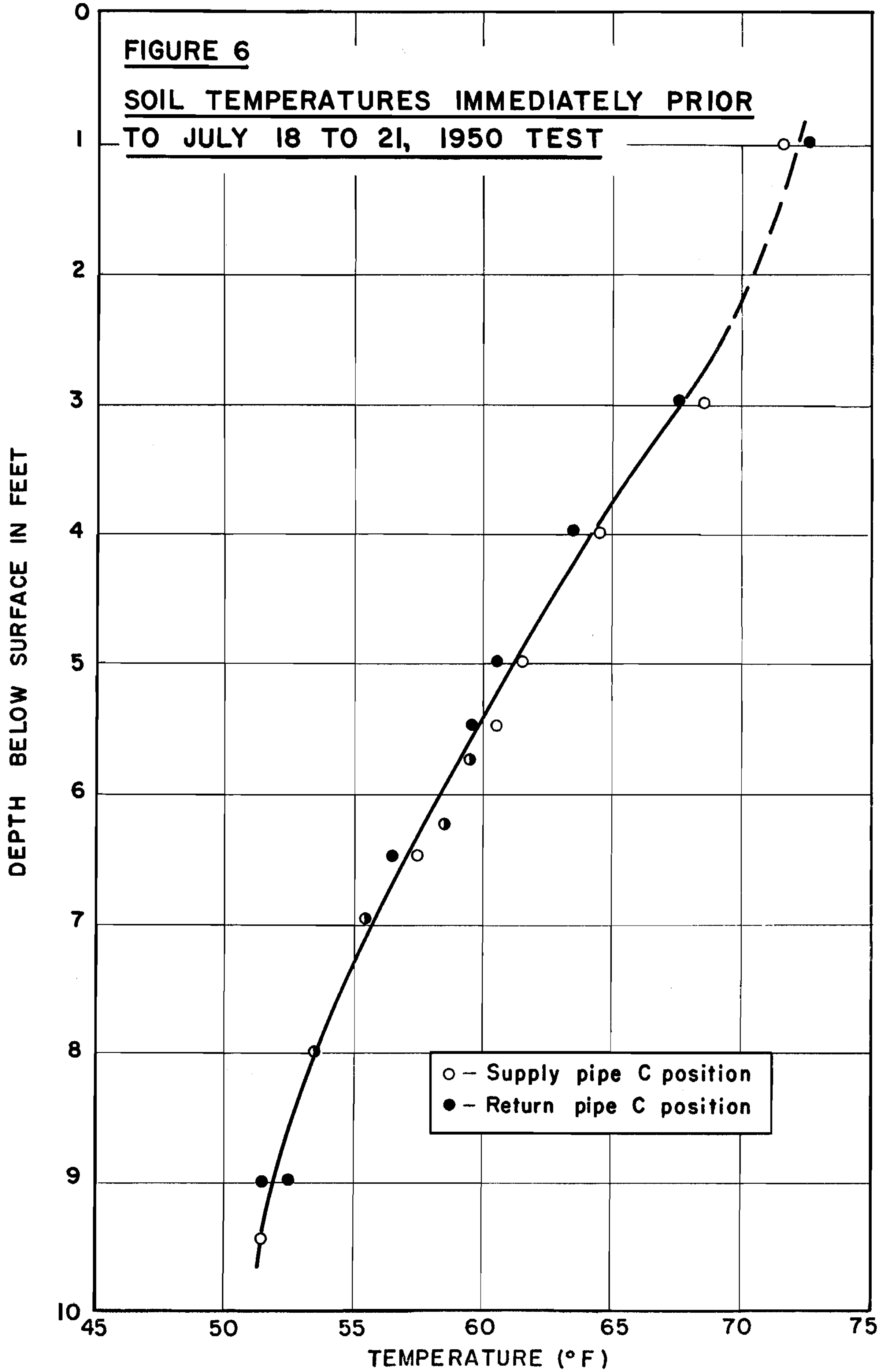


FIGURE 5

PIPE LOOP C SURFACE TEMPERATURES IN PRELIMINARY TESTS

(Average for supply and return pipe)



INTERNAL RPT. 129

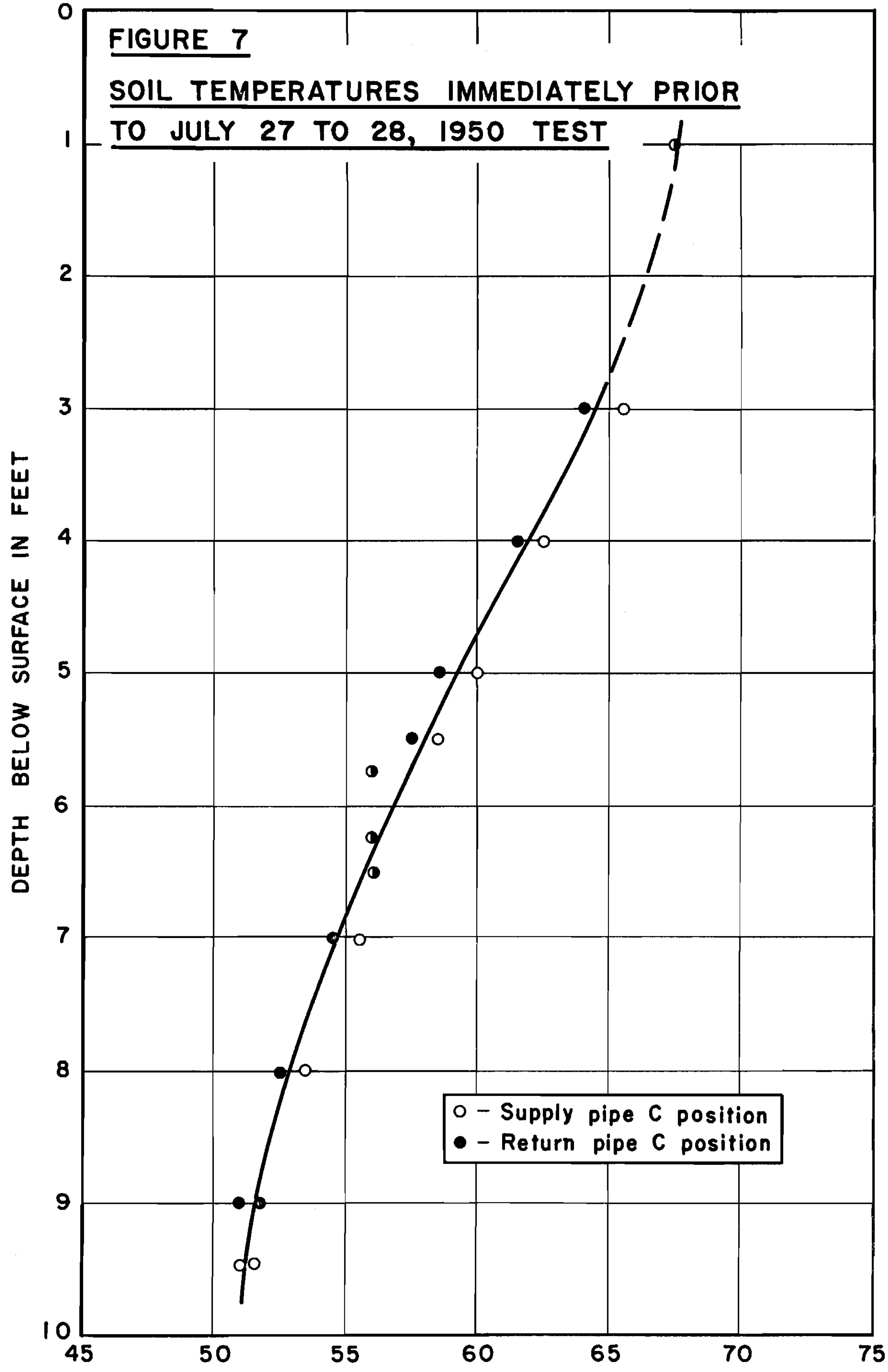


FIGURE 7
SOIL TEMPERATURES IMMEDIATELY PRIOR TO JULY 27 TO 28, 1950 TEST
DEPTH BELOW SURFACE IN FEET
0
1
2
3
4
5
6
7
8
9
10
45
50
55
60
65
70
75
○ – Supply pipe C position
● – Return pipe C position

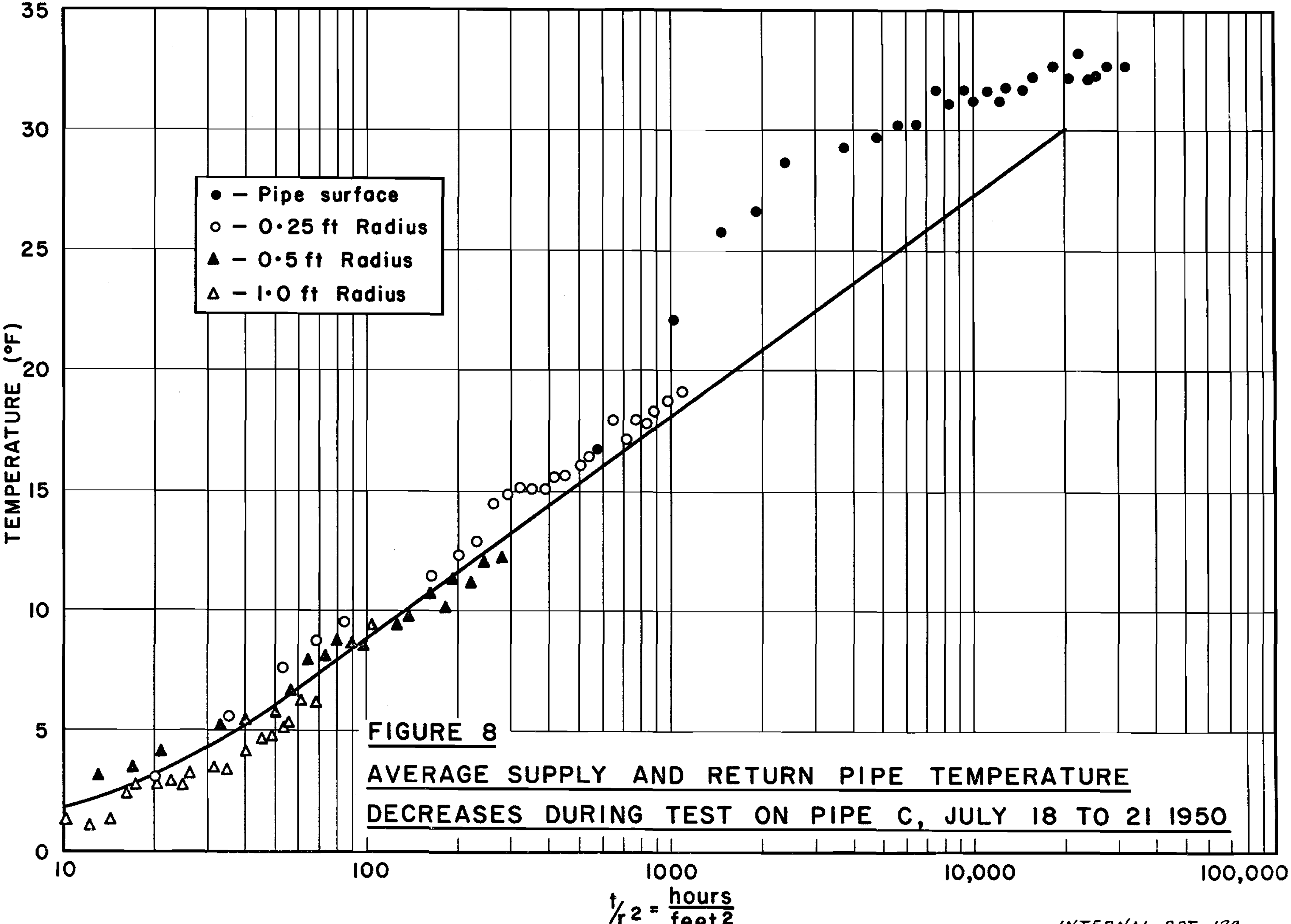


FIGURE 8

AVERAGE SUPPLY AND RETURN PIPE TEMPERATURE DECREASES DURING TEST ON PIPE C, JULY 18 TO 21 1950

INTERNAL RPT. 129

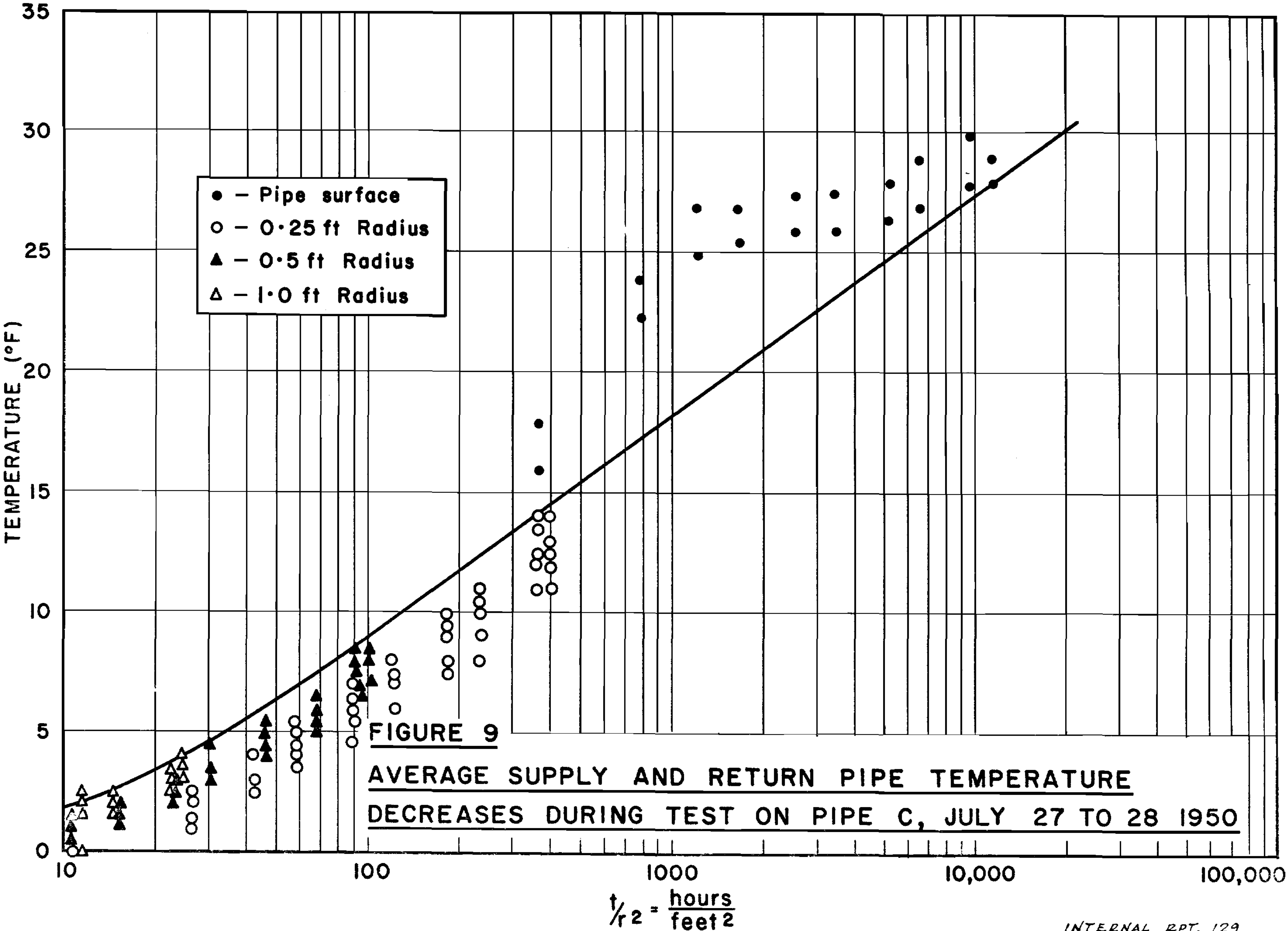


FIGURE 9

AVERAGE SUPPLY AND RETURN PIPE TEMPERATURE DECREASES DURING TEST ON PIPE C, JULY 27 TO 28 1950

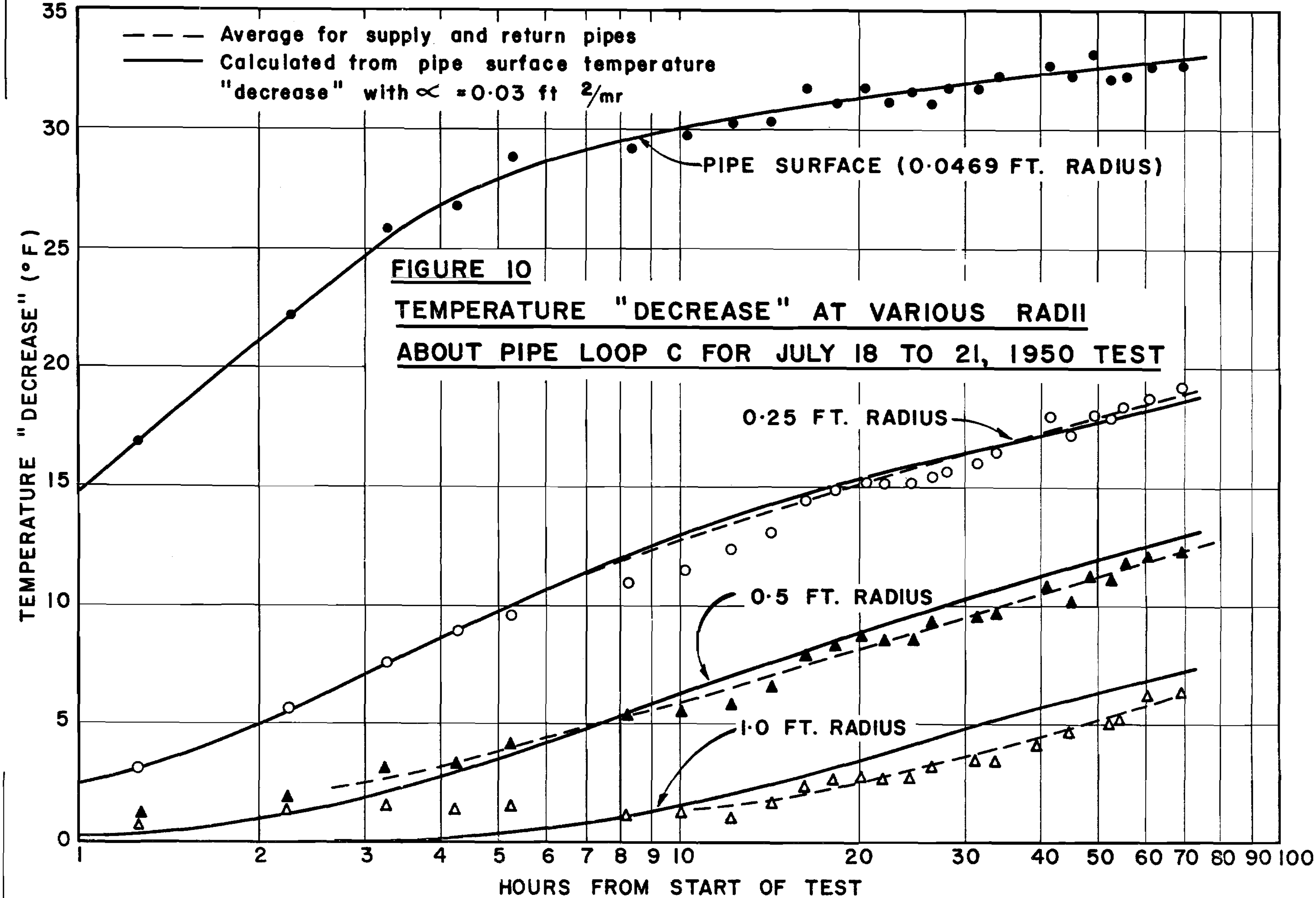


FIGURE 10

TEMPERATURE "DECREASE" AT VARIOUS RADII ABOUT PIPE LOOP C FOR JULY 18 TO 21, 1950 TEST

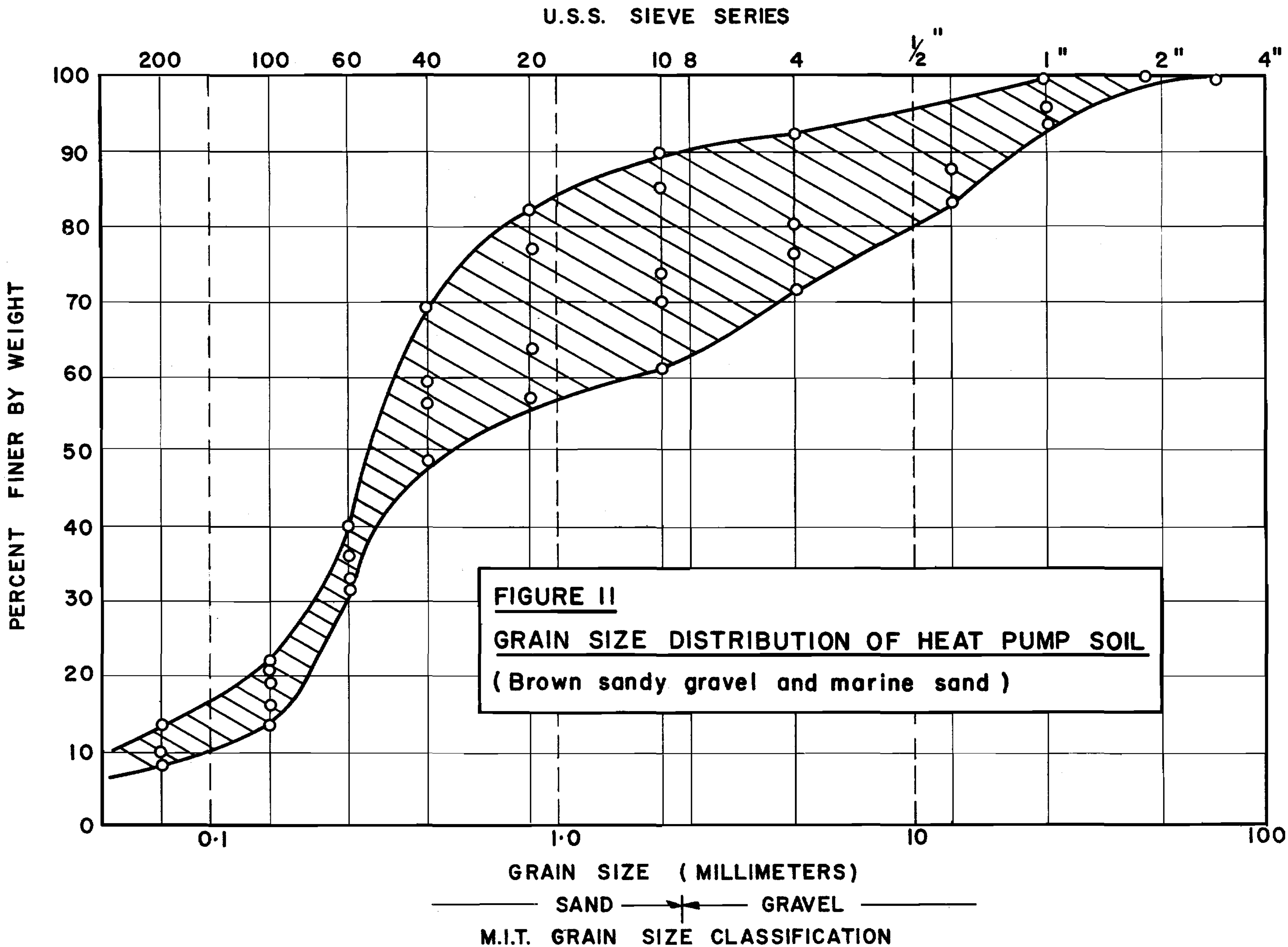


FIGURE 11

GRAIN SIZE DISTRIBUTION OF HEAT PUMP SOIL

(Brown sandy gravel and marine sand)

INTERNAL RPT. 129

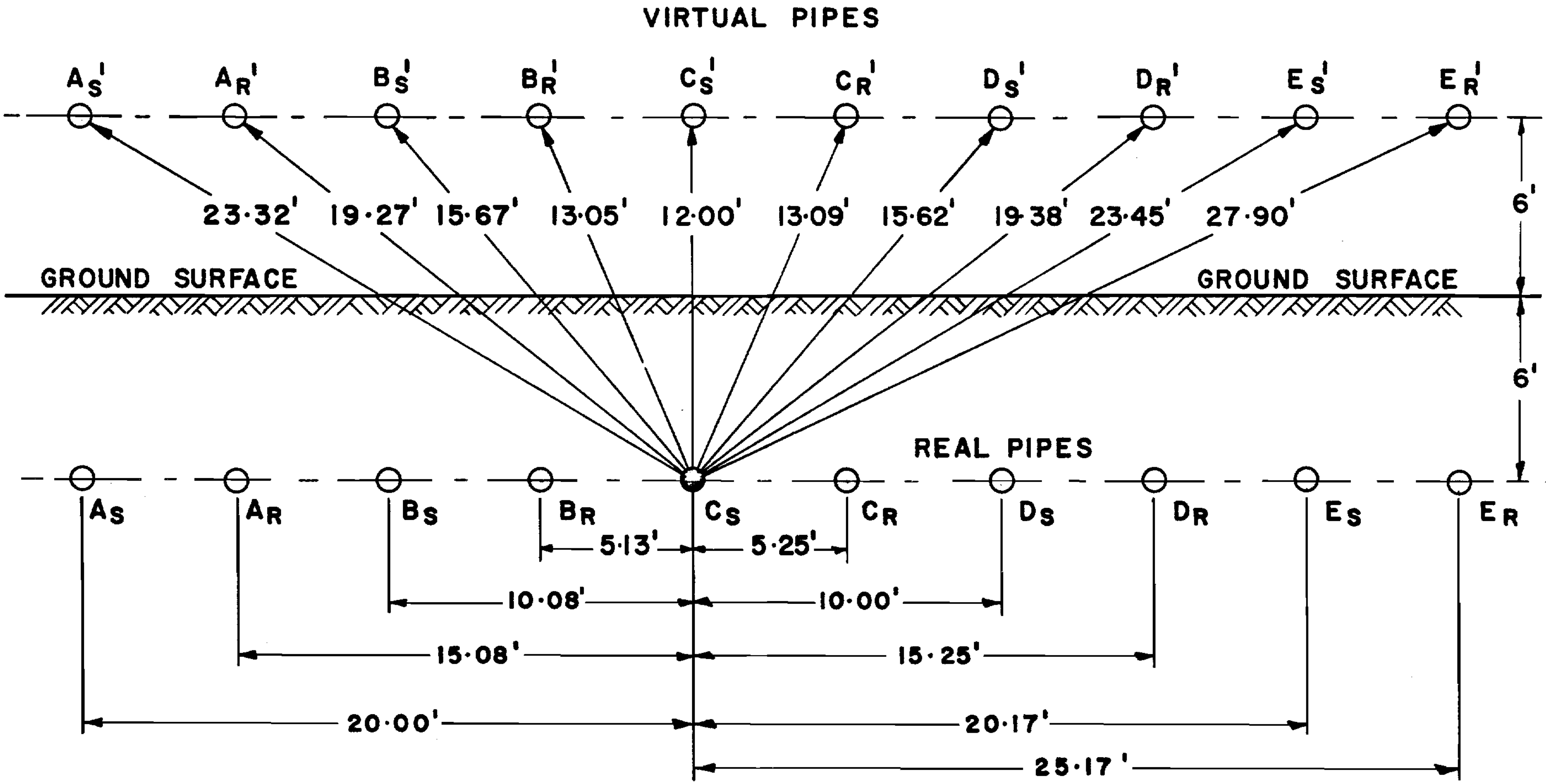


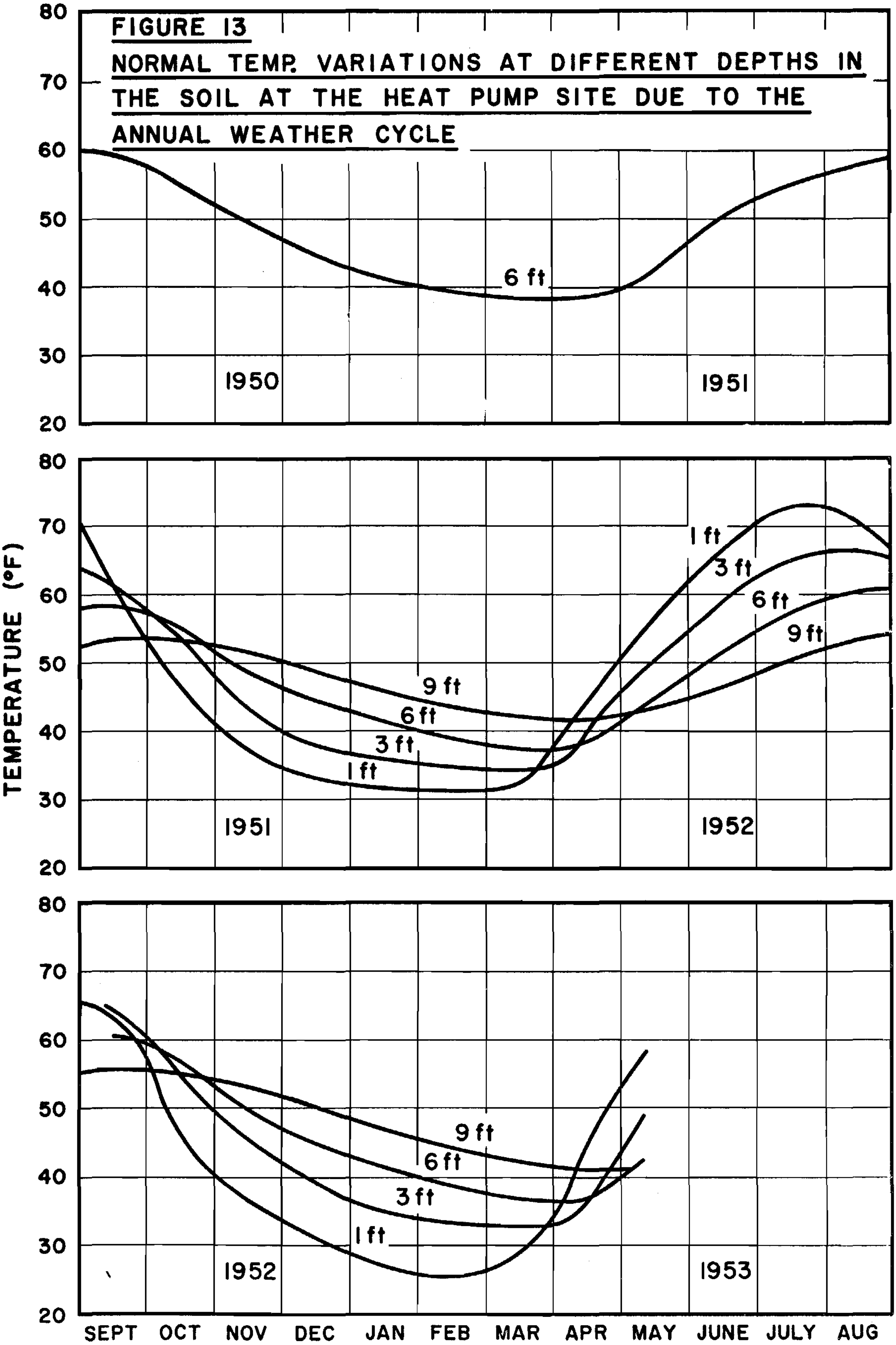
FIGURE 12

DISTANCES OF REAL AND VIRTUAL PIPES FROM SUPPLY PIPE C_S

(For use in calculation of C_S surface temperature)

FIGURE 13

NORMAL TEMP. VARIATIONS AT DIFFERENT DEPTHS IN THE SOIL AT THE HEAT PUMP SITE DUE TO THE ANNUAL WEATHER CYCLE



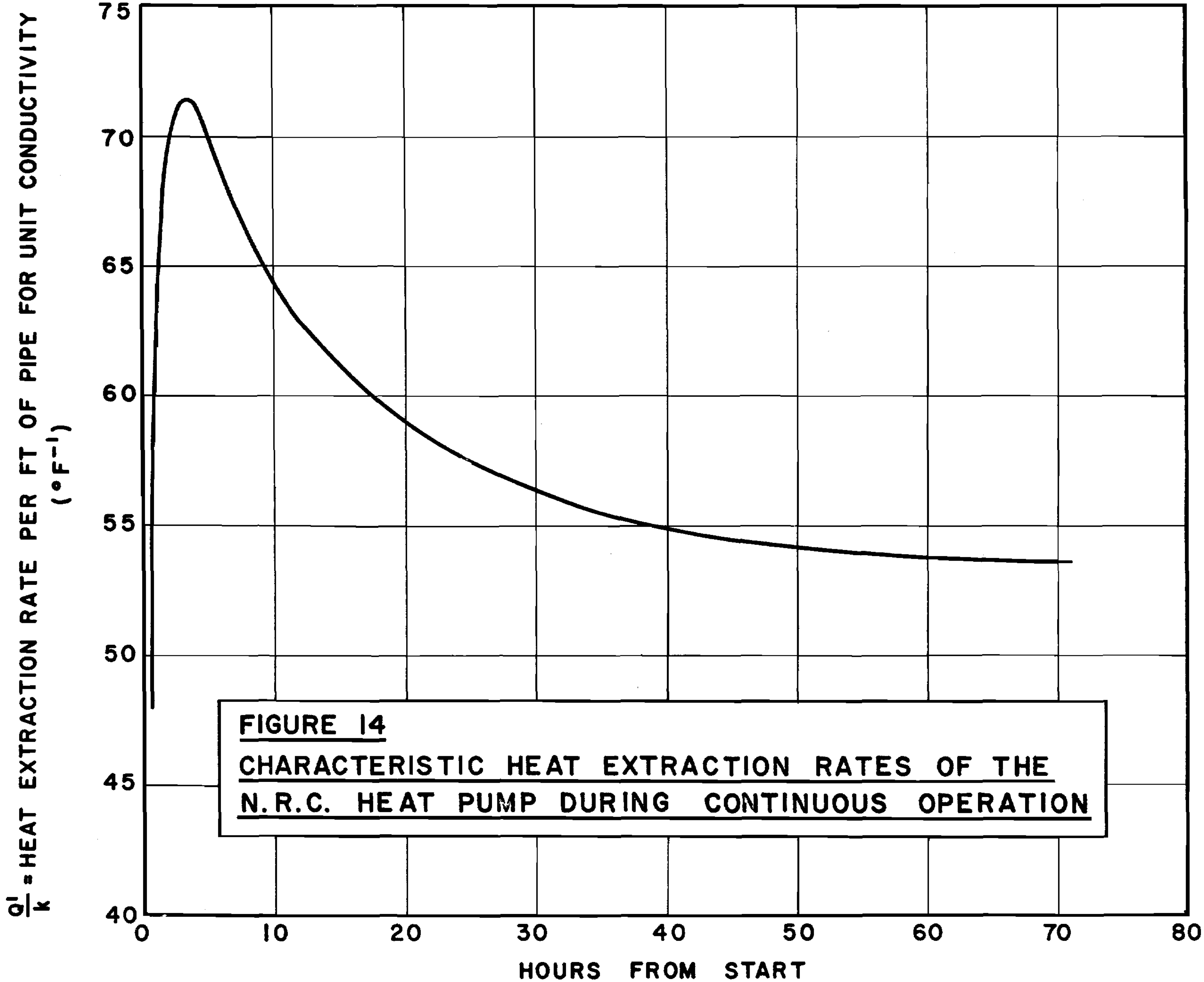


FIGURE 14

CHARACTERISTIC HEAT EXTRACTION RATES OF THE N.R.C. HEAT PUMP DURING CONTINUOUS OPERATION

APPENDIX I

DETERMINATION OF THERMAL CONDUCTIVITY AND DIFFUSIVITY OF THE SOIL - FIRST APPROXIMATION ASSUMING CONSTANT HEAT EXTRACTION RATES

Theory of the line heat source (1) shows that for constant heat extraction rates and $\frac{\alpha t}{r^2}$ greater than 6.25, the temperature "decrease" is,

$$\Delta T = \frac{Q^1}{2\pi k}\left[\ln \frac{2\sqrt{\alpha t}}{r} + \frac{r^2}{8\alpha t} - \frac{r^4}{128\alpha^2 t^2} - 0.2886\right] \qquad \text{(a)}$$

Hence with two different values of t/r^2 subtraction yields,

$$\Delta T_1 - \Delta T_2 = \frac{Q^1}{2\pi k}\left[\ln\sqrt{\frac{t_1}{r_1^2}\cdot\frac{r_2^2}{t_2}} + \frac{1}{8\alpha}\left(\frac{r_1^2}{t_1} - \frac{r_2^2}{t_2}\right) - \frac{1}{128\alpha^2} \times \left(\frac{r_1^4}{t_1^2} - \frac{r_2^4}{t_2^2}\right)\right]$$

Using the July 18 to 20, 1950 test data, assume α - 0.04, and solve for $\frac{Q^1}{k}$.

t/r^2 must be greater than $\frac{6.25}{.04} = 156.$

From Fig. 8; at $t_2/r_2^2 = 10{,}000$, $\Delta T_2 = 27.4°F$

at $t_1/r_1^2 = 200$, $\Delta T_1 = 11.7°F$

$$\therefore\ 11.7 - 27.4 = -15.7 = \frac{1}{2\pi}\left(\frac{Q^1}{k}\right)\left[\ln\sqrt{\frac{200}{10{,}000}} + \frac{1}{.32}\,(.005 - .0001) - 0\right]$$

$$\text{or, } \frac{Q^1}{k} = -\frac{15.7 \times 6.28}{(-1.95 + .02)} = 50.8°F^{-1}$$

Solve equation (a) for α

$$27.4 = \frac{50.8}{2\pi}\left(\ln 2\sqrt{10{,}000} + .0003 - 0 - 0.2886\right)$$

$$\text{or } \ln 2\sqrt{10{,}000\alpha} = 3.68$$

$$\therefore\ 2\sqrt{10{,}000\alpha} = 39.7$$

and $\alpha = 0.04$

Thermal conductivity, diffusivity, density ρ and specific heat Cp are related by the following equation:

$$k = \alpha \rho \ Cp \quad ((1)\ p.4) \qquad (b)$$

From Table IA, the average dry density at the coil depth was 102 lb/cu ft and from Table IB the average moisture content during the test was 12.4 per cent. Although the specific heat was not measured, Kersten (3) shows it can be accurately determined from the following equation,

$$Cp - \frac{100C + M}{100 + M} \text{ B.t.u./lb/°F}$$

where, C is the specific heat of dry soil and M is the moisture content.

Kersten's data also show that C varies only between 0.16 and 0.17 B.t.u./lb/°F for all types of soil. Assuming an average value of 0.165 for the heat pump soil, gives:

$$Cp - \frac{16.5 + 12.4}{112.4} = 0.26 \text{ B.t.u./hr/°F}$$

Hence, from equation b,

$$k = 0.04 \times 102 \times 1.124 \times 0.26 = 1.2 \text{ B.t.u./hr/ft/°F}$$

Since $\frac{Q^1}{k} = 50.8$, the average heat extraction rate Q^1 is

$$1.2 \times 50.8 = 61 \text{ B.t.u./hr/ft.}$$

APPENDIX II

DETERMINATION OF HEAT FLOW RATES FROM RADIAL SOIL TEMPERATURES

When heat extraction rates vary with time, equation 2

$$\Delta T = \frac{Q^1}{2\pi k} \quad I\left(\frac{r}{2\sqrt{\alpha t}}\right),$$

can be used to determine the value of thermal diffusivity α from ΔT data at various radii from a buried pipe. With this α the value of $\frac{Q^1}{k}$ can then be determined at all times.

The temperature "decrease" ΔT, at radius r at any time t_1 can be written as the sum of increments due to constant Q^1 for short periods of time i.e.,

$$\Delta T = \frac{Q_1^{\ 1}}{2\pi k} \ I\left(\frac{r}{2\sqrt{\alpha t}}\right) + \frac{\left(Q_2^{\ 1} - Q_1^{\ 1}\right)}{2\pi k} \ I \ \frac{r}{2\sqrt{\alpha\left(t - t_1^{\ 1}\right)}} \qquad (c)$$

$$+ \text{----} \frac{\left(Q_3^{\ 1} - Q_2^{\ 1}\right)}{2\pi k} \ I\left(\frac{r}{2\sqrt{\alpha\left(t - t_2\right)}}\right) + \text{--} \text{ etc.}$$

For authority see (1) p. 261.

Where $Q_1^{\ 1}$, $Q_2^{\ 1}$, $Q_3^{\ 1}$, are the average heat extraction rates in the first, second and third periods, and t_1 and t_2 are the times to the end of the first and second periods.

Equation c was used with the temperature "decrease" data for the pipe surface (Fig. 10) to calculate the values of $\frac{Q^1}{k}$ at various times for the different values of α. The equation was then used to calculate ΔT at 0.25-, 0.5- and 1.0-ft radii to determine which value of α gave the best solution. For the first 10 hours of test 1-hr increments $t_1^{\ 1}$ were used then 5-hr increments between 10 and 20 hours, and finally 10-hour increments for the remainder of the test period. Values of the integral I were obtained from (1), p.297.

The value of α which gave the best agreement of calculated and actual temperature "decreases" at all radii was 0.03 ft^2/hr. The calculated "decreases" at all radii are given in Figure 10. Values of $\frac{Q_1 1}{k}$ are given in Fig. 14.

The true thermal conductivity would be $k = \alpha\rho$ Cp, (Appendix I),

$= 0.03 \times 102 \times 1.124 \times 0.26 = 0.89$ B.t.u./hr/ft/°F.

Values of Q^1 calculated from Fig. 14 with this value of thermal conductivity are given in Fig. 4.

Example of determination of $\frac{Q^1}{k}$

For the first hour:

ΔT (Fig. 10, pipe surface) = 14.8°F

α (assumed) = 0.03 ft^2/hr

pipe radius, r = 0.0469 ft.

$$\therefore \frac{r}{2\sqrt{\alpha t}} = \frac{0.0469}{2\sqrt{0.03 \times 1.0}} = 0.135$$

I (0.135) = 1.72 ((1) p.297).

From equation c, for one period,

$$\frac{Q_1^{\,1}}{k} = \frac{2 \times 14.8}{1.72} = 54.0°F^{-1}.$$

For the second hour:

ΔT (Fig. 10) = 21.2°F

$$\frac{r}{2\sqrt{\alpha t}} = \frac{0.0469}{2\sqrt{0.03 \times 2}} = 0.096$$

I (0.096) = 2.06,

$\therefore$ From equation c,

$$21.2 = \frac{54.0}{2\pi} \times 2.06 + \frac{1}{2\pi}\left(\frac{Q_2^{\,1} - Q_1^{\,1}}{k}\right) \times 1.72$$

$$\text{or } \left(\frac{Q_2^{\,1} - Q_1^{\,1}}{k}\right) = \frac{2\pi \times 21.2 - 54.0 \times 2.06}{1.72} = 12.7°F^{-1}$$

$$\therefore \frac{Q_2^{\,1}}{k} = 54.0 + 12.7 = 66.7°F^{-1}$$

APPENDIX III

ESTIMATE OF THE EFFECT OF FREEZING ON PIPE SURFACE TEMPERATURE DURING THE JULY 18 TO 21, 1950 PRELIMINARY TEST.

For constant soil properties and no moisture migration the equation relating heat flow rate, freezing temperature and latent heat is,

$$\left[\frac{Q^1}{2\pi} - k\frac{(T_o - T_f)}{I\left(\sqrt{\frac{b}{\alpha}}\right)}\right] e^{-b/\alpha} = 2Lb; \qquad (d)$$

and the equation giving the temperature "decrease" at any time is,

$$\Delta T = T_o - T_f + \frac{Q^1}{2\pi k} \quad I\left(\sqrt{\frac{b}{\alpha}}\right)\left[\frac{I\left(\frac{r}{2\sqrt{\alpha t}}\right)}{I\left(\sqrt{\frac{b}{\alpha}}\right)} - 1\right], \qquad (e)$$

for $r \leqslant$ freezing radius.

where: T_o = initial pipe surface temperature = 59°F

T_f = freezing temperature = 32°F

L = latent heat of fusion per cu ft of soil = 144 x .124 x 102 = 1820 B.t.u./cu ft.

b = a constant

Q^1 = heat extraction rate per ft of coil = 61 B.t.u./hr/ft (Appendix II)

k = thermal conductivity = 1.2 B.t.u./hr/ft/°F (Appendix II)

α = thermal diffusivity = 0.04 ft^2/hr (Appendix II)

r = pipe radius = 0.0469 ft.

t = test duration

With the above conditions equation d becomes,

$$\frac{61}{2\pi} - 1.2\,\frac{(59 - 32)}{I\left(\sqrt{\frac{b}{0.04}}\right)} \quad e^{\left(\frac{-b}{0.04}\right)} = 2 \times 1820\ b.$$

Solve for b. This gives $b = 2.75 \times 10^{-5}$, with $I\left(\sqrt{\frac{b}{0.04}}\right) = 3.36$, (values of I given in (1), p.257)

Without freezing the temperature decrease would be,

$$\Delta T_1 = \frac{Q^1}{2\pi k} \quad I\left(\frac{r}{2\sqrt{\alpha t}}\right) \qquad (f)$$

$\therefore$ From equations e and f the error due to neglect of freezing which is independent of time is:

$$\Delta T_1 - \Delta T = -(T_o - T_f) + \frac{Q^1}{2\pi k} \quad I\left(\sqrt{\frac{b}{\alpha}}\right)$$

$$= 32 - 59 + \frac{61}{2\pi \text{x} 1.2} \text{ x } 3.36$$

$$= -27 + 27.2 = 0.2°F.$$

APPENDIX IV

CALCULATION OF PIPE SURFACE TEMPERATURES DURING THE HEATING SEASON

The calculations involved in determining the pipe surface temperatures at any time during the heating season, although not difficult, were very extensive. Consequently the data used in determining only one temperature (supply pipe Cs for 2:15 p.m. February 2, 1951), will be given here as an example.

Calculations (Tables A1, A2, A3,) involved only the methods of (1) and use of the line-source relationship, equation 2

$$\Delta T = \frac{Q^1}{2\pi k} \quad I\left(\frac{r}{2\sqrt{\alpha t}}\right), \tag{2}$$

with appropriate substitutions for Q^1, r, and t. (k was taken as 1.0 B.t.u./hr/ft/°F and α as 0.04 ft^2/hr.) The contribution of each pipe to the cooling of pipe Cs was calculated from equation 2 using the distance between the pipe in question and Cs (Fig. 12), and the difference between the time when any particular period of operation began or ended and the final time, (Feb.2, 2:15 p.m.). The value of the function I was first calculated, then the difference between I for the beginning and end of the operating period was multiplied by the appropriate value of $\frac{Q^1}{2\pi k}$ to obtain the net contribution to temperature decrease. For the virtual or imaginary pipes above the ground the values of $\frac{Q^1}{2\pi k}$ were the negatives of those in the corresponding real pipes, hence their temperature "decrease" contributions were negative.

For times greater than 2000 hours the derivative of equation 2, with respect to time was used to obtain better computation accuracy. Since the operating periods of the heat pump were small compared to 2000 hours the differential form of the derivative could be used, i.e.,

$$\Delta(\Delta T) = \frac{Q^1}{4\pi k} \left(\frac{\Delta t}{tm}\right) e^{\frac{-r^2}{4\alpha tm}}$$

Where: $\Delta(\Delta T)$ is the contribution to temperature "decrease" caused by heat extraction at rate Q^1 for time period Δt; and tm is the mean time from the period in question to the final time. (Note: in Table A, $\Delta(\Delta T)$ is written simply ΔT.)

The total contributions of all pipes to the cooling of pipe Cs (Table A), were as follows:

Pipe C	17.87°F
Pipe D	0.93°F
Pipe B	1.53°F
Pipe E	≈ 0
Total	20°F

In the calculations it was necessary to have high precision due to the additive effect of the temperature reduction elements. The final total was rounded off to two figures, however, since the factor Q^1 had no greater precision.

The calculated temperature at Cs was determined by subtracting the total temperature "decrease" from the normal ground temperature at the pipe depth. This normal temperature was taken as the average of the measured temperatures 6 ft beyond the outermost pipes A and E, (Fig. 1). A small correction of 1.1°F (calculated by the methods above), was added to the normal temperature beyond pipe E to account for the cooling due to the pipe. No correction was necessary for the normal temperature beyond pipe A since this pipe had not been in use. The two corrected normal temperatures were 40.5° and 41.4°F, thus the calculated temperature of Cs was 41 - 20 = 21°F.

APPENDIX IV - TABLE AI

CALCULATED CONTRIBUTION OF PIPE LOOP C TO COOLING OF ITS SUPPLY PIPE Cs FOR FEB. 2, 1951

Date (see also Table III)	t Hours to Feb. 2, 2:15 pm	Q^1 B.t.u./hr/ft for pipe C (see Table III)	Pipe Cs r = 0.0469 ft			Pipe Cs' r = 12.00 ft			Pipe C_R r = 5.25 ft			Pipe C_R' r = 13.09 ft		
			$\frac{r}{2\sqrt{\alpha t}}$	I	ΔT (°F)	$\frac{r}{2\sqrt{\alpha t}}$	I	ΔT (°F)	$\frac{r}{2\sqrt{\alpha t}}$	I	ΔT (°F)	$\frac{r}{2\sqrt{\alpha t}}$	I	ΔT (°F)
Feb. 2	0													
Jan. 29	100.0	+ 21	.0117	4.17	+13.95	3.00	-	-	1.31	.036	0.12	3.27	-	-
" 26	165.5	- 20	.00910	4.41	0.80	2.33	.0003	-.01	1.02	.102	0.33	2.54	-	-
" 22	268.3	+ 20	.00714	4.66		1.83	.004		.802	.207		2.00	.0015	
" 19	333.8	- 20	.00638	4.77	0.41	1.63	.010	-.04	.717	.273	0.26	1.78	.005	-.03
" 15	435.9	+ 20	.00560	4.90		1.43	.023		.629	.355		1.56	.013	
" 12	501.8	- 27	.00523	4.97	0.56	1.34	.033	-.08	.587	.403	0.29	1.46	.020	-.05
" 8	605.2	+ 27	.00476	5.06		1.22	.051		.535	.470		1.33	.031	
" 5	669.6	- 24	.00453	5.11	0.23	1.16	.063	-.06	.509	.521	0.13	1.27	.043	-.05
" 2	748.7	+ 24	.00428	5.17		1.10	.079		.480	.554		1.20	.055	
Dec. 29	837.9	- 19	.00405	5.22	0.09	1.04	.097	-.03	.454	.600	0.08	1.13	.071	-.03
" 27	892.8	+ 19	.00392	5.25		1.00	.108		.440	.625		1.10	.080	
" 22	1007.7	- 21	.00369	5.32	0.13	.945	.132	-.07	.415	.678	0.12	1.03	.098	-.06
" 18	1108.3	+ 21	.00353	5.36		.904	.152		.396	.715		.985	.115	
" 15	1173.7	- 20	.00342	5.39	0.13	.877	.165	-.06	.384	.740	0.12	.955	.128	-.04
" 11	1275.4	+ 20	.00328	5.43		.841	.185		.368	.779		.916	.141	
" 8	1341.7	- 21	.00320	5.46	0.10	.820	.197	-.07	.359	.800	0.12	.893	.157	-.05
" 4	1444.7	+ 21	.00308	5.49		.789	.217		.346	.835		.862	.172	
" 1	1509.7	- 22	.00302	5.51	0.14	.774	.228	-.07	.339	.850	0.11	.844	.182	-.06
Nov. 27	1613.2	+ 22	.00292	5.55		.748	.248		.328	.880		.817	.199	
" 24	1677.5	- 22	.00286	5.57	0.11	.733	.259	-.06	.321	.900	0.09	.799	.210	-.06
" 20	1780.5	+ 22	.00278	5.60		.713	.276		.312	.925		.777	.226	
" 17	1845.5	- 20	.00273	5.61	0.13	.700	.286	-.05	.307	.940	0.08	.764	.235	-.05
" 13	1943.6	+ 20	.00266	5.65		.682	.303		.299	.965		.744	.250	
					+16.78 Total			-0.60 Total			+1.85 Total			-0.48 Total

Date (see also Table III)	$\frac{\Delta t}{t_{av.}}$	Q^1 B.t.u./hr/ft for pipe C (see Table III)	Pipe Cs r = 0.0469 ft			Pipe Cs' r = 12.00 ft			Pipe C_R r = 5.25 ft			Pipe C_R' r = 13.09 ft		
			$\frac{r^2}{4\alpha t}$	$e^{\frac{-r^2}{4\alpha t}}$	ΔT (°F)	$\frac{r^2}{4\alpha t}$	$e^{\frac{-r^2}{4\alpha t}}$	ΔT (°F)	$\frac{r^2}{4\alpha t}$	$e^{\frac{-r^2}{4\alpha t}}$	ΔT (°F)	$\frac{r^2}{4\alpha t}$	$e^{\frac{-r^2}{4\alpha t}}$	ΔT (°F)
Nov. 6-10	.0498	17	↑	↑	.067	0.44	0.64	-.043	.084	0.92	.062	0.52	0.59	-.040
Oct.30-Nov.3	.0469	39			.145	0.40	0.67	-.097	.077	0.92	.134	0.48	0.62	-.090
Oct.23-27	.0430	46			.157	0.37	0.69	-.108	.072	0.93	.146	0.45	0.64	-.101
" 19-20	.0125	55			.055	0.36	0.70	-.039	.068	0.93	.051	0.42	0.66	-.036
" 18	.0027	56			.012	0.35	0.70	-.008	.067	0.93	.011	0.42	0.66	-.008
" 17	.0028	37			.008	0.35	0.70	-.006	.067	0.93	.007	0.41	0.66	-.005
" 16	.0027	40			.009	0.34	0.71	-.006	.066	0.93	.008	0.41	0.66	-.006
" 13	.0027	50			.011	0.34	0.71	-.009	.064	0.94	.010	0.40	0.67	-.007
" 12	.0027	42	0.00	1.00	.009	0.33	0.72	-.006	.064	0.94	.008	0.39	0.68	-.006
" 11	.0028	46			.010	0.33	0.72	-.007	.063	0.94	.010	0.39	0.68	-.007
" 10	.0027	42			.009	0.33	0.72	-.006	.063	0.94	.008	0.39	0.68	-.006
" 6	.0025	40			.008	0.32	0.73	-.006	.060	0.94	.008	0.37	0.69	-.006
" 5	.0025	44			.009	0.31	0.73	-.006	.060	0.94	.008	0.37	0.69	-.006
" 4	.0025	44			.009	0.31	0.73	-.006	.059	0.94	.008	0.37	0.69	-.006
" 3	.0025	43			.008	0.31	0.73	-.006	.059	0.94	.007	0.37	0.69	-.005
" 2	.0024	38	↓	↓	.007	0.30	0.74	-.005	.058	0.94	.007	0.36	0.70	-.005
					+0.53 Total			-0.36 Total			+0.49 Total			-0.34 Total

Net total contribution of coil C = 17.87°F

APPENDIX IV - TABLE A2

CALCULATED CONTRIBUTION OF PIPE LOOP D TO COOLING OF SUPPLY PIPE Cs FOR FEB. 2, 1951

Date (see also Table III)	t Hours to Feb. 2, 2:15 pm	Q^1 B.t.u./hr/ft for pipe D (see Table III)	Pipe Ds r = 10.00 ft			Pipe Ds' r = 15.62 ft			Pipe D_R r = 15.25 ft			Pipe D_R' r = 19.38 ft		
			$\frac{r}{2\sqrt{\alpha t}}$	I	ΔT (°F)	$\frac{r}{2\sqrt{\alpha t}}$	I	ΔT (°F)	$\frac{r}{2\sqrt{\alpha t}}$	I	ΔT (°F)	$\frac{r}{2\sqrt{\alpha t}}$	I	ΔT (°F)
Feb. 2	0													
Jan. 29	100.0	+ 21	2.56	-	-	4.00	-	-	3.89	-	-	4.94	-	-
" 26	165.5	- 19	1.98	.002		3.10	-		3.02	-		3.83	-	
" 22	268.3	+ 19	1.57	.013	.03	2.46	-	-	2.40	-	-	3.05	-	-
" 19	333.8	- 18	1.39	.027		2.18	.001		2.12	.001		2.69	-	
" 15	435.9	+ 18	1.22	.051	.07	1.91	.003	-.01	1.86	.004	.01	2.36	-	-
" 12	501.8	- 26	1.14	.068		1.79	.005		1.74	.006		2.21	.001	
" 8	605.2	+ 26	1.04	.095	.11	1.63	.010	-.02	1.58	.013	.03	2.01	.002	-
" 5	669.6	- 24	.990	.113		1.55	.014		1.51	.016		1.92	.003	
" 2	748.7	+ 24	.935	.136	.09	1.46	.020	-.02	1.42	.024	.03	1.80	.005	-
Dec. 29	837.9	- 22	.883	.162		1.38	.028		1.34	.032		1.70	.008	
" 27	892.8	+ 22	.855	.178	.06	1.34	.032	-.01	1.31	.036	.01	1.66	.009	-
" 22	1007.7	- 22	.807	.205		1.26	.044		1.23	.049		1.56	.014	
" 18	1108.3	+ 22	.770	.231	.09	1.21	.054	-.04	1.18	.060	.04	1.49	.018	-.01
" 15	1173.7	- 20	.746	.249		1.17	.062		1.14	.068		1.45	.021	
" 11	1275.4	+ 20	.716	.273	.08	1.12	.073	-.03	1.09	.081	.04	1.39	.028	-.02
" 8	1341.7	- 21	.698	.287		1.09	.081		1.06	.089		1.35	.031	
" 4	1444.7	+ 21	.674	.310	.08	1.05	.092	-.04	1.02	.101	.04	1.30	.038	-.02
" 1	1509.7	- 22	.659	.324		1.03	.098		1.01	.107		1.28	.041	
Nov. 27	1613.7	+ 22	.638	.345	.07	1.00	.109	-.04	.974	.120	.05	1.24	.048	-.02
" 24	1677.5	- 23	.624	.360		.975	.119		.950	.130		1.21	.054	
" 20	1780.5	+ 23	.607	.379	.07	.950	.130	-.04	.925	.142	.04	1.17	.061	-.03
" 17	1845.5	- 21	.596	.392		.932	.138		.908	.149		1.15	.065	
" 13	1943.6	+ 21	.580	.411	.06	.906	.150	-.04	.884	.161	.04	1.12	.073	-.03
					+.83 Total			-0.29 Total			+0.33 Total			-0.13 Total

Date (see also Table III)	$\frac{\Delta t}{t_{av.}}$	Q^1 B.t.u./hr/ft for pipe C (see Table III)	Pipe Ds r = 10.00 ft			Pipe Ds' r = 15.62 ft			Pipe D_R r = 15.25 ft			Pipe D_R' r = 19.38 ft		
			$\frac{r^2}{4\alpha t}$	$e^{\frac{-r^2}{4\alpha t}}$	ΔT (°F)	$\frac{r^2}{4\alpha t}$	$e^{\frac{-r^2}{4\alpha t}}$	ΔT (°F)	$\frac{r^2}{4\alpha t}$	$e^{\frac{-r^2}{4\alpha t}}$	ΔT (°F)	$\frac{r^2}{4\alpha t}$	$e^{\frac{-r^2}{4\alpha t}}$	ΔT (°F)
Nov.6-10	.0498	18	0.30	0.74	.053	0.74	0.48	-.034	0.70	0.50	.036	1.14	0.32	-.023
Oct.30-Nov.3	.0469	39	0.28	0.76	.111	0.68	0.51	-.074	0.65	0.52	.076	1.05	0.35	-.051
Oct.23-27	.0430	44	0.26	0.77	.116	0.64	0.53	-.080	0.61	0.54	.081	0.99	0.37	-.056
" 19-20	.0125	55	0.25	0.78	.043	0.60	0.55	-.030	0.57	0.57	.031	0.92	0.40	-.022
" 18	.0027	56	0.25	0.78	.009	0.60	0.55	-.007	0.57	0.57	.007	0.92	0.40	-.005
" 17	.0028	39	0.24	0.79	.007	0.58	0.56	-.005	0.55	0.58	.005	0.89	0.41	-.004
" 16	.0027	38	0.24	0.79	.006	0.58	0.56	-.005	0.55	0.58	.005	0.89	0.41	-.003
					+0.35 Total			-0.24 Total			+0.24 Total			-0.16 Total

Net total contribution of coil D = 0.93°F

APPENDIX IV - TABLE A3

CALCULATED CONTRIBUTION OF PIPE LOOP B TO COOLING OF SUPPLY PIPE Cs FOR FEB. 2, 1951

Date (see also Table III)	t Hours to Feb. 2, 2:15 pm	Q^1 B.t.u./hr/ft for pipe B (see Table III)	Pipe Bs r = 10.08 ft			Pipe Bs' r = 15.67 ft			Pipe B_R r = 5.13 ft			Pipe B_R' r = 13.05 ft		
			$\frac{r}{2\sqrt{\alpha t}}$	I	ΔT (°F)	$\frac{r}{2\sqrt{\alpha t}}$	I	ΔT (°F)	$\frac{r}{2\sqrt{\alpha t}}$	I	ΔT (°F)	$\frac{r}{2\sqrt{\alpha t}}$	I	ΔT (°F)
Feb. 2	0													
Jan. 29	100.0	+ 23	2.52	-	-	4.00	-	-	1.28	.041	.15	3.27	-	
" 26	165.5	- 23	1.96	.002	.05	3.10	-	—	.995	.111	.41	2.54	-	-.01
" 22	268.3	+ 23	1.54	.015		2.46	-		.781	.223		2.00	.0015	
" 19	333.8	- 21	1.37	.029	.08	2.18	.001	-.01	.688	.297	.26	1.78	.005	-.03
" 15	435.9	+ 21	1.21	.053		1.91	.003		.613	.373		1.56	.013	
" 12	501.8	- 31	1.13	.077	.11	1.79	.005	-.02	.572	.421	.35	1.46	.020	-.05
" 8	605.2	+ 31	1.03	.100		1.63	.010		.521	.492		1.33	.031	
" 5	669.6	- 30	.975	.119	.11	1.55	.014	-.03	.496	.528	.22	1.27	.043	-.06
" 2	748.7	+ 30	.921	.143		1.46	.020		.468	.574		1.20	.055	
					+0.35 Total			-0.06 Total			+1.39 Total			-0.15 Total

Net total contribution of coil B = 1.53°F

APPENDIX V

ESTIMATE OF RECOVERY TIMES OF BURIED PIPE TEMPERATURE

At the conclusion of the heating season the pipe surface temperatures were considerably lower than the temperatures in the surrounding ground. An estimate of the times required for the temperatures to recover to near-normal can be made for a single pipe using the line source theory, equation 2, assuming constant heat extraction rates over the whole heating season. The calculation method is the same as that already described. The temperature "decrease" below normal at any time t after the heating season has ended is:

$$\Delta T = \frac{Q^1}{2\pi k}\left[I\left(\frac{r}{2\sqrt{\alpha(t+t')}}\right) - I\left(\frac{2s}{2\sqrt{\alpha(t+t')}}\right) - I\left(\frac{r}{2\sqrt{\alpha t}}\right) + I\left(\frac{2s}{2\sqrt{\alpha t}}\right)\right]$$

Where t' is the duration of the heating season, s is the depth of bury, and other symbols are as previously defined.

Assuming a 1-in. pipe buried 6 ft deep with an average heat extraction rate of 20 B.t.u./hr/ft over a heating season of 8 months in a soil having thermal properties of k = 1.0 B.t.u./hr/°F/ft and α = 0.04 ft^2/hr the following values for various times were obtained.

t (months)	ΔT (°F)
4	0.3
3	0.4
2	0.7
1	1.4

APPENDIX VI

COMPARISON OF A HEAT PUMP DESIGN (CONSIDERING PRIOR OPERATING HISTORY) WITH A DESIGN ASSUMING STEADY-STATE OPERATION

Comparison of ground coil design accounting for prior operating history, and design using the simple equation 6 can be illustrated by an example using degree-day data available for the Ottawa area, and the ground thermal properties as recorded in this report. Data for the coefficient of performance of a Carrier 7K4 Freon compressor (7) are given below for a condenser temperature of 98°F.

Evaporator temperature (°F)	C.OP.
50	5.85
40	4.80
30	4.02
20	3.43
10	2.99
0	2.62

For a house having a heat loss of 90,000 B.t.u./hr with an indoor air temperature of 70°F and a design outdoor air temperature of -20°F, the heat loss/deg temperature difference is 1000 B.t.u./°F/hr. The normal number of degree-days (below 65°F) for September through December is 3277 as compiled by the Dominion Department of Transport. For an indoor temperature of 70°F the degree-days for the 122 days of this period would be 3887. Hence the average heat loss of the house during this period would be

$\frac{1000 \times 3887}{122} = 31,900$ B.t.u./hr. If 2000 ft of pipe are proposed for the ground grid the heat extraction rate during this period is $\frac{31,900}{2000} = 15.5$ B.t.u./hr/ft. The average normal ground temperature for the 6 ft depth during the period is about 50°F (Fig. 13). Since a temperature "decrease" of about 10°F is expected, the coefficient of performance of the heat pump will be 4.8. The heat extraction rate from the ground is, therefore,

$$15.5 \times \frac{(4.8 - 1)}{4.8} = 12.3 \text{ B.t.u./hr/ft.}$$

To check the assumption of the 10°F temperature "decrease" the actual value can be calculated by using equation 2 and accounting for the effect of the depth of bury s,

$$\text{i.e. } \Delta T = \frac{Q^1}{2\pi k}\left[I\left(\frac{r}{2\sqrt{\alpha t}}\right) - \left(\frac{2s}{2\sqrt{\alpha t}}\right)\right]$$

Using a thermal conductivity of 1.0 B.t.u./hr/°F/ft and diffusivity of 0.04 ft^2/hr the calculated temperature drop for 122 days is 10.6°F. Proceeding now to the month prior to the peak load, the maximum number of degree-days for January for the period 1947 to 1955 was 1942 for an indoor temperature of 70°F. Thus, the design heat rate for this period would be $\frac{1000}{2000} \times \frac{1942}{31} = 31.4$ B.t.u./hr/ft of coil.

The average normal ground temperature at a 6-ft depth for this month is about 41°F and if the total temperature "decrease" is about 20°F the C.O.P. of the heat pump will be about 3.43. Hence the rate at which heat must be extracted from the ground is $31.4 \left(\frac{3.43 - 1}{3.43}\right) = 22.2$ B.t.u./hr/ft.

Calculation of the temperature "decrease" for comparison with the assumed 15°F now requires that the effect of the previous months as well as of the month of January be accounted for, i.e.,

$$\Delta T = \frac{Q^1_j}{2\pi k}\left[I\left(\frac{r}{2\sqrt{\alpha t}}\right) - I\left(\frac{2s}{2\sqrt{\alpha t}}\right)\right.$$
$$\left. + \frac{Q^1_a}{2\pi k}\left[I\,\frac{r}{2\sqrt{\alpha t'}} - I\left(\frac{2s}{2\sqrt{\alpha t'}}\right) - I\left(\frac{r}{2\sqrt{\alpha t}}\right) + I\left(\frac{2s}{2\sqrt{\alpha t'}}\right)\right]\right]$$

Where, Q^1_j = the heat extraction rate during January

Q^1_a = the average heat extraction rate for previous months

t = the duration of January = 31 days

t' = the time between September 1 and January 31 = 153 days.

Calculations give ΔT = 18.7 °F. For design purposes this is sufficiently good agreement with the assumed value of ΔT = 20°F.

For the peak load period (the first 2 weeks in February), the design heat load is $\frac{90,000}{2,000} = 45$ B.t.u./hr/ft. With a normal ground temperature of 37°F and an assumed temperature decrease of 25°F, the C.O.P. will be 3.1 and the heat extraction rate from the ground is $45 \times \left(\frac{3.1 - 1}{3.1}\right) = 30.5$ B.t.u./hr/ft.

The equation for the temperature "decrease" at the end of the peak load period is:

$$\Delta T = \frac{Q^{l}_{p}}{2\pi k}\left[I\left(\frac{r}{2\sqrt{\alpha t}}\right) - I\left(\frac{2s}{2\sqrt{\alpha t}}\right)\right] + \frac{Q^{l}_{j}}{2\pi k}\left[I\left(\frac{r}{2\sqrt{\alpha t'}}\right) - I\left(\frac{2s}{2\sqrt{\alpha t'}}\right) - I\left(\frac{r}{2\sqrt{\alpha t}}\right) + I\left(\frac{2s}{2\sqrt{\alpha t}}\right)\right] + Q^{l}_{a}\left[I\left(\frac{r}{2\sqrt{\alpha t''}}\right) - I\left(\frac{2s}{2\sqrt{\alpha t''}}\right) - I\left(\frac{r}{2\sqrt{\alpha t'}}\right) + I\left(\frac{2s}{2\sqrt{\alpha t'}}\right)\right]$$

where, Q^{l}_{p} = the heat extraction rate during the peak period.

t = duration of peak period = 14 days.

t' = time between January 1st and mid-February = 45 days.

t'' = time between September 1st and mid-February = 167 days.

The temperature "decrease" calculated from this equation is 25.2°F.

To summarize, the design method taking some account of the variation of the heat load during the heating season suggests that the temperature "decrease" of a buried pipe grid used to heat a house having a design or peak load of 30.5 B.t.u./hr/ft of pipe would be about 25°F. The temperature decrease using the simplified equation 6 with a constant heat extraction rate of 30.5 B.t.u./hr/ft of pipe is,

$$\Delta T = \frac{30.5}{6.28 \times 1.0} \ \ln\left(\frac{12.0}{0.0469}\right)$$

$$= 4.86 \times 5.55$$

$$= 27°F.$$

Hence both methods of design give very nearly the same result and the simpler method appears to be fully justified. In practice the actual performance of the heat pump system may be considerably poorer due to the use of auxiliary heat exchangers, but this would not be expected to have any effect on the relative merits of various design methods.