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Management of failure risk in large-diameter buried pipes using fuzzy-based techniques

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Abstract

Effective management of failure risk of large-diameter water mains requires knowledge of their current condition, their rate of deterioration, the expected consequences of failure and the owner's risk tolerance. By far the greatest obstacle to formulating an effective strategy is the lack of sufficient historical data on the deterioration of these buried pipes. The National Research Council of Canada (NRC), with the support of the American Water Works Association Research Foundation (AwwaRF) is developing a new approach, which is largely based on fuzzy-based techniques.

Fuzzy-based techniques seem to be particularly suited to modeling the deterioration of buried infrastructure assets, for which data are scarce, cause-effect knowledge is imprecise and observations and criteria are often expressed in vague (linguistic) terms (e.g., 'good', 'fair' 'poor' condition, etc.). The use of fuzzy sets and fuzzy-based techniques helps to incorporate the inherent imprecision, uncertainty and subjectivity of available data, as well as to propagate these attributes throughout the model, yielding more realistic results.

Earlier publications, reporting on the same research effort, introduced two new concepts: (a) modeling the deterioration of a buried pipe as a fuzzy Markov process, and (b) combining the possibility of failure with the fuzzy consequences to obtain fuzzy risk of failure throughout the life of the pipe. In this paper a method is presented to use the fuzzy deterioration model and the fuzzy risk for the effective management of failure risk. These decisions include when to renew a deteriorated pipe, or alternatively, when to schedule the next inspection and condition assessment, and if renewal is required, what renewal alternative should be selected.

Keywords

Large-diameter pipes; deterioration; fuzzy Markov; risk; decision-making.

1 Introduction

Low rate of failure, combined with the high cost of inspection and condition assessment, are the main reasons why there is a dearth of historical data on the condition of large-diameter buried pipes. The risk needs to be evaluated and managed as the failure of such pipes can have disastrous consequences. Managing this failure risk requires a deterioration model that enables the forecast of the asset condition as well as the 'possibility' of failure in the future. However, the historical data that are required to understand and model this deterioration are scarce.

The modeling of infrastructure deterioration has received much attention in the last two decades. The Markov deterioration process (MDP) is one approach that has gained prominence as exemplified by Madanat *et al.* (1997), Wirahadikusumah *et al.*

(2001), Mishalani and Madanat (2002), Kleiner (2001) and others. Examples of other types of statistical models include those proposed by Lu and Madanat (1994), Ramia and Ali (1997), Flourentzou *et al.* (1999), Ariaratnam *et al.* (2001) and others.

In recent years, the application of soft computing methods to assess infrastructure deterioration has gained popularity. Soft computing methods include techniques such as artificial neural network, genetic algorithms, belief networks, fuzzy sets and fuzzy-based techniques. Fuzzy-based techniques seem to be particularly suited to modeling the deterioration of infrastructure assets for which data are scarce and cause-effect knowledge is imprecise. Some examples from the literature of applications of these techniques to infrastructure systems include: Chao and Cheng (1998) used a fuzzy-based pattern recognition model to diagnose cracks in reinforced concrete structures; Liang *et al.* (2001) developed a multiple-layer fuzzy method for concrete bridge health monitoring; Sadiq *et al.* (2004) employed a fuzzy-based technique to determine soil corrosivity as a surrogate measure for breakage/corrosion rate in cast iron pipes.

The Markovian deterioration process requires that the condition of the deteriorating system be encoded as an ordinal condition state. The condition assessment of a large buried pipe comprises two steps. The first step involves inspection of the pipe using direct observation (visual, video) and/or non-destructive evaluation (NDE) techniques (radar, sonar, ultrasound, sound emissions, eddy currents, etc.), which reveal distress indicators. The second step involves interpretation of these distress indicators to determine the condition state of the pipe. This interpretation process is dependent upon the inspection technique. The interpretation of the visual inspection results, although based on strict guidelines, can often be influenced by subjective judgment. The interpretation of NDE results on the other hand, is often complex (at times proprietary) and imprecise. Fuzzy-based techniques seem therefore uniquely suited to addressing this encoding process as well.

Kleiner *et al.* (2004a) introduced a new approach to modeling the deterioration of buried pipes, using a fuzzy rule-based, non-homogeneous Markov process. This deterioration model yields the possibility (as opposed to probability) of failure at every point along the life of the pipe. Kleiner *et al.* (2004b) expanded this approach by expressing the possibility of failure, as a fuzzy number, and then coupled it with the failure consequence (also expressed as a fuzzy number) to obtain the failure risk as a function of pipe age. In this paper the approach is expanded further, to include methods to assess post-renewal deterioration rate and subsequently make rational decisions on when to schedule the subsequent condition assessment of a pipe, when to renew a deteriorated pipe, and how to select the most economical renewal alternative, assuming it is technically feasible and appropriate.

2 Fuzzy sets and the fuzzy rule-based algorithm

A fuzzy set describes the relationship between an uncertain quantity x and a membership function μ , which ranges between 0 and 1. A fuzzy set is an extension of the traditional set theory (in which x is either a member of set A or not) in that an x can be a member of set A with a certain degree of membership μ . Fuzzy techniques help address deficiencies inherent in binary logic and are useful for propagating uncertainties through models. Several textbooks on fuzzy sets, e.g., Dubois and Prade (1985), are available that describe the techniques in detail.

The proposed models use triangular fuzzy numbers (TFN), as these are often used for representing linguistic variables due to their simplicity (Lee, 1996). Figure 1 illustrates

the concept for the linguistic definition of pipe age. In this example, it can be seen that for a pipe of age 50 years the membership values are 0.52 and 0.40 to *Medium* and *Old* grades respectively, and zero membership to all other grades. The fuzzy set representing the buried pipe at age 50 can be written as a 5-element vector (0, 0, 0.40, 0.52, 0), in which each element (tuple) depicts the pipe's membership value to the corresponding subset of aging grade (from *new* to *very old*).

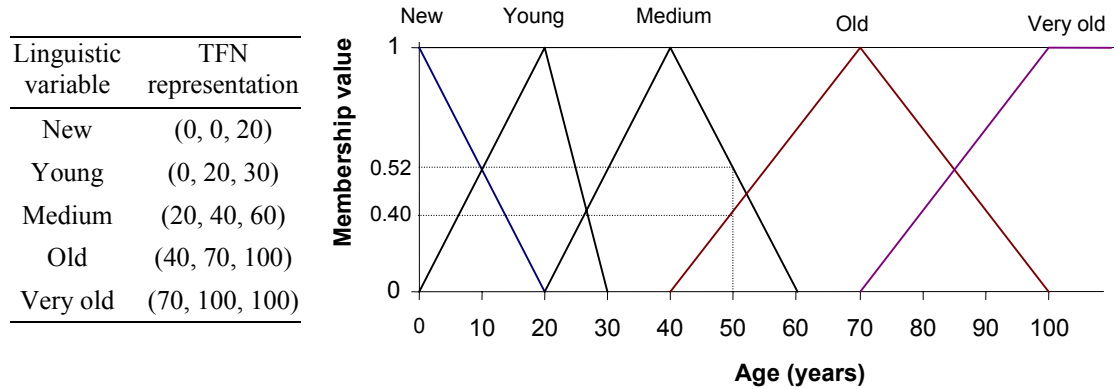


Figure 1. Example of a fuzzy set representing linguistic variables for pipe age.

In fuzzy rule-based models, the relationships between variables are represented by means of linguistic *if-then* rules of the form “If antecedent (fuzzy) proposition then consequent (fuzzy) proposition”. An example rule can be “if the pipe is *young* and the pipe condition is *poor* than the deterioration rate of the pipe is *fast*”. The Mamdani (1977) algorithm enables the making inferences based on such rules. In the example above, the algorithm will calculate for instance what the deterioration rate is if the pipe age is somewhere between *young* and *medium* (e.g., the pipe age has membership to both grades). This method enables the capture of qualitative and highly uncertain knowledge, forms it into the *if-then* rules, and subsequently creates a complete inference system. In modeling a system as a fuzzy entity, the fuzzy sets and the rules that define their relationships are referred to as the system’s underlying knowledge base.

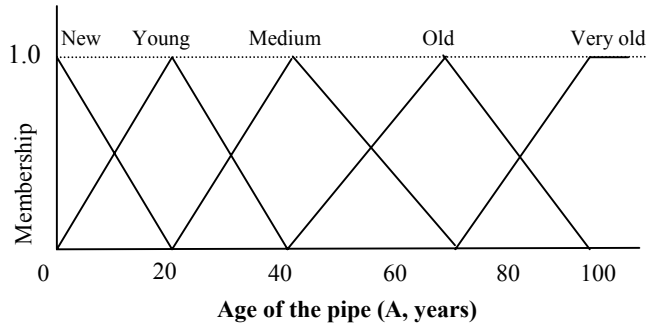
3 Fuzzy rule-based Markovian deterioration process

Figure 2 depicts the knowledge base for the proposed deterioration model, including fuzzy sets for pipe age, condition and deterioration rate, as well as a rule set that governs their relationships. Detailed explanations are provided in Kleiner *et al.* (2004a,b).

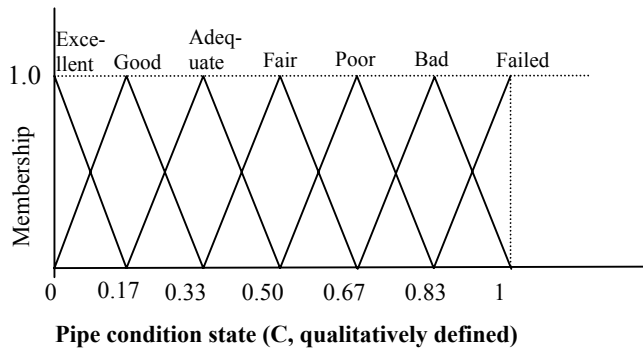
The deterioration process is modeled as a “flow” of membership from one condition state to the next lower condition state. Figure 3 illustrates how the condition of a pipe changed from (0.14, 0.59, 0.27, 0, 0, 0, 0) in year 20 to (0, 0.09, 0.38, 0.52, 0, 0, 0) in year 40. The parameters that control the shape and scale of the deterioration curves are obtained by least-square regression.

Knowledge-base

Age	Min	MLV	Max
New	0	0	20
Young	0	20	40
Medium	20	40	70
Old	40	70	100
Very old	70	100	100

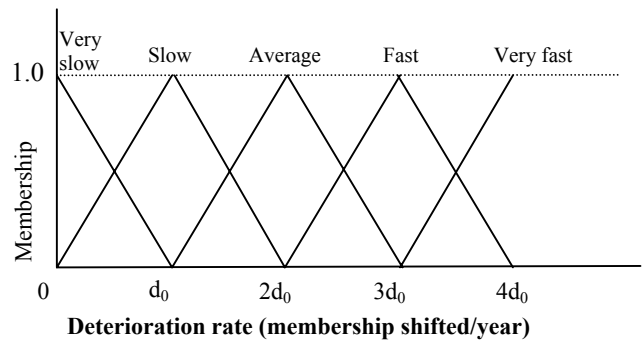


Condition state	Min	MLV	Max
Excellent	0	0	0.17
Good	0	0.17	0.33
Adequate	0.17	0.33	0.50
Fair	0.33	0.50	0.67
Poor	0.50	0.67	0.83
Bad	0.67	0.83	1
Failed	0.83	1	1



Deterioration rate	Min	MLV	Max
Very slow	0	0	d_0
Slow	0	d_0	$2d_0$
Average	d_0	$2d_0$	$3d_0$
Fast	$2d_0$	$3d_0$	$4d_0$
Very fast	$3d_0$	$4d_0$	$4d_0$

MLV - :most likely value



Fuzzy rule-set R_p

R_i = If pipe age is “A” and pipe condition state is “C” then deterioration rate is “D” (at time = t)

Pipe condition (C):	Excellent	Good	Adequate	Fair	Poor	Bad	Failed
Age (A): New	Slow	Average	Fast	Very fast	Very fast	Very fast	Very fast
Young	Slow	Average	Fast	Fast	Fast	Very fast	Very fast
Medium	Very slow	Slow	Average	Average	Fast	Fast	Very fast
Old	Very slow	Very slow	Slow	Slow	Average	Average	Fast
Very old	Very slow	Very slow	Very slow	Slow	Slow	Average	Average

Figure 2. Fuzzy rule-base for the Markovian deterioration process.

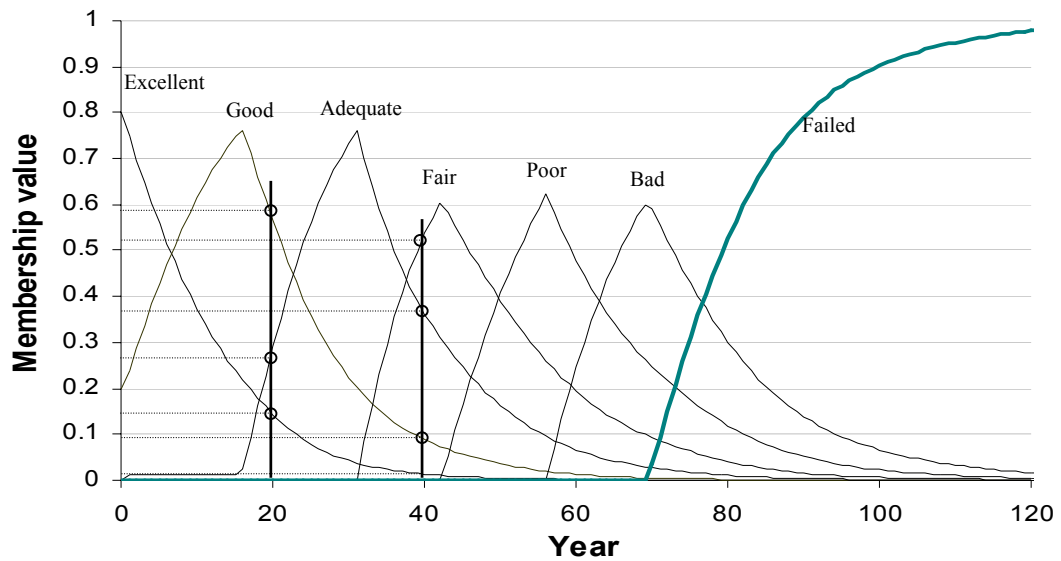


Figure 3. Deterioration curves.

4 Post-renewal condition improvement and subsequent deterioration

In the following, the term “repair” is used for a very localized intervention that does not improve the general condition of the pipe by a noticeable amount, and is not likely to change the deterioration rate of the pipe. The terms “renewal” or “rehabilitation” are used interchangeably, and refer to the type of intervention that improves the condition of the pipe and possibly modifies its deterioration rate as well. Consequently, a repair event does not warrant any special attention in the deterioration model. In considering a renewal alternative k , it is assumed to have three specific attributes. The first is a condition improvement matrix P_k , which determines how much the condition of the pipe will improve immediately after renewal. The second is a post renewal deterioration rate, which determines how fast the pipe will continue to deteriorate after renewal. The third is the cost of renewal S_k .

Table 1. Expert input to construct condition improvement matrix P_k .

Confidence to get condition shift		To condition					
From condition	Excellent	Good	Adequate	Fair	Poor	Bad	Failed
Excellent	Highest						
Good	Highest	Lowest					
Adequate	Medium	Highest	Lowest				
Fair	Medium	Highest	Medium				
Poor	Lowest	Highest	Medium				
Bad		Medium	Highest	Lowest			
Failed		Lowest	Highest	Medium			

There are insufficient field data to assign deterministic or even probabilistic values to P_k . Consequently the condition improvement matrix, P_k , is constructed based on expert opinion, which is extracted linguistically as shown in Table 1. The relative levels of confidence *highest*, *medium*, *lowest* are designed to capture the uncertainty of the prediction. These values are subsequently assigned relative weights, say, 0.7, 0.4 and 0.1, for *highest*, *medium*, *lowest*, respectively to obtain the condition improvement matrix.

The post-renewal deterioration rate must also be evaluated based on expert opinion for lack of sufficient field data. Similar to the condition improvement matrix, expert opinion on post-renewal deterioration rate is extracted using a matrix similar to that in Table 2. The relative levels of confidence *highest*, *medium* and *lowest* are again designed to capture the uncertainty of the prediction (or belief), and are assigned relative weights, which can be the same as those assigned to extract the condition improvement matrix i.e., 0.7, 0.4 and 0.1 for *highest*, *medium* and *lowest*, respectively.

Table 2. Expert input for evaluating the post-renewal deterioration rate.

Confidence that post-intervention deterioration rate will be, compared to the current deterioration rate				
Much lower	Lower	Same	Higher	Much higher
	Medium	Highest	Lowest	

The values in Table 2 are converted to a fuzzy deterioration rate that is expected after pipe renewal. An example of resulting deterioration curves is illustrated in Figure 4. In this example, a renewal was implemented at year 54, when the pipe condition was approximately (0, 0, 0.13, 0.52, 0.31, 0, 0). After renewal the pipe condition shifted to approximately (0.19, 0.46, 0.33, 0, 0, 0, 0).

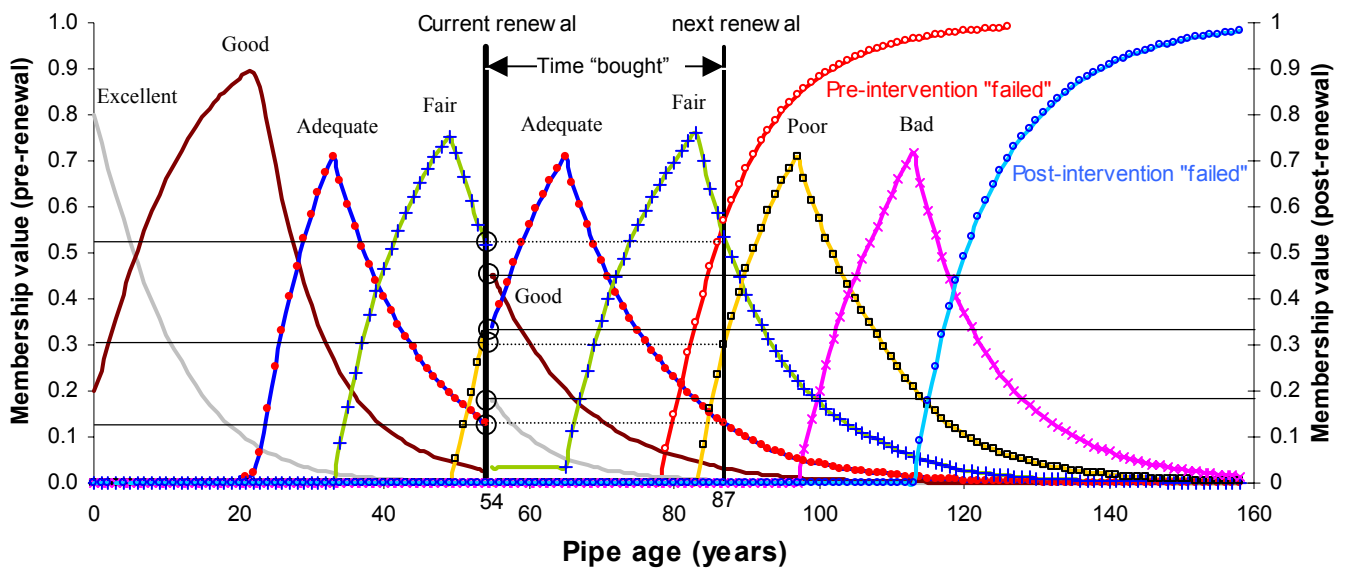


Figure 4. Pre and post-renewal deterioration curves.

It can also be seen that the condition of the renewed pipe at year 87 is expected to be the same as just before the renewal was implemented. This means that, based on the assessment regarding post-renewal condition and post-renewal deterioration rate, the renewal action “bought” the pipe 33 years of additional service.

5 Fuzzy rule-based risk

Lawrence (1976) defines risk as *a measure of probability and severity of negative adverse effects*. A complex system that involves various contributory risk items with uncertain sources and magnitudes, often cannot be treated with mathematical rigor during the initial or screening phase of decision-making (Lee, 1996). In the realm of buried pipes failures, not only is the likelihood of failure difficult to quantify, but failure consequences as well.

Fuzzy sets are often interpreted as possibility distributions in contrast to probability distributions (Klir and Yuan, 1995). It follows that the membership value to the *failed* condition can be viewed as the possibility of failure. Kleiner *et al.* (2004b) showed how these membership values can be mapped onto a secondary fuzzy linguistic scale, comprising nine grades including *extremely low*, *very low*, *quite low*, *moderately low*, *medium*, *moderately high*, *quite high*, *very high* and *extremely high*, to express possibility of failure. Similarly, a nine-grade fuzzy set was constructed for the severity of the pipe failure consequences. This set comprised the linguistic variables *extremely low*, *very low*, *quite low*, *moderately low*, *medium*, *moderately severe*, *quite severe*, *very severe* and *extremely severe*. An additional nine-grade fuzzy set was constructed to represent fuzzy risk. This set had nine linguistic variables from *extremely low* to *extremely high*. A set of rules was constructed to govern the relationships between these three fuzzy sets representing the possibility, the consequences and the risk of failure. This set of rules comprised 81 rules (9 possibility levels by 9 consequence levels) such as “if the possibility of failure is *quite low* and the consequences of failure are *very severe* then the risk of failure is *medium*”. Here too, the Mamdani (1977) algorithm is used to make the proper inferences.

The possibility of failure can be inferred for each year in the life of the pipe from the deterioration model described earlier. If the fuzzy failure consequences are provided, the fuzzy risk level can be inferred as well, as illustrated in Figure 5. The intensity of the gray levels represents the membership values to the respective risk levels such that darker colours represent higher membership values.

Figure 5 also illustrates the concept of α -cuts and its use in the context of confidence bands. The α -cut of a fuzzy set is defined as the range for which the membership values are equal or higher than the value of α . Imagine that the gray scale in Figure 5 (representing membership values) is replaced by a height dimension (darker colours correspond to larger heights). The profile (cross-section A-A) of the fuzzy risk can be plotted as a triangular fuzzy number for every year in the risk plot. A given α value can define a range of risk values for which membership is greater than α . This range can be viewed as a possibility interval, which is akin to the concept of confidence interval in probability theory. Plotting this range for each year, will result in a possibility band – analogous to confidence band. The high side of this band depicts the conservative attitude, while the low side depicts the optimistic attitude. The selection of an α value to make decisions is a matter of the decision maker’s policy.

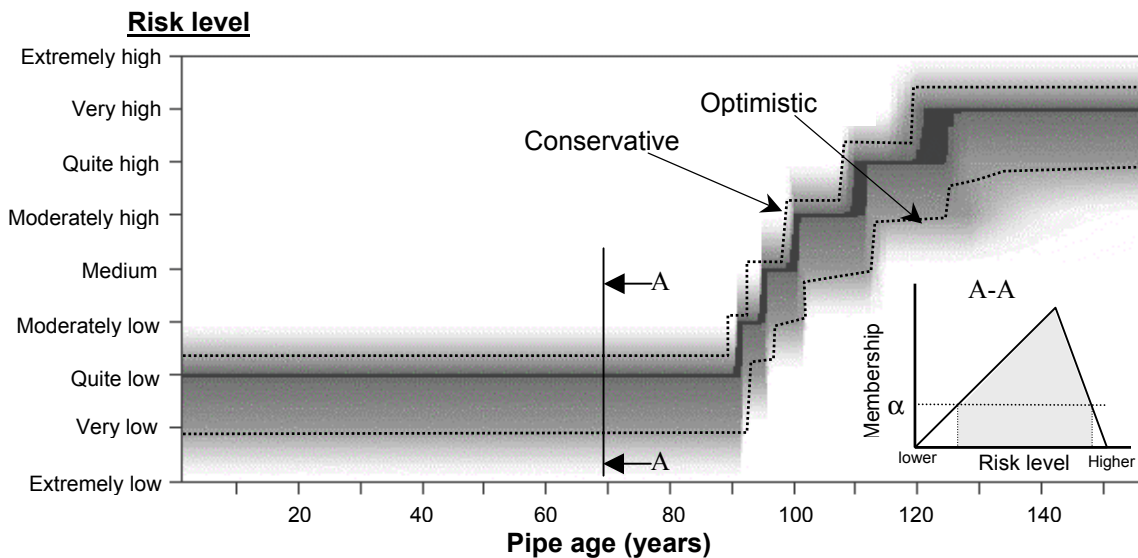


Figure 5. Fuzzy risk levels over the life of a pipe

6 Making decisions

Ideally, the optimal strategy of renew/repair/inspect will be the one which minimizes the present value of the total life-cycle costs (including direct, indirect and social costs of failure) that are associated with the pipe. This requires accurate forecasting of the pipe deterioration and the probability of failure over its life cycle (before and after renewal), as well as the expected consequences of failure.

Since the useful life of large buried pipes is usually measured in decades, the issue of discount rate is very important. In general, the higher the discount rate the greater the economic drive to delay expenditure as much as possible and visa versa. The choice of an appropriate discount rate for public investment is an issue that has been debated and researched extensively. Steiner (1969) mentions two major approaches to address this issue: (a) the discount rate should reflect some kind of marginal cost of capital, and (2) the discount rate should reflect some kind of explicit measure of social time preference (hence the term "social discount rate").

Importance of discount rate notwithstanding, there is a difficulty in applying it to fuzzy quantities representing linguistic variables. For example, how does one discount fuzzy failure consequences that have 50% membership to *moderately high* and 50% membership to *quite high*? Theoretically, it is possible to construct a set of fuzzy discount rates and relate them to the fuzzy consequences through a specific set of rules. This approach, however, requires further research.

Since various alternatives cannot be directly compared without discounting the life-cycle costs, a maximum acceptable risk value is proposed as a decision criterion. A water utility, through a consensus-building process like Delphi, will define the maximum acceptable risk z_{max} for its large-diameter transmission mains. Remembering that the term *risk* is a composition of both the possibility of failure and the failure consequences, it is possible that one z_{max} will be applicable to the entire inventory of large-diameter transmission mains. At the same time, special consideration, which

might not be readily integrated into the set of factors that determine failure consequences, may render more than one z_{max} necessary.

It is assumed that any decision concerning an intervention in a pipe will always be preceded by an inspection and condition assessment. Thus, if the deterioration model of a pipe predicts that z_{max} is going to be reached at year t , it follows that at year t an inspection/condition assessment will be scheduled. This inspection/condition assessment can have one of two outcomes: either the observed condition of the pipe is better than predicted (the model overestimated the deterioration rate) or the observed condition of the pipe is the same or worse than the model predicted. In case the former outcome is encountered, the deterioration model is re-calibrated to include the newly acquired data, then re-applied to obtain a new time, t , for inspection/condition assessment. If the latter outcome is encountered, renewal work has to be planned immediately and implemented as soon as possible.

The decision maker can use two inputs to define tolerance. The first is an explicit measure of risk tolerance z_{max} . The second is the α -level of the possibilistic confidence limit. The lower the α -value the wider the possibility band. For example, Figure 6 illustrates that for $z_{max} = moderately\ high$, the next inspection/condition assessment will be scheduled at age $t = 90-92$ years or $99-101$ years for α -values of 80% and 50% respectively, when the decision maker prefers the conservative approach.

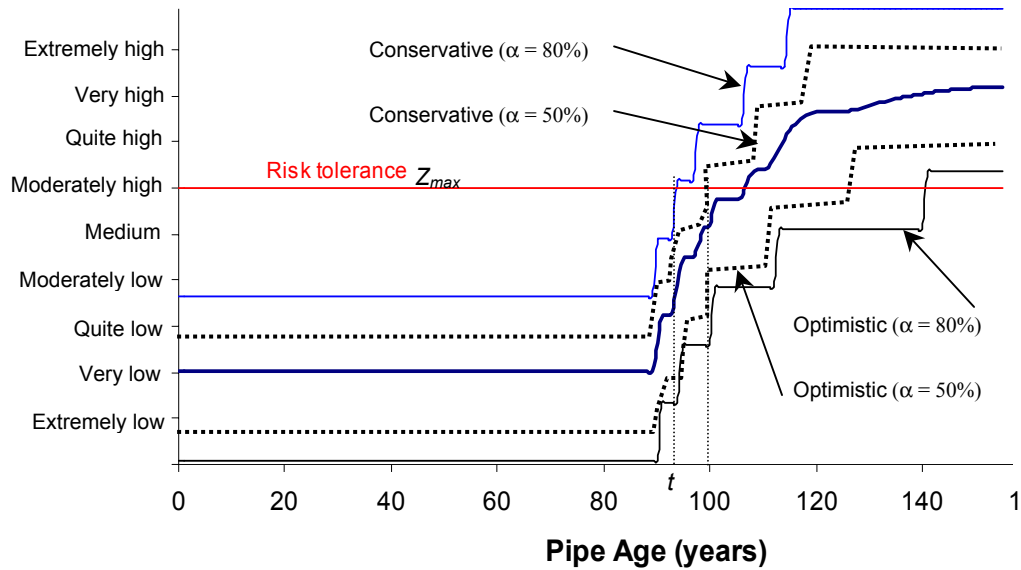


Figure 6. Renewal/inspection decision

As described earlier in Section 4, a renewal alternative k is assumed to have three specific attributes, namely the condition improvement matrix P_k , the post renewal deterioration rate and the cost of renewal S_k . Recall that a renewal alternative k essentially “buys” time T_k until the next expected renewal. The magnitude of T_k is determined by how much the pipe condition is expected to improve and by the expected post-renewal deterioration rate. Assuming that all renewal alternatives will perform equally well during their respective expected T_k periods, the only selection criterion can be cost versus time “bought”, or more precisely, the preferred renewal

alternative will be that for which the ratio S_k/T_k is the lowest. If equality of performance cannot be assumed, then performance criteria need to be defined and quantified for all the renewal alternatives.

7 Summary

The scarcity of data on the deterioration rates of buried infrastructure assets, coupled with the imprecise and often subjective nature of the assessment of pipe condition merits the usage of fuzzy techniques to model the deterioration of these assets. The deterioration process is modeled as a fuzzy rule-based non-homogeneous Markov process. As the deterioration process progresses, the pipe gradually “flows” from high membership in good condition states to high membership in worse states. Expert opinion is used to determine post-renewal pipe condition as well as post-renewal deterioration rate.

Consequences of pipe failure are also modeled as fuzzy sets and are coupled with the possibility of failure, using fuzzy based rules, to provide the fuzzy risk of failure. The concept of α -cuts is used to construct a possibilistic confidence band for pipe failure risk. A decision on the next inspection schedule is based on the decision maker’s specific risk tolerance. Renewal alternatives are compared based on the ratios of their costs and their expected capability to defer subsequent renewal.

Acknowledgement

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