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#### **Publisher's version / Version de l'éditeur:**

*Proceedings. The 63th Annual Technical Conference and Exhibition, 2005-05-01*

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# A NEW ABSORPTION BASED MODEL FOR SHEET REHEAT IN THERMOFORMING

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## Abstract

Thermoforming is the process of manufacturing plastic parts by preheating a sheet and then draping it onto a mould. At present, the heater temperature set-points for sheet preheat are typically chosen by trial and error. Very little work has been published regarding the control of the sheet surface temperature. Such a control system will improve the quality of the product, reduce scrap and allow for temperature zoning. However, before a suitable control can be successfully designed, an accurate model of the process is required. In this paper, we present such a model of the sheet reheat process which includes the effect of radiation absorption inside the sheet. The inclusion of the radiation absorption effect corrects errors which were found in some previously published controllers. The model presented here is a nonlinear, temperature-dependent multivariable representation of the temperature variation through the sheet thickness.

## Introduction

Models of the reheat phase in the thermoforming process have been presented over the last few years [1-2],[4-5],[8-10]. The underlying assumption in these previous works was that the energy within the sheet is solely transferred through conduction from the surface. Along this line Moore [1] developed a comprehensive model of plastic sheet reheat. However the suggested model failed to match the experimental results. The major reason for this discrepancy was the above-mentioned assumption. However, later work proved that some of the radiant energy striking the sheet actually penetrates the surface before being absorbed by the plastic [6]. The present paper presents an improved model taking into account the absorption of radiant energy into the sheet.

The first section of this paper presents the model developed by Moore [1], which is the starting model in our development. The second section adds to the model the absorption of the radiant energy. Experimental validation is then presented, showing a better correlation with the model. Finally, concluding remarks are put forward.

## Moore's model of the reheat of a plastic sheet

In Moore's model, the plastic sheet surface is divided into  $S$  zones, while the thickness is divided into  $N$  layers with each a corresponding node. Nodes are evenly spaced by a distance  $\Delta z$  (see Figure 1). Node 1 is located at the upper surface of the sheet, and node  $N$  at the lower surface. It is standard practice to have a node at the centre of each isothermal layer [3].

Moore assumed that the primary mode of heat transfer occurring at the sheet surface was radiant heat transfer, with additional effects due to natural convection and conduction. The energy balance equation for the surface nodes, for example node 1 of zone  $i$ , was given as:

$$\frac{dT_{i1}}{dt} = \frac{1}{\rho V C_p} \left[ Q_{R_i} + q_{conv_i} - \frac{kA}{\Delta z} (T_{i1} - T_{i2}) \right] \quad (1)$$

where  $\rho$ ,  $C_p$  and  $k$  are respectively, the density, the specific heat and the heat conduction. Letter  $V$  represents the volume of the section including layer 1. Note that volume  $V$  for the upper and lower layers is equal to the area  $A$  of the surface multiplied by  $\Delta z/2$ , as the thickness of these layers is half that of the inner layers.

In (1), conduction between nodes 1 and 2 is given by the last term, while the radiant energy term  $Q_{R_i}$  can be written as:

$$Q_{R_i} = \sigma \epsilon_{eff} A_h \left[ \sum_{j=1}^M (T_{h_j}^4 - T_{i1}^4) F_{ij} \right] \quad (2)$$

where  $\sigma$  is the Stefan Boltzman constant,  $\epsilon_{eff}$  is the effective emissivity,  $A_h$  is the heater bank area,  $T_{h_j}$  is the  $j^{th}$  heater bank temperature and  $F_{ij}$  is the view factor between the  $i^{th}$  zone and the  $j^{th}$  heater bank.  $M$  is the number of independent heater banks in the thermoforming oven and  $q_{conv_i}$  is the convection energy exchange and is given by:

$$q_{conv_i} = h(T_{\infty} - T_{i1}) \quad (3)$$

where  $h$  is the convection factor,  $T_\infty$  is the temperature of the ambient air. Equations (1), (2) and (3) can be applied to  $N$ .

As mentioned before, conduction was assumed by Moore to be the single mode of heat transfer within the sheet. A one-dimensional conduction equation was then used for the interior nodes of the sheet and led to:

$$\frac{dT_{ij}}{dt} = \frac{k}{\rho C_p} \left( \frac{T_{i(j-1)} - 2T_{ij} + T_{i(j+1)}}{\Delta z^2} \right), j \in \{2, \dots, N-1\} \quad (4)$$

Absorption of radiant energy inside the plastic sheet is left out in equations (1)-(4). In the next section radiant energy absorption is added to the model.

### Model including radiant energy absorption

Inside a thermoforming oven, the heating elements radiate energy towards the plastic sheet. The largest part of this energy is absorbed by the sheet (95% for a typical plastic – [2]) and a small portion is reflected. Of that 95%, some is absorbed by the sheet itself, and some is transmitted inside the sheet. This means that the transmissivity  $\tau$  cannot be assumed to be zero. Therefore, the energy balance equation for the interior nodes becomes [4-5]:

$$\frac{\partial T_i}{\partial t} = \frac{k}{\rho C_p} \frac{\partial^2 T_i}{\partial x^2} + \dot{q}_{abs} \quad (5)$$

According to the Beer-Lambert Law [6], the transmissivity of a material depends on two main parameters: the spectral absorption coefficient of the material, as well as the material thickness. The law is stated as:

$$T = e^{-A\Delta z} \quad (6)$$

where  $A$  is the average absorptivity of the material across its spectrum in units of  $m^{-1}$ . Next, we need to discretize the continuous transmittivity equation, so that the amount of energy dissipated in each layer can be calculated.

Looking at the discretized sheet from Figure 1, it is noted that the exterior thicknesses are equal to  $\Delta z/2$ , and the interior ones equal to  $\Delta z$ . When the net radiant heat flux of intensity  $q_{rad}$  strikes the sheet surface, a portion of  $q_{rad}$  is absorbed inside the first zone and the remainder is transmitted through the zone. By definition, the absorbed fraction of  $q_{rad}$  is  $\beta(z)$ , where  $z$  is the depth within the sheet. The Beer-Lambert law is applied to the calculation of  $\beta(z)$ .

Thus, when the thickness of the zone is  $\Delta z$ , the total absorbed energy becomes equal to the integral sum of all energies absorbed over that distance:

$$\beta(\Delta z) = \int_0^{\Delta z} A_\lambda e^{-A_\lambda z} dz = \left[ 1 - e^{-A_\lambda \Delta z} \right] = \beta_2 \quad (7)$$

Since for external layers the thickness is  $\Delta z/2$ , the absorbed energy becomes:

$$\beta(\Delta z/2) = \int_0^{\Delta z/2} A_\lambda e^{-A_\lambda z} dz = \left[ 1 - e^{-A_\lambda \Delta z/2} \right] = \beta_1 \quad (8)$$

If we define  $Q$  as the incident radiant energy on the surface, we can obtain the absorbed energy in zone 1 as:

$$Q_{a1} = \beta_1 Q \quad (9)$$

The radiant energy transmitted through zone 1 is simply the non-absorbed fraction:

$$Q_{t1} = (1 - \beta_1) Q \quad (10)$$

The absorbed energy in zone 2 is:

$$Q_{a2} = \beta_2 Q_{t1} = \beta_2 (1 - \beta_1) Q \quad (11)$$

and the transmitted part through zone 2 is:

$$Q_{t2} = (1 - \beta_2) Q_{t1} = (1 - \beta_2)(1 - \beta_1) Q \quad (12)$$

For zone  $n$ , the equations for the absorbed and transmitted energy can be similarly derived:

$$Q_{an} = \beta_n (1 - \beta_2)^{n-2} (1 - \beta_1) Q \quad (13)$$

$$Q_{tn} = (1 - \beta_2)^{n-1} (1 - \beta_1) Q \quad (14)$$

Then, for the bottommost zone (zone  $N$ ), the equations become:

$$Q_{aN} = \beta_1 (1 - \beta_2)^{N-2} (1 - \beta_1) Q \quad (15)$$

$$Q_{tN} = (1 - \beta_2)^{N-1} (1 - \beta_1)^2 Q \quad (16)$$

The last equation corresponds to the radiant energy transmitted through the entire thickness of the plastic sheet. So the absorbed energy inside the plastic sheet is the difference between the energy entering in the plastic sheet and the energy transmitted:

$$Q_{ABS} = Q - Q_{tN} = \left\{ 1 - (1 - \beta_2)^{N-1} (1 - \beta_1)^2 \right\} Q \quad (17)$$

Now, we need to revise equations (1) and (4). For a given sheet zone  $i$ , the energy balance with the absorption terms is:

$$\frac{dT_{i1}}{dt} = \frac{2}{\rho C_p \Delta z} \left\{ (Q_{Tn} + Q_{Bn}) + h(T_\infty - T_{i1}) \right\} + \frac{k}{\Delta z} (T_{i2} - T_{i1}) \quad (18)$$

Equation (18) is similar to Equation (1), except the radiant energy absorbed by the top layer appearing in the first term which will be defined later.

The energy balance equation with absorption being included is:

$$\frac{dT_{ij}}{dt} = \frac{1}{\rho C_p \Delta z} \left\{ \frac{k}{\Delta z} [T_{i(j-1)} - 2T_{ij} + T_{i(j+1)}] + [Q_{T_i} + Q_{B_i}] \right\} \quad (19)$$

Again, (19) is similar to (4), with the absorbed radiant energy appearing in the last term added.

First, we need to evaluate the incident energy hitting the  $i^{\text{th}}$  zone. The top node of the  $i^{\text{th}}$  zone receives:

$$Q_{RT_i} = \sigma \varepsilon_{\text{eff}} \sum_{j=1}^{P_T} F_{T_j} (T_{S_{T_j}}^4 - T_{i1}^4) \quad (20)$$

and the bottom one receives:

$$Q_{RB_i} = \sigma \varepsilon_{\text{eff}} \sum_{j=1}^{P_B} F_{B_j} (T_{S_{B_j}}^4 - T_{i5}^4) \quad (21)$$

where,  $P_T$  and  $P_B$  represent the number of heating element zones on the top and bottom of the oven, respectively.  $T_{S_{T_j}}$  and  $T_{S_{B_j}}$  are the surface temperatures of each heating zone.

Equations (20) and (21) can be rewritten as:

$$Q_{RT_i} = \sigma \varepsilon_{\text{eff}} \left\{ \left[ \sum_{j=1}^{P_T} F_{T_j} T_{S_{T_j}}^4 \right] - F_{T_i} T_{i1}^4 \right\} \quad (22)$$

$$Q_{RB_i} = \sigma \varepsilon_{\text{eff}} \left\{ \left[ \sum_{j=1}^{P_B} F_{B_j} T_{S_{B_j}}^4 \right] - F_{B_i} T_{i5}^4 \right\} \quad (23)$$

where,

$$F_{T_i} = \sum_{j=1}^{P_T} F_{T_j} \quad (24)$$

and

$$F_{B_i} = \sum_{j=1}^{P_B} F_{B_j} \quad (25)$$

The absorption term of the radiant energy coming from the top of the sheet is:

$$Q_{T_{i1}} = \beta_1 Q_{RT_i} \quad (26)$$

$$Q_{T_j} = \beta_2 (1 - \beta_2)^{j-2} (1 - \beta_1) Q_{RT_i}, \quad j \in \{2, 3, 4, 5\} \quad (27)$$

Similarly, the absorption term of the radiant energy coming from the bottom of the sheet is:

$$Q_{B_{i5}} = \beta_1 Q_{RB_i} \quad (28)$$

$$Q_{B_{j2}} = \beta_2 (1 - \beta_2)^{j-2} (1 - \beta_1) Q_{RB_i}, \quad j \in \{2, 3, 4, 5\} \quad (29)$$

Before writing these equations into state-space form, the notation is simplified by defining the following variables:

$$a = \frac{1}{\rho C_p \Delta x} \quad (30)$$

$$b = \frac{k}{\Delta x} \quad (31)$$

$$c_1 = \sigma \varepsilon_{\text{eff}} \beta_1 \quad (32)$$

$$c_j = \sigma \varepsilon_{\text{eff}} \beta_2 (1 - \beta_2)^{j-2} (1 - \beta_1), \quad j \in \{2, 3, 4, 5\} \quad (33)$$

Using Equations (18) to (21), the state-space matrix equation for zone  $i$  can be written as shown in (34).

The temperatures  $T_{ij}$  of all nodes are the states  $x_j^i$  of the system defined in (34). The superscript  $i$  denotes the sheet zone, and the subscript  $j$  the node.

For instance, if the number of sheet zones  $M$  is equal to 6, each zone having 5 layers, there is a total of 30 state variables.

One of the reasons for developing this model is to be able to simulate how the system behaves with different types of controllers. One difficulty in designing a controller for this system is the nonlinearity induced in the last term of Equation (34) by the 4<sup>th</sup> power dependence of the radiation heat transfer on the temperature at the sheet surface.

The model must first be validated before a controller can be designed. In other words, the model must be tuned experimentally to the parameters of the actual sheet-heater system. Only then can this tuned model be used to simulate the actual plant of the system [7].

## Experimental results

This section will briefly present an experimental validation of the model described in this paper.

Testing was done using a thermoforming oven to heat HDPE plastic sheets. In order to determine properly the conduction and absorption properties of HDPE, a sheet thick enough to accommodate several thermocouples was needed. For this reason, a 12mm thick sheet was used. The size of the sheet was 205mm by 230mm. The test setup is shown in Figure 2.

Three tests were performed, with only the lower heater banks active. For the first test, the lower heater banks were held at 280°C. For the two other tests, we held them at 320°C and 420°C. We also did the tests with the simulation model, while tuning the parameters of this model.

$$\begin{bmatrix} \dot{x}_1^i \\ \dot{x}_2^i \\ \dot{x}_3^i \\ \dot{x}_4^i \\ \dot{x}_5^i \end{bmatrix} = \begin{bmatrix} -2a(h+b) & 2ab & 0 & 0 & 0 \\ ab & -2ab & ab & 0 & 0 \\ 0 & ab & -2ab & ab & 0 \\ 0 & 0 & ab & -2ab & ab \\ 0 & 0 & 0 & 2ab & -2a(h+b) \end{bmatrix} \begin{bmatrix} x_1^i \\ x_2^i \\ x_3^i \\ x_4^i \\ x_5^i \end{bmatrix} + \begin{bmatrix} 2ah & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 2ah \end{bmatrix} \begin{bmatrix} T_{\infty_{\text{top}}} \\ T_{\infty_{\text{bottom}}} \end{bmatrix} \quad (34)$$

$$- a \begin{bmatrix} 2c_1 F_T & 2c_5 F_B \\ c_2 F_T & c_4 F_B \\ c_3 F_T & c_3 F_B \\ c_4 F_T & c_2 F_B \\ 2c_5 F_T & 2c_1 F_B \end{bmatrix} \begin{bmatrix} (x_1^i)^4 \\ (x_5^i)^4 \end{bmatrix} + a \begin{bmatrix} 2c_1 & 2c_5 \\ c_2 & c_4 \\ c_3 & c_3 \\ c_4 & c_2 \\ 2c_5 & 2c_1 \end{bmatrix} \begin{bmatrix} \sum_{j=1}^{P_T} F_{T_j} T_{S_{T_j}}^4 \\ \sum_{j=1}^{P_B} F_{B_j} T_{S_{B_j}}^4 \end{bmatrix}$$

The model parameters that can be tuned are the following: upper and lower ambient air temperatures, convection coefficient, heat capacity, density, conduction coefficient, and absorption coefficient. The latter four are strictly material dependent, and only the conduction coefficient  $k$  is not temperature dependent. However, to keep the model as simple as possible, these temperature dependencies were not taken into account.

We present here the results of the simulation and experimental data at depths of 3 mm and 9 mm for the 280°C test in Figure 3. Figure 4 shows the difference between those data sets, i.e., the simulation error. For this test, the model parameters were tuned so that the resulting temperature error curves remained within 6°C of their corresponding experimental data.

For the other two tests (320°C and 420°C), the material parameters obtained via simulation are close to the nominal values for HDPE. Similarly, the physical parameters of the system approach those measured during other testing (ambient air temperatures). Then, nearly all parameters were on the expected nominal ranges. The only parameter not within nominal range is the convection coefficient. Because of the large openings at either end of the oven, there may be unseen air drafts causing major conduction effects at the sheet surfaces, such that the upper sheet surface temperature is higher than the lower sheet surface temperature, contrary to what could be expected as usually heat is trapped under the bottom sheet surface.

## Conclusion

The proposed model of the thermoforming process with absorption gives promising results. The error between the modeled and measured temperatures was relatively small when we defined parameters corresponding to the actual HDPE plastic properties.

We will use this model to test and tune different control approach, in cycle control and cycle to cycle control. In fact

we had already used this model to test the design of cycle to cycle control. And, the results were very promising as some preliminary tests on iterative learning control have shown.

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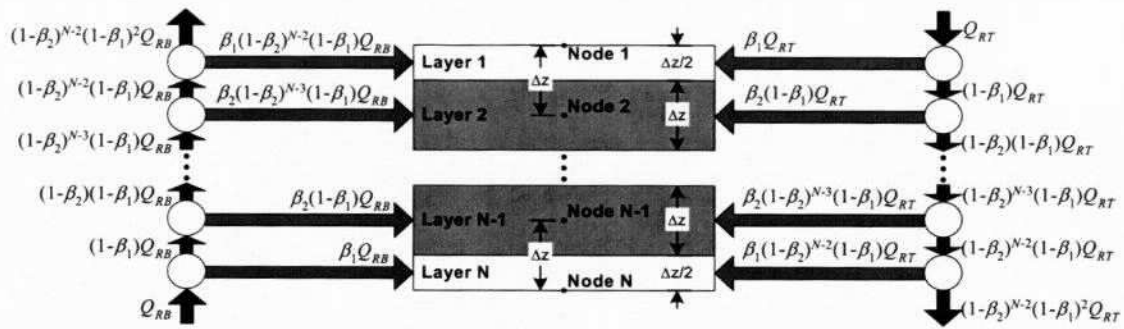


Figure 1 : Definition of layers, nodes and radiation transfers

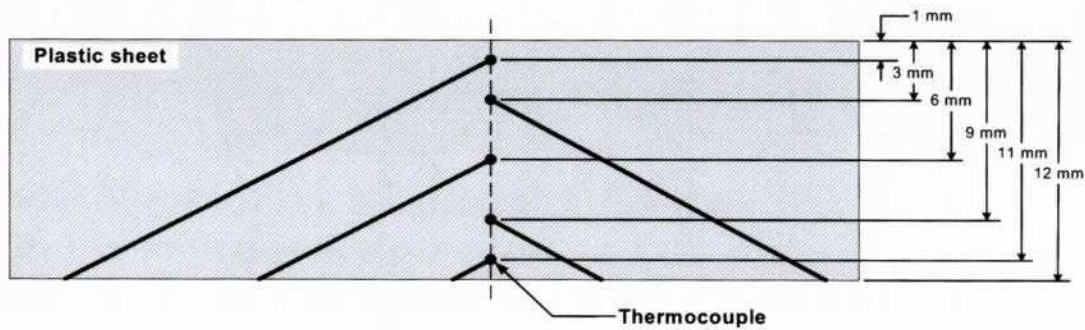


Figure 2 : Experimental setup of thermocouples on the sheet

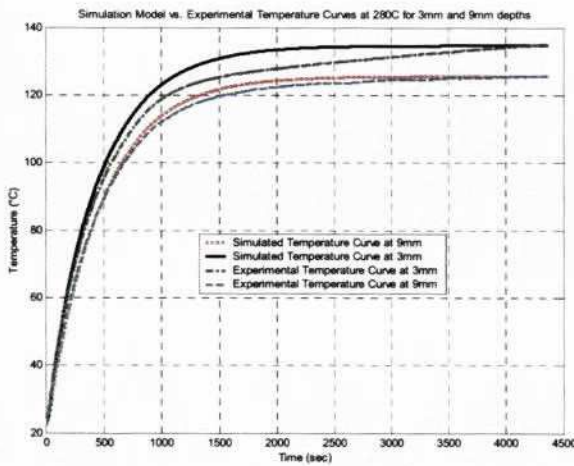


Figure 3 : Experimental and simulated temperature response for 280°C test

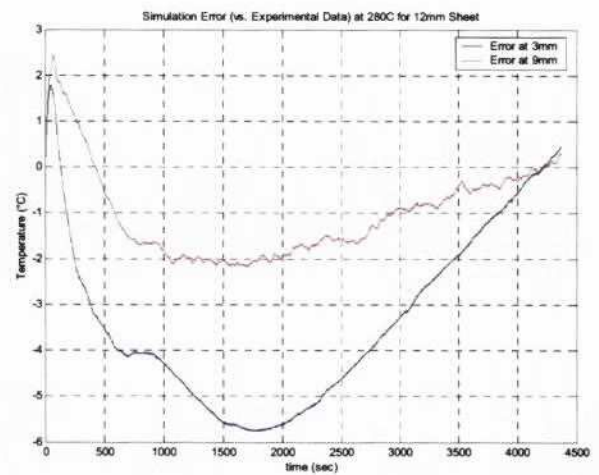


Figure 4 : Error between experimental and simulated temperatures for the 280°C test