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A Composite Fitting Model of Discrete Handbook Data for Peripheral End Milling

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Abstract

Machining data handbooks are important reference books in the machining industry as they provide recommended process parameter values for common machining operations. The machining data, although cover a wide range of relevant cutting conditions, are only listed under discrete cutting conditions. Rough interpolation-based calculations are often needed in order to estimate the process parameter values at the desired cutting condition. In this work, a composite fitting model is presented to fit a composite functional curve through the discrete handbook data of recommended cutting speeds and feeds with respect to the cutting condition of radial depth of cut for peripheral end milling. The objective is to establish a functional relationship from the handbook data such that recommended cutting speed and feed can be obtained for any given radial depth of cut. According to the tabulated layout of the machining data, the entire range of the radial depth of cut is divided into three segments having distinctive formulations and trends. Constraints are then imposed to preserve the trends and smoothly connect the adjacent segments. As a possible application of the presented model, a case study of machining a rectangular pocket is provided. Machining time of a potential process plan is readily evaluated based on the cutting speeds and feeds obtained from the composite model.

Keywords: Machining handbook data; Cutting speed; Feed; Composite fitting model; Peripheral end milling

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1. Introduction

In the modern computer-aided manufacturing (CAM) environment, selection of optimal process parameter values for a particular machining operation is a routinely tedious task. A conservative selection leads to a safe but under-performing machining operation, whereas an aggressive selection tends to produce scrapped parts and thus compromises the machining quality. Existing process parameter selection methodologies can be categorized into four groups [1]: data retrieval, optimization model, empirical model, and also their combinations [2]. As the two important process parameters in end milling, cutting speed is mostly selected based on Taylor's tool life equation [3,4] and feed is selected based on radial depth of cut [5-7], volume of material removed [8] and permissible cutting forces [8,9].

In practice, handbooks such as the Machining Data Handbook [10] have been widely used to select the cutting speed and feed for a particular machining operation, considering the workpiece and cutter material and cutting conditions. It is a data retrieval methodology where prior test results are compiled and used to provide valid process parameter recommendations. However, the discrete nature of these data raises challenges in their direct applications in the CAM environment, which necessitates continuous process parameter recommendations in the cutting condition domain. It is thus important to develop an appropriate curve fitting technique to establish relevant functional relationships from the handbook data.

The technique of curve fitting has been extensively used to characterize and analyze discrete experimental data [11]. Compared with non-parametric fitting models, parametric models provide more insight [12,13], are based on well-defined objective functions and known physical interpretations of the associated coefficients [14], and can work with constraints [15,16]. The most common fitting tool is the least-squares method [17] due to its simplicity in implementation and reduced demand of statistical information [18,19]. The unique approach of dividing the entire range of the independent variable into segments and generating individual best-fitted lines was proposed to accommodate significant variation in the fitted data [20,21]. For end milling, some software tools are now commercially available to automatically select the cutting speed and feed for any radial depth of cut. Nonetheless, the underlying principles of these software tools are mostly unknown and in particular, no insight into their respective fitting approach has been clearly documented.

In this work, a composite fitting model has been developed with the objective of fitting a composite curve through the discrete machining data provided in handbooks. In Section 2, the characteristics of the data available from the Machining Data Handbook [10] are presented and the involved research issues are identified when attempting to fit the composite curve from the listed discrete data. The objectives, specific components, and procedure of the proposed composite fitting model are discussed in Section 3. Components of the fitting models for both the cutting speed and feed data are presented in detail in Sections 4, 5, and 6. The typical fitted results of the cutting speed and feed data along with their evaluation are given in Section 7. A case study of a possible application of the proposed model is presented in Section 8, in which the fitted results aid in the evaluation of the machining time for a rectangular pocket.

2. Machining handbook data

Handbooks are widely used as conventional industrial references and readily provide recommendations of applicable process parameters that have been collected from industrial practices and machining tests. A typical such handbook, the Machining Data Handbook [10], provides discrete cutting speed and feed data according to the type of milling operation, workpiece material (hardness), radial depth of cut d_r, and cutter diameter D. In this work, the end-milling operation is considered since it is the most versatile milling operation. If $d_r < D$, it is referred to as peripheral milling and if $d_r = D$, it is slot milling.

A typical table containing the machining data for end milling for free machining carbon steels (wrought) with low and medium carbon (leaded) taken from the handbook is shown in Table 1, which includes the machining data for both peripheral and slot milling. The slot milling data is included as a mere guidance for the data fitting in the range of $0.5D \le d_r \le D$. According to the table layout, the machining data can be divided into two categories (I and II) as shown in Table 1, based on how the value of d_r is listed. It can be seen that in Category I, d_r is an absolute value of either 0.50 or 1.50 mm, whereas in Category II, d_r is a relative value with respect to *D* (*D*/4, *D*/2 or *D*). In other words, the cutting speed data, for example, are independent and dependent on *D* for Category I and II, respectively. This introduces ambiguities when trying to relate the cutting speed with the radial depth of cut, which is undesirable for successful modeling attempts. Moreover, large variations exist in the data listed in the handbook. As an example of the machining data shown in Table 1, a maximum variation of 70% is seen in the cutting speed and 200% in feed for different d_r, indicating the requirement of the fitting model to accommodate large data variations.

3. Composite fitting model

A composite fitting model is proposed in order to fit composite curves through the discrete machining data for peripheral end milling listed in the Handbook [10]. It is done such that automatic selection of the related process parameters of cutting speed and feed, in particular for CAM applications, is facilitated. The objectives of the fitting model are given below:

- Definition of the independent variable (radial depth of cut) and its relationship with the listed cutting speed and feed data should be non-ambiguous;
- Large variations in the cutting speed and feed data should be consistently accommodated for any value of the independent variable with the considered range; and
- The trend of the fitting model should have a valid physical basis.

In order to achieve the objectives outlined above, a number of components of the composite fitting model are proposed as follows:

Independent Variables: To avoid the ambiguities due to the inconsistent definition of d_r with respect to D for the cutting speed and feed data, new independent variables are introduced.

Segmentation: To accommodate the large variations in the cutting speed and feed data, the entire range of the independent variable d_r is divided into three segments having individual characteristic trend based on valid physical consideration.

Constraints: To fit a composite curve through the three data segments such that the characteristic trend of each segment and the continuity between the segments are preserved, constraints are imposed on the coefficients of the fitting model for each segment.

4. Independent variables

The independent variable d_r for the cutting speed and feed data in the handbook is respectively replaced by chip length ratio r_{length} and chip area ratio r_{area}. These new independent variables are defined as follows:

Chip Length Ratio is defined as the ratio of the arc length l between the entry and exit angle of the cutter engagement with respect to the arc length l_{ref} corresponding to a slot cut (Fig. 1):

$$r_{\text{length}} = \frac{l}{l_{\text{ref}}} = \frac{\cos^{-1}\left(1 - \frac{2d_r}{D}\right)}{\pi} \tag{1}$$

Chip Area Ratio is defined as the ratio of the chip area A, the area swept by the cutter in one tooth period [7] with respect to the ship area A_{ref} corresponding to a slot cut (Fig. 1):

$$r_{\text{area}} = \frac{A}{A_{\text{ref}}} = \frac{d_r}{D} \tag{2}$$

The variable r_{length} is a measure of the cutter contact length with the workpiece during cutting and directly related to the cutting speed v, thus used for its fitting. The maximum chip thickness is correlated to the chip area and thus r_{area} has been employed as a dimensionless parameter for machining research [9]. Table 2 lists their values for the entire range of d_r: 0.5 mm, 1.5 mm, $D/4$, $D/2$, D (D: 10 mm, 12 mm, 18 mm). It can be seen from this table that r_{length} and r_{area} for Category II have the same value for different D, because both of them are functions of the ratio d_r/D in Eqs. (1) and (2), which remains constant in this category. The values of r_{length} and r_{area} lie within the range of 0 and 1, which corresponds to no cutting and slot milling, respectively.

5. Segmentation

The typical v-r_{length} and f-r_{area} plots for a particular cutter from the machining data and using the proposed independent variables are shown in Fig. 2. It can be observed that the trend of the plot changes significantly. This indicates the need to divide the entire range of the machining data into segments with individual characteristic trend.

5.1. Data ranges

It is observed in Fig. 2a that the plot of v-r_{length} is monotonically decreasing which can be accounted for considering the well-known Taylor's tool life equation. Data point 5 (at $r_{\text{length}} = 1$) corresponds to slot milling, and according to the handbook [10], the plot between points 4 and 5 does not represent peripheral milling. However the trend of the plot between the points 4 and 5 would give a rough guidance regarding the change in the machining data between peripheral and slot milling. In Fig. 2b, a more complicated bell-shaped trend is observed between data points 1 and 3, and followed by a monotonically decreasing trend between points 3 and 5. This can be accounted for by the fact that when d_r is approaching zero (the same for r_{area}), feed can be increased to improve the material removal rate. There is, however, a limit and at a certain point the feed has to be reduced in order to compensate for the higher tool wear and possibly surface inaccuracies caused by the increased cutting speeds.

The entire range of the machining data is thus to be divided into three different segments as illustrated in Fig. 2 with Segment 1, 2 and 3 covering data points 1-3, 3-4 and 4-5, respectively. The d_r ranges along with the corresponding r_{length} and r_{area} ranges for $D = 10\,\text{mm}$, are given in Table 3.

5.2. Fitting equations

The individual trends of the three data segments are different, which will require different fitting equations. Polynomials are selected as the fitting equations and the lowest possible orders are preferred in order to avoid unwanted undulations. Segment 1 is the most important segment for both v and f data as the data trends are most complicated; thus a cubic polynomial is proposed to fit through the associated data points 1, 2 and 3. A quadric curve is proposed to fit through data points 3 and 4 for Segment 2, which would continue the trend of the later part of Segment 1. A linear Segment 3 is proposed to represent the trend between data points 4 and 5, the last data point of peripheral milling and that of slot milling, because of the lack of available machining data within this range. The fitting equations of the three data segments are:

Segment 1: $y_1(x) = \sum_{j=0}^{3} A_j x^j = A_0 + A_1 x + A_2 x^2 + A_3 x^3$ (3a)

Segment 2: $y_2(x) = \sum_{j=0}^{2} B_j x^j = B_0 + B_1 x + B_2 x^2$ (3b)

Segment 3: $y_3(x) = \sum_{j=0}^{1} C_j x^j = C_0 + C_1 x$ (3c)

where x is the independent variable of r_{length} or r_{area} for a particular cutter diameter D, y is the dependent variable of v or f, and A_j, B_j and C_j are the fitting coefficients to be determined.

6. Constraints

The proposed composite fitting model needs to preserve the characteristic trends of the individual data segments as well as smoothly join the adjacent segments. For this purpose, constraints are imposed on the fitting coefficients of each data segment and these constraints are essential to the success of the proposed fitting model. The imposed constraints can be classified into two types according to their specific purposes and are discussed below in more detail.

6.1. Trend constraints

The trend constraints are defined as the constraints imposed on the coefficients of the fitting equations for the individual segments, such that the fitted curve will preserve the individual characteristic trend. Analyzing the machining data listed in the Machining Data Handbook [10] and considering the cutting mechanism of the end milling operation, a practical basis of the trend for each data segment can be obtained. The characteristic trends of the machining data should be similar for different D but with different magnitudes. This suggests that the fitting coefficients A_j, B_j and C_j in Eq. (3) are better expressed as a dimensionless cutter diameter term using coefficients a_{jk}, b_{jk} and c_{jk}:

$$A_j(D) = \sum_{k=0}^{2} a_{jk} \left(\frac{D}{18} \right)^k \qquad j = 0,1,2,3 \tag{4a}$$

$$B_j(D) = \sum_{k=o}^{2} b_{jk} \left(\frac{D}{18} \right)^k \qquad j = 0,1,2 \tag{4b}$$

$$C_j(D) = \sum_{k=o}^{2} c_{jk} \left(\frac{D}{18} \right)^k \qquad j = 0,1 \tag{4c}$$

The dimensionless cutter diameter term is formulated as $D/18$ and has a maximum value of 1, as the maximum D is 18 mm. A second order function is selected to allow the presence of a maximum or minimum value for the variation of the coefficients A_j, B_j and C_j with respect to D.

From the plot of $v\text{-}r_{\text{length}}$ in Fig. 2a, it is observed that the trend of the cutting speed in Segment 1 is monotonically decreasing and of a concave shape, primarily due to the tool life considerations. As a result, mathematically a negative slope as well as a positive curvature should be associated with its fitting equation. This leads to a non-negative value of A_2, which is proposed as the trend constraint for Segment 1. Similarly, the monotonically decreasing trend of Segment 2 and 3 requires a non-negative B_2 and a non-positive C_1.

It is observed in Fig. 2b that the trend of the feed data in Segment 1 is bell shaped and this results in a more complex mathematical condition of an unconstrained (negative, zero or positive) slope and a negative curvature. No trend constraint is thus imposed on A_j for fitting the feed data. A non-negative B_2 and a non-positive C_1 are needed to accommodate the monotonically decreasing trend in Segment 2 and 3, respectively. The imposed trend constraints for fitting the cutting speed and feed data are different for only Segment 1 but the same for Segment 2 and 3. The imposed trend constraints are summarized in Table 4.

6.2. Boundary constraints

The boundary constraints are defined as the constraints imposed on the coefficients of the fitting equations for the individual data segments, such that the fitted composite curve is smooth at each junction of adjacent segments. To ensure smooth junctions, continuity of the highest possible order across the adjacent curve segments is considered. As a result, a C^2 boundary constraint is imposed at the junction of the cubic Segment 1 and the quadric Segment 2 ($r_{\text{length}} = 0.33$ and $r_{\text{area}} = 0.25$), whereas a C^1 boundary constraint is imposed at the junction of the quadric Segment 2 and the linear Segment 3 ($r_{\text{length}} = 0.50$ and $r_{\text{area}} = 0.50$). The imposed boundary constraints are summarized in Table 5.

7. Fitted results and evaluation

Discrete machining data of thirty-four different grades of materials from the handbook were examined and each was fitted with a composite curve based on the proposed composite fitting model. The materials and their grades were selected such that the most popular materials used for end milling were included. The fitted results of the proposed composite fitting model were plotted to show the intermediate machining data of cutting speed and feed between the discrete data points. For evaluating the fitted results, quantitative measures like the correlation coefficient of determination R^2 and relative maximum deviation (RMD) were employed, which are discussed in detail in the following subsections.

7.1. Evaluation measures

Correlation coefficient of determination R^2 is a standard statistical measure used to quantify the correlation between the actual and fitted data. It is defined as the ratio of the regression sum of squares (*SSR*) to total sum of squares (*SST*) [22]:

$$R^2 = \frac{SSR}{SST} = \frac{\sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2} \tag{5}$$

where y_i denotes the *i*th actual data in a set of *n* data, $\hat{y}_i$ the corresponding fitted data, and $\bar{y}$ the mean of the *n* actual data. The R^2 values of the fitted results for all the examined materials were calculated and evaluated to validate the effectiveness of the fitting model.

Unlike the feed data, the cutting speed data in Segment 2 and 3 are not dependent on *D*, as can be seen in Tables 1 and 2. Thus the fitted curves of the cutting speed data for different *D* in Segment 2 and 3 are expected to be coincident. A percentage measure named relative maximum deviation (RMD) was introduced to quantify the maximum deviation of the fitted cutting speed data with respect to the average of the associated fitted cutting speed data:

$$\mathrm{RMD}_m = \frac{div_m^{\max}}{(v_{D1} + v_{D2} + v_{D3})/3} \times 100\% \qquad m = 2, 3 \tag{6}$$

where $div_m^{\max}$ is the maximum deviation of the three fitted mth curve segments for the three different cutter diameters $D1$, $D2$ and $D3$. v_{D1}, v_{D2} and v_{D3} are the fitted cutting speed data for the three cutter diameters at the r_{length} corresponding to $div_m^{\max}$. The resulting RMD values were also evaluated to validate the effectiveness of the fitting model.

7.2. Cutting speed data

Although there exist numerous mathematically possible values for the fitting coefficients, only the practically viable ones were considered. Adjustments to the coefficients a_{jk}, b_{jk} and c_{jk} in Eq. (4) may be required in order that the fitting coefficients A_j, B_j and C_j in Eq. (3) satisfy the constraints summarized in Table 4, which will be the same for any D. For the thirty-four sets of machining data from the handbook, not an adjustment case was encountered for C_1. Some typical adjustments of the fitting curves due to the A_2 and B_2 constraints are outlined below.

For some materials, the trend constraint $A_2 \geq 0$ for Segment 1 may not be satisfied and this would cause the fitted curve to be convex which is practically not meaningful. An adjustment of the coefficients a_{2k} in Eq. (4a) in order to adjust A_2 is required to satisfy the constraint for all the three cutter diameters. Fig. 3 illustrates the fitted curve before and after the adjustment of A_2. A change in the concavity for Segment 1 is clearly observed in the figure.

Similarly, an adjustment of the coefficients b_{2k} in Eq. (4b) is made in order that the trend constraint $B_2 \geq 0$ for Segment 2 would be satisfied. The effect of such an adjustment is shown in Fig. 4. In the figure, it is clearly seen that the concavity of Segment 2 is maintained as that of Segment 1 when the trend constraint was imposed on the fitting model.

A very high correlation between the actual and fitted cutting speed data has been obtained from the composite fitting model, with all the calculated R^2 values lying in the range of 0.986 – 0.993. Apart from the robustness of the proposed fitting model, the high R^2 values can also be attributed to the relatively low variation and simple trend of the cutting speed data. The resulting RMD values for Segment 2 and 3 are presented with an average of 3.28% and 3.81%, respectively, for all the thirty-four data sets. This indicates that the fitted curves for the cutting speed data are very close of being coincident for Segment 2 and 3. The worst RMD value of 5.75% is observed for Aluminum alloys where the variation in the discrete cutting speed data is significantly higher than that for the other materials.

7.3. Feed data

The composite fitting model was applied to the feed data for the thirty-four different grades of materials selected from the handbook, which were the same as those selected for fitting the cutting speed data. For the thirty-four machining data sets, not an adjustment case was encountered for B_2; thus only a typical adjustment case for C_1 is presented. Fig. 5a shows an impractical positive slope in the fitted curves for Segment 3 and this needs to be adjusted according to the constraint listed in Table 4. The fitted results after the adjustment are shown in Fig. 5b, where the practical monotonically decreasing trend of feed with increasing r_{area} in Segment 3 is clearly observed.

Similar to the fitting of cutting speed data, R^2 values are used to evaluate the goodness of fit between the fitted results and actual data. The calculated R^2 values mostly lie in the range of 0.781 – 0.999 except for the material of Alloy Steels (Cast), Medium Carbon and cutter diameter of 10 mm, the R^2 value is 0.446. The relative low R^2 values for the feed data compared to those for the cutting speed data, can be attributed to the fact that the feed data for Segment 1 are fluctuating and not as regular.

8. A case study

As a possible application of the presented composite fitting model, a typical process planning task for pocket machining was examined: a rectangular pocket as depicted in Fig. 6a was to be machined by a flat-end mill. A commonly used zig-zag cutting strategy was selected to machine the pocket. The associated machining time was calculated on the basis of the cutting speed and feed recommendations obtained from the composite fitting model, corresponding to any given radial depth of cut d_r and cutter diameter D. The total machining time is estimated by summing the machining times for different sections of the machining tool path, segmented according to the type of the end milling operation:

Slot Cut (machining time T_1): the initial cut in pocket machining (Fig. 6b)

Peripheral Cut (machining time T_2): the zig-zag part of the tool path having a constant radial depth of cut (Fig. 6c)

Intermediate Slot Cut (machining time T_3): the intermediate slot cut associated with the side move of the end mill in-between the individual peripheral cuts (Fig. 6c)

Cleanup Cut (machining time T_4): the last cut to remove the residual material left behind by the zig-zag tool path (Fig. 6d)

The total machining time T_{total} for machining this pocket was obtained by summing all the above machining times (considering a single axial cut and only when the end mill was in contact with the workpiece):

$$T_{\text{total}} = \sum_{i=1}^{4} T_i = \frac{L_0 - D}{f_{r1}} + \frac{L_0 - D}{f_{r2}} n_c + \frac{W_0 - D}{f_{r3}} + 2\frac{W_0 - D}{f_{r4}} \tag{7}$$

where L_0 is the pocket length, W_0 the width, f_{ri} the feed rates for the four types of end milling cut, and $n_c = W_0 - D/d_r$ the number of peripheral cuts needed to completely machine the pocket.

The expression of Eq. (7) was applied to calculating the total machining time of machining the pocket for the workpiece material of Aluminum Alloys (Cast), Sand and Permanent Mold with hardness of 40 to 100 Bhn (v = 69 – 245 m/min and f = 0.05 – 0.20 mm/tooth). Fig. 7 illustrates the total machining time corresponding to five cutters with diameters of 10 mm, 12 mm, 14 mm, 16 mm and 18 mm. The required cutting speed and feed data for any combination of d_r and D were calculated by using the composite fitting model. For the five cutter diameters, the cutting speed and feed data for D = 14 mm and 16 mm (shown as dashed lines in Fig. 7) were not directly available in the handbook but calculated through the expressions of Eq. (4).

It can be observed in Fig. 7 that cutters with smaller diameters will require higher number of peripheral cutting passes, thus increasing the tool path length and machining time. For a particular D, tool path length decreases with an increase in d_r, but at the same time the permissible feed rate also decreases. This results in a trade-off in the selection of d_r in order to achieve a lower machining time. However, it is seen that for higher values of D, the relatively flat portion of the machining time curves for higher values of d_r indicates a tempering of the trade-off. With the aide of such machining time curves, applicable recommendations for the selection of d_r and D to achieve maximum productivity for machining the pocket can be identified.

9. Conclusions

A composite fitting model has been presented in this paper in order to establish a functional relationship from the handbook data for peripheral end milling such that recommended cutting speed and feed can be obtained for any given radial depth of cut. The model uses the chip length and chip area ratio as the respective independent variables for the cutting speed and feed data. This is to deal with the ambiguity observed in the handbook about the original independent variable of radial depth of cut being tabulated as a combination of absolute and relative values with respect to the cutter diameter. A composite polynomial curve is fitted through segments of the discrete cutting speed and feed data. Trend and boundary constraints have been imposed on the coefficients of the fitting equations for the individual data segments to preserve their characteristic trends and smoothly connect the adjacent segments, respectively.

The fitted results of machining data of thirty-four different work materials show high correlation with the actual data. This validates the ability of the composite fitting model in accommodating high variations in the machining data for different work materials. For the cutting speed data, the coincidence of the fitted curves for Segment 2 and 3 has been confirmed by the resulting low RMD values. As an example of a possible application of the proposed fitting model, a case study of machining a pocket has been presented. It demonstrates the capability of the model in determining the intermediate cutting speed and feed data not available in the handbook, which are needed in order to calculate and evaluate the associated machining time.

The primary drawback of the presented composite fitting model is that its applicability heavily depends on the applicability of the machining data obtained from a handbook such as the Machining Data Handbook [10]. Much like all the handbooks, the calculated cutting speed and feed data by the composite fitting model only provide first recommendations from which the optimized cutting speed and feed can be determined.

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List of Tables

Table 1 Typical machining data taken from the Handbook [10].

Material	Hardness (Bhn)	Radial Depth of Cut (mm)	Speed (m/min)	Feed (mm/tooth) Cutter Diameter 10 mm	12 mm	18 mm	
Free Machining Carbon Steels (Wrought), Low Carbon (Leaded)	100 to 150	0.5	76	0.025	0.050	0.102	Category I
		1.5	58	0.050	0.075	0.130	Category I
		$D/4$	52	0.025	0.050	0.102	Category II
		$D/2$	46	0.018	0.025	0.075	Category II
		D	32	0.013	0.025	0.050	Category II
Free Machining Carbon Steels (Wrought), Medium Carbon (Leaded)	125 to 175	0.5	70	0.025	0.050	0.102	
		1.5	53	0.050	0.075	0.130	
		$D/4$	47	0.025	0.050	0.102	
		$D/2$	41	0.018	0.025	0.075	
		D	27	0.013	0.025	0.050	

Table 2 r_{length} and r_{area} for different d_r and D.

d_r	D = 10 mm r_{length}	10 mm r_{area}	12 mm r_{length}	12 mm r_{area}	18 mm r_{length}	18 mm r_{area}	
0.5 mm	0.1436	0.0500	0.1309	0.0417	0.1066	0.0278	Category I
1.5 mm	0.2532	0.1500	0.2301	0.1250	0.1864	0.0833	Category I
$D/4$	0.3333	0.2500	0.3333	0.2500	0.3333	0.2500	Category II
$D/2$	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	Category II
D	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	Category II

Table 3 Ranges of the three data segments for $D = 10\,\text{mm}$.

Segment	d_r	Data Ranges: Fitting of v data	Data Ranges: Fitting of f data
1	$0.5\,\text{mm} \le d_r \le D/4$	$0.14 \le r_{\text{length}} \le 0.33$	$0.05 \le r_{\text{area}} \le 0.25$
2	$D/4 \le d_r \le D/2$	$0.33 \le r_{\text{length}} \le 0.50$	$0.25 \le r_{\text{area}} \le 0.50$
3	$D/2 \le d_r \le D$	$0.50 \le r_{\text{length}} \le 1.00$	$0.50 \le r_{\text{area}} \le 1.00$

Table 4 Trend constraints imposed on the fitting coefficients.

Segment	Fitting of v data	Fitting of f data
1	$A_2 \ge 0$	None
2	$B_2 \ge 0$	$B_2 \ge 0$
3	$C_1 \le 0$	$C_1 \le 0$

Table 5 Imposed boundary constraints.

Junction	Fitting of v data: r_{length}	Fitting of v data: Boundary Constraints	Fitting of f data: r_{area}	Fitting of f data: Boundary Constraints
Segment 1 and 2	0.33	$v_1''(0.33) = v_2''(0.33)$ $v_1'(0.33) = v_2'(0.33)$ $v_1(0.33) = v_2(0.33)$	0.25	$f_1''(0.25) = f_2''(0.25)$ $f_1'(0.25) = f_2'(0.25)$ $f_1(0.25) = f_2(0.25)$
Segment 2 and 3	0.50	$v_2'(0.50) = v_3'(0.50)$ $v_2(0.50) = v_3(0.50)$	0.50	$f_2'(0.50) = f_3'(0.50)$ $f_2(0.50) = f_3(0.50)$

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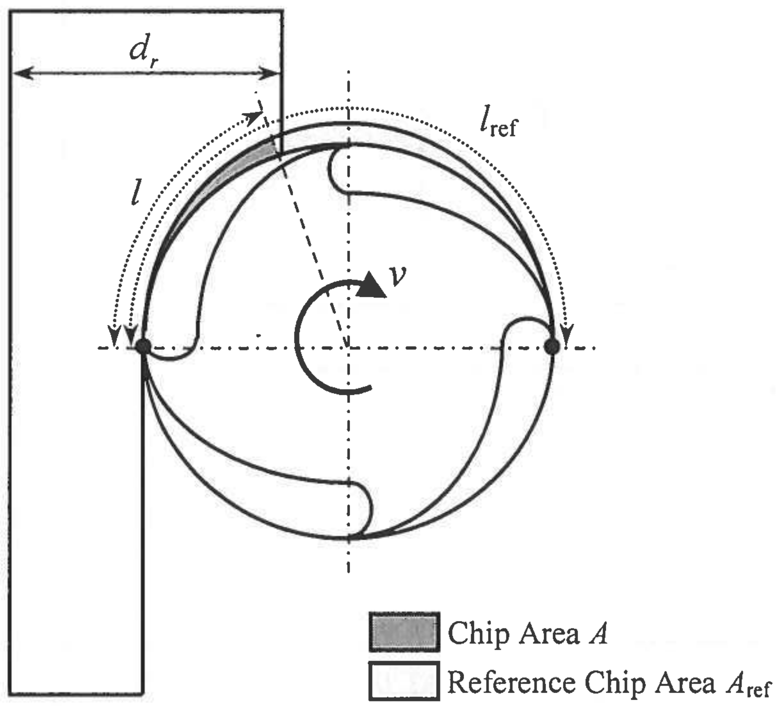
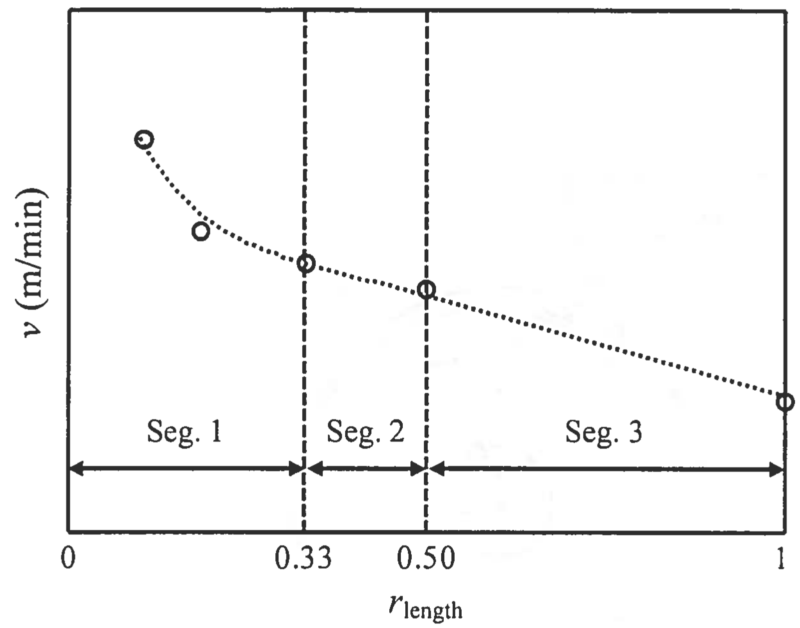
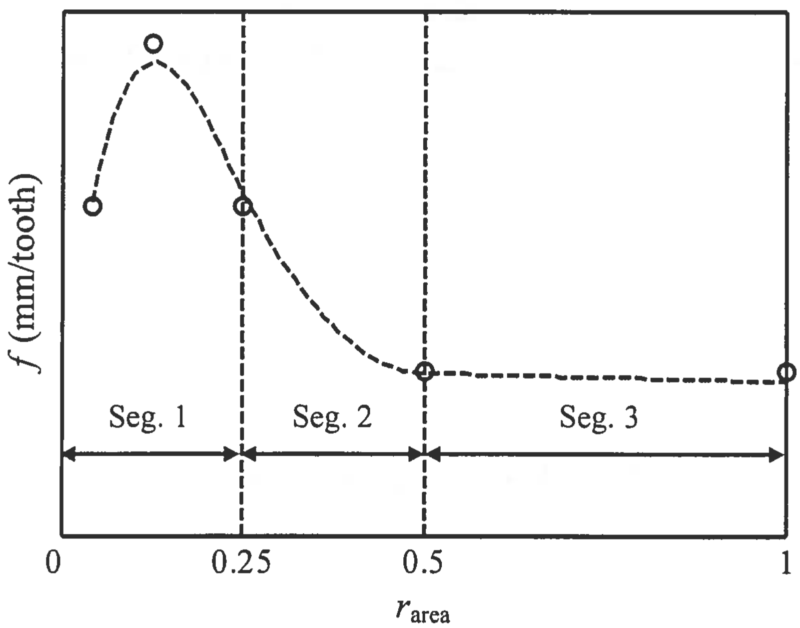


Figure 1 Chip length and area.



(a)



(b)

Figure 2 Typical plots: (a) cutting speed vs. r_{length}; and (b) feed vs. r_{area}.

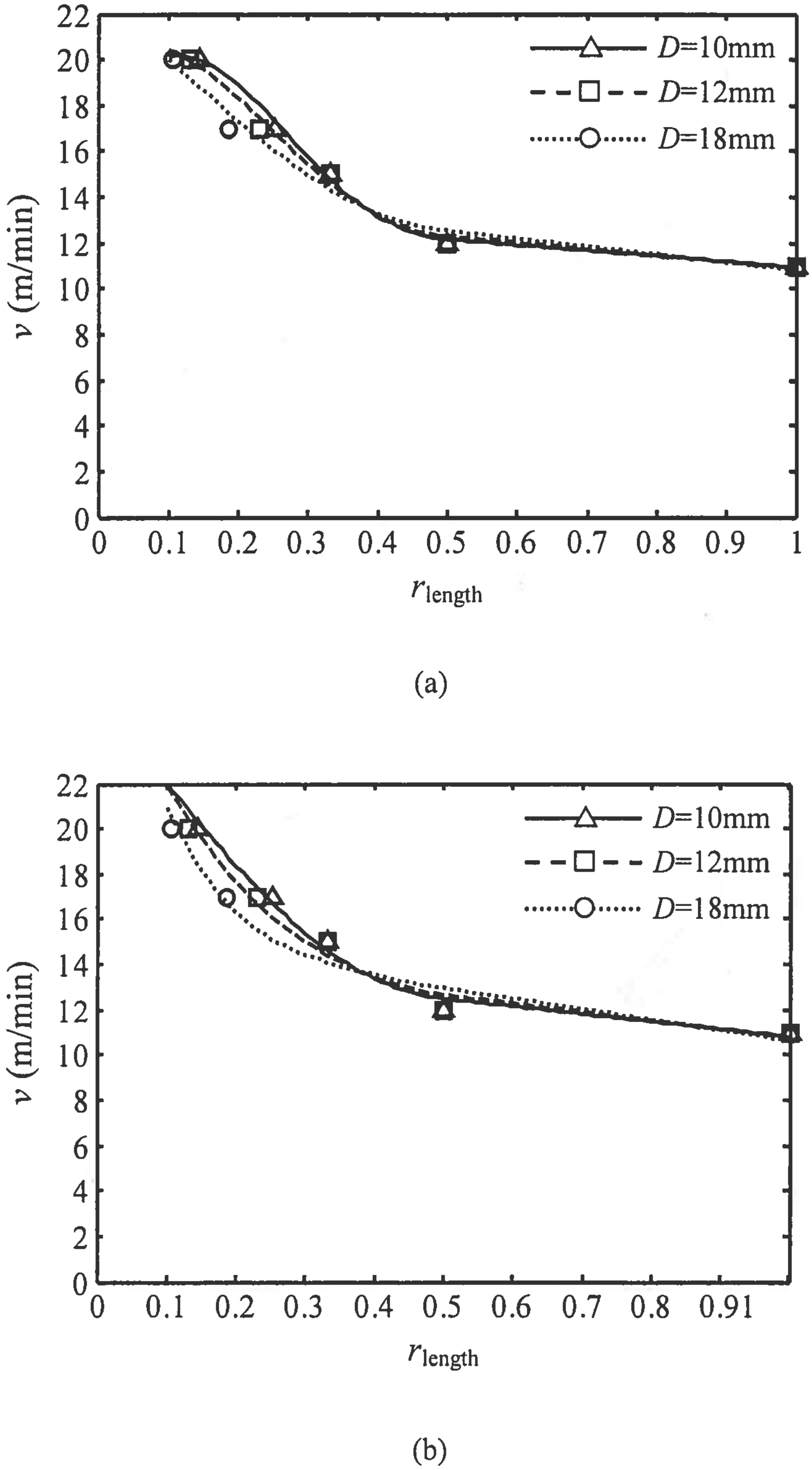
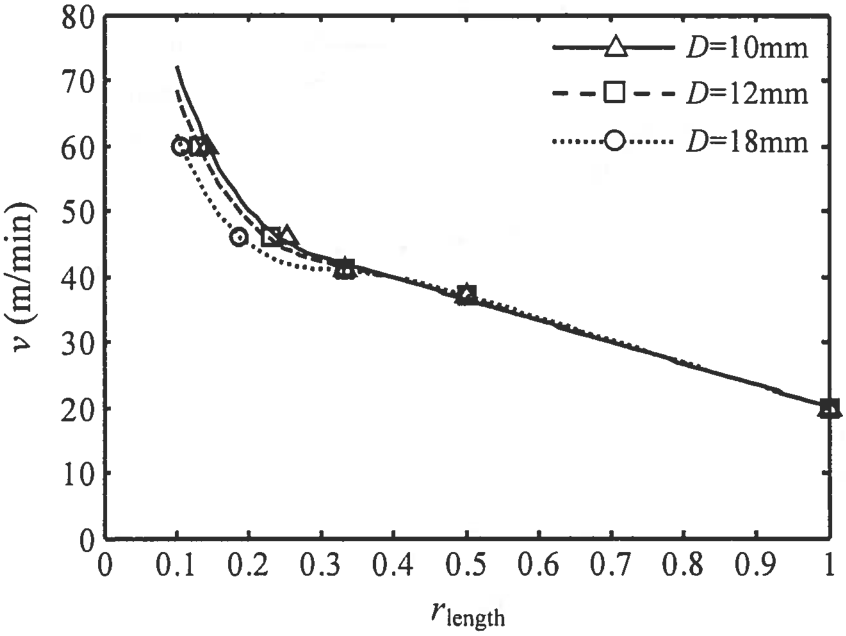
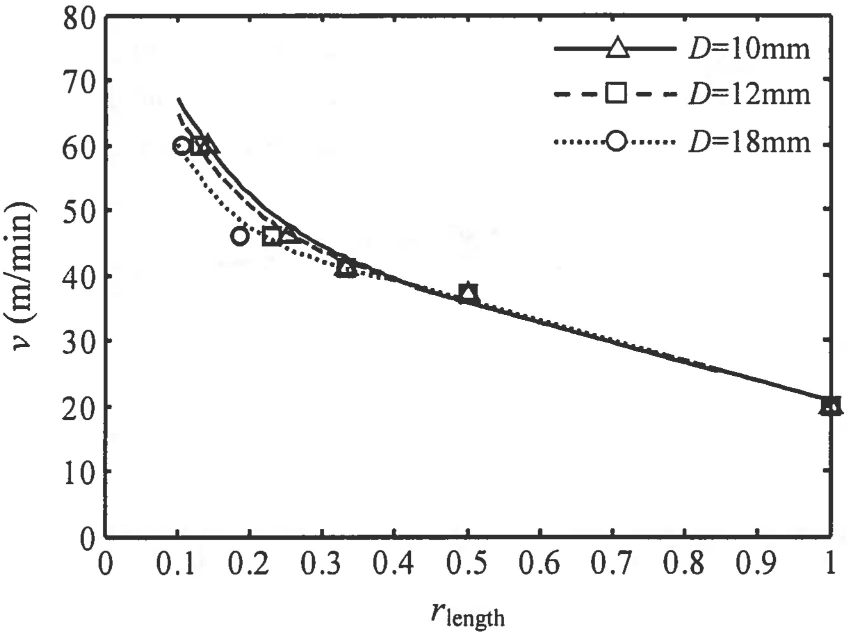


Figure 3 Effect of the trend constraint $A_2 \geq 0$ for Segment 1 of cutting speed data: (a) before; and (b) after imposing the constraint.



(a)



(b)

Figure 4 Effect of the trend constraint $B_2 \geq 0$ for Segment 2 of cutting speed data: (a) before; and (b) after imposing the constraint.

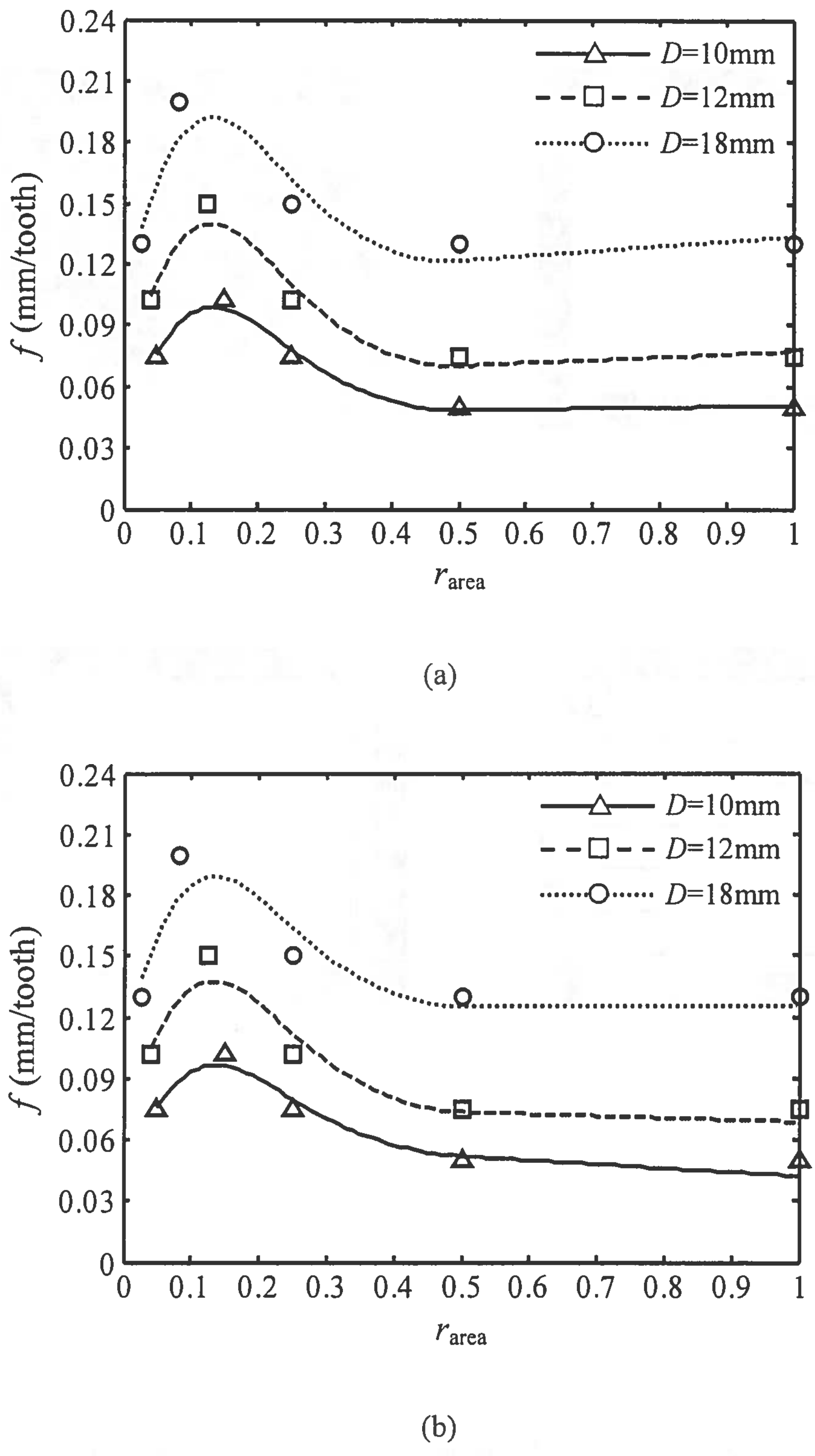


Figure 5 Effect of the trend constraint $C_1 \leq 0$ for Segment 3 of feed data: (a) before; and (b) after imposing the constraint.

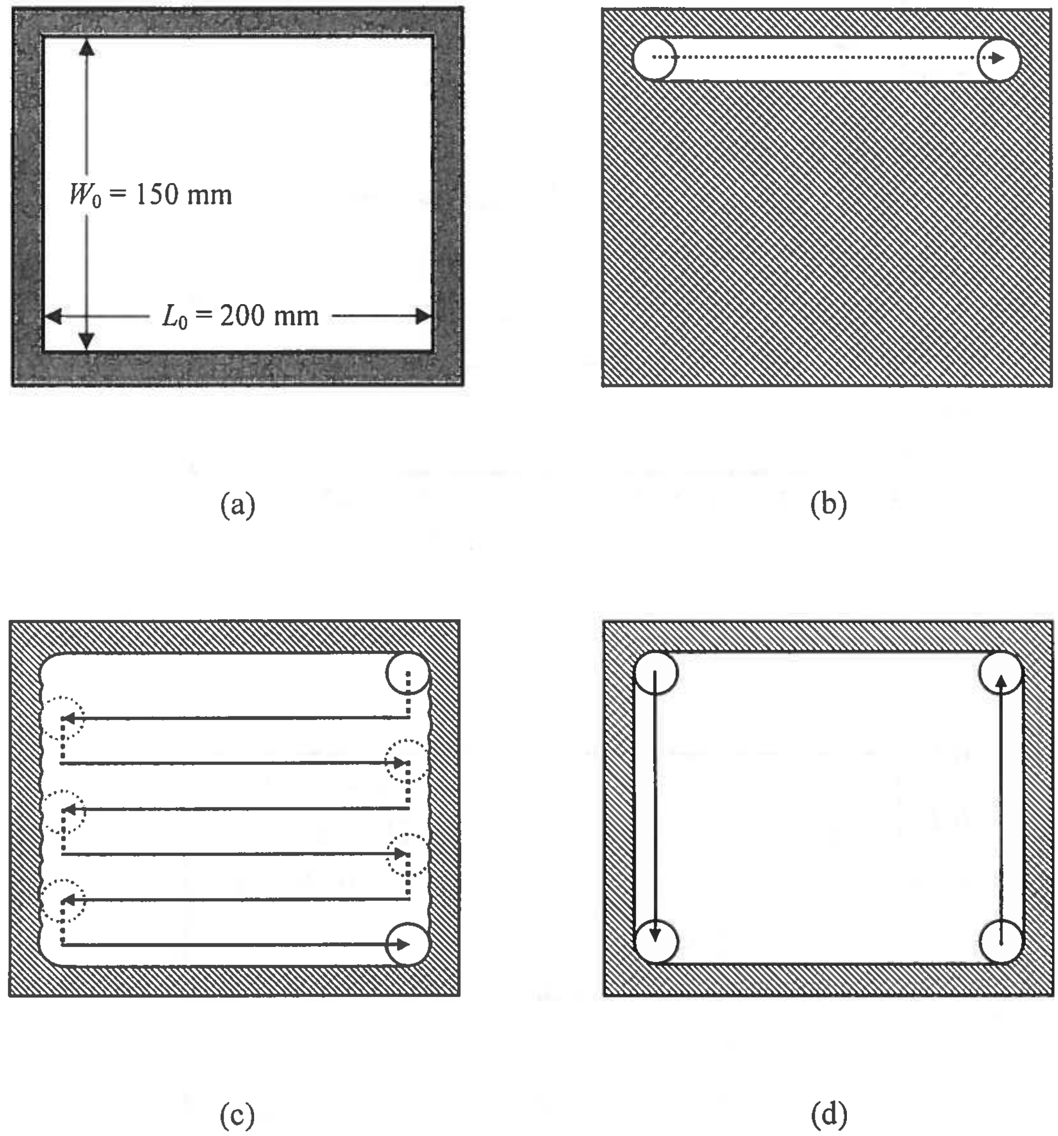


Figure 6 Pocket machining: (a) pocket geometry; (b) initial slot cut; (c) peripheral and intermediate slot cut; and (d) cleanup cut.

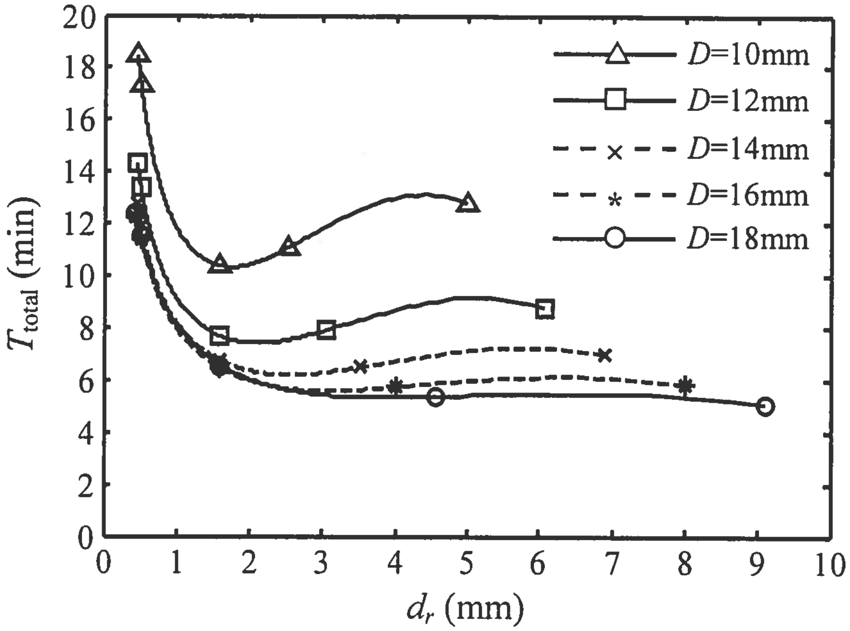


Figure 7 Total machining time for machining the pocket in Fig. 6.