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Effect of Location of Transmitting Boundary on Seismic Hydrodynamic Pressures on Gravity Dams

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Résumé

L'auteur utilise la méthode des équations intégrales (MÉI) constantes pour déterminer l'effet de l'emplacement d'une limite de transmission sur les pressions hydrodynamiques sismiques dans un réservoir d'eau. Il se sert à cette fin d'un modèle mathématique 2D d'un système barrage-réservoir-fondation. Les résultats révèlent que la géométrie de la partie finie du réservoir est un facteur déterminant pour ce qui est des pressions hydrodynamiques s'exerçant sur les barrages, l'emplacement de la limite de transmission dans le réservoir modifiant la géométrie de cette partie. Dans le cas des réservoirs presque rectangulaires, les résultats de l'utilisation de la MÉI dépendent moins de l'emplacement. L'auteur a aussi évalué d'autres facteurs : le nombre d'éléments employés pour modéliser la limite de transmission, le nombre total d'éléments et la longueur de l'élément relativement à la plus courte longueur d'onde induite dans le réservoir.

EFFECT OF LOCATION OF TRANSMITTING BOUNDARY ON SEISMIC HYDRODYNAMIC PRESSURES ON GRAVITY DAMS

Alexander M. Jablonski [I]

ABSTRACT

The constant boundary element method is used to assess the effect of the location of a transmitting boundary on seismic hydrodynamic pressures in a water reservoir. A 2D mathematical model of a dam-reservoir-foundation system is used. The results indicate that the geometry of the finite part of the reservoir is a governing factor of hydrodynamic pressures acting on dams, where the location of the transmitting boundary changes the geometry of that part. For almost rectangular reservoirs the BEM results are less dependent on the location. Additional factors evaluated are the number of elements used to model the transmitting boundary, the overall number of elements and the length of the element with respect to the induced shortest wave length in the reservoir.

INTRODUCTION

A concrete gravity dam is designed to be stable under its own load as well as hydrostatic and hydrodynamic water pressures. The dam cross-section is usually kept constant throughout the length of the dam. In this case a 2D geometrical model may be sufficient.

Extensive research on the seismic response of gravity dam-reservoir-foundation systems has been pursued during the last decade. The finite element method and boundary element method have been used to evaluate hydrodynamic pressures acting in these systems [1,2,3,4,5,6,7]. A 2D model of the dam-reservoir system, based on the model first introduced by Chopra and Hall [1,2,3], was used in this study. Their infinite radiation condition has been recently incorporated in the 2D boundary element solution [6,7]. The foundation damping was accounted for by a simplified boundary condition [1,2]. The effects of infinite radiation were modelled by assuming that beyond a certain length upstream of the dam, the reservoir has a uniform rectangular section. For a regular but infinite region a continuum or a one-dimensional finite element solution was employed. Compatibility of pressures and pressure gradients was then enforced at the so-called transmitting boundary at the interface of the irregular finite and the rectangular infinite regions of the reservoir.

In this paper, the effect of the location of the transmitting boundary on hydrodynamic pressures is studied. The results are presented to cover three specific cases: a rectangular

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infinite reservoir; a trapezoidal finite reservoir region connected to an infinite rectangular region; and a finite region with a partially trapezoidal shape for which the boundary is moved to an infinite rectangular region.

The reservoir was subjected to harmonic upstream-downstream and/or vertical motion. The analytical technique can, however, be extended to the case of an earthquake type of motion using the Fast Fourier Transformation.

BOUNDARY ELEMENT RESERVOIR MODEL

The development of a 2D model of a gravity dam-reservoir-foundation system subjected to ground motion has been presented by Liu and Cheng [5], Humar and Jablonski [6,7]. The water of the reservoir is assumed to be compressible and non-viscous. The effect of the surface waves is considered to be negligible and water motion is limited to small amplitudes. The model includes dam-reservoir interaction and reservoir-foundation soil interaction. The base of the rigid dam and the reservoir bottom (foundation) may undergo a prescribed acceleration due to horizontal or/and vertical components of ground motion. The interaction between dam and foundation is not considered here. The bottom of the reservoir can be rigid or flexible. The governing linearized Navier-Stokes equations for harmonic motion of the dam and foundation may be reduced to the well-known Helmholtz equation

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + k^2 p = 0 \qquad (1)$$

where $k = \frac{\omega}{c}$ is the wave number.

Equation 1 is then solved numerically using a constant boundary element formulation. At first it is transformed into an integral equation and solved by the BEM technique using the so-called fundamental solution. Some different geometrical models of the system are shown in Fig. 1, 2 and 3. Each model of the reservoir consists of two regions: the irregular finite and the regular infinite section. An essential component of the model is the so-called transmitting boundary at the interface of the finite and infinite regions. The discretized version of the system of boundary element equations can be derived, as presented in detail in previous studies [6,7]. After the appropriate boundary conditions are applied, these equations may be solved for unknown values of pressure and pressure gradient at the boundary.

MODELLING OF INFINITE RADIATION USING A TRANSMITTING BOUNDARY

For an infinite reservoir, the transmitting boundary is located at the interface of the irregular finite and regular infinite regions of the reservoir . Its goal is to assure perfect energy loss in the outgoing waves. Beyond this boundary, the reservoir is assumed to have a regular rectangular section extending to infinity. The transmitting boundary takes the form of a special boundary condition as presented earlier by Hall and Chopra [2,3].

The cross-sections of the reservoirs studied are shown in Figs. 1, 2 and 3. The special boundary condition for the transmitting boundary is derived using a one-dimensional finite element discretization of the infinite region. Within an element, the pressure and pressure gradient are assumed to vary linearly in the y-direction. When constant boundary elements are used for the finite region, it is required to match the values of pressure and pressure gradient at the transmitting boundary. This can be done by introducing a transformation matrix between constant boundary element nodal values and finite element nodal values. One of the options has been discussed in Refs. 6 and 7.

Then the transmitting boundary condition together with an appropriate foundation damping condition is incorporated into the set of boundary equations [6,7]. The eigenvalue problem associated with the transmitting boundary is solved by means of the QZ algorithm developed by Moler and Stewart [8]. These eigenvalues are real when the foundation is treated as rigid; otherwise they are complex.

The earlier studies have indicated that the hydrodynamic pressures depend on the geometry of the finite portion of the reservoir. The present study is aimed at investigating how different geometries of the finite region, combined with the regular geometry of the infinite region, influence the results. In addition, the effect of the number of finite elements used in modelling the transmitting boundary is also briefly addressed.

ANALYTICAL STUDIES

The analytical studies cover the effect of the location of the transmitting boundary on hydrodynamic pressures induced by harmonic excitation.

Case 1: Effect of location in a rectangular infinite reservoir

Six reservoirs with different length have been studied: $H/L = 0.5, 1.0, 1.5, 2.0, 2.5$ and 3.0, for four chosen frequencies of excitation in upstream-downstream direction only: $\omega/\omega_1 = 0.8, 1.2, 1.6, 2.0$, where ω_1 = the fundamental frequency of the reservoir. The number of finite elements used to model infinite radiation is kept constant in all reservoir models and is equal to 10. Figure 1 shows a schematic view of these six models. Some results are presented for one excitation frequency $\omega/\omega_1 = 1.2$ in Table 1. Other results are presented in Ref. 7. Results from the classical solution are listed in Table 2. The BEM results show that the difference in the absolute values is generally less than 5% w.r.t. available classical solutions. The model seems to represent the geometry quite faithfully. The transmitting boundary need not be located too far from the dam. Thus moving the transmitting boundary away, from 0.5 H to 1.0 H or farther, does not necessarily improve the results. All results are for a rigid reservoir foundation with a coefficient of reflection $\alpha_R = 1.0$ [3,7].

Case 2: Transmitting boundary is moved to an infinite regular section

This part of the study has the objective of evaluating hydrodynamic pressures for one chosen shape of the reservoir with different locations of the transmitting boundary. However, the geometry of the irregular finite part is also changed. Five reservoir models are used. Their schematic view is presented in Fig. 2. The number of constant boundary elements changes from one model to another, but the number of finite elements at the transmitting boundary remains the same. The results for one excitation frequency $\omega/\omega_1 = 1.2$ and $\alpha_R = 1.0$ are presented in Table 3. The results do not exhibit any particular trend for this frequency and seem to depend only slightly on the location of the transmitting boundary in the rectangular infinite region of the reservoir. Further studies may be required to screen the behaviour for the representative frequency range.

Case 3: Trapezoidal finite reservoir region is connected to an infinite regular region

Five reservoirs with different lengths but with the same angles of the inclined bottoms have been studied. Thus, they have different heights of infinite section, while the actual dam height remains the same. The results are presented for one chosen excitation frequency $\omega/\omega_1 = 1.2$ and the rigid reservoir bottom with $\alpha_R = 1.0$. Figure 3 shows a schematic view of the geometry of five models. Each model has a different number of constant boundary elements. The results are presented in Table 4. The results show an increase in the absolute values of hydrodynamic pressure with increase in length of the irregular finite portion of the reservoir. The results clearly depend on the overall geometry of the reservoir. The large increases in hydrodynamic pressures could be a function of specific parameters chosen and likely result from shifts in the true natural frequency of the considered reservoir with respect to ω_1, the natural frequency of the infinite rectangular reservoir.

Modelling of transmitting boundary

Earlier studies have shown that for an excitation frequency greater than ω_1 the numerical results of the analysis depend strongly on the modelling of the transmitting boundary [7]. The results of these studies showed that even a fairly coarse division of the transmitting boundary did not introduce any appreciable error in hydrodynamic pressure values on the dam face, as long as the element size was not more than about 1/10 of the shortest wave length in the reservoir [7]. To obtain a direct measure of the effect of discretization used on the transmitting boundary (also called the far boundary), the first four eigenvalues (λ) together with respective eigenfrequencies (ω) are calculated, for four different one-dimensional meshes. The results obtained are presented in Table 5. Even with a 2-element mesh, the difference between the first frequency obtained from a one-dimensional finite element solution and that obtained from a continuum solution is remarkably small. The difference is very small when eight or more elements are used.

CONCLUSIONS

The following conclusions can be drawn:

1. The location of the transmitting boundary, which divides a reservoir into a finite part near a dam and the infinite rectangular part, can result in changes in the acting hydrodynamic forces when the geometry of the reservoir is changed.
2. If the irregular part is nearly rectangular the BEM results depend only slightly on the location of the transmitting boundary, while for a rectangular reservoir the results are virtually independent of the boundary location.
3. For the rigid case (full reflection) a relatively small number of elements may be used

for modelling the transmitting boundary provided that their length is less than 1/10 of the shortest induced wave length in the reservoir.

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Table 1. Hydrodynamic pressure ($\frac{p}{\gamma H}$ × 100) on dam in rectangular infinite reservoirs with different location of the transmitting boundary - $\frac{\omega}{\omega_1}$ = 1.2 (γ = unit weight of water and H = dam height) - Case 1.

	30 Elem. L = 0.5 H			40 Elem. L = 1.0 H		
y, m	Re Part	Im Part	Abs. Val.	Re Part	Im Part	Abs. Val.
95.	0.6656	-0.8701	1.0955	0.7713	-0.9041	1.1883
85.	0.9465	-2.7480	2.9064	1.2600	-2.9535	3.2110
75.	0.8025	-4.5384	4.6088	1.3115	-4.9205	5.0923
65.	0.4501	-6.2105	6.2268	1.1411	-6.7608	6.8564
55.	-0.0010	-7.7289	7.7289	0.8560	-8.4296	8.4729
45.	-0.4776	-9.0597	9.0723	0.5267	-9.8849	9.8989
35.	-0.9255	-10.1730	10.2150	0.2052	-11.0900	11.0919
25.	-1.3038	-11.0430	11.1197	-0.0695	-12.0150	12.0152
15.	-1.5854	-11.6520	11.7594	-0.2714	-12.6340	12.6369
5.	-1.7801	-11.9860	12.1175	-0.4107	-12.9280	12.9345
	50 Elem. L = 1.5 H			60 Elem. L = 2.0 H		
y, m	Re Part	Im Part	Abs. Val.	Re Part	Im Part	Abs. Val.
95.	0.7457	-0.8970	1.1664	0.7371	-0.8985	1.1621
85.	1.1181	-2.9111	3.1184	1.1153	-2.9047	3.1114
75.	1.1825	-4.8418	4.9841	1.1350	-4.8262	4.9579
65.	0.9653	-6.6476	6.7173	0.9000	-6.6231	6.6840
55.	0.6377	-8.2854	8.3099	0.5562	-8.2530	8.2717
45.	0.2712	-9.7153	9.7191	0.1757	-9.6766	9.6782
35.	-0.0813	-10.9020	10.9023	-0.1884	-10.8590	10.8606
25.	-0.3804	-11.8150	11.8241	-0.4764	-11.7710	11.7806
15.	-0.5994	-12.4320	12.4464	-0.7212	-12.3900	12.4110
5.	-0.7483	-12.7340	12.7560	-0.8728	-12.6980	12.7280
	70 Elem. L = 2.5 H			80 Elem. L = 3.0 H		
y, m	Re Part	Im Part	Abs. Val.	Re Part	Im Part	Abs. Val.
95.	0.6899	-1.0218	1.0241	0.7658	-0.8699	1.1590
85.	0.9433	-3.2560	3.3899	1.2591	-2.8315	3.0988
75.	0.7625	-5.3897	5.4434	1.3172	-4.7127	4.8933
65.	0.3729	-7.3832	7.3926	1.1538	-6.4725	6.5745
55.	-0.1100	-9.1923	9.1930	0.8748	-8.0683	8.1156
45.	-0.6087	-10.7750	10.7922	0.5501	-9.4608	9.4768
35.	-1.0651	-12.0950	12.1418	0.2311	-10.6150	10.6175
25.	-1.4350	-13.1200	13.1982	-0.0441	-11.5020	11.5021
15.	-1.6875	-13.8280	13.9306	-0.2499	-12.0990	12.1016
5.	-1.8277	-14.2020	14.3191	-0.3977	-12.3060	12.3124

Table 2. Hydrodynamic pressure ($\frac{p}{\gamma H} \times 100$) on dam by classical solution for rectangular infinite reservoir - $\frac{\omega}{\omega_1} = 1.2$ - Case 1.

y, m	Re Part	Im Part	Abs. Value
95.	0.7521	-0.9772	1.2331
85.	1.1920	-2.9077	3.1425
75.	1.1809	-4.7665	4.9106
65.	0.9526	-6.5080	6.5773
55.	0.6155	-8.0892	8.1126
45.	0.2409	-9.4713	9.4744
35.	-0.1185	-10.6200	10.6207
25.	-0.4223	-11.5070	11.5147
15.	-0.6411	-12.1111	12.1279
5.	-0.7554	-12.4170	12.4399

Table 5. Comparison of first four eigenfrequencies in 2D case for different finite element meshes on the transmitting boundary.

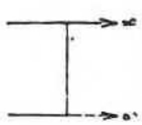
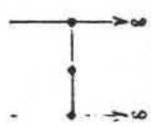
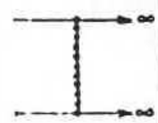
	Classical Solution	1-d. FEM 2 Elem.	1-d. FEM 4 Elem.	1-d. FEM 8 Elem.	1-d. FEM 10 Elem.
λ_1^2	0.000247	0.000260	0.000250	0.000247	0.000247
λ_2^2	0.002221	0.003169	0.002487	0.002286	0.002262
λ_3^2	0.006168	-	0.008207	0.006678	0.006492
λ_4^2	0.012090	-	0.017163	0.014081	0.013350
ω_1, s^{-1}	22.62	23.20	22.77	22.66	22.64
ω_2, s^{-1}	67.86	81.06	68.84	68.84	68.49
ω_3, s^{-1}	113.10	-	130.45	117.67	116.02
ω_4, s^{-1}	158.34	188.65	170.87	170.87	166.38
Mesh Sketch					

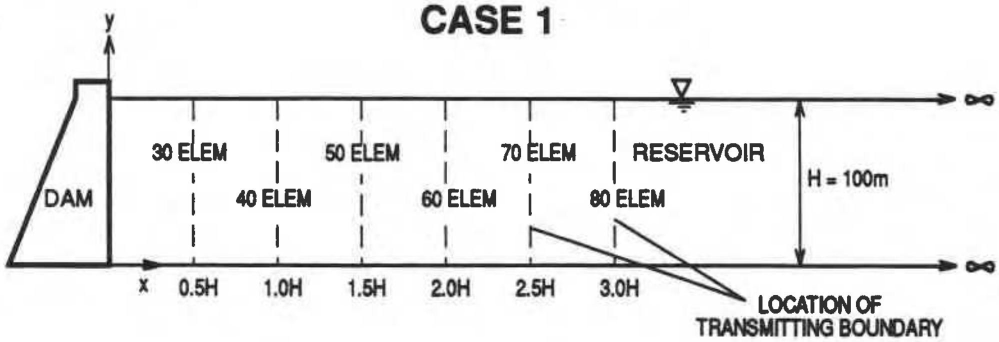
Table 3. Hydrodynamic pressure ($\frac{p}{\gamma H} \times 100$) on dam in partially trapezoidal reservoirs in which the transmitting boundary is moved to an infinite region - $\frac{\omega}{\omega_1} = 1.2$ - Case 2.

	55 Elem. L = 200 m			65 Elem. L = 250 m		
y, m	Re Part	Im Part	Abs. Val.	Re Part	Im Part	Abs. Val.
95.	7.7825	does not exist	same as real	8.8256	does not exist	same as real
85.	23.9630			27.2400		
75.	38.9980			44.3940		
65.	52.7540			60.1190		
55.	64.9880			74.1260		
45.	75.4550			86.1320		
35.	83.9490			95.8970		
25.	90.3170			103.2500		
15.	94.4730			108.0800		
5.	96.3672			110.3600		
	75 Elem. L = 300 m			85 Elem. L = 350 m		
y, m	Re Part	Im Part	Abs. Val.	Re Part	Im Part	Abs. Val.
95.	6.6508	does not exist	same as real	7.6518	does not exist	same as real
85.	20.4170			23.5570		
75.	33.1590			38.3300		
65.	44.7870			51.8430		
55.	55.1020			63.8570		
45.	63.3030			74.1330		
35.	71.0170			82.4660		
25.	76.3180			88.7090		
15.	79.7300			92.7750		
5.	81.2010			94.6140		
	95 Elem. L = 400 m			105 Elem. L = 450 m		
y, m	Re Part	Im Part	Abs. Val.	Re Part	Im Part	Abs. Val.
95.	8.1251	does not exist	same as real	11.3850	does not exist	same as real
85.	25.0410			35.2560		
75.	40.7730			57.5870		
65.	55.1770			78.1170		
55.	67.9930			96.4550		
45.	78.9650			112.2200		
35.	87.8750			125.1000		
25.	94.5620			134.8500		
15.	98.9370			141.3500		
5.	100.9500			144.5800		

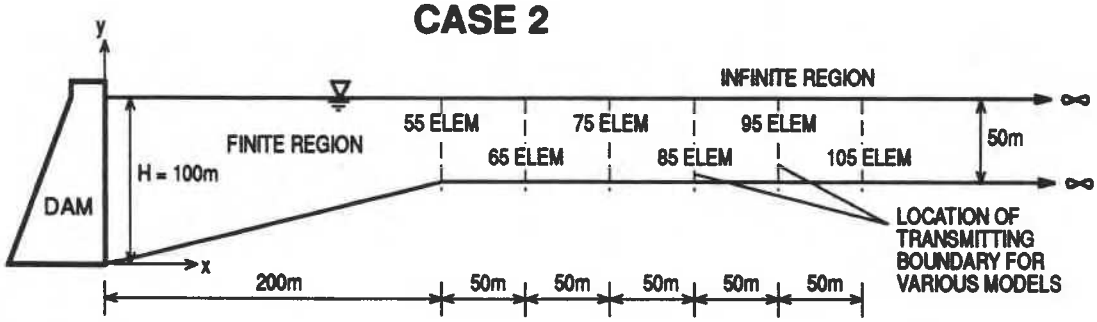
Table 4. Hydrodynamic pressure ($\frac{p}{\gamma H} \times 100$) on dam in reservoirs with different heights of infinite region - $\frac{\omega}{\omega_1} = 1.2$ - Case 3.

	Classical Sol., H = 100 m			27 Elem. L = 40 m		
y, m	Re Part	Im Part	Abs. Val.	Re Part	Im Part	Abs. Val.
95.	0.7521	-0.9772	1.2331	0.3723	-1.2318	1.2868
85.	1.1920	-2.9077	3.1425	0.0003	-3.9359	3.9359
75.	1.1809	-4.7665	4.9106	-0.7594	-6.5111	6.5552
65.	0.9526	-6.5080	6.5773	-1.6717	-8.9025	9.0581
55.	0.6155	-8.0892	8.1126	-2.6098	-11.0530	11.3569
45.	0.2409	-9.4713	9.4744	-3.4870	-12.9100	13.3526
35.	-0.1185	-10.6200	10.6207	-4.2405	-14.4310	15.0411
25.	-0.4223	-11.5070	11.5147	-4.8276	-15.5880	16.3184
15.	-0.6411	-12.1111	12.1279	-5.2271	-16.3640	17.1786
5.	-0.7554	-12.4170	12.4399	-5.4712	-16.7600	17.6304
	34 Elem. L = 80 m			41 Elem. L = 120 m		
y, m	Re Part	Im Part	Abs. Val.	Re Part	Im Part	Abs. Val.
95.	5.9799	does not exist	same as real	7.2462	does not exist	same as real
85.	8.8170			22.3550		
75.	30.7290			36.3800		
65.	41.5800			49.1980		
55.	51.1510			60.5800		
45.	59.2310			70.2980		
35.	65.6450			78.1570		
25.	70.2650			84.0160		
15.	73.0110			87.7910		
5.	73.8090			89.4280		
	48 Elem. L = 160 m			55 Elem. L = 200 m		
y, m	Re Part	Im Part	Abs. Val.	Re Part	Im Part	Abs. Val.
95.	7.4040	does not exist	same as real	7.7825	does not exist	same as real
85.	22.7820			23.9630		
75.	37.0560			38.9980		
65.	50.1050			52.7540		
55.	61.7020			64.9880		
45.	71.6150			75.4550		
35.	79.6500			83.9490		
25.	85.6620			90.3170		
15.	88.7400			94.4730		
5.	89.5690			96.3672		

CASE 1



CASE 2



CASE 3