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STATISTICAL PROPERTIES OF ENERGY SPECTRAL RESPONSE AS A FUNCTION OF DIRECT-TO-REVERBERANT ENERGY RATIO

by W.T. Chu

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RESUME

On a dérivé des expressions analytiques de propriétés statistiques de la courbe spectrale dans une salle de réverbération, en fonction du rapport de l'énergie directe à l'énergie de réverbération. On compare avec des résultats obtenus par d'autres méthodes numériques.

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Statistical properties of energy spectral response as a function of direct-to-reverberant energy ratio[a)]

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(Received 15 May 1980; accepted for publication 4 July 1980)

Analytical expressions for statistical properties of the spectral response curve in a reverberation room have been derived as a function of direct-to-reverberant energy ratio. Comparison with results obtained by other numerical methods is given.

PACS numbers: 43.55.Br, 43.45.Dn

INTRODUCTION

In a recent article,[1] Jetzt proposed a new method for measuring the critical distance of a room, based on computing the standard deviation of the spectral response from its mean. Applying a Monte Carlo simulation with 20 000 samples taken at each direct-to-reverberant energy ratio, he obtained "theoretical" values of the standard deviation of the spectral response from its mean as a function of direct-to-reverberant energy ratio. These values can, however, be obtained more readily by analytical analysis of the spectral response curve.

For pure-tone excitation, Diestel[2,3] has shown that the probability distribution of sound pressure at different locations in a room is related to the ratio of the direct-to-reverberant sound. Bowers and Lubman[4] gave corresponding expressions for sound pressure levels. These results might well be adopted for the present investigation. As no detailed derivation was given in these reports, it seemed instructive to derive the results from first principles.

I. THEORY

Consider a fixed speaker–microphone pair in a reverberation room with the speaker emitting a pure-tone signal of frequency ω. The instantaneous sound pressure picked up by the microphone can be written as

$$p(t)=p_d(t)+p_r(t)\,, \tag{1}$$

where p_d and p_r represent the direct and the reverberant sound field, respectively. The mean-square pressure is given by

$$\overline{p}^2=\overline{p}_d^2+\overline{p}_r^2+2\overline{p_d p_r}\,, \tag{2}$$

where the overbar denotes the time average value. If the direct field and the reverberant field are statistically independent, the cross-product term will be zero. For pure-tone excitation, however, the reverberant field is also a coherent field and p_d and p_r are definitely correlated.

For the direct field,

$$p_d(\omega,t)=A\cos\omega t\,, \tag{3}$$

where the phase angle has been set to zero as the origin of reference for other phase angles. For the reverberant field,[5]

$$p_r(\omega,t)=B\sum_{i=1}^{n}\cos(\omega t+\alpha_i)\,, \tag{4}$$

assuming it to be composed of many plane traveling waves of equal amplitude crisscrossing in space. This results in a set of n phase angles α_i distributed at random over the range 0 to 2π, and a fixed value of $\overline{p}_r^2(\omega_1)$. At a different source frequency there will be another set of random phase angles, assumed to be statistically independent of the first set, and another value $\overline{p}_r^2(\omega_2)$. Although this seems to be a rather simple representation of the reverberant sound field, it does provide correct results for many statistical quantities, as verified by a more detailed analysis using the eigenmode approach.[6] This provides confidence in this model for the present analysis.

Substituting Eqs. (3) and (4) in Eq. (2) gives

$$\overline{p}^2(\omega)=(B^2/2)\left[\left(A/B+\sum^n\cos\alpha_i\right)^2+\left(\sum^n\sin\alpha_i\right)^2\right]. \tag{5}$$

Waterhouse[5] has shown that both $\sum^n\cos\alpha_i$ and $\sum^n\sin\alpha_i$ are normally distributed variables with mean value zero and variance $n/2$; in addition, they are statistically independent.

Let

$$u=\left(A/B+\sum^n\cos\alpha_i\right)^2,$$

$$v=\left(\sum^n\sin\alpha_i\right)^2, \tag{6}$$

so that

$$\overline{p}^2(\omega)=(B^2/2)(u+v)\,.$$

Using the general rules for probability distribution,[7] it can be shown that the probability density functions, P, for u and v are

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a) After this work was completed, it was learned that an identical result had been reported independently by K. J. Ebeling in another Letter to the Editor in this issue [J. Acoust. Soc. Am. 68, 1206–1207 (1980)]. It was felt, however, that the somewhat different derivation presented here provides a sufficiently different view of the subject to warrant the apparent duplication.

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$$P(u)=\begin{cases}\exp[-(u+A^2/B^2)/n] \\ \times\cosh(2A\sqrt{u}/nB)/(\pi n u)^{1/2}, & \text{for } u\geq 0, \\ 0, & \text{for } u<0,\end{cases}$$

$$P(v)=\begin{cases}e^{-v/n}/(\pi n v)^{1/2}, & \text{for } v\geq 0, \\ 0, & \text{for } v<0.\end{cases} \quad (7)$$

The mean value of the spectral response curve is then given by

$$\langle\bar{p}^2\rangle_\omega=(B^2/2)(\langle u\rangle+\langle v\rangle),$$

where $\langle u\rangle$ and $\langle v\rangle$ can be computed using Eq. (7), i.e.,

$$\langle u\rangle=\int_0^\infty u\,P(u)du.$$

The result is

$$\langle\bar{p}^2\rangle_\omega=(A^2+nB^2)/2. \quad (8)$$

Define

$$I=\bar{p}^2(\omega)/\langle\bar{p}^2\rangle_\omega$$

so that

$$I=B^2(u+v)/(A^2+nB^2). \quad (9)$$

From Eqs. (3) and (4),

$$\bar{p}_d^2=A^2/2,$$

$$\bar{p}_r^2=B^2\left[\left(\sum^n\cos\alpha_i\right)^2+\left(\sum^n\sin\alpha_i\right)^2\right]/2, \quad (10)$$

and

$$\langle\bar{p}_r^2\rangle_\omega=nB^2/2.$$

For a fixed separation of speaker and microphone, $\bar{p}_d^2$ can be considered as independent of ω. Using the usual definition for direct-to-reverberant energy ratio,

$$D=\bar{p}_d^2/\langle\bar{p}_r^2\rangle_\omega=A^2/nB^2. \quad (11)$$

Equation (9) can therefore be rewritten as

$$I=(u+v)/n(1+D). \quad (12)$$

As $\sum\cos\alpha_i$ and $\sum\sin\alpha_i$ are statistically independent, u and v can also be considered as independent. Knowing the probability density functions for u and v, that for I can readily be derived[7] and the result is

$$P(I)=\begin{cases}(1+D)\exp\{-[(1+D)I+D]\} \\ \times I_0\{2[D(1+D)I]^{1/2}\}, & \text{for } I\geq 0, \\ 0, & \text{for } I<0,\end{cases} \quad (13)$$

where I_0 is the modified Bessel function. Equation (13) can also be derived from the result given by Diestel.[2,3]

In terms of sound pressure level, i.e., defining

$$y=10\log I=\ln(I)/k, \quad (14)$$

where $k=\ln(10)/10$, one can also show that

$$P(y)=k(1+D)\exp\{-[(1+D)e^{ky}+D]+ky\} \times I_0\{2[(1+D)De^{ky}]^{1/2}\}, \quad \text{for } -\infty<y<\infty. \quad (15)$$

An equation identical to Eq. (15) was given by Bowers and Lubman[4] without proof, and when $D=0$,

$$P(y)=ke^{ky-e^{ky}}.$$

This agrees with Schroeder's result[8] also.

It is clear that the spectral response curves are different for different locations of this speaker–microphone pair of fixed separation. The statistical properties of the different spectral response curves, however, are independent of location of the speaker–microphone pair provided they are not close to the boundaries of the room. For a different separation of the pair, there will be a different value for D; the probability density function is still given by Eq. (15).

II. RESULTS AND DISCUSSION

Once the probability density function of the sound pressure level of the spectral response curve is known, it is an easy matter to compute the standard deviation from the mean using

$$\langle y^m\rangle=\int_{-\infty}^{\infty}y^m P(y)dy. \quad (16)$$

No serious attempts have been made to obtain analytical solutions for Eq. (16) for $m=1$ and 2. Numerical integration was used to solve for the mean and mean-square values of y for the standard deviation of y from the mean. Results are tabulated in Table I for different direct-to-reverberant energy ratio. They compare very well with results given by Jetzt,[1] who asserted that the direct and reverberant pressures are independent, although such an assumption was in fact not utilized in the Monte Carlo simulation. Thus his "computed" curve for the standard deviation of the spectral response applies, in fact, to cases with pure-tone excitation.

TABLE I. Comparison of standard deviation of spectral response from its mean between current results and those of Jetzt (Ref. 1).

Direct-to-reverberant energy ratio (dB)	Standard deviation (dB)	
	Jetzt's results	Current results
−18.0	5.57	5.55
−12.0	5.57	5.55
−6.0	5.53	5.51
0.0	5.19	5.08
3.1	4.35	4.33
6.0	3.28	3.28
8.0	2.57	2.56
12.0	1.57	1.57
18.0	0.77	0.77

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