A Statistical Method of Design of Building Structures
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PREFACE

Safety factors now in use in structural design have the important function of ensuring safety in structures at an economically tolerable level. These safety factors have been arrived at through a long process of trial and experience with little application of any scientific analysis. The Division of Building Research, NRC, through its Building Structures Section, has undertaken an investigation of structural safety and thus is reviewing not only the safety factors themselves but also the procedure in which they are applied in practice. The five articles comprising this Technical Translation are particularly relevant to this part of the project.

Because of the extreme need for additional building construction in the USSR, special efforts have been made in that country to review, and if necessary revise, design rules for structural safety and performance. As a result, a new approach to the design of structures — design by "limit states" — was tried and adopted. This approach is gaining acceptance in Europe for design of reinforced concrete structures. Briefly, this method defines the conditions that determine the design of a structure (strength, deflection, cracking, etc.) and attempts to base safety factors on probability of failure, i.e. of realizing the "limit state".

Even for limit state design, safety factors are still partly based on experience rather than on probability of failure. The following first four articles describe a method of determining safety factors more directly on the basis of probability of failure and statistics. Many simplifying assumptions were made by the authors, but because the method attempts a more rational basis, it deserves consideration by engineers and code writers. The fifth article is a review of the development of probability methods for the design of structures in the USSR.

The translations were prepared by a member of the DBR Building Structures Section, Dr. D. E. Allen. The assistance of Mr. G. Belkov of the Translations Section of the National Research Council of Canada in checking the translations is gratefully acknowledged.

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Robert F. Legget
Director

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FIVE RUSSIAN ARTICLES ON A STATISTICAL METHOD OF DESIGN OF BUILDING STRUCTURES

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IT IS NECESSARY TO IMPROVE THE STANDARDS OF DESIGN OF BUILDING STRUCTURES

(Neobkhodimo sovershenstvovat' normy rascheta stroitel'nykh construktsii)

A.R. Rzhanitzyn

Stroitel'naya Promyshlennost'

(8): 29-32, 1957

The method of designing building structures by limit states is based on a number of advanced ideas concerning the reasons for, and the nature of, violations of the safety conditions of the structure. In particular, one of the principle factors, subject to calculation in fixing the safe dimensions of the structure and safe loads, is the statistical scatter of the values used in the design. This scatter depends on causes of technological sequence or on the conditions of use of the structure and results in the fact that each design quantity can take different values, which can be determined beforehand only with some degree of probability.

Furthermore, in the method of design by the limit state, conditions unacceptable in the performance of the structure are formulated and are called the design or conditional limit states. Finally, in this method, instead of one safety coefficient, three calculation coefficients are introduced: one for overload, one for uniformity and one for the performance conditions of the structure.

The transition to the new method of design of structures helped the engineer to give up antiquated habitual ideas on design by the allowable stress and put before him a number of new questions. At the same time, one should not overlook some inconsistency of the design method by the limit state, as a consequence of its incomplete development.

The first shortcoming of the design method by the limit state is that all the limit states are considered identically inadmissable for all types of buildings and structures and for all elements of the structure. The definition itself "limit states" is inaccurate and scholastic. It considers that the structure in the limit state loses the capacity to resist external action or ceases to satisfy its use requirement. However, the concept of use requirement is very vague in so far that the use of the structure can be obtained with different degrees of convenience in different conditions as determined by the degree of safety of the structure. Moreover, one cannot consider absolutely inadmissable interruptions of the normal use of the structure when repairs have to be made. Therefore, the limit states are not equally dangerous, and their occurrence should be avoided much more when there may be severe consequences, and less when the limit state threatens nothing but small inconveniences or troubles.

For example, the limit state determined by exceeding the carrying capacity of the main members of the building, i.e. columns or roof trusses which threatens catastrophe, cannot be set on the same level as the limit state of a truss for skylights, in which case the only possibility is its repair or replacement. However, according to the existing standards the design in both these cases must be made with the same design coefficients and design values.

Thus, the first shortcoming which must be overcome in working out a new edition of the standards is ignoring the consequences of passing beyond the calculated limit states.

The second important shortcoming of the approved standards is the incorrect procedure in accounting for the joint action of the statistical scatter of several quantities taking part in the deisgn formula. It is clearly evident that the coefficient of material uniformity corresponds to a particularly unfavourable case of material of the very lowest quality occurring in the structure. In an overwhelming majority of cases the quality of the material in the structure will be better. In precisely the same way, the overload coefficient corresponds to the same exceptional occurrence, i.e. when the load has an extremely high value, whereas during its normal use, as a rule it will not exceed the standard value. In combining several variable quantities in the same design formula, the introduction of limit values of the uniformity and overload coefficients for all initial design quantities corresponds to the highly improbable case of simultaneous coincidence of all unfavourable events, i.e. an occurrence of the greatest stress in parts of the structure where the material is of the lowest quality and simultaneously all loads are at their maximum values.

If the standard design quantities, i.e. the coefficients of uniformity and overload, were determined on the basis of rigorous methods of mathematical statistics, the application of the described method would lead in the majority of cases to a highly over-engineered structure.

The way out of the situation that was found was, unfortunately, not prompted by science. Instead of rejecting the principle of joint consideration of the design coefficients given in the standards, and using the well-known methods of mathematical statistics which have justified themselves in other areas of technology, the coefficients of uniformity and overload and the corresponding normative quantities were adjusted, so that the results of design calculations by the new standards corresponded approximately (with some minimum economy) to the results of design by the old standards, on the assumption that they were justified in practice. As an explanation to the method of design by the limit state, one of the authors of this method says "Thus, the method of establishing the design coefficients is based on the following:

(1) Distribution curves of the indices of strength, the magnitude of loads, etc. are constructed.

(2) On the basis of these curves, preliminary values of the coefficients are calculated by the formulae of mathematical statistics.

(3) On the basis of the coefficients calculated in paragraph (2), a test design is made.

(4) The coefficients obtained combine with each other since they act jointly. A good method of combination is formulating a test design.

The results of the test design are compared with the data of use of different structures. The results of the test design are not compared with separate data, but with all the available data in aggregate on the use of any form of building or structure.

(5) On the basis of the analysis made the final values of the design coefficients are established."

Such a method of establishing the numerous coefficients taken in the standards in practice means that they are adjusted to the results obtained by designing structures according to the old standards. The data of use of the structures require introduction of some correctives, which were always made earlier in the process of a more precise definition of the old standards.

The complexity of fixing the design coefficients renders impossible a critical appraisal of standard data and of introducing corrections and a more precise definition of the standards based on new results on the scatter of design values in the transition to a more advanced construction technology or to an improved use of structures. At the same time, this method does not provide identical safety of structures, since the method of limit states, being corrected for one number of variable design quantities of definite degree of variability must without fail lead to decreased safety of structures when a smaller number of variable factors are considered in the design, and to an uneconomic solution when a larger number of variable factors are present in the design.

The standards also contain other principle shortcomings, in particular the presence of coefficients of performance conditions which must consider all other favourable and unfavourable factors without any kind of procedure or scientific basis for fixing these coefficients.

Nevertheless, it is fully possible to use the theory of calculation of structures in its present state to formulate standards, based on a strict scientific procedure of calculating the statistical properties of design quantities, which can provide in all cases the necessary safety of structures at the maximum economy of building materials. Such a procedure has already been worked out long ago in the Laboratory of Building Mechanics TsNISK, where a plan of basic principles was drawn up for new standards of the design of structures. This plan provides a simpler definitive procedure of design, based on the application of general safety coefficients, which have different values for each type of element of the structure working under specific conditions of use. The plan gives a procedure for computing the safety coefficients, differentiated not only by the type of structure and construction, but also according to the elements of design of these structures. The principle of this methodology consists in the following.

The possible combinations of design quantities, which have to do with the external actions (loads, temperature, settlement of the structure, etc.) and the parameters determining the geometric dimensions and mechanical properties of the material of the structure, must be defined as the most unfavourable for the calculated element of the structure. If, as usually occurs, a number of design quantities are quantities without a definite value which obey the statistical distribution laws, then an unfavourable combination of the design quantities are taken so that the sum probability of occurrence of this combination and even more unfavourable combinations in the course of the selected period of service of the given element of the structure, must be equal to a sufficiently small magnitude, determined as depending on the character of the consequences of the occurrence of these unfavourable combinations.

In some cases the violation of design conditions of the performance of the structure leads to the destruction of the whole structure, which must not be allowed in practice. Thus the theoretical probability of destruction is taken to be a very small probability. In other cases, such a violation requires only the repair of the structure. Thus the probability that the state of the structure is such as to require repairs, V, can be determined from the condition that the quantity

$$C = C_o + V C_p , \qquad (1)$$

is at a minimum, where C_o is the original cost of the structure and C_p is the cost of its repair, including loss from interruption of the normal use of the structure.

The quantity C is the mathematical expectation of expenditure, connected with the erection of the structure and its maintenance in a satisfactory state during its use.

Since design by formula (1) can be shown difficult because of insufficient development of the economic calculations in the construction, at first, conditional values of the probability of exceeding the limit state assumed in the design can be recommended as tentative data. Considering that the distribution of the resulting quantity of any calculation formula, usually differs little from the normal distribution, instead of the probability of the resulting quantity exceeding its limiting value, the quantity γ , functionally connected with probability, and equal to the number of standard deviations of the limiting deviation of the resulting design quantity from the average value, can be used. The quantity γ is called the safety characteristic; in the existing standards it corresponds to the number of standard deviations away from the mean value of the variable to its greatest or smallest design value, and preferably assumed to be three.

As tentative initial data for the safety characteristic, the following values can be named:

(1) For load-bearing elements of large buildings and structures, where their failure in a system of these

elements signifies destruction of a catastrophic character:

(a) for the usual buildings and structres $\gamma = 4$

(b) for especially important structures $\gamma = 4.5$

(2) For protecting structures and such load-bearing elements, whose failure does not signify catastrophe but only requires urgent repairs:

(a) for the usual buildings and structures $\gamma = 3$

(b) for important structures $\gamma = 4$

(c) for structures of the temporary and light type $\gamma = 2.5$.

(3) For such elements of buildings and structures, damage of which does not cause serious conseqeuences, in which the damaged elements can be replaced without difficulty, $\gamma = 2$.

To evaluate the sufficiency of the proposed values of γ, we find the corresponding probability V of collapse in the case of the normal distribution law: for $\gamma = 4.5$, 4, 3, 2.5 and 2, V is respectively 3×10^{-7}, 3.2×10^{-5}, 1.35×10^{-3}, 6.2×10^{-3} and 2.28×10^{-2}.

The cited classification of elements of structure can be afterwards somewhat modified, and the values of the safety characteristics must be made more refined.

After this each element of the structure is put into one category or the other and the corresponding safety characteristic is assigned to it; the design of this element for safety is made by the usual methods of mathematical statistics. The following calculation system can be proposed.

I. The overload coefficient is determined for all loads, acting on the given element, according to the formula

$$k_p = 1 + \gamma \frac{s_q}{q} . \tag{2}$$

Here k_p is the overload coefficient; $\frac{s_q}{q}$ is the variability of the load, equal to the standard deviation of the load divided by the average (expected) value. For several loads acting on the element the average value of the total load is determined by way of summation:

$$q = q_1 + q_2 + q_3 + \ldots, \tag{3}$$

and the standard deviation of the total load is taken as the average quadratic of the standard deviations of the separate loads

$$s_q = \sqrt{s_{q_1}^2 + s_{q_2}^2 + s_{q_3}^2 + \ldots} \tag{4}$$

II. The repetition or duration of action of the load can be calculated separately. It is evident that for repeated or long-acting variable loads the probability of reaching their maximum value is increased. Therefore, in these cases in place of the average value and the standard deviation of a singly-applied load, the average value and the standard deviation of the maximum of the load is taken for the considered period of time. This can be given by the approximate formulae

$$q = q_0 + 3{,}5\, s_{q_0}\left(1 - \frac{1}{\sqrt[4]{n}}\right); \quad s_q = \frac{s_{q_0}}{\sqrt[4]{n}}. \tag{5}$$

where n is the number of applications of the load; q_0 and s_{q_0} are the average value and the standard deviation of the singly-applied load. For continuously variable load the singly-applied load must be considered as the maximum during a specific interval of time, and the number of loads, n, is given by the particular division of the term of service of the structure into this interval. This interval must not be too small otherwise the values of the load in two neighbouring intervals may not be independent(2).

III. The uniformity coefficient for the strength characteristics of elements of the structure is determined by the formula

$$k_0 = 1 + \gamma \frac{s_r}{r}. \tag{6}$$

where k_0 is the uniformity coefficient; r is the average value and s_r is the standard deviation of the strength characteristic.

The quantity r is determined by the usual formulae of the theory of structures, using the mean values of the initial design quantities. The standard deviation of the strength can be determined by the approximate formula

$$s_r = \sqrt{s_a^2\left(\frac{\partial r}{\partial a}\right)^2 + s_b^2\left(\frac{\partial r}{\partial b}\right)^2 + s_c^2\left(\frac{\partial r}{\partial c}\right)^2 + \cdots}\ , \tag{7}$$

where $s_a, s_b, s_c \ldots$ are the standard deviations of the initial design

(2) For the basis and derivation of formula (5), see A.R. Rzhanitzyn "Design of Structures Taking Account of the Plastic Properties of Materials", Grosstroiizdat, izd.I, 1949.

quantities; a, b, c, ... are the initial design quantities entering the formula of the theory of structures for the strength r.

The partial derivatives $\frac{\partial r}{\partial_a}, \frac{\partial r}{\partial_b}, \frac{\partial r}{\partial_c}$ are taken at the average values of the quantities a, b, c ...... The basis of this formula is also in the book mentioned.

IV. In many cases the strength of the structure is determined by the strength of the weakest element, where destruction of this element means destruction of the whole structure, as for example, in statically determinate systems. Thus it is necessary to find the average value and the standard deviation of the minimum strength of the elements of the structure, which can be given by formulae analogous to those which are recommended for the finding of the average value and standard deviation of the maximum load from a number of repeated loads

$$R = r - 3{,}5\, s_r\left(1 - \frac{1}{\sqrt[4]{n}}\right); \quad s_R = \frac{s_r}{\sqrt{n}}. \tag{8}$$

Here R is the mean strength of the whole structure; s_r is the standard deviation of this strength; n is the number of completely stressed elements of the structure, failure of which leads to failure of the whole system.

It is assumed that the strength of all these elements has the same average value r and the same standard deviation s_r, which is correct in the case of identical material of the elements and the same design stress in them. The understressed elements need not be considered in the number n, because the probability of their failure is very small.

V. Having found the overload coefficient k_p and the uniformity coefficient k_o from the final values of variability of the strengths and loads by formulas (2) and (6), it is possible to obtain the necessary safety coefficient, ξ, for the examined element of the structure by the formula

$$\xi = \frac{1 + \sqrt{1 - k_0 k_n (2 - k_0)(2 - k_n)}}{k_0(2 - k_0)} \tag{9}$$

The safety coefficients, determined by the described method for separate but similar elements and structures, will differ very little and they can be readily grouped in order to standardize the averaged safety coefficients for each group of elements and structures of any type.

STATISTICAL METHOD OF DETERMINING STANDARD STRESSES FOR STEEL STRUCTURES

(Statisticheskii metod opredeleniya normativnikh napryazhenii dlya stal'nikh konstruktsii)

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At a conference held to discuss the method of design of building structures by limit states, organized under the authority of the Praesidium of the Academy of Sciences, USSR at the end of 1953, it was noted that in this method the breakdown of the general safety factor into three separate safety factors - homogeneity of material, overloads, and performance conditions - makes it possible to evaluate more precisely the influence of the different factors allowed for by these coefficients on the carrying capacity of structures than design by allowable stresses.

However, a simple multiplication of the separate safety factors does not give a correct idea of the joint influence of the factors on the total reserve of strength of elements of the structure.

Prof. A.R. Rzhanitsin, in the article "Application of statistical methods in the design of structures for strength and safety", Stroitel'naya Promyshlennost', No. 6, 1952, showed that the combination of two of these factors - overload and uniformity of material - must occur according to the laws of mathematical statistics and the theory of probability. This circumstance was also given attention at the conference of the Academy of Sciences USSR.

For the determination of stress in elements of steel structures a number of physical and geometrical quantities should be considered: the loads P_1, P_2, ... P_n, the yield point of steel σ_T, the geometrical parameters of the cross-section of the element - area F, section modulus W, static area-moment of the section S, the coefficient of decrease ϕ of the design stress in longitudinal bending of centrally loaded bars, etc.

According to the present method, the coefficient ϕ is a function of the flexibility and initial curvature of the element, and also of unavoidable eccentricity in the application of the force not considered in the design.

All these quantities are in essence random (in mathematical conception), i.e. for a number of reasons they can have different values. It can be considered that their deviations from the average values obey the normal distribution.

The application of the normal distribution for the yield point of steel is confirmed by repeated statistical processing of the results of mechanical tests of steel (in factory laboratories) used for steel structures.

According to the data of TsNIPS for steel St3 manufactured in recent years, the average value of the yield point is 2900 kg/cm^2 and the standard deviation is 223 kg/cm^2.

The distribution curve of the greatest values of the depth of snow cover, given in the book "Steel Structures" edited by N.S. Strelestkii (p.32), is established without difficulty as being near to the normal law with average thickness of snow cover 46 cm. and standard deviation 17.5 cm. Undoubtedly, the same type of distribution also fits the wind loads.

On the basis of design information it is possible, for example, to take for the pressure on the roller of an electric bridge crane with carrying capacity 20/5 T and span 16.5 m, a distribution curve with average value 17.1T and standard deviation 3.7T.

The values of the geometric characteristics (the area, section modulus, static moment) of sections composed of rolled profiles, obey the normal law.

The random character of the value of initial crookedness of the element and uncalculable eccentricities in the application of axial forces in centrally loaded columns is confirmed by a number of investigations.

In connection with the circumstances set forth, to obtain a proper definition of design stresses the method of the theory of probability should be applied.

In the method of design of steel structures by the limit states, only the yield point of steel is considered as a random quantity, and the loads as quantities having definite values (standard values, increased by overload coefficients).

Thus, using the method of design by the limit state in its present form, just as with the method of design by allowable stress, it is impossible to solve the problem of designing steel structures having identical reserve of strength in all elements, and consequently without excessive expenditure of material.

It is possible to correctly solve this problem if the determination of the amount of standard stresses proceeds from the fact that the majority of the quantities, entering into our static calculation, are random.

The theoretical part of the problem is now examined.

1. <u>Axial Tension or Compression (without consideration of longitudinal bending)</u>.

On a rectilinear column act independent loads, P_1, P_2, ... P_n in the form of single or clustered forces. The stress in the column, $\sigma = \frac{\Sigma P}{F}$.

According to the method of design by the limit state for strength, we take as a design limit state of each element of the structure the reaching of the yield point of steel by the local stress in the dangerous section.

We introduce the auxiliary quantity

$$R = \sigma_1 - \sigma = \sigma_r - \frac{\Sigma P}{F}, \tag{1}$$

which characterizes the fraction of the carrying capacity used by the material of the bar.

R is a function of random quantities and is itself a random function with mean value equal to zero.*

The deviation of the quantity R, whose exceedance has the same probability as exceedance of the deviation of the yield point $\Delta\sigma_T$, the load ΔP and the cross-section area ΔF, can be found by the equation:

$$\Delta R = \sqrt{\left(\frac{\partial R}{\partial \sigma_T}\right)^2 \Delta\sigma_T^2 + \sum\left(\frac{\partial R}{\partial P}\right)^2 \Delta P^2 + \left(\frac{\partial R}{\partial F}\right)^2 \Delta F^2}. \tag{2}$$

From equation (1) we find:

$$\frac{\partial R}{\partial \sigma_T} = 1; \frac{\partial R}{\partial P_1} = \frac{\partial R}{\partial P_2} = \ldots = \frac{\partial R}{\partial P_n} = \frac{1}{F};$$
$$\frac{\partial R}{\partial F} = -\frac{\Sigma P}{F^2}$$

and

$$\Delta R = \sqrt{\Delta\sigma_T^2 + \sigma^2 \frac{\Sigma\Delta P^2}{(\Sigma P)^2} + \sigma^2 \frac{\Delta F^2}{F^2}}$$

We introduce the designation $\sigma = k\sigma_T$. It is evident that* $\Delta R = \sigma_T - \sigma = \sigma_T(1-k)$, therefore

$$\sigma_T(1-k) = k\sigma_T \sqrt{\frac{\Delta\sigma_T^2}{k^2\sigma_T^2} + \frac{\Sigma\Delta P^2}{(\Sigma P)^2} + \frac{\Delta F^2}{F^2}}.$$

The equation for the determination of the coefficient k will have the final form:

$$(1-k)^2 = k^2\left[\frac{\Sigma\Delta P^2}{(\Sigma P)^2} + \frac{\Delta F^2}{F^2}\right] + \frac{\Delta\sigma_T^2}{\sigma_T^2} \tag{3}$$

where k is the inverse of the coefficient of safety, guaranteeing that in elements of a steel structure used for an indefinitely long period, the stress from the loads does not reach the yield

* Translator's note: The reasoning given on p.20 gives the same result, equation (3).

point of steel with the same probability that deviations of σ_T, P and F can exceed their largest possible values.

2. Transverse Bending

In the general case of bending, the moment in the design section of a bar or beam is the result of the action of independent loads P_1, P_2, P_3, ... P_n. Each of these loads can consist of a group of concentrated or distributed forces.

The fibre stress at the calculation section

$$\sigma = \frac{\Sigma M}{W}.$$

As in the above, we introduce the quantity

$$R = \sigma_T - \sigma = \sigma_T - \frac{\Sigma M}{W}. \tag{4}$$

Analogously to the axial tension or compression,

$$\Delta R = \sqrt{\Delta\sigma_T^2 + \frac{\sigma^2 \Sigma\Delta M^2}{(\Sigma M)^2} + \frac{\sigma^2 \Delta W^2}{W^2}}.$$

The section modulus of the section W = sF (s is the kernel distance), and therefore it is possible to take

$$\Delta W = s\Delta F.$$

Then

$$\Delta R = \sqrt{\Delta\sigma_T^2 + \frac{\sigma^2 \Sigma\Delta M^2}{(\Sigma M)^2} + \frac{\sigma^2 \Delta F^2}{F^2}},$$

and introducing $\sigma = k\sigma_T$, we obtain for determining the coefficient k the equation

$$(1-k)^2 = k^2\left[\frac{\Sigma\Delta M^2}{(\Sigma M)^2} + \frac{\Delta F^2}{F^2}\right] + \frac{\Delta\sigma_T^2}{\sigma_T^2} . \tag{5}$$

Combined Action of Tension or Compression Force and Bending Moment

(a) The bending moments and longitudinal forces are independent quantities. In this case,

$$\sigma = \frac{\Sigma P}{F} + \frac{\Sigma M}{W} = \frac{s\,\Sigma P + \Sigma M}{W}$$

where s is the kernel distance.

The quantity

$$R = \sigma_T - \frac{s\,\Sigma P + \Sigma M}{W}. \tag{6}$$

We find ΔR by equation (2)

$$\frac{\partial R}{\partial P_1} = \frac{\partial R}{\partial P_2} = \dots = \frac{\partial R}{\partial P_n} = \frac{s}{W};$$

$$\frac{\partial R}{\partial M_1} = \frac{\partial R}{\partial M_2} = \dots = \frac{\partial R}{\partial M_n} = \frac{1}{W};$$

$$\frac{\partial R}{\partial W} = -\frac{s\Sigma P + \Sigma M}{W^2};$$

$$\Delta R = \sqrt{\Delta\sigma_T^2 + \frac{s^2\Sigma\Delta P^2 + \Sigma\Delta M^2}{W^2} + \frac{(s\Sigma P + \Sigma M)^2\Delta W^2}{W^4}}$$

Introducing $\sigma = k\sigma_T$ and $\sigma W = s\Sigma P + \Sigma M$, we obtain the equation for determining k:

$$(1-k)^2 = k^2\left[\frac{s^2\Sigma\Delta P^2 + \Sigma\Delta M^2}{(s\Sigma P + \Sigma M)^2} + \frac{\Delta F^2}{F^2}\right] + \frac{\Delta\sigma_T^2}{\sigma_T^2}. \tag{7}$$

(b) The bending moment and the longitudinal force are dependent quantities.

Each bending moment,

$$M_n = f_n(P_n);$$

$$M_1 = a_1P_1,\ M_2 = a_2P_2, \dots,\quad M_n = a_nP_n,$$

where a_1, a_2 ..... a_n are constants. Then

$$(s^2\Sigma\Delta P^2 + \Sigma\Delta M^2) = \Sigma(a^2 + s^2)\Delta P^2;$$
$$s\Sigma P + \Sigma M = \Sigma(s + a)\, P.$$

By analogy with the previous case we obtain:

$$(1 - k)^2 = k^2 \left\{ \frac{\Sigma(a^2 + s^2)\Delta P^2}{[\Sigma(a + s)P]^2} + \frac{\Delta F^2}{F^2} \right\} + \frac{\Delta\sigma_T{}^2}{\sigma_T{}^2}. \quad (8)$$

4. <u>Tension and Longitudinal Bending of Bars with Random Initial Curvature of the Bar (Deflection y_1)and Ecentricity y_2 in the Application of Force P</u>

Random crookedness and eccentricity of the applied force are possible for both compression and tension bars, and therefore their influence on the reserve of strength must be considered in both these cases. In the tension bar the fibre stress at the calculation section can be expressed as:

$$\sigma = \sigma_0 + \sigma_0\alpha\lambda\frac{e}{\rho}\left(1 - \frac{1}{\frac{\pi^2 E}{\lambda^2\sigma_0} + 1}\right) + \sigma_0\beta\left(1 - \frac{\frac{4}{\pi}}{\frac{\pi^2 E}{\lambda^2\sigma_0} + 1}\right).$$

In this formula: $\sigma_o = \frac{P}{F}$ is the stress from the longitudinal force P; e is the distance from the centre of gravity to the edge of the section; $\alpha = \frac{y_1}{l}$; $\beta = \frac{y_2}{s}$; $\lambda = \frac{e}{\rho}$ is the flexibility of the bar.

The quantity

$$R = \sigma_T - \frac{P}{F}\left[1 + \alpha\lambda\frac{e}{\rho}\left(1 - \frac{1}{\frac{\pi^2 E}{\lambda^2\sigma_0} + 1}\right) + \beta\left(1 - \frac{\frac{4}{\pi}}{\frac{\pi^2 E}{\lambda^2\sigma_0} + 1}\right)\right]. \quad (9)$$

The deviation ΔR is found by formula (2). For mean values $\alpha = \beta = 0$

$$\frac{\partial R}{\partial P} = \frac{1}{F}; \qquad \frac{\partial R}{\partial F} = -\frac{P}{F^2}; \quad \frac{\partial R}{\partial \alpha} = -\sigma_0 \lambda \frac{c}{\rho}\left(1 - \frac{1}{\frac{\pi^2 E}{\lambda^2 \sigma_0} + 1}\right); \quad \frac{\partial R}{\partial \beta} = -\sigma_0\left(1 - \frac{\frac{4}{\pi}}{\frac{\pi^2 E}{\lambda^2 \sigma_0} + 1}\right);$$

$$\Delta R = \mp\sqrt{\Delta\sigma_T^2 + \frac{\sigma_0^2 \Delta P^2}{P^2} + \frac{\sigma_0^2 \Delta F^2}{F^2} + \sigma_0^2 \lambda^2 \left(\frac{e}{\rho}\right)^2 \left(1 - \frac{1}{\frac{\pi^2 E}{\lambda^2 \sigma_0} + 1}\right)^2 \Delta\alpha^2 + \sigma_0^2 \left(1 - \frac{\frac{4}{\pi}}{\frac{\pi^2 E}{\lambda^2 \sigma_0} + 1}\right)^2 \Delta\beta^2};$$

and after substituting $\sigma_0 = k'\sigma_T$,

$$\Delta R = k'\sigma_T \sqrt{\frac{\Delta\sigma_T^2}{k'^2 \sigma_T^2} + \frac{\Delta P^2}{P^2} + \frac{\Delta F^2}{F^2} + \lambda^2 \left(\frac{c}{\rho}\right)^2 \left(1 - \frac{1}{\frac{\pi^2 E}{\lambda^2 k' \sigma_T} + 1}\right)^2 \Delta\alpha^2 + \left(1 - \frac{\frac{4}{\pi}}{\frac{\pi^2 E}{\lambda^2 k' \sigma_T} + 1}\right)^2 \Delta\beta^2}$$

or

$$\left(1 - k'\right)^2 = k'^2 \left[\frac{\Delta P^2}{P^2} + \frac{\Delta F^2}{F^2} + \lambda^2 \left(\frac{c}{\rho}\right)^2 \left(1 - \frac{1}{\frac{\pi^2 E}{\lambda^2 k' \sigma_T} + 1}\right)^2 \Delta\alpha^2 + \left(1 - \frac{\frac{4}{\pi}}{\frac{\pi^2 E}{\lambda^2 k' \sigma_T} + 1}\right) \Delta\beta^2\right] + \frac{\Delta\sigma_T^2}{\sigma_T^2}. \quad (10)$$

According to TU (specifications), in the manufacture of steel structures $\Delta\alpha$ must not exceed .001; this requirement is often not met. Therefore it is safe to consider $\Delta\alpha = 0.002$.

The mean value of the random eccentricity was determined by a number of investigators as (.06 to .07) s; $\Delta\beta$ can thus be taken equal to :

3 x 1.2533 (.06 to .07) = .225 to .263 ≈ 0.25.

In this calculation the number 1.2533 is the transition coefficient from the simple average to the standard deviation for a quantity obeying the normal distribution.

<u>Compression and Longitudinal Bending with Random Initial Curvature (Deflection y_1) of a Bar and Eccentricity y_2 in the Application of the Longitudinal Force</u>

The fibre stress at the calculation section of the bar,

$$\sigma=\sigma_0+\sigma_0\alpha\lambda\frac{e}{\rho}\left(1+\frac{1}{\frac{\pi^2E}{\lambda^2\sigma_0}-1}\right)+\sigma_0\beta\left(1+\frac{\frac{4}{\pi}}{\frac{\pi^2E}{\lambda^2\sigma_0}-1}\right).$$

The designation in this formula is the same as in the previous case.

By analogy with tension and longitudinal bending with random eccentricities and initial curvature of the bar, we obtain

$$\left(1-k''\right)^2=k''^2\left[\frac{\Delta P^2}{P^2}+\frac{\Delta F^2}{F^2}+\lambda^2\left(\frac{e}{\rho}\right)^2\left(1+\frac{1}{\frac{\pi^2E}{\lambda^2k''\sigma_T}-1}\right)^2\Delta\alpha^2+\left(1+\frac{\frac{4}{\pi}}{\frac{\pi^2E}{\lambda^2k''\sigma_T}-1}\right)^2\Delta\beta^2\right]+\frac{\Delta\sigma_T^2}{\sigma_T^2}, \qquad (11)$$

where

$$k''=\frac{\sigma_0}{\sigma_T}.$$

6. Determination of the Standard Stress

Making use of the above formulae, we find the magnitude of the standard stress for different cases of the influence of forces on elements of the structure.

For random quantities, obeying the normal distribution, the largest practically possible deviation is usually taken as $3(\sigma)$, where (σ) is the standard deviation.

The probability of exceeding such a deviation both above and below the average is sufficiently small - 0.00272, and in one direction only - half as much, i.e. 0.00136.

As stated above, the yield point of steel St3 presently has an average value of 2900 kg/cm^2 and a standard deviation of 228 kg/cm^2. For the following calculations we have

$$\Delta\sigma_T = 3 \times 228 = 684 \text{ kg/cm}^2 \quad \text{and}$$

$$\frac{\Delta\sigma_T}{\sigma_T} = 0.236.$$

The greatest deviation of the cross-section area of different rolled profiles, calculated according to the negative tolerance of the dimensions of the sections, is given in Table 1.

According to the data of this Table, it is possible to take

$$\frac{\Delta F}{F} = 0.14.$$

For different types of loads the ratio $\frac{\Delta P}{P}$ has different values. In designing industrial structures by the limit state method, the overload coefficient was established for dead load as $n = 1.1$, and for live load as 1.1 to 1.4. For the majority of live loads $n = 1.4$.

In our equations the deviation is the greatest possible from the average value of the load.

In design by the limit state method the overload coefficient determines the deviation of the amount of the load from its standard value, which, as a rule, will be higher than the average value.

It is easy to show in specific examples that in design by the examined method it is possible to consider both the average value of the loads and their greatest deviation, and also, the standard values of the loads with overload coefficients.

The deviation of the loads ΔP from their average or standard value must account not only for possible changes of the magnitude of the loads themselves (the forces), but also for inaccuracy in the determination of the forces because of the discrepancy between the actual performance of the structure and its performance according to the design plan.

From equation (3) it follows that the standard stress must be different depending on the ratio between the magnitudes of loads acting on the structures, i.e. depending on the cited ratio

$$\frac{\Delta P}{P} = \frac{\sqrt{\Sigma \Delta P^2}}{\Sigma P}.$$

Table 2 gives the values of the coefficient k, by which the design force for the element of the structure must be decreased, and the cited overload coefficient n', by which the value of the standard stress at $\frac{\Delta P}{P} = 0$ must be divided. For comparison with n', the value of the overload coefficient n for design by the limit state method is also given.

Figure 1 shows curves of the values of the coefficient k' for tensile elements in the presence of random eccentricities and initial curvatures for the following values of the quantities entering into equation (8).

$$\frac{\Delta\sigma_T}{\sigma_T} = 0{,}236; \quad \frac{\Delta P}{P} = 0{,}1,\ 0{,}4 \text{ and } \text{[illegible]}; \quad \frac{\Delta F}{F} = 0{,}14;$$

$$\Delta\alpha = 0{,}002; \quad \frac{e}{\rho} = 3{,}0 \text{ and } \Delta\beta = 0{,}25.$$

From the figure it is evident that for any value $\frac{\Delta P}{P}$ the value of the coefficient k' depends very little on the flexibility of the rod. In all cases k' has the smallest value for the average rod flexibility ($\lambda \approx 100$).

Figure 1 also gives curves of the values of the coefficient k" for compressive elements in the presence of random eccentricities and initial curvatures, and Figure 2 shows the value of the coefficient ϕ, obtained as a ratio of the coefficient k" at the given flexibility to the value of this coefficient for a flexibility $\lambda = 0$. Also drawn on this figure is the curve for the value of ϕ according to the existing standards. From the diagram it is evident that in the examined method of design the coefficient ϕ depends not only on the flexibility of the rod, but also on the deviation $\frac{\Delta P}{P}$.

Calculation for Loads with Different Probabilities of Occurrence

In the previous calculations it was supposed that all loads could act on the structure equally often; but actually different loads act in time with different frequency. The probability of the action of dead load is equal to one.

The probability of action of bridge cranes on a structure depends on the intensity of their performance. The probability of action of large snow and wind loads is usually significantly smaller than for crane loads. Some loads, as for example erection loads, act on the structure only once during the whole period of its construction and use.

This difference in the frequency of action of separate forms of the loads can be accounted for by the value of the largest deviation ΔP, taken in the calculation.

It was shown above that for ΔP we take $3(\sigma)$, i.e. three standard deviations. The probability that the regularly acting loads deviate (both signs) more than this amount is $p_1 = 0.0027$. If the probability of occurrence of other loads is x times less, then the magnitude of their deviation from the mean value which can be exceeded with the same probability $p_1 = 0.0027$ will be found according to the probability $p = xp_1$.

Table 3 gives the values of the deviations in the number of standard deviations (σ) and the coefficient of reduction μ for loads with different probabilities of action p.

The same table also gives data of the lowering of the overload coefficients for standard loads with the snow load examined above as an example.

The different probability of action of separate loads must be considered in setting the standard overload coefficients. For erection loads $\frac{\Delta P}{P}$ can be taken as zero.

Conclusions and Practical Applications

(i) The method of design of steel structures examined above makes it possible to design for equal strength, and consequently also for the most economical structures.

(ii) The method of design according to the limit state is a particular case of the examined method of design, when the loads are not random quantities ($\frac{\Delta P}{P} = 0$). Actually, for any forces, when $\frac{\Delta P}{P}$ equals zero,

$$(1-k)^2 = k^2 \frac{\Delta F^2}{F^2} + \frac{\Delta \sigma_T^2}{\sigma_T^2};$$

For $\frac{\Delta F}{F} = 0.14$ and $\frac{\Delta \sigma_T}{\sigma_T} = 0.236$:

$$k = 0.746$$

$$\text{and } \sigma_a = 0.74 \times 2900 = 2160 \text{ kg/cm}^2$$

This corresponds to the guaranteed yield point of steel St3 - 2400 kg/cm², multiplied by the uniformity coefficient 0.9, i.e. the design strength of steel St3 in design by the limit state method.

It is known that design by allowable stress in turn is a particular case of design by the limit state method, where for all loads, a single overload coefficient n = 2100/1600 = 1.3 is taken.

Thus, the method of design examined at the beginning is a general method, which includes the two other methods as particular cases.

(iii) For the general method of design it is convenient to fix the basic standard stresses for cases of action of the dead loads (i.e. for the loads with the smallest overload coefficient):

(a) for design of transverse bending and for combined tension or compression and bending the standard stress can be taken as $\sigma_1 = 0.733 \times 2900 = 2120 \simeq 2100$ kg/cm^2.

(b) for design for compression and tension under a relatively central application of the longitudinal force the basic standard stress can be taken

$$\sigma_2 = 0.683^* \times 2900 = 1970 \simeq 1950 \text{ kg/cm}^2.$$

The expediency of lowering the basic standard stress for compressive and tensile elements in comparison with bending is confirmed because bending elements in comparison with compressive or tension elements have additional reserve of strength. This is because the section modulus of the section W_T for plastic stage of performance is always higher than the section modulus of the section W_y for the elastic stage.

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* The coefficient $k_1 = 0.683$ is obtained from Fig. 1. $\frac{\Delta P}{P} = 0.1$ and $\lambda = 0$.

(iv) In design for combined action of live and dead load for case (iii)(a) the basic standard stress must be decreased by the coefficient ϕ_1, a quantity which depends on the ratio

$$\frac{\Delta P}{P} = \frac{\sqrt{\Sigma \Delta P^2}}{\Sigma P}.$$

Values of the coefficient ϕ_1 are presented in Table 4.

(v) In design of compressive elements for live and dead loads the basic standard stress must be decreased by the coefficient ϕ_2, some values of which depend on $\frac{\Delta P}{P}$ and λ as given in Table 5. In the design of tension elements for the same loads the basic standard stress must be decreased by the coefficient ϕ_2 with $\lambda = 0$.

(vi) The data of Table 5 show that in design by the limit state method, a structure calculated for loads with high overload coefficients will have larger reserve of strength than structures calculated under loads with low overload coefficients. This difference will be especially significant for structures performing under several independent loads.

For example, a structure on which act three equal but independent loads with overload coefficients 1.1, 1.3 and 1.4 (dead, crane and snow respectively) by the limit state method must be calculated for the mean overload coefficient

$$n = \frac{1,1 + 1,3 + 1,4}{3} = 1,23.$$

The area or section modulus of the cross-section of an element of this structure

$$F_1 = \frac{n\Phi(\Sigma P)}{\sigma_H} = \frac{1{,}23\Phi(\Sigma P)}{2\,100} = 5{,}87 \times 10^{-4}\,\Phi(\Sigma P).$$

According to the method herein

$$\frac{\Delta P}{P} = \frac{\sqrt{(0{,}1^2 + 0{,}3^2 + 0{,}4^2)}}{3} = 0{,}17;$$

From Table 4, ϕ_1 = 0.97 and the area or section modulus of the section of the element

$$F_2 = \frac{\Phi(\Sigma P)}{0{,}97 \cdot 2100} = 4{,}92 \cdot 10^{-4}\,\Phi(\Sigma P).$$

Thus the given structure, calculated by the limit state method, will have an excess reserve of strength of

$$\left(\frac{F_1 - F_2}{F_2}\right)100 = 19\%.$$

<u>Translator's Note</u> (Cont'd. from p.6)

R, a function of random variables, is itself a random variable approximated by the normal curve with mean $\bar{R} = \bar{\sigma}_t - \frac{\Sigma \bar{P}}{F} = \bar{\sigma}_t(1 - k)$, and standard deviation $S_R = \frac{\Delta R}{\gamma}$ where γ is called the safety characteristic (see article by A.R. Rzhanitzyn), and ΔR the "greatest possible" deviation of R, is defined equal to γ standard deviations. For $\gamma = 3$ adopted later, the probability of exceeding ΔR is 0.14%. Failure occurs when $R < 0$. Therefore, if we choose $\Delta R = \bar{R} = \bar{\sigma}_t(1 - k)$, which results in equation (3), the values of k obtained corresponds to the probability of failure 0.14%. The values for P, F and σ_T to be used in equation (3) are the average values.

TABLE 1

Profile	Deviation in %		Profile	Deviation in %	
	From	To		From	To
Angles of equal sides	7.3	13.3	Ship Steel	4.0	8.3
Angles of unequal sides ...	8.4	13.9	Round Steel of Diameter > 26 mm	3.3	5.8
I-beams	4.3	6.4	2 Equal-sided Angles	5.1	9.5
Channels	5.4	7.3	2 Unequal-sided Angles	6.0	9.8
Universal Sections > 6 mm	2.6	8.7	I-beam from universal sections.	1.5	5.1
Sheet Steel > 6 mm	2.6	10.0	The same, from sheet steel	1.5	5.8

TABLE 2

$\frac{\Delta P}{P}$	k	n'	n
0,6	0,575	1,30	1,6
0,5	0,606	1,23	1,5
0,4	0,641	1,16	1,4
0,3	0,676	1,10	1,3
0,2	0,708	1,05	1,2
0,1	0,733	1,02	1,1
0	0,746	1,00	1,0

TABLE 3

x	p	$1-p$	$\frac{\Delta P}{[\sigma]}$	μ	Snow load			
					$\Delta P = 58\,\mu$	P_{max}	$n = \frac{P_{max}}{P_H}$	μ'
1	0,0027	0,9973	3,00	1,00	58,0	104,0	1,48	1,00
2	0,0054	0,9946	2,78	0,93	54,0	100,0	1,43	0,97
3	0,0081	0.9919	2,65	0,87	50,5	96,5	1,38	0,93
5	0,0135	0,9364	2,47	0,82	48,0	94,0	1,34	0,91
7	0,0182	0,9811	2,34	0,78	45,0	91,0	1,30	0,88
10	0,0270	0,9730	2,20	0,74	43,0	89,0	1,27	0,86
50	0,1350	0,8650	1,50	0,50	29,0	75,0	1,07	0,72
100	0,2700	0,7300	1,11	0,37	21,5	66,7	1,00	0,68

TABLE 4

$\frac{\Delta p}{p}$	k	φ_1
0,6	0,575	0,78
0,5	0,606	0,83
0,4	0,641	0,88
0,3	0,676	0,92
0,2	0,708	0,96
0,1	0,733	1,00
0	0,746	1,02

TABLE 5

$\frac{\Delta p}{p}$	λ				
	0	50	100	150	200
0,5	0,86	0,76	0,55	0,32	0,21
0,4	0,90	0,79	0,56	0,32	0,21
0,3	0,94	0,82	0,56	0,33	0,21
0,2	0,97	0,84	0,56	0,34	0,21
0,1	1,00	0,86	0,57	0,34	0,21

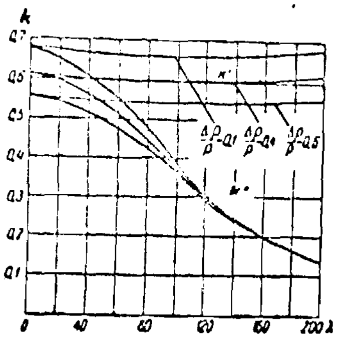
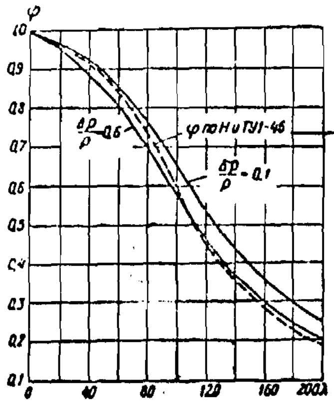


Fig.1



∅ according to NiTYI-46

Fig.2

STATISTICAL METHOD OF DESIGN OF REINFORCED CONCRETE STRUCTURES

(Statisticheskii metod rascheta zhelebetonnikh konstruktsii)

B.I. Belyaev

Stroitel'naya Promylshlennost'

(8): 32-37, 1957

The perfecting of methods of designing cross-sections of structural elements, in particular the working out and the introduction in practice of the statistical method of design, is one of the most important ways of reducing the consumption of building materials in short supply, and consequently reducing construction costs. In the first place this article treats the design of reinforced concrete structures, whose application in building is increasing from year to year.

As in the case of steel structures, the statistical method of designing reinforced concrete structures makes it possible to design load-bearing structures of buildings and engineering structures for equal strength, without excessive reserve strength, and thereby to economize on a substantial quantity of materials - cement and steel - being consumed as a result of inaccuracy of the existing design methods.

Some specialists doubt the possibility of calculating reinforced concrete structures by the statistical method because of the complexity of design formulas. However, design formulas in the statistical method are only slightly more complicated compared to formulas in design by the method of limit states (NiTU 123-55).

In the following calculations, in full accordance with reality, the loads and consequently the internal forces and bending moments, the resistance of concrete and steel, the area of the concrete section and steel reinforcing are taken as statistical quantities whose deviations from the average values obey the normal distribution law.

In deriving the formulas for the design of sections of reinforced concrete elements, the same method is used as in the derivation of the corresponding formulas for steel structures(1).

It is also shown that the method of design set forth can be applied for structures made from all building materials: steel, reinforced concrete, concrete and wood.

Let us examine the typical cases of designing reinforced concrete elements with coefficients of performance conditions t = 1.

1. Centrally Compressed Elements with Longitudinal Reinforcing

The stress from the standard loads in such an element

$$\sigma = \frac{\Sigma N}{F_6 + \frac{R_a}{R_6} F_a}. \tag{1}$$

According to the strength conditions of the element this stress must be equal to or less than $X_1 \phi R_6$, where ϕ is the coefficient of longitudinal bending, determined by an empirical formula of the form:

$$\varphi = \frac{1}{0{,}7 + 0{,}00012 \left(\frac{l}{\rho}\right)^2} \left(\varphi < 1\right).$$

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(1) B.I. Belyaev, Stroitel'naya Promyshlennost', No.3, 1954. (See previous article)

X_1, the inverse of the general safety coefficient, is the ratio of the resistance of concrete in compression to the average (standard) value, which guarantees that in elements of reinforced concrete structures during their period of use, the stress from the loads does not reach the least possible value of the resistance of concrete (or the yield point of steel) with the same probability with which deviations of random quantities entering into the formula can exceed their greatest or smallest possible values as taken in the calculation.

We introduce the auxiliary quantity

$$D_1 = \varphi\left(R_{\text{б}} - \frac{\Sigma N}{F_{\text{б}} + \frac{R_{\text{а}}}{R_{\text{б}}} F_{\text{а}}}\right). \qquad (2)$$

The quantity D_1 characterizing the degree of use of the load-bearing capacity of the material of the element as a function of random quantities, is itself a random quantity with mean value equal to zero*.

The size of the deviation of D_1, whose exceedance has the same probability as the exceedance of the deviation of the resistance of concrete and steel, the loads and the sectional areas of the concrete and steel taken in the design, is found by the formula

$$\Delta D_1^2 = \left(\frac{\partial D_1}{\partial R_{\text{б}}}\right)^2 \Delta R_{\text{б}}^2 + \left(\frac{\partial D_1}{\partial R_{\text{а}}}\right)^2 \Delta R_{\text{а}}^2 + \left(\frac{\partial D_1}{\partial F_{\text{б}}}\right)^2 \Delta F_{\text{б}}^2 + \left(\frac{\partial D_1}{\partial F_{\text{а}}}\right)^2 \Delta F_{\text{а}}^2 + \Sigma\left(\frac{\partial D_1}{\partial N}\right)^2 \Delta N^2. \qquad (3)$$

Making use of this formula and considering that

$D_1 = \varphi(1 - X_1) R_{\text{б}}$ and $\sigma = X_1 \varphi R_{\text{б}},$ we find*

* See translator's note on p. 6 of the previous article.

$$\left(1-X_1\right)^2=-\frac{2X_1\mu\frac{R_a}{R_б}b_б^2}{\left(1+\mu\frac{R_a}{R_б}\right)}+X_1^2\left[\frac{\Sigma a^2N^2}{(\Sigma N)^2}+\frac{c_б^2+\mu^2\left(\frac{R_a}{R_б}\right)^2\left(b_a^2+b_б^2+c_a^2\right)}{\left(1+\mu\frac{R_a}{R_б}\right)^2}\right]+b_б^2. \quad (4)$$

In this formula:

a is the coefficient of load variation,

$$a=\frac{\Delta N}{N}=\left(n-1\right).$$

where n is the overload coefficient;

$b_a=\frac{\Delta R_a}{R_a}$ is the coefficient of strength variation (the yield point) of the steel;

$b_б=\frac{\Delta R_б}{R_б}$ is the same for concrete;

$c_a=\frac{\Delta F_a}{F_a}$ is the coefficient of variation in reinforcements;

$c_б=\frac{\Delta F_б}{F_б}$ is the same for the sectional area of the concrete.

Depending on the investment and responsibility of the structure, and also the degree of danger if the separate elements of the structure exceed the limit state, the coefficients of load variation and of the resistance of steel and concrete can have different values. In general these coefficients are determined by the standard deviation of the variable quantities and the safety characteristic γ - the number of standard deviations guaranteeing that the greatest (or smallest) value of this quantity can occur only with the probability taken in the calculation(2). For example, the coefficient of load variation

$$a = \frac{\Delta N}{N} = \frac{\gamma s_N}{N}$$

and for γ the values 2.5 - 4.5 are proposed.

With very small error formula (4) can be brought to the more convenient form

$$\left(1 - X_1\right)^2 = -2X_1A_1b_6' + X_1^2\left[\frac{\Sigma a^2N^2}{(\Sigma N)^2} + A_1^2\left(b_a' + b_6' + c_a' + c_6'\right)\right] + b_6', \qquad (5)$$

where

$$A_1 = \frac{\mu\frac{R_a}{R_6}}{\left(1 + \mu\frac{R_a}{R_6}\right)}.$$

For the above variation coefficients the following values will be taken (for γ = 3): $c_a = c_6 = 0{,}1$; $b_a = 0{,}25$ and $b_6 = 0{,}4$; Then the final formula for the determination of the quantity X_1 will have the form

$$\left(1 - X_1\right)^2 = -0{,}32\,X_1A_1 + X_1^2\left[\frac{\Sigma a^2N^2}{(\Sigma N)^2} + 0{,}24\,A_1^2\right] + 0{,}16. \qquad (6)$$

The theoretical quantity A_1 can vary from 0 at μ = 0 to a value near to 1 at large values of μ, i.e. essentially for a solid metallic section.

Table 1 gives the values X_1 for different values of $\frac{\sqrt{\Sigma a^2N^2}}{\Sigma N}$ and A_1.

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(2) See the article by A.R. Rzhanitzyn "It is Necessary to Improve the Standards of Design of Structures"

The data of Table 1 show that even at $A_1 = 0$ the statistical method of design gives a more economical section than design by limit states. This economy increases with an increase of the overload coefficient (the coefficient of load variation) reaching 24% at $a=0{,}5$ $(n=1{,}5)$

The design formula for a centrally compressed element has the following form:

$$\Sigma N \leqslant X_1 \varphi R_б\left(F_б + \frac{R_а}{R_б} F_а\right) = X_1 \varphi R_б F_б\left(1 + \mu \frac{R_а}{R_б}\right). \tag{7}$$

2. Bending Elements

Such elements include rectangular beams (slabs) and T-sections with a flange in the compression zone of the section, when the neutral axis is within the flange, with single reinforcing (Fig. 1).

The design formula is

$$\Sigma M \leqslant X_2 R_б\,[bx\,(h - 0{,}5\,x)],\ \ R_а F_а = R_б bx,$$

whence

$$\Sigma M \leqslant X_2 R_б h\left(\frac{R_а}{R_б} F_а - 0{,}5\, \frac{R_а^2 F_а^2}{R_б^2 F_б}\right). \tag{8}$$

The stress in the beam from the design bending moment is

$$\sigma = \frac{\Sigma M}{h\left(\frac{R_а}{R_б} F_а - 0{,}5 \frac{R_а^2 F_а^2}{R_б^2 F_б}\right)}.$$

and the auxiliary quantity is

$$D_2 = R_б - \sigma = R_б - \frac{\Sigma M}{h\left(\frac{R_а}{R_б} F_а - 0{,}5 \frac{R_а^2 F_а^2}{R_б^2 F_б}\right)}.$$

The coefficient X_2 is found by the same method as for compressive elements, from the equation

$$- X_2\Big)^2 = -2X_2 \frac{\left(1 - \mu \frac{R_а}{R_б}\right) b_б^2}{\left(1 - 0{,}5\mu \frac{R_а}{R_б}\right)} + X_2^2 \left[\frac{\Sigma a^2 M^2}{\left(\Sigma M\right)^2} + \frac{\mu^2 \frac{R_а^2}{4R_б^2}}{\left(1 - 0{,}5\mu \frac{R_а}{R_б}\right)^2} c_б^2 + \frac{\left(1 - \mu \frac{R_а}{R_б}\right)^2}{\left(1 - 0{,}5\mu \frac{R_а}{R_б}\right)^2}\left(b_а^2 + b_б^2 + c_а^2\right)\right] + b_б^2 \qquad (9)$$

We designate the expression $\frac{\left(1 - \mu \frac{R_а}{R_б}\right)}{\left(1 - 0{,}5\mu \frac{R_а}{R_б}\right)}$ by A_2; then approximately

$$\left(1 - X_2\right)^2 = -2X_2 A_2 b_б^2 + X_2^2 \left[\frac{\Sigma a^2 M^2}{(\Sigma M)^2} + A_2^2\left(b_а^2 + b_б^2 + c_а^2 + c_б^2\right)\right] + b_б^2, \qquad (10)$$

and for the adopted values of the above coefficients,

$$\left(1 - X_2\right) = -0{,}32\, X_2 + X_2^2\left[\frac{\Sigma a^2 M^2}{(\Sigma M)^2} + 0{,}24\, A_2^2\right] + 0{,}16.$$

A_2 can vary from 0.62 at $\frac{x}{h} = \frac{\mu R_а}{R_б} = 0{,}55$ to 0.99 at the smallest allowable per cent of reinforcing μ = 0.15%.

The quantity A_2 and consequently the quantity X_2 are increased with a decrease of the coefficient of reinforcing.

3. Examination of a Section of Bent-Up Rods in Bending Elements Under a Transverse Force

This is an example when the calculation is made not according to the strength of the concrete but according to the strength of the reinforcing (Fig. 2).

The design formula is

$$\Sigma Q < X_3 R_a \left(F_0 \sin \alpha_1 + \sqrt{0{,}6 \frac{F_\text{б} F_x R_\text{б} h}{R_a a_1}} \right) \tag{11}$$

In this formula: F_o is the area of section of the bent-up rods; α_1 is the angle of inclination of bent-up rods to the longitudinal axis of the element; F_x is the sectional area of the stirrups in one section, a_1 is the distance between the stirrups.

The auxiliary quantity

$$D_3 = R_a - \frac{\Sigma Q}{F_0 \sin \alpha_1 + \sqrt{0{,}6 \frac{F_\text{б} F_x R_\text{б} h}{R_a a_1}}}$$

and the coefficient X_3 are determined approximately from the equations

$$\begin{aligned} \left(1 - X_3\right)^2 &= -2 X_3 A_3 b_a^2 + X_3^2 \left[\frac{\Sigma a^2 Q^2}{(\Sigma Q)^2} + A_3^2 \left(b_a^2 + b_\text{б}^2 + c_\text{б}^2 + c_0^2 + c_x^2 \right) \right] + b_a^2 = \\ &= -0{,}125 X_3 A_3 + X_3^2 \left[\frac{\Sigma a^2 Q^2}{(\Sigma Q)^2} + 0{,}25 A_3^2 \right] + 0{,}0625, \end{aligned} \tag{12}$$

where

$$A_3 = \frac{\sqrt{0{,}15 \frac{F_\text{б} F_x R_\text{б} h}{F_a^2 R_a a_1}}}{\left(\sin \alpha_1 + \sqrt{0{,}6 \frac{F_\text{б} F_x R_\text{б} h}{F_a^2 R_a a_1}} \right)} .$$

The value of the quantity A_3 changes from 0 at $F_X = 0$ to 0.5 at $F_O = 0$.

4. Eccentrically Compressed Elements (Rectangular Section) with Double Reinforcing (Fig. 3)

We cite only the final formulas

(a) Case 1. $x \leqslant 0{,}55 h \geqslant 2a'$. The design formula is

$$\Sigma N < X_4 R_\text{б} \left(\frac{x}{h} F_\text{б} + \frac{R_a}{R_\text{б}} F_a' - \frac{R_a}{R_\text{б}} F_a \right); \tag{13}$$

where x is determined from the conditions:

$$R_6 F_6 \frac{x}{h}\left(e-h+0{,}5x\right)+R_a F'_a e'-R_a F_a e=0,$$

and the coefficient X_4 for the numerical values of the variation coefficients taken above is determined from the equation

$$\left(1-X_4\right)^2=-0{,}32\,X_4 A_4+X_4^2\left[\frac{\Sigma\,a^2N^2}{(\Sigma\,N)^2}+0{,}24\,A_4^2\right]+0{,}16. \tag{14}$$

In this formula, $A_4=\dfrac{\left(\mu'-\mu\right)\frac{R_a}{R_6}}{\left[\alpha+(\mu'-\mu)\frac{R_a}{R_6}\right]}$ can change from zero (at $\mu' = \mu$) to nearly 1 (at $\frac{x}{h}=\frac{2a'}{h}$).

(b) Case 2. $x > .55h$. The design formula is

$$\Sigma\,Ne\leqslant X_5 R_6 h\left(0{,}5\,F_6+\frac{R_a}{R_6}F'_a\right).$$

The value of the coefficient X_5 is found from the equation

$$\left(1-X_5\right)^2=-0{,}32\,X_5+X_5^2\left[\frac{\Sigma\,a^2N^2e^2}{(\Sigma\,Ne)^2}+0{,}24\,A_5^2\right]+0{,}16, \tag{15}$$

in which $A_5=\dfrac{\mu'\frac{R_a}{R_6}}{0{,}5+\mu'\frac{R_a}{R_6}}$: theoretically A_5 can change from zero (at $\mu' = 0$) to nearly 1.

5. Standard and Design Characteristics of Materials

In the statistical method of design, the average value of the material strength - the ultimate strength for concrete and the yield point for steel - must be used for the standard material characteristics.

In the standards and technical specifications for design of concrete and reinforced concrete structures (NiTU 123-55) different coefficients of uniformity of concrete K_b are given for different types and different methods of their preparation. For simplification of the statistical method of design it is expedient to retain one coefficient of concrete variation, and the stated influences of the type and method of preparation are considered in the magnitude of the standard resistance of the concrete.

The greatest errors in the values of the coefficients X will therefore not exceed 2 - 2.5%.

The standard resistance of reinforcing of different steel types must be determined by means of statistical processing of material tests. It can be assumed that the standard resistance of reinforcing hot-rolled steel R_a^n will be near to the following values: St.3 - 2900 kg/cm^2; St.5 - 3400 kg/cm^2, 25G25 - 4800 kg/cm^2.

For the purpose of simplifying calculations in the statistical method it is also convenient to retain the idea of a design resistance of concrete and steel reinforcing.

The design resistance of concrete and steel in this case will be equal to the standard resistance, decreased by a coefficient X_n at $\frac{\sqrt{\Sigma a^2P^2}}{\Sigma P}=0$ and $A_n=0$.

Then in place of the coefficient X_n in the design formula the coefficient

$$\varphi_n = \frac{X_n}{X_0},$$

where $\bar{X}_0 = 0.6$, must be introduced.

Tables 2, 3 and 4 gives values of the design resistance of materials and the coefficient φ_n.

The design resistance of reinforcing steel R_a for St.3 is 1750 kg/cm², for St.5 is 2000 kg/cm², and for 25G25 is 2900 kg/cm².

In the design of a section for R_a (bent-up bars and stirrups in bending elements, reinforcing for torsion) the value of φ_n varies little for different values A_n; therefore, these calculations (in reserve of strength) can be obtained by using the coefficient $\bar{\varphi}_n$ when $A_n = 0$, according to Table 4.

In conclusion we examine some examples of the calculation of sections of elements by the method of limit states and by the statistical method.

1. Columns of Middle Row of a Shop of Span 24 m

In both adjacent spans are two cranes under average conditions of work of carrying capacity 20/5 T. The height from the bottom of the crane beam to the base of the column is 9.4 meters. The roof is composed of large span slabs and metallic roof trusses, the crane beam is reinforced concrete; the spacing of the columns is 6 m.

The loads acting on the column are: dead load from the roof, P_p = 0.3 x 6 x 24 = 43.1 T; dead load from the weight of the beam and column, $P_k = 14T$, snow load $P_c = 0.1 \times 6 \times 24 = 14.4$ T. The loads from the crane: from two cranes according to $P_1 = 1.1 \times 21 \times \frac{6 + 1.55}{6} = 29.0$ T; from two cranes according to $P_2 = 1.1 \times 21 \times \frac{4.3}{6} = 16.6$ T.

(a) Design by the method of limit states:

$\Sigma nP = 1.1 \times 57.2 + 1.4 \times 14.4 + 1.3\ (2 \times 29 + 2 \times 16.6) = 209.7$ T.

For a column section of 36 x 90 cm, $F_a = 32$ cm^2, $\mu = 0.01$. The type of concrete is 200; the reinforcing steel is St.5. The design formula

$$\Sigma nN < \varphi R_{\text{пр}} \left(F_б + \frac{R_а}{R_{\text{пр}}} F_а \right),$$

$$209{,}7 < 0{,}57 \times 90 \left(3\,240 + \frac{2\,400}{90} \times 32 \right) = 210 \text{ T}.$$

(b) Design by the statistical method:

$$\frac{\sqrt{\Sigma a^2 P^2}}{\Sigma P} = \frac{\sqrt{0{,}1^2 \times 57{,}2^2 + 0{,}4^2 \times 14{,}4^2 + 2 \times 0{,}3^2 (29{,}0^2 + 16{,}6^2)}}{162{,}8} = \frac{16{,}3}{162{,}8} = 0{,}1.$$

The design formula is $\Sigma N \leqslant \varphi_1 \varphi R_{\text{пр}} \left(F_б + \frac{R_а}{R_{\text{пр}}} \cdot F_а \right).$

For a column section of 35 x 73 cm, $F_a = 25$ cm^2, $\mu = 0.01$,

$\mu \frac{R_а}{R_{\text{пр}}} = 0{,}01 \times \frac{2\,000}{90} = 0{,}22$, $A_1 = \frac{0{,}22}{1{,}22} = 0{,}18$, $\varphi_1 = 1{,}08$ (Table 3)

$$162{,}8 < 0{,}55 \times 1{,}08 \times 90 \left(2\,550 + \frac{2\,000}{90} \times 25 \right) = 163{,}5 \text{ T}.$$

For the statistical method of design we obtain a saving in concrete and steel of $100\ (1 - \frac{2510}{3240}) = 22\%$.

2. Crane Beam with Span 6 Meters for two cranes of load capacity 30/5 T under average conditions. The span of the crane is 28.5 m; P_1 = 31 T.

The bending moment from the cranes

$$M_{cr} = 1.1 \times 31 \times \frac{6}{4}\left(1 + \frac{1.05}{3.00}\right) = 51.2 + 17.9 = 69.1 \text{ T.m.}$$

The bending moment from the dead load is

$$M_p = \frac{0.8 \times 6^2}{8} = 3.6 \text{ T.m.}$$

(a) Design by the Method of Limit States

$$\Sigma nM = 1.3 \times 69.1 + 1.1 \times 3.6 = 93.8 \text{ T.m.}$$

Type 300 concrete, reinforcing from steel St.5, $\mu = 0.02$

$$\alpha = \frac{x}{h} = \frac{\mu R_a}{R_u} = \frac{2\,400}{170} \times 0{,}02 = 0{,}28,$$

$$F_a = 0.02 \times 50 \times 75 = 75 \text{ cm}^2 \text{ (Fig. 4)}$$

The design formula $\Sigma nM \leqslant R_6 bh^2 \alpha (1 - \alpha),$

$$93.8 \text{ T.m.} < \frac{170 \times 50 \times 75^2 \times 0.28 \times 0.72}{10{,}000} = 96.3 \text{ T.m.}$$

(b) Design by the Statistical Method

$$\frac{\sqrt{a^2M^2}}{M} = \frac{\sqrt{0{,}1^2 \times 3{,}6^2 + 0{,}3(51{,}2^2 + 17{,}9^2)}}{72{,}7} = \frac{16{,}2}{72{,}7} = 0{,}22;$$

$$\frac{\mu R_a}{R_u} = 0{,}02 \times \frac{2\,000}{170} = 0{,}235,\ A_2 = 0{,}87,\ \varphi_2 = 1{,}19,$$

$$F_a = 0{,}02 \times 44 \times 69 = 61 \text{ см}^2$$ (Fig. 5)

The design formula is

$$\Sigma M \leqslant \varphi_2 R_6 bh^2 \alpha (1 - \alpha);$$

$$72.7 \text{ T.m.} < \frac{1.79 \times 170 \times 44 \times 69^2 \times 0.765 \times 0.0235}{10{,}000} = 75.8 \text{ T.m.}$$

For the beam an economy of 100 $(1 - \frac{1880}{2430})$ = 22% in the concrete and 100 $(1 - \frac{61}{75})$ = 18% in the steel is obtained by the statistical method.

3. Fixed Slab of Span 5 Meters

The dead load g = 250 kg/m², the live load q = 400 kg/m² with overload coefficient n = 1.4,(coefficient of load variation a = 0.4).

(a) Design by the Method of Limit States

$$p = 1.1 \times 250 + 1.4 \times 400 = 835 \text{ kg/m}^2$$

$$M_r = 0.835 \times 5^2/16 = 1.3 \text{ T.m.}$$

The concrete is type 22, the steel is type St.5.

The design formula is

$$\Sigma nM \leqslant R_6 h^2 \alpha (1 - \alpha); \quad \alpha = \mu \frac{R_a}{R_6};$$

$$\alpha = \frac{2\,400 \times 0{,}014}{110} = 0{,}305; \quad h = 7{,}5 \text{ см};$$

$$1.3 \text{ T.m.} \leq 1100 \times .075^2 \times .305 \times .695 = 1.31 \text{ T.m.}$$

The full thickness of the slab h_o = 7.5 + 1.5 = 9 cm

$$F_a = 0.014 \times 7.5 \times 100 = 10.5 \text{ cm}^2$$

(b) Design by the Statistical Method

$$\frac{\sqrt{\Sigma a^2 P^2}}{\Sigma P} = \frac{\sqrt{0{,}2^2 \times 0{,}250^2 + 0{,}4^2 \times 0{,}400^2}}{0{,}650} = \frac{0{,}162}{0{,}650} = 0{,}25.$$

The design formula is $\Sigma M \leqslant \varphi_2 R_6 h^2 \alpha (1 - \alpha);$

$$\Sigma M = \frac{0.65 \times 5^2}{16} = 1.015 \text{ T.m.}$$

$$\alpha = \frac{\mu R_a}{R_н} = 0{,}014 \times \frac{2\,000}{110} = 0{,}255; \quad A_2 = 0{,}85; \quad \varphi_2 = 1{,}18; \quad h = 6{,}5 \text{ см};$$

The full slab thickness $h_o = 6.5 + 1.5 = 8$ cm.

$$F_a = 0.014 \times 6.5 \times 100 = 9.1 \text{ cm}^2.$$

The economy in concrete and steel is $100\,(1 - \frac{8}{9}) = 11\%$.

Thus, in all the examined cases the statistical method of design of reinforced concrete structures gives significant economy of concrete and steel as compared to the method of limit states.

TABLE 1

$\frac{\sqrt{\Sigma a^2 N^2}}{\Sigma N}$	A_r						$\frac{k_6}{n}$
	0	0,2	0,4	0,6	0,8	1	
0	0,6	0,65	0,7	0,74	0,75	0,75	0,6
0,1	0,59	0,64	0,69	0,73	0,73	0,74	0,546
0,2	0,58	0,63	0,67	0,7	0,71	0,72	0,5
0,3	0,57	0,61	0,65	0,67	0,69	0,69	0,462
0,4	0,55	0,58	0,61	0,63	0,64	0,66	0,428
0,5	0,53	0,54	0,57	0,59	0,61	0,62	0,4

Note: k_6 is the uniformity coefficient of concrete.

TABLE 2

Design Resistance of Concrete R_6 in kg/cm^2

Type of Resistance	Type of Concrete					
	100	150	200	300	400	500
Axial Compression A	48	70	90	140	190	230
B	44	65	80	130	170	210
Compression during bending A	60	85	110	170	230	280
B	55	80	100	160	210	260

Note: The design resistance in line A is for concrete manufactured in factories with automatic and semi-automatic proportioning, and under systematic strength control, line B is for other cases.

TABLE 3

Value of Coefficient ϕ_p for Calculation According to $R_б$

$\frac{\sqrt{\Sigma a^2 p^2}}{\Sigma P}$	A_n					
	0	0,2	0,4	0,6	0,8	1
0	1	1,09	1,16	1,22	1,24	1,25
0,1	0,99	1,08	1,15	1,21	1,23	1,24
0,2	0,98	1,05	1,12	1,17	1,21	1,22
0,3	0,94	1,01	1,07	1,11	1,14	1,16
0,4	0,91	0,97	1,02	1,05	1,07	1,1
0,5	0,87	0,91	0,95	0,98	1,01	1,03

TABLE 4

Value of Coefficient ϕ_p for Calculation According to R_a

$\frac{\sqrt{a^2 p^2}}{\Sigma P}$	0	0,1	0,2	0,3	0,4	0,5
φ_n	1,25	1,24	1,2	1,14	1,08	1,02

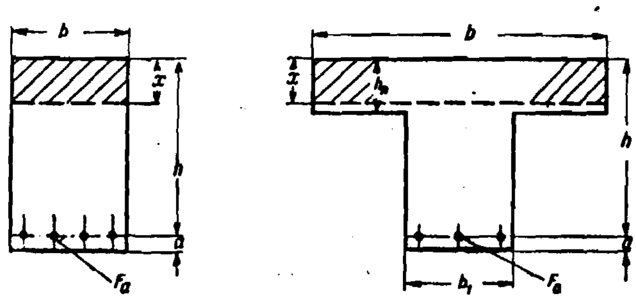

Fig. 1

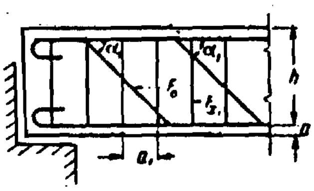

Fig. 2

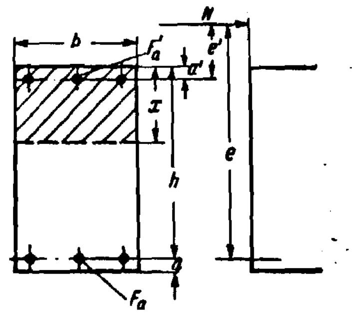

Fig. 3

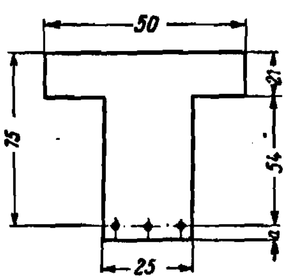

Fig. 4

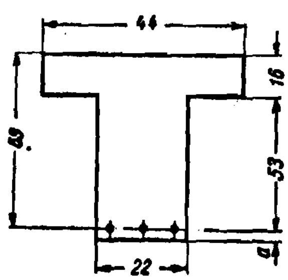

Fig. 5

ONCE MORE ON THE STATISTICAL METHOD OF DESIGN OF BUILDING STRUCTURES

(Esche raz o statisticheskom metode rascheta straitel'nikh konstruktsii)

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Promyshlennoe Stroitel'stvo

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In the statistical method of design of building structures[1], a considerable portion of the quantities (loads, resistance of materials, geometric parameters of sections of elements, etc.) are considered in full accordance with reality as random quantities, and their combined influence on the structure or on its carrying capacity is evaluated according to the rules of the theory of probability. It is often mentioned that the statistical method is a natural development and a deepening of the present method of design of building structures according to limit states, through a closer approximation of the calculated premises to the real performance condition of the structure.

It is known that the method of design according to the limit state is also built on the theory of probability in estimating the combined influence of some design quantities. However, this is not done fully and in some cases it is not fully consistent.

In order to be convinced of this, it is enough to analyze the formula of the safety condition of the structure - the basic requirement in the design of the structure according to the first limit state (strength)

$$a_p P_p n_p + c_1 (\Sigma \alpha q_i Q_i n_{qi} + \Sigma \alpha d_i D_i n_{di}) \leqslant mkFR.$$

(1) "Stroitel'naya Promyshlennost" 1929, No. 10; 1932, No. 1; 1947, No. 8; 1952, No. 6; 1954, No. 3; 1957, No. 8.

Here: P_p is the standard dead load; Q_i are the standard live loads (basic); D_i are the additional loads; n are the coefficients of overload; c_i is the coefficient of combination under additional loads; α is the coefficient of the load effects for the considered element of the structure or for its section; m is the coefficient for the performance conditions; k is the coefficient of homogeneity of material; F is a geometric parameter of the section of the element; R is the design resistance of the material.

In this inequality, the quantity N = FR (the load-carrying capacity of the element's section) is considered by itself as a function of two random quantities F and R and according to this the values of the coefficient of homogeneity k are established. But according to the ratio on the left side of the inequality, the product kFR is not a random quantity. Also the coefficient m, the basic loads (dead and live), without the additional and special loads, are also considered not as random quantities. However the same basic loads (excluding the dead load) and all additional and special loads are now considered as a system of random quantities, since the coefficients of combination c are introduced.

There are some principle objections to the statistical method of design, objectively restraining its introdution into the practice of designing building structures. These objections reduce to the following: for confident use of the statistical method of design the law of distribution of the random design quantities, in the first place the load, must be known precisely. Also to establish the empirical laws of the distribution of the design quantities it is necessary to have a very large number of their values, because the distribution curve of the calculated values particularly interests us in the area of very small probabilities (i.e., the "tail" of the distribution). The opinion is expressed that long-term work is necessary to perfect the distribution laws of calculated values; without this accurate definition the practical use of the statistical method of design would seem to be impossible.

It is also considered that all random quantities determining the performance of the elements of the structure, or of all the building, can be divided into two groups:

(a) the resistance of materials, geometric parameters of the section (area, section modulus, moment of inertia, etc.) and the dead load. Each element of the structure takes one quite definite value of every quantity, retaining it during all periods of the operation of the building, provided such factors are neglected as: the decrease of the area of the cross-section by corrosion of material; the change of resistance of material due to some physical or chemical condition (decay, ageing); the change with time of the density of building materials, for example, in its drying up (wood) or impregnation of water in the process of operation (wood, insulation), etc.

(b) the live load, which acts during each period of operation of the structure although of short duration, but repeatedly (wind, snow, ice, crane loads, groups of people, etc.) and each time has, as a rule, a new value.

There is the opinion that in view of the differences in principle between the two groups of random quantities, it is impossible to use the law of the theory of probability for the determination of their combined influence on the structure.

We note the fact that for the statistical method, in the design formula there is a single (by value) characteristic of safety γ (usually equal to 3); however for different distribution laws of the random quantities a different probability of exceeding the design values of the unknown random quantities (stress, force) will correspond to this value 3.

Finally, some of the opponents of the statistical method attempt to prove that such a method in general does not apply to the engineering design of structures.

This leads to the consideration[1] that "random events must carry a mass character permitting repeated occurrence in practical and identical conditions, and the probability at which the mathematical operations are made, must be confirmed by the proper quantity of experimental data"

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(1) V. V. Bolotin, "Statistical Methods in Structural Mechanics", p.9.

and that "the goal of engineering design is the choice of a structure, in which destruction would be a highly improbable event". However, far from these correct principles the unexpected conclusion is made. "Therefore, in essence, the failure of a structure can not be a mass event and the statistical interpretation of the probability loses its meaning. Moreover, the uniform conditions for the performance of building and construction rarely occurs".

On the grounds of this objection it is necessary to state that the statistical method of design of building structures in not built on the statistics of structural failures. Moreover, analysis of the causes of damages shows that most of them in general could not be averted by means of engineering calculation. The statistical method of design is based on the study of the performance of the structure, constructed from materials possessing definite physical and mechanical properties, and subjected to different categories of loads whose values obey definite statistical laws peculiar to each load category. The performance of the structure under the influence of loads is undoubtedly a mass phenomenon, for there are no unloaded structures on earth. This mass phenomenon in its own particulars is repeatedly realized in practically uniform conditions (of materials, loads).

Therefore it is reasonable to suppose that in principle, the use of statistical methods in engineering design of building structures is completely justified.

We will consider a way of overcoming the other three objections, which appear to prevent the application of the statistical method.

Distribution Laws of Design Random Quantities

For the practical use of the statistical method a knowledge of the distribution laws of the design quantities is not necessary. It is necessary to know only the numerical characteristics (average value and standard deviation) of the distributions. Such characteristics can be established with the necessary precision by a relatively limited number of values of these random quantities. The asymptotic laws of the distribution of the majority of design quantities are also sufficiently clear.

The distribution of a considerable number of design quantities is undoubtedly near to the normal distribution (NR). Examples are: the resistances of materials, the volume weights of materials, the geometric parameters of the section, the random warping of bar elements, etc. This conclusion can be made not only from the results of a study of numerous distribution curves of the values of these quantities, but also mainly from analysis of the general conditions on which the normal distribution must arise. It is known that a sufficient condition for this is the possibility of considering the deviation of the given random values from its mean value as the sum of a large number of mutually independent items, none of which is characterized by an exclusively large dispersion (as compared to the others). Such a condition is satisfied by all the above enumerated random quantities.

On the other hand, the values of many live loads, in the first place atmospheric (wind, snow, ice), temperature influences and also crane loads can be considered as a statistical population of the greatest values of these loads, determined on the basis of sufficiently numerous routine measurements at definite intervals of time (month, quarter-year, half-year, year). In such an approach to the design values of many live loads, it is possible to use the theory of distribution of extreme values to establish the theoretical distribution law.

According to this theory(1), for a sufficiently large number of selections, the distribution of the largest terms in the case when the least value of these terms cannot be less than a, asymptotically approaches the distribution according to the model law (PR).

$$F(x) = e^{-\left(\frac{x-a}{b}\right)^{\alpha}}. \qquad (1)$$

The function F(x) gives the probability of occurrence of all values of the random quantity X exceeding its value x.

If the origin of coordinates is transferred to the point where x = a, and it is considered that the coefficient b

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(1) N. V. Smirnov, I. V. Dudin - Barkovskii. Short Course on Mathematical Statistics for Technical Applications. Fizmatgiz. 1959, page 339.

does not influence the form of the distribution curve (it changes only the "scale" of values), then it is possible for further investigation to write the function F(x) in a simpler form, i.e.

$$F(x) = e^{-x^{\alpha}}. \tag{2}$$

Hence the function of the probability density distribution

$$f(x) = \frac{dF(x)}{dx} = \alpha e^{-x^{\alpha}} x^{(\alpha-1)}. \tag{3}$$

It is very important that the distribution of the largest values follow the model law every time the distribution $F(x_i)$ of each of the quantities $X_i (i = 1, 2, \ldots, n)$, approaches a sufficiently quickly. This condition is satisfied in particular by all empirical distributions of atmospheric loads. For the rest the function F(x) can be completely arbitrary.

According to the information of the Voikov Main Geophysical Observatory in Leningrad, the practical distribution of wind speed in the present limited region on the ground surface of the U.S.S.R., measured four times daily, also agrees well with the theoretical distribution according to the model law.

The integral function of the distribution of wind speeds, i.e., the probability that the speed of wind is larger than u, is expressed by the meteorological formula

$$F(u) = e^{-\left(\frac{u}{\beta}\right)^{\alpha}}. \tag{4}$$

This formula is completely identical in form with formula (1) for a = 0; in order to show this, it is sufficient to introduce the substitution $b = \beta^*$ *.

In formula (4) α and β are parameters depending on the wind regime of the given region; their values are found by construction of a graph of the function F(u) in logarithmic coordinates.

$$\lg[-\lg F(u)] = \alpha(\lg u - \lg \beta) + \lg \lg e.$$

The practical value of the parameter α has been established within the confines of the comparatively narrow interval, $1 < \alpha < 3$.

For example, according to the observations of the meteorological stations (Gur'evsk, Berezovsk, Mare-Sal and Dikson), values of the power index α = 1.6; 2.2; 2.8; 2.9 are obtained.

The velocity pressure of the wind is connected with its speed by the relation

$$q = \frac{u^2}{16},$$

from which

$$u = 4\sqrt{q},$$

and the distribution function of the speed pressure values

*Translator's note: This is probably a misprint in the Russian article. It should be b = β.

will have the form

$$F(q) = e^{-\left(\frac{4\sqrt{q}}{\beta}\right)^{\alpha}} = e^{-\left(\frac{q}{\beta_1}\right)^{\alpha'}}$$

that is, the values of the speed pressures of the wind are distributed according to the same model law as the wind speed, but with the changed parameters

$$\alpha' = 0{,}5\,\alpha; \quad \beta' = 0{,}25\,\beta.$$

The average value of the argument x and its standard deviation are easily calculated by the formulas

$$\bar{x} = \Gamma\left(1 + \frac{1}{\alpha}\right); \quad \sigma = \bar{x}\sqrt{\frac{\Gamma\left(1+\frac{2}{\alpha}\right)}{\Gamma^2\left(1+\frac{1}{\alpha}\right)} - 1},$$

where Γ is the gamma function.

Table 1 gives values $\bar{x}$ and σ for different values of the parameter α.

Figure 1 shows curves of the probability density of f(x) for different values α of the model distribution (PR). For comparison curves of f(x) for the normal (NR) distribution are shown for the same numerical characteristics $\bar{x}$ and σ as for the model distribution when α = 2 and 3.

From Table 1 and Figure 1 it is seen that with an increase of α the value $\bar{x}$ changes little: initially it decreases slightly, but beginning with α = 3.5 it increases once again.

The standard deviation σ sharply decreases with an increase of the parameter α. The relative dispersion (coefficient of variation υ) also decreases with increase of α.

Such an analysis removes the first objection against the statistical method of the calculation of building loads (i.e., the absence of sufficient knowledge of the laws of distribution of the loads). This distribution is well-known provided the design values of the loads are established as the values of the extreme (largest) selected terms. In this case the distribution of values of the loads will asymptotically follow the model law. The values of the empirical parameters α and β in the majority of cases (for atmospheric loads) can be obtained from available meteorological observations; the necessary distribution of extreme values for crane loads by contemporary measuring techniques (automatic recording of pressure in a column, or on a wheel) also can be obtained without special difficulty.

Consideration of the Action of Repeated Loadings of Structures by Live Loads

The repeatedly-acting load can be replaced by an equivalent singly-acting load with average value and standard deviation of an appropriately altered form.

As such a load, the greatest value of repeatedly-acting loads is taken from N of its actions during the planned period of operation of the building. This value of the load in turn is considered as a random quantity.

For loads with a normal or nearly normal distribution the numerical characteristics of equivalent singly-acting loads can be determined according to an approximate formula(1). The mean value is

$$P_N = P_1 + 3{,}5\,\sigma_1\left(1 - \frac{1}{\sqrt[4]{N}}\right);$$

the standard deviation is

$$\sigma_N = \frac{\sigma_1}{\sqrt[4]{N}}; \qquad (5)$$

the coefficient of variation is

$$v_N = \frac{\sigma_N}{P_N} = \frac{v_1}{3{,}5\,v_1\left(\sqrt[4]{N} - 1\right) + \sqrt[4]{N}}.$$

In the case where the load is given by the distribution function of the form $F(x) = e^{-\left(\frac{x}{\beta}\right)^{\alpha}}$ and the numerical values of its parameters are obtained by statistical processing of the results of regular observations of the quantity X, n times for each definite interval of time (month, quarter, year), then the greatest value of this load $\bar{x}_n$ in T intervals of time (the planned period of building operation) is found according to the function*

$$F(x) = \frac{1}{nT}, \qquad (6)$$

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(1) A. R. Rzhanitzyn, "Design of Structures by Consideration of the Plastic State of Materials"; 1954.

*Translator's note: The average $\bar{x}_n$ of the greatest loads in N years is determined from the model distribution of the singly-applied loads by putting $F(x) = \frac{1}{nT}$. The explanation of this formula is given subsequently.

and the standard deviation of this greatest load can, for practical designs, be taken equal to:

$$\sigma_N = \sigma_1 - \frac{\alpha(\bar{x}_N - \bar{x}_1)}{8.5}, \tag{7}$$

where $\bar{x}_1$ is the mean value of the acting load; $\alpha \leq 2.5$. For large values of α the model distribution approaches the normal distribution; and in this case formula (5) becomes valid for the determination of the numerical characteristics of the equivalent load.

Table 2 shows the results of calculating the numerical characteristics of the equivalent singly-applied load, the maximum yearly value of which is distributed according to the model law with value of the index $\alpha = 2$, $\bar{P}_1 = 0.89$, $\alpha_1 = 0.47$ (Table 1).

The numerical characteristics of the load are determined by formulas (6) and (7) for the model distribution, and also by (5) for the normal distribution. The agreement of calculated results in both cases is very satisfactory, for which the model distribution $\alpha = 2$ greatly differs from the normal distribution (see Figure 1).

It is known that the probability of exceeding a calculated value of the live load for an unlimited range of dispersion (although on one side - zero to infinity) increases with an increase of the number of loadings.

If p(x) is designated as the probability of exceeding the calculated value x of load acting once, then the probability of exceeding this value of the load under repeated application is determined by the formula

$$p_N(x) = 1 - [1 - p(x)]^N, \tag{8}$$

where N is the number of loadings.

Since the probability $p_1(x)$ is taken sufficiently small in the calculations, then, approximately [the more precise, the smaller $p_1(a)$],

$$p_N(x) = 1 - [1 - Np_1(x)] = Np_1(x).$$

Thus for the design value of repeated loads, a value must be taken corresponding to the probability which is N times smaller than the case where the given load would act on the structure only once:

$$p_N(x) = \frac{p_1(x)}{N}.$$

Its standard deviation is

$$\sigma_N = \frac{\gamma_N}{\gamma_1}\sigma_1,$$

where γ_N and γ_1 are the number of standard deviations of the actual load corresponding to the probabilities $P_N(x)$ and $p_1(x)$. The average value of the equivalent load remains the same as for the acting load.

To verify how close the results of the two methods of substitution of repeatedly-acting loads by singly-acting load are comparative calculations were carried out.

The stress, which can be assumed in the section of a tension bar from the average value of the singly-applied equivalent load,

$$\sigma = y \frac{P_N}{F},$$

where y is the design coefficient, determined by the formula

$$y = \frac{1-\gamma\sqrt{v_p^2+v_T^2+v_F^2-\gamma^2 v_p^2\,(v_T^2+v_F^2)}}{1-\gamma^2 v_p^2}. \qquad (9)$$

Here v_p, v_T, v_F are coefficients of load variation*, resistance variation* (yield point or tentative resistance) and the area variation* of the section; γ is the characteristic of safety.

The results of calculating the value of the coefficient y for a tension bar from steel type St. 3 with a coefficient of load variation, $\upsilon_p = .25$, yield point $\upsilon_T = 0.1$ and area of section $\upsilon_F = 0.25$, number of load applications N = 1, 10, 10^2, 10^3 and 10^4 and safety coefficient $\gamma = 3$, are given in Table 3. It is easy to show that the calculation according to both methods in all cases gives practically identical results (the ratio $\frac{y_1 \bar{P}_1}{y_2 \bar{P}_N}$ changes from 1. to 0.94).

Consequently, any repeatedly-acting load can be replaced by an equivalent one through the effect on the structure

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*Translator's note: The coefficient of load variation υ means the coefficient of variation ($\sigma_X/\bar{X}$) of the load times γ.

of a singly-applied load, and the principle difference between the two groups of design random quantities is eliminated.

The value of the coefficient of live load variation should be fixed differentially, depending on the planned period of service of the designed construction, i.e., on its importance.

On the Values of the Safety Characteristics

In the statistical method of design we are concerned with the value of the safety characteristic γ only for the auxiliary design quantity

$$D = ARF - B \sum_{1}^{n} P_i, \tag{10}$$

where R = resistance of the material;

F = a geometric parameter of the section (area, section modulus);

$\sum_{1}^{n} P_i$ = sum of the force effects on the given element in the design section from different loads;

A = a coefficient, taking into account the difference between the resistance of the material determined by standard specimens, and the calculated resistance of the material in the rolled profiles, and also the influence of the technological processes involved in manufacture and erection, on the strength of material going into the structure;

B = a coefficient, taking into account the difference of the distribution of the forces between the design diagram and the real structure. The coefficients A and B are random quantities, in the same way as R, F and P_i.

In a series of investigations by the statistical method the auxiliary quantity D is taken in a different form, namely,

$$D_1 = AR - \frac{B \sum_{1}^{n} P_i}{F} \quad \text{or} \quad D_2 = \frac{ARF}{B \sum_{1}^{n} P_i}.$$

However, this has no significance in principle.

From the theory of probability (the theory of composition of distributions) it is known that in the case where a generalized random quantity is the algebraic sum of a sufficiently large number of particular random quantities with different laws of distribution, then in the majority of cases the distribution of this generalized random quantity must approach the normal distribution asymptotically.

In our case the significant portion of the design random quantities (material resistance, geometrical parameters of the section, certain loads), as shown above, has a distribution near to the normal.

Table 4 gives values of the safety characteristic for loads distributed according to the model law with different parameters, having the same value of the function $F(x) = 0.00135$, which corresponds to $\gamma = 3$ for the normal distribution.

From Table 4 it is seen that for the model law of distribution at values of the parameter $\alpha = 1.5$—4, the safety characteristic γ for which the value of the function $F(x) = 0.00135$, is 2.8 to 4.3, i.e., near to 3 for the normal distribution.

If the random quantity is the product of a number of particular random quantities, then its distribution tends to the logarithmic-normal[1].

The differential function of this distribution is

$$f(x) = \frac{e^{-\frac{\ln x}{2b^2}}}{b\sqrt{2\pi}\cdot x}.$$

In this case, if the coefficient of variation of this random quantity is relatively small (less than 0.25), its distribution also approaches the normal.

Table 5 gives values of $\bar{x}$, σ and γ for the logarithmic-normal distribution with coefficient $b = 0.1 - 0.25$ for $F(x) = 0.00135$.

From the above one can assume that the safety characteristic in formula (9) for the determination of the

(1) Kh. B. Kordanskii, "Applications of the Theory of Probability to Engineering Works"; Gosfizmatizdat, 1963.

design coefficient y can be taken without substantial error in conformity with the value of the function F(x) for the normal distribution.

It is hoped that the objection against the statistical method of calculation of building structures is removed and this advanced method finds application in design practice.

TABLE 1

	1	1,5	2	2,5	3	3,5	4
$\bar{x}$	1	0,9	0,89	0,89	0,89	0,9	0,91
$v=\frac{\sigma}{\bar{x}}$	1	0,68	0,53	0,42	0,36	0,31	0,28
σ	1	0,61	0,47	0,37	0,32	0,28	0,25
γ for $F(x)= 0{,}00135$	5,63	4,32	3,6	3,35	3,13	2,93	2,81

TABLE 2

Period of use of structure (years) or number of load repetition (N)	Model Distribution				Normal Distribution				Ratio		
	$F(x)$	P_N	σ_N	v_N	$\sqrt[4]{N}$	$\bar{P}'_N$	σ'_N	v'_N	$\frac{\bar{P}'_N}{P_N}$	$\frac{\sigma'_N}{\sigma_N}$	$\frac{v'_N}{v_N}$
25	0,04	1,8	0,25	0,138	2,23	1,8	0,21	0,117	1	0,84	0,84
50	0,02	1,98	0,21	0,105	2,67	1,92	0,18	0,094	0,97	0,85	0,89
75	0,0134	2,07	0,19	0,096	2,94	1,98	0,16	0,081	0,96	0,85	0,88
100	0,01	2,14	0,18	0,084	3,16	2,04	0,15	0,073	0,96	0,84	0,87

TABLE 3

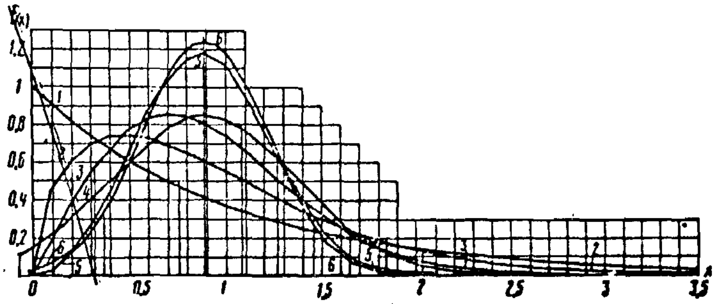
v_p	N	y_1	y_2	$\frac{\bar{P}_N}{\bar{P}_1}$	$\frac{y_2\bar{P}_1}{\bar{P}_N}$	$\frac{y_1\bar{P}_1}{y_2\bar{P}_N}$
0,25	1	0,5	0,5	1	0,5	1
0,25	10	0,46	0,61	1,38	0,45	0,98
0,25	10^2	0,42	0,65	1,6	0,41	0,98
0,25	10^3	0,4	0,66	1,72	0,38	0,95
0,25	10^4	0,39	0,66	1,78	0,37	0,95
0,15	10^4	0,49	0,67	1,47	0,46	0,94
0,333	10^4	0,34	0,66	2,05	0,32	0,94

TABLE 4

	Model Distribution (PR)							NR
	α							
	1	1,5	2	2,5	3	3,5	4	
x for $F(x) =$ $= 0{,}00135$ . .	6,61	3,52	2,57	2,13	1,88	1,74	1,61	
$\bar{x}$	1	0,9	0,89	0,89	0,89	0,9	0,91	
$x - \bar{x}$	5,61	2,62	1,68	1,24	0,99	0,82	0,7	
σ	1	0,61	0,47	0,37	0,32	0,28	0,25	
γ for $F(x) =$ $= 0{,}00135$ . .	5,6	4,3	3,6	3,4	3,1	2,9	2,8	3

TABLE 5

b	0.25	0,2	0,18	0,16	0,15	0,14	0,12	0,1
$\bar{x} = e^{-\frac{b^2}{2}}$	1,03	1,02	1,02	1,01	1,01	1,01	1,01	1,01
$\sigma = \sqrt{e^{b^2}(e^{b^2} - 1)}$	0,262	0,206	0,185	0,163	0,154	0,142	0,121	0,1
$x_A = x - \bar{x}$	1,09	0,8	0,69	0,6	0,56	0,51	0,42	0,34
γ	4,2	3,8	3,7	3,6	3,66	3,55	3,5	3,4



Curves of probability density

1. ПР —— $\alpha = 1$
2. ПР —— $\alpha = 1{,}5$
3. ПР —— $\alpha = 2$
4. НР —— $\bar{x} = 0{,}89$; $\sigma = 0{,}47$, as for PR —— $\alpha = 2$
5. ПР —— $\alpha = 3$
6. НР —— $\bar{x} = 0{,}89$; $\sigma = 0{,}32$, as for PR —— $\alpha = 3$

FIGURE 1

DEVELOPMENT OF PROBABILITY METHODS FOR THE DESIGN OF STRUCTURES IN THE USSR

(Razvitie v SSSR veroyatnostnykh metodov rascheta sooruzhenii)

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Stroitel'naya Mekhanika i Raschet Sooruzhenii

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For a long time investigations in the area of structural design were aimed at the maximum precision of the initial data, thinking that this approach brought the design nearer to the actual performance of the structure.

Only gradually did it become evident that the statistical nature of the majority of design quantities had to be considered and a change from deterministic to random dependence had to be made. In the theory of structures all this was first revealed in the analysis of structures and in the maintaining of safety factors in the strength calculations.

The safety factor was always determined by consideration of the experience of erection and use of a large number of structures, but without any calculation basis. A rough determination of the safety factor contradicted the applied precise and laborious methods of deterministic calculation of strength and often did not provide the necessary economy of expenditure of materials, and in some cases did not provide the required safety.

Since the safety factor depends on many factors, repeated attempts were made to break it down into a series of coefficients, each of which would determine the influence of any one factor. Usually these particular coefficients were proposed in the form of factors whose product must give the full amount of the safety factor. Such a method was, in particular, recommended for designs in mechanical engineering by I. A. Oding, and for the design of building structures by P. Ya. Kamentsev. In this case the particular coefficients were determined by eye and their product, through the accumulation of errors, was even less accurate than the full safety factor obtained by a direct intuitive estimate.

The essence of the safety factor, as a function of the possible deviations of the properties of materials and loads from their expected values, was revealed at the end of the twenties in a series of articles by N. F. Khotsialov (27-28) and N. S. Streletskii (25-26). N. F. Khotsialov was the first to suggest determining the safety factor from the condition of not exceeding a definite but very small probability of structural collapse, taking the strength properties as random, and the load as deterministic.

N. S. Streletskii in 1935 suggested considering the random deviations of both the strength characteristics and the loads. In this case he introduced a quantity, called the safety guarantee, which depended on functions of the distribution of strengths and loads.

For the determination of this quantity the distribution curves of strength and load were constructed on the same axis, expressed in identical units, for example in Kg/cm^2 (Fig. 1). A vertical passing through the point of intersection of these curves, cut off two areas ω_1 and ω_2, whose product established the safety guarantee $\Gamma = \omega_1 \omega_2$.

From the fixed quantity Γ the correlation of the centres of the distributions of strength and loads was determined, i.e., the safety factor sought for.

It should be noted that the safety guarantee did not determine synominously the probability of collapse (16).

More recently the method of probability theory was applied to the given problem in the articles (17-18). The principle position and conclusions of these investigations consist in the following. The conclusive stage of the design of the structure for strength and deformability must be the estimation of their possible deviations from the expected values. In view of the statistical character of the basic deviations both for the characteristic of strength of the elements and for the loads on the structure, the method of the theory of random quantities can be used. The value of the required safety factor must be based on the law of the distribution of the final results of the calculation, which is also a random variable, and on the fixed safety characteristic, which for a normal curve of the distribution of calculation results is equal to the number of standard deviations cut off in this curve from the centre of the distribution. In the case of a distribution curve of different form the value of the safety factor can be determined from a fixed sufficiently small value of the probability of collapse. Such a method made it possible to consider the various aspects of the performance of the structure - its set-up and the character of the load. These investigations were conducted simultaneously and independently of some analogous investigations by Freudenthal (U.S.A.).

In 1945, in connection with the development of new standards of design and planning by the newly organized Narkomtyazhstroi Commission for the unification of methods of design, a conditional system of calculation coefficients, suggested by I. I. Gol'denblat, M. G. Kostyukovskii and A. N. Popov was adopted. According to this system the general safety factor was broken down into three groups: the uniformity coefficient, accounting for possible deviations of strength characteristics; the overload coefficient, accounting for random increases of the loads; and the coefficient of the performance conditions. This system fixed the basis of design calculations, adopted by the new SNiP*, in which the definition of what should be called failure, was given according to newly proposed terminology, the limit states of the structure (2). The commission mentioned above, composed of N. S. Streletskii, V. M. Keldysn, A. A. Gvozdev, V. A. Baldin, I. I. Gol'denblat, and others, defined three forms of the limit states: based on strength, deformation and the amount of opening of cracks (in reinforced concrete and masonry structures). The limit state was defined as the condition of the structure, where its further serviceability became

*Rules and standards of construction

impossible or inadvisable. In spite of some insufficient development, this method was an advance, since it allowed one to estimate separately the influence of the random character of the strength properties of materials and of the load. The short-coming of the method of limit states is that the coefficients of uniformity and overload were determined for each design factor independently of the scatter in the other factors. This led to an overstated reliability, i.e., insufficiently economic design when there are a large number of random factors, and a small reliability in those cases when only one factor is random (for example, in metal reservoirs, where the load is determined and the deviation can only be in the magnitude of the steel strength). In recent years an attempt was undertaken to introduce in the method of limit states the so-called coefficients of combination.

At the present time the method of limit states is applied in the design of all building structures and analogous methods were introduced in a number of other countries. However, the time has come to work out new standards for the design of structures. These standards must be based on a stricter use of the theory of probability and random functions (17).

In the probability approach a vulnerable area for a long time was the principle of fixing the standard probability of collapse (or the number of standard deviations from the average value of the calculated quantity). It is true to say that the volitional approach here was not removed, but was only transferred to the earlier stages of the calculation.

A. R. Rzhanitzyn suggested an approach to the determination of the safety factor (20) based on the minimum expected full cost, including the cost of erecting the structure and of liquidation as a result of collapse or damage multiplied by the probability of collapse during the period of service of the structure.

There is no objection to this method for cases where there are purely economic consequences of collapse and it requires only correct calculations of expected expenditures.

In the article (22) formulae were given for determining optimum dimensions of the sections of structures on the assumption that the cost of erection and restoration are non-uniform linear functions of the cross-sectional areas. The normal law of distribution was taken for the limits of strength and for the load. In recent times B. I. Snarskii gave a number of generalizations and gave recommendations for the practical application of this method with a different law of the distributions of loads and strengths. He also made an attempt to substantiate the economic approach for any structure.

The investigation of statistical strength of structures appeared partly in connection with the problem of the safety factor. The development of the theory of statistical strengths of materials began much earlier. In 1940 the work of T.A. Kontorov and Ya. I. Frenkel' (13) was published. Their work was closely connected to the work of the Swedish scientist W. Weibull and was based on the concept of ideally brittle materials, whose collapse was determined by the maximum reduction of the tentative strength in any point of the body. The correlation between the presence of defects at sufficiently close points of the body was not considered. In spite of the formal character and also disregarding the possible redistribution of forces in the body at partial collapse, this theory made it possible to describe such qualitative phenomena as, for example, scale effect.

Analogous methods were used by N.N. Afanas'ev in the theory of fatigue strength (1). Subsequently this theory was developed in the works of V.V. Bolotin (7-8).

The first consideration on a different approach to the strength of statically determinate and statically indeterminate systems was expressed in the paper (18). Thus for statically determinate systems the strength is determined by the weakest element and the problem in this case is to find the probability of occurrence of the minimum value of some quantity in several statistical sets.

In statically indeterminate systems the case is more complicated. For the simplest statically indeterminate systems the statistical strength was determined by L. G. Sedranyan (23). A number of considerations regarding the problem of the statistical strength of concrete can be found in reference (14).

An original approach to the statistical strength of materials was applied in (29), where instead of defects, the statistical set of the three-dimensional forces was examined, equivalent in their action to the presence of defects.

The statistical approach is especially necessary when small chance deviations in the initial values lead to large quantitative and qualitative changes in the results of the calculation. Such a condition takes place in problems of stability of compressive structures. The first calculation of stability by statistical methods, evidently are the articles (18) and (19). The author proposed a method of constructing curves of coefficients of decrease in the allowable stresses in compression depending on the random errors in the centering of members and from possible elementary wow of the member axis. The method was based on calculating eccentrically loaded members with respect to boundary stresses. In calculating the dispersion of the critical forces, which depend non-linearly on the initial data, a linearization of these dependencies was made which led to simple final formulae. The method of linearization is sufficiently accurate in cases of a comparatively small change of the random factors, when large deviations of the latter have a small probability.

The statistical method was applied by V. V. Bolotin (5) to the problem of stability of compressed cylindrical shells based on the non-linear theory of shell stability. In this case the random factor was considered only the amount of elementary flexure in the centre of the panel of the shell. This method was used by B. P. Makayov (15), who examined the stability of cylindrical shells under lateral normal pressure, under all around compression and under torsion. Analogous investigations were conducted also by I. I. Vorovich (11), V. N. Goncharenko (12) and A. S. Vol'mir (10).

Of theoretical interest and of practical importance in the application of probability methods was the investigation on settlements of buildings in unevenly compressible soil. The problem of a beam on a statistically non-uniform elastic base was suggested and solved by D. N. Sobolev (24). The coefficient of subgrade here was considered a stationary random function of length, computed along the beam. For the solution of the problem a method was applied in which the equation containing the random function is reduced to an equation containing a random quantity. This is achieved by integrating the equation over the length of the beam with a fixed weight by a method close to that of Bubnov-Galerkin. Simultaneously V. V. Bolotin offered a different solution to the given problem. He used (8) the concept of a small random component of the coefficient of subgrade in comparison with its mathematical expectancy. This solution was applied to calculating the strength of a pipeline lying on a non-uniform elastic base.

More general problems posed more recently are problems of the theory of elasticity or the theory of plasticity of continuous bodies with random physical characteristics, for example, with coefficients of elasticity as random functions of the coordinates. In this area little has been done yet. The article of V. V. Bolotin (9) on the calculation of reinforced plastics with random irregularities should be mentioned. A number of investigations are now working in this area.

The random character of the actions on a structure occurs perhaps more often than the random nature of strength. This action in the majority of cases is better considered not as random quantities but as random functions of time. A typical action in this respect is wind load. The concept of wind load as a stationary random process was used in a paper by M. F. Barshtein (3) for the design of tower structures under wind load. The problem reduced to the calculation of vibrating systems during random perturbations. The apparatus of the spectral theory of random functions was used. Later on M. F. Barshtein gave analogous calculations for the action of sea waves and for seismic vibrations (4), which were examined also as stationary random processes. In reality the vibrations of the ground in earthquakes are non-stationary random processes which was mentioned somewhat earlier by V. V. Bolotin. He examined this problem using rather general notions concerning the many components of seismic action, i.e., the presence of three random translational displacements and three random rotations (6).

In problems of the action of random processes the probability of collapse is determined as a function of the time of their action, for which the theory of ejections is used. In the majority of cases the ejections can be considered as sufficiently rare, non-correlated events, which makes it possible simply to solve the problem as depending on the given period of service of the structure.

The maximum values of the snow load reached towards the end of the winter can be considered as repeated random values. The calculation here reduces to the problem of the probability of the occurrence of the specific maximum value of repeated random values for which various approximate methods can be used (21 and 16).

The loads obtained also carry a random character, but their regularity is sometimes very difficult to catch. There is a great deal of statistical data for crane loads in industrial buildings with roadway cranes. However, there are deterministic dependencies embedded in the random factors, and this requires more care in the application of statistical methods. The same phenomenon occurs in the pay loads in dwellings and public buildings.

It must be mentioned that in the existing standards of the design of structures the random character of the loads is not reflected sufficiently completely.

The whole set of probability calculations of structures for strength is sometimes called the theory of reliability of buildings and structures, including the change of fitness with time owing to corrosion, ageing of materials, etc. The theory of reliability in the construction area has not yet received much development, but is applied with success for the solution of problems of machinery construction and for the performance of various devices.

An important contribution to the theory of reliability of building structures was made by V. V. Bolotin, who gave special attention to the problem of accumulation of damage in repeated overloads (8).

At the present time investigations in the area of design of structures with the application of probability methods are being carried out on broad fronts. Their expediency and efficiency are no longer in doubt. The development of these methods promotes advanced refinement of the mathematical apparatus and promotes its application in other branches of technology and physics which could not be dealt with in this paper.

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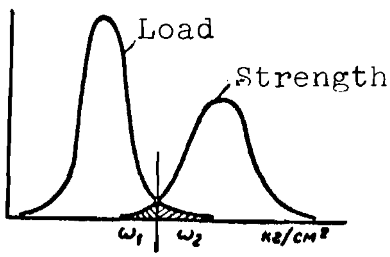

Figure 1