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Use of Streamwise Periodic Boundary Conditions for Problems in Heat and Mass Transfer

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Fully developed periodic boundary conditions have frequently been employed to effect performance calculations for heat and mass exchange devices. In this paper a method is proposed, which is based on the use of primitive variables combined with the prescription of slip values. Either pressure difference or mass flow rate may be equivalently prescribed. Both constant wall temperature (Dirichlet) and constant heat flux (Neumann) conditions may be considered, as well as the intermediate linear (Robin) boundary condition. The example of an offset-fin plate-fin heat exchanger is used to illustrate the application of the procedure. The mathematical basis by which the method may be extended to the consideration of mass transfer problems with arbitrary boundary conditions, and associated continuity, momentum, and species sources and sinks is discussed. [DOI: 10.1115/1.2709976]

Keywords: computational fluid dynamics, periodic boundary conditions, primitive variables, heat and mass transfer

Introduction

In the application of computational fluid dynamics (CFD) to the analysis of heat and mass exchange devices, much computational effort may be avoided by considering elements deep within the design where the flow-field is "fully developed," [1–3]. Let it be assumed the domain has been tessellated with a structured mesh with associated finite-volume equations [4],

$$\sum a_{NB}(\phi_{NB} - \phi_P) + S = 0 \tag{1}$$

where $\phi = p', u, v, w, T, m$, and the compass notation [4] is employed for the neighbor values, NB = W (west), E (east), S (south), N (north), L (low), and H (high). The source term in Eq. (1) is frequently linearized according to

$$S = C(V - \phi_P) \tag{2}$$

where C is a source-term coefficient and V is a source-term value. If the flow is fully developed

$$u(0,y,z) = u(l,y,z) \tag{3}$$

$$p(0,y,z) = p(l,y,z) + \Delta p_0 \tag{4}$$

For constant heat flux, the temperature field is piecewise linear, $T(0,y,z) = T(l,y,z) + c$, whereas for constant wall temperature $\theta(0,y,z) = \theta(l,y,z)$, where $\theta = (T - T_w)/(T_0 - T_w)$, and T_0 is a reference temperature, often chosen as the bulk value. These may be combined to obtain [5]

$$T(0,y,z) = c_1 T(l,y,z) + c_2 \tag{5}$$

$$c_1 = \begin{cases} \dfrac{T_0(0) - T_w}{T_0(l) - T_w} & \text{constant } T_w \\ 1 & \text{constant } \dot{q}''_w \end{cases} \tag{6}$$

$$c_2 = \begin{cases} T_w(1 - c_1) & \text{constant } T_w \\ T_0(l) - T_0(0) & \text{constant } \dot{q}''_w \end{cases} \tag{7}$$

Patankar Liu and Sparrow [6] transformed the state variables to obtain a set of cyclic equations

$$\phi(x,y,z) = \phi(x + l, y, z) \tag{8}$$

This was achieved by defining a reduced pressure, $\tilde{p} = p - \beta x$. Murthy and Mathur [7] proposed a rationale whereby β is adjusted until a desired mass flux is obtained. For constant wall flux, a reduced temperature, $\tilde{T} = T - \gamma x$ may also be defined. The reduced-variable approach was subsequently adopted by numerous researchers, for example Patankar and Prakash [8]. The user must introduce volumetric sources into the transformed equations, and also modify the wall boundary conditions. For constant T_w, the nondimensional temperature, θ, is the state variable. The θ system of equations is, however, complex as it involves the solution for the local reference temperature. Kelkar and Patankar [9] subsequently proposed a primitive-variable formulation in temperature (only) for the constant T_w problem. The present author also worked with primitive variables, p, T (not $\tilde{p}$, $\tilde{T}$, θ) in previous work [5,10,11]. Periodic boundary conditions were implemented by the addition of a slab of halo cells downstream [12–14].

Present Approach

The problem to be addressed is the means whereby periodic boundary conditions may be reduced to cyclic conditions in the primitive-variable formulation, without the introduction of either transformed variables or halo cells. This is achieved by imposing slip conditions in the x direction.

Fluid Flow. For streamwise u momentum, an impulse or pressure "shock" condition is imposed

$$S = A_P \Delta p_0 \tag{9}$$

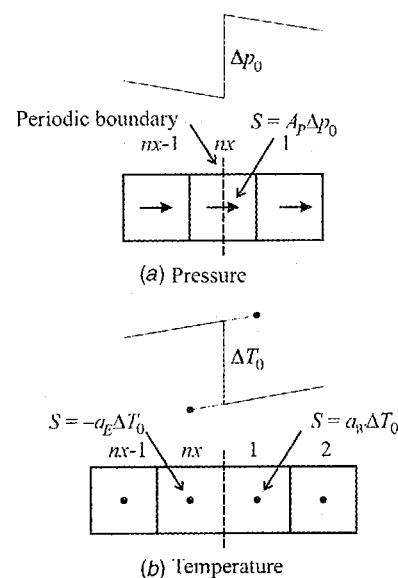along a single y-z plane of cells. Figure 1(a) illustrates the notion

(a) Pressure

(b) Temperature

Fig. 1 Slip boundary conditions for a staggered scheme, constant wall flux

schematically. If it is desired that the bulk velocity, u_0, or Reynolds number be prescribed, Δp_0 may be corrected according to $\Delta p_0 = \Delta p_0^* + \Delta p_0'$, where Δp_0^* is the value at the previous iteration, and $\Delta p_0'$ is a correction factor. Neglecting streamwise diffusion and applying an order-of-magnitude analysis yields

$$\Delta p_0' = \rho|u_0|(u_0 - u_0^*) \tag{10}$$

which is similar to, but simpler than that given in Ref. [7].

Heat Transfer. For the temperature field there is a slip in the values across the boundary faces, and it is necessary to prescribe a source/sink pair. The slip values are the differences between actual and apparent temperatures that would arise if the field were truly periodic. They are not necessarily equal and opposite. If the user has access to the neighbor values, ϕ_{NB} in Eq. (1), in the CFD solver, it is possible to directly add/subtract the ΔT slip values from the neighbors, T_E and T_W. If the user does not have access to the solver, it is more convenient to introduce a pair of source terms say, $S = a_W \Delta T_W$ at $i=1$, and $S = a_E \Delta T_E$ at $i=nx$. For the constant $\dot{q}_w''$ (Neumann) condition, $\Delta T_W = -\Delta T_E = \Delta T_0$. Figure 1(b) illustrates schematically the source–sink pair. Although it is possible to directly set $\Delta T_0 = \dot{q}_w/\dot{m}c_p$ [6], as a fixed flux (source), the author chooses to adjust the source term iteratively, until the upstream in-cell temperature reaches a desired reference value, $T_0(0)$, with

$$\Delta T_0 = T_0(0) - T_0(l) \tag{11}$$

where $T_0(l)$ is a fetched value, e.g., the bulk downstream value, and $T_0(0)$ is a prescribed value. This rationale works well under most circumstances, however, situations can arise where the choice of $T_0(0)$ is not arbitrary, but is a function of the boundary values.

For the constant T_w (Dirichlet) condition, the implementation is similar to that in Ref. [9]. The source terms are $a_W[(c_1-1)T_W + c_2]$ at $i=1$ and $a_E[(1-c_1)T_E - c_2]$ at $i=nx$, where T_W is the west neighbor at $i=1$ i.e., T_P at $i=nx$, and T_E is the east neighbor at $i=nx$, i.e., T_P at $i=1$, and $\Delta T_E \neq -\Delta T_W$. The boundary conditions for both Dirichlet and Neumann problems may be written as

$$S = \pm a_{NB}[(c_1 - 1)T_{NB} + c_2] \tag{12}$$

where c_1 and c_2 are specified in Eqs. (6) and (7), the subscript NB refers to the east or west neighbor, and the linking coefficients a_E and a_W must be computed by the same scheme employed in the CFD solver.

Mass Transfer. The unit cell approach has also been applied to mass transfer problems [15–17], and may be used in heat transfer problems with injection/suction at the boundaries, where temperature (or enthalpy) replaces mass fraction, denoted here by ϕ. A constant transformed substance state (Robin) boundary condition is appropriate

$$S = \dot{m}(\phi_\infty - \phi_w) \tag{13}$$

where ϕ_∞ denotes ϕ in the transferred substance state [18,19]. Comparison of Eqs. (2) and (13) reveal that if $\phi_w \approx \phi_P$, the mass transfer problem is another instance of the linearized source term with $C = \dot{m}$ and $V = \phi_\infty$. Differences between nodal and wall values are easily dealt with using harmonic averaging of the coefficients; e.g., with $S = a_w(\phi_w - \phi_P)$, Eq. (2) is enumerated with $C = \dot{m}a_w/(\dot{m} + a_w)$, and $V = \phi_\infty$. For low Péclet numbers $a_w = \Gamma A/|P - w|$; while at high Péclet numbers both convection and diffusion must be accommodated [17]. The constant transferred substance state boundary condition differs from constant value or flux. However in the limit $\dot{m} \to 0$, it reduces to a Neumann condition and in the limit $\dot{m} \to \infty$, it reduces to a Dirichlet condition. The reader will note that the linear wall condition does not necessarily have to assume the form, Eq. (13), but could also be expressed as

$$S = g_\infty A(\phi_\infty - \phi_w) \tag{14}$$

where g_∞ is a heat/mass transfer coefficient.

Equations (5)–(7) may thus be conveniently generalized for the linear boundary condition

$$\phi(0) = c_1\phi(l) + c_2 \tag{15}$$

$$c_1 = \frac{\phi_0(0) - \phi_w(0)}{\phi_0(l) - \phi_w(l)} \tag{16}$$

$$c_2 = \phi_0(0) - c_1\phi_0(l) \tag{17}$$

The upstream wall value must be computed

$$\phi_w(0) = \frac{\phi_0(0) + B(l)\phi_\infty}{1 + B(l)} \tag{18}$$

$$B(x) = \frac{\phi_0(x) - \phi_w(x)}{\phi_w(x) - \phi_\infty} \tag{19}$$

and it is tacitly assumed that the driving force [18,19] is cyclic, $B(l) = B(0)$.

For low mass flow rates, no continuity modifications are required. At higher mass flow rates, injection/suction rates have a significant impact on the crosswise v velocity, pressure gradients, and scalar transport, and these can be captured only by including continuity changes. A similarity velocity profile, $u(0) = cu(l)$, may be presumed [17]. In the continuity/pressure–correction equation, a mass sink equal and opposite to the source at the wall (or vice-versa) is required, $S = -\rho A_P(c-1)u(l)$. In the u momentum equations it is also necessary to introduce momentum slip terms, $V = \pm a_{NB}(c-1)u_{NB}$. The equations are further complicated because the a_{NB} coefficients include convection terms and must also be modified. High mass transfer rate problems will be considered in future work.

Example: Offset-Fin Heat Exchanger

The problem considered is conjugate laminar-flow heat transfer in a three-dimensional offset-fin plate-fin heat exchanger as illustrated in Fig. 2. Solutions to this problem were provided in Refs. [5,10] together with details of the geometry and fluid properties. A domain of $2l \times p/2 \times b$ was tessellated with a mesh of $56 \times 37 \times 57$ (118, 104) cells. Calculations were performed with a segregated solver, staggered grid, and hybrid scheme [4], using the CFD code PHOENICS [20]. Figure 3 shows velocity vectors for plan and elevation views of the heat exchanger design midway between the fins at $y=p/4$ and the plates at $z=b/2$, respectively. Figures 4 and 5 are contours of pressure, p, and temperature, T,
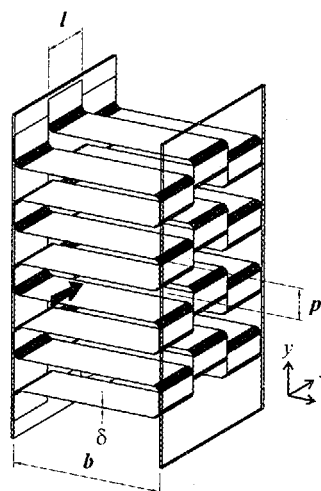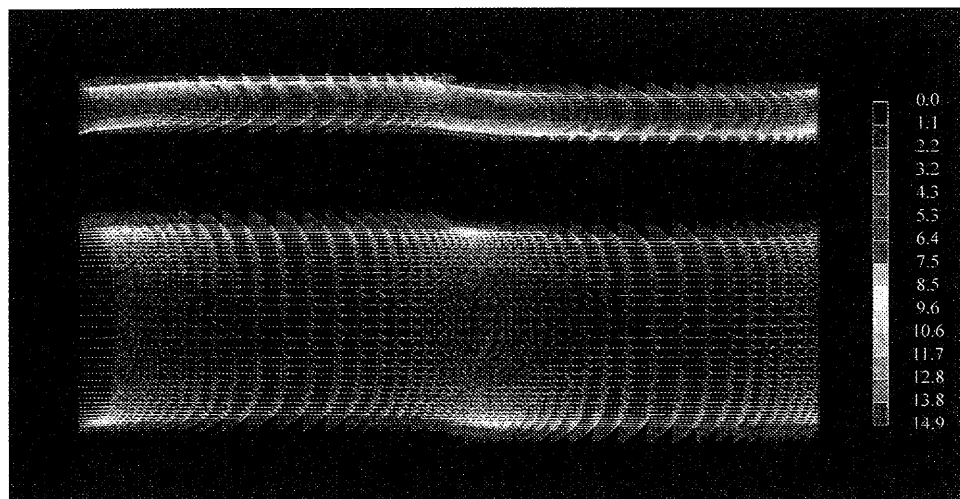

Fig. 2 Offset-fin plate-fin geometry

Fig. 3 Velocity vectors (m/s) for Re=500

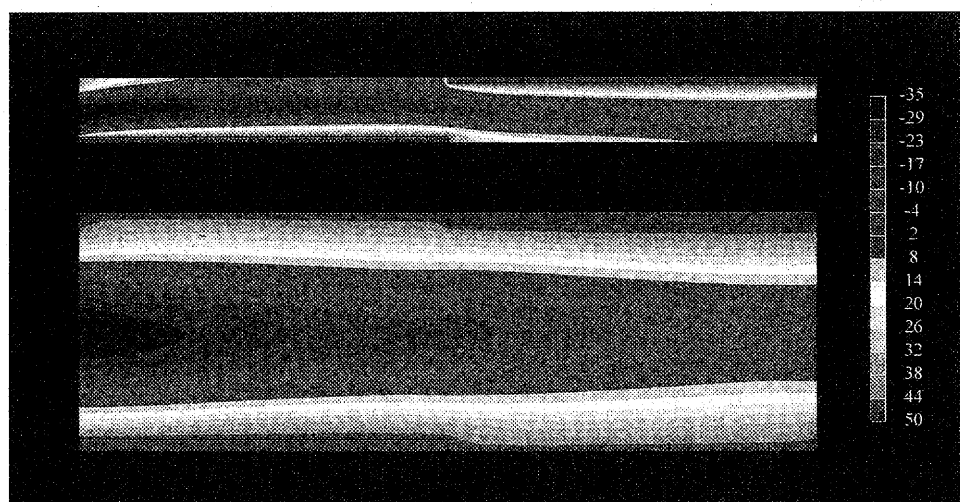

Fig. 4 Pressure (Pa) distribution for Re=500
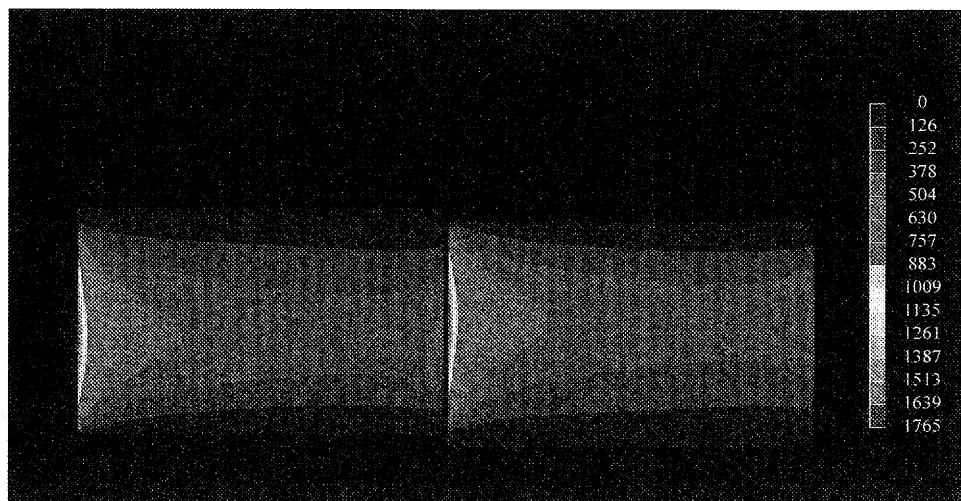

Fig. 5 Temperature (°C) distribution for Re=500

Fig. 6 Local Nusselt number for Re=500

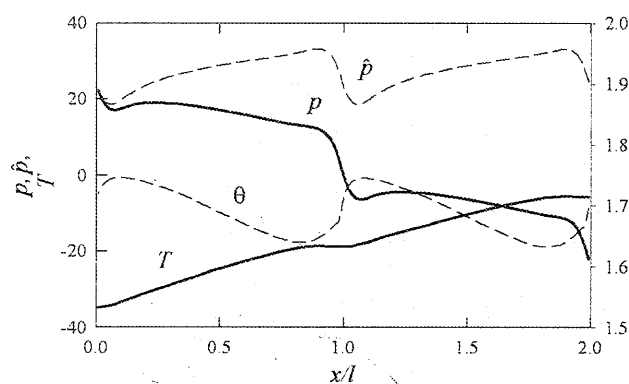**Fig. 7 Dimensional pressure p (Pa) and temperature T (°C), as well as reduced pressure $\bar{p}$ (Pa) and nondimensional temperature θ, for Re=500 along the symmetry line $y=p/4$, $z=b/2$**
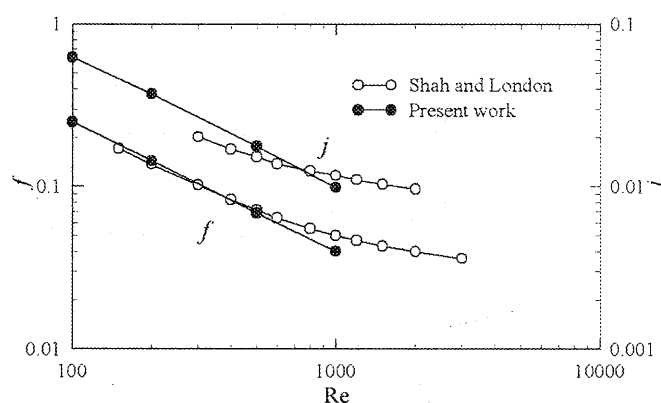

Fig. 8 Friction and heat transfer factors compared with experimental data

respectively, in the symmetry planes. Figure 6 shows the local Nusselt number on the fin surface. It is a maximum at the front center of the fin, where shear is greatest and decreases rapidly as the boundary layer develops. Figure 7 shows a comparison of p and T with values of $\bar{p}$ and θ, computed at the end of the calculations, along the center line $y=p/4$, $z=b/2$. It can readily be seen that these are cyclic. Overall friction and heat transfer factors were also computed [5,10], and are compared to experimental values in Fig. 8. It can be seen that agreement is good for friction factor, f, with some error apparent in heat transfer factor, j, consistent with values in Refs. [5,10].

Discussion and Conclusions

Periodic boundary conditions may readily be prescribed for problems in fluid mechanics, heat, and mass transfer, in primitive form with cyclic boundary conditions, by associating periodic boundary conditions with the slip values, introduced as source terms. No equation transformations or new field variables are required, and standard wall boundary conditions may readily be employed. Either the mean pressure gradient, or the reference velocity/flow Reynolds number may be prescribed. The method devised to adjust the pressure jump based on a desired reference bulk velocity or Reynolds number, Eq. (10), was found to work well. The formulation is advantageous, not only for the constant T_w (Dirichlet) condition, but also for the generalized heat/mass transfer (Robin) problem. Equations (15)–(19) are quite general and can be applied to problems containing a mix of different

linear coefficients and values. If two or more different conditions are prescribed on different wall sections, driving force and upstream wall values are simply computed individually using Eqs. (18) and (19). Thus, cell-by-cell variations in the boundary values and coefficients across the domain may be accommodated. A method for modifying the continuity and momentum equations for high rates of mass transfer was detailed, and will be implemented in the future. Although the work presented here is based on a finite-volume method, with structured mesh and staggered velocity scheme, the technique is quite general and may be readily extended to situations where unstructured meshes, co-located variables, and finite-element analysis techniques are used to perform flow-field calculations.

Nomenclature

A = area, m^2
a = coefficient in finite-volume equations
B = driving force
b = heat exchanger plate width, m
C = source term coefficient
C_p = specific heat, J/kg K
f = friction factor

g = heat/mass transfer coefficient, W/m^2 K or kg/m^2 s

j = heat transfer factor

l = streamwise characteristic length, pitch, m

$\dot{m}$ = mass flow rate. kg/s

T = temperature, K

$\dot{q}$ = heat transfer rate, W

q'' = heat flux rate, W/m^2

u = streamwise velocity component, m/s

v = crosswise velocity component, m/s

w = crosswise velocity component, m/s

p = pressure. Pa, fin pitch. m

$\hat{p}$ = reduced pressure, Pa

Re = Reynolds number

V = source term value

V_p = volume of cell P, m^3

T = temperature, K

$\hat{T}$ = reduced temperature, K

x = streamwise displacement component, m

y = crosswise displacement component, m

z = crosswise displacement component, m

Greek Symbols

β = volumetric term, Pa/m

Γ = exchange coefficient, kg/m s

γ = volumetric term, K/m

δ = fin thickness, m

θ = nondimensional temperature

ρ = density, kg/m^3

ϕ = generalized state variable

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