Skin friction on a single pile to bedrock

Poorooshasb, H. B.; Bozozuk, M.

Publisher's version / Version de l'éditeur:

Proceedings, Third Panamerican Conference on Soil Mechanics and Foundation Engineering, pp. 1-11, 1968-03-01

IN FRICTION ON A SINGLE PILE TO BEDROCK

by H. B. Poorooshasb and M. Bozozuk

Presented at the Third Panamerican Conference on Soil Mechanics and Foundation Engineering (Division II), Caracas, Venezuela, July 1967

DBR NRC

Ottawa

March 1968

Price 25 cents

NRC 10024

NATIONAL RESEARCH COUNCIL OF CANADA
CONSEIL NATIONAL DE RECHERCHES DU CANADA

SKIN FRICTION ON A SINGLE PILE TO BEDROCK

by

H. B. Poorooshasb

Professor of Civil Engineering
University of Waterloo, Waterloo, Canada
(Visiting Scientist, DBR/NRC, Ottawa, 1966-67)

and

M. Bozozuk

Research Officer, Soil Mechanics Section
Division of Building Research
National Research Council, Ottawa, Canada

Presented at the
Third Panamerican Conference
on
Soil Mechanics and Foundation Engineering (Division II)
Caracas, Venezuela, 8 - 15 July 1967

Research Paper No. 354
of the
Division of Building Research

OTTAWA

March 1968

SKIN FRICTION ON A SINGLE PILE TO BEDROCK

by

H. B. Poorooshasb and M. Bozozuk

SUMMARY

A kinematically admissible displacement field describing the movement of the particles of a clay body surrounding a single prebored end-bearing vertical pile is used to obtain the nature and variation of the tangential component of the traction vector acting on the pile surface. The clay is assumed to be saturated, uniform and underlain by bedrock having a surface loading of $p\,\tilde{1}(t)$ where $\tilde{1}(t)$ is the unit step function.

* * * * * * *

A single vertical pile is embedded in a clay layer of thickness H, drainage being provided at both upper and lower surfaces of the layer. Bedrock underlies the clay stratum and supports the tip of the pile. After pile installation, say at time $t = 0$, the upper surface of the clay layer is subjected to an extended uniformly distributed vertical load of intensity p which causes the consolidation of the soil, hence transmitting skin friction to the pile.

An analysis to provide an estimate of the value and variation of the skin friction as a function of time and position along the pile, forms the study presented in this paper.

ANALYSIS

Polar cylindrical coordinates (r, θ, z) will be employed. The origin is taken at the tip of the pile and the z axis points upward in a vertical direction. Dimensionless parameters will be used:

$$\bar{r} = \frac{r}{H}\ , \ \bar{\theta} = \theta\ , \ \bar{z} = \frac{z}{H}\ , \ \sigma_{ij} = \frac{\sigma_{ij}}{p}\ , \ \bar{u} = \frac{u}{p}\ , \ \bar{t} = \frac{t}{H^2} \cdot c_v$$

where σ_{ij} is the stress, u the pore water pressure and c_v the coefficient of consolidation of the clay. Throughout the remainder of this paper the bar over the symbols will be eliminated but it should be borne in mind that all parameters are dimensionless according to the above scheme.

Assume a displacement field in the form:

$$V_r = V_\theta = 0, \quad V_z = -R(r,t)Z(z,t) \qquad \ldots \quad (1)$$

where V_i are the components of the displacement vector at (r,z) since time $t=0$, and where $R = R(r,t)$ is independent of z and $Z(z,t)$ is independent of r. The form of R and Z are to be determined considering the boundary conditions and equilibrium requirements.

When $r \to \infty$, R must tend to a limit, say $\bar{R}$. It will be assumed, temporarily, that $\bar{R}$ is independent of time. Once the displacement field is determined it must be verified that this assumption is legitimate.

At $r \rightarrow \infty$ the consolidation of the clay layer is not influenced by the pile:

$$V_z = -\bar{\bar{R}}\, Z(z,t) =$$

$$-RL_1 \left\{ \log \frac{k}{k-z} - \sum_m \frac{2}{m} \left[\cos mk\ \mathrm{Si}\ (mk-mz) - \sin\ (mk)\ \mathrm{Ci}\ (mk-mz) + N_m \right] \cdot \exp\ (-m^2 t) \right\} \qquad \ldots \qquad (2)$$

where $\quad m = \pi/2,\ 3\pi/2\ , \ldots\ldots$

and $\quad N_m = \sin\ (mk)\ \mathrm{Ci}\ (mk) - \cos\ (mk)\ \mathrm{Si}\ (mk).$

In deriving Eq. (2) a constitutive equation in the form

$$\epsilon_z = L_1 \frac{\sigma_z}{k-z} = \left(\frac{\Delta V_z}{\Delta z} \right)_r \qquad \ldots \qquad (3)$$

where L_1 and k are constants, has been used in conjunction with the expression

$$u = \sum_m \frac{2}{m} \sin\ (mz)\ \exp\ (-m^2 t) \qquad \ldots \qquad (4)$$

representing the value of pore water pressure at $(z,t)^*$.

---

* Eq. (4) assumes the soil to be of constant compressibility; Eq. (3) indicates an increase in stiffness with depth. Although the two assumptions appear to be incompatible at first sight, theoretical studies on the mode of dissipation of pore water pressure in clay layers have shown the effect of such variation of stiffness to be insignificant (Le Lievre 1967).

The very complexity of the system permits linearizations implied in Eq. (3) in which constant k describes the effect of surface crust hardness and must be slightly larger than unity. The appearance of z in the denominator expresses the change in clays "stiffness" with depth.

Substitution for z from Eq. (2) in Eq. (1) results in:

$$V_z = \frac{-R}{\bar{R}} L_1 \{ \log \frac{k}{k-z} - \sum_m \frac{2}{m} [\cos (mk)Si(mk-mz) - \sin(mk)Ci(mk-mz) + N_m] \exp (- m^2 t) \} \quad . . . (5)$$

Function R will be determined from the over-all equilibrium of the system which may be expressed according to

$$\int_0^1 (\sigma_z - z \frac{\partial \tau_{rz}}{\partial r} - z \frac{\tau_{rz}}{r}) dz - 1 = 0 \quad . . . \quad (6)$$

Equation (6) represents an equilibrium condition suited to the displacement field (1) (Hill 1963).

Now anologous to Eq. (3) let

$$L_2 \tau_{rz} = (k-z) \epsilon_{rz} = (k-z) (\frac{\Delta V_z}{\Delta r})_z \quad . . . (7)$$

then
$$\sigma'_z = \frac{(k-z)}{L_1} \epsilon_z = \frac{R}{\bar{R}} [1 - \sum_m \frac{2}{m} \operatorname{Sin}(mz) \exp (-m^2 t)] \quad . . .(8)$$

and
$$\tau_{rz} = \frac{L_1}{L_2} (k-z) \frac{1}{\bar{R}} \frac{\partial R}{\partial r} Y (z,t) \quad . . . (9)$$

where σ'_z is the effective stress component and

$$Y (z,t) = \log \{\frac{k}{k-z} - \sum_m \frac{2}{m} [\cos(mk)Si(mk-mz) - \sin(mk)Ci(mk-mz) + N_m] \exp (-m^2 t) \}$$

To obtain σ_z, the total stress component, Eqs. (8) and (4) may be combined:

$$\sigma_z = \sigma'_z + u = \frac{R}{\bar{R}} + \left(1 - \frac{R}{\bar{R}} \right) \sum_m \frac{2}{m} \text{ Sin } (mz) \exp (-m^2 t) \quad \ldots \qquad (10)$$

Note that in deriving this last equation it has tacitly been assumed that the presence of the pile has little effect on the dissipation of the pore water pressure. Experience with full size pile groups has shown such an assumption to be fairly reasonable. (Instrumentation of some 270 ft. piles to bedrock revealed a variation of pore water pressure of less than 0.15 lb/in^2 from a point in the vicinity of the pile to another about 20 ft. away from it. Both piezometers were installed at elevations of 50 ft. below ground surface.) Substitutions for σ_z and τ_{rz} from Eqs. (9) and (10) in Eq. (6) results in the equality.

$$\rho + (1 - \rho) \sum_m \frac{2}{m^2} \exp (-m^2 t) - \frac{L_1}{L_2} \left\{ \lambda(k) - \sum_m \lambda_m \exp (-m^2 t) \right\} \cdot$$

$$\cdot \left\{ \frac{\partial^2 \rho}{\partial r^2} + \frac{1}{r} \frac{\partial \rho}{\partial r} \right\} = 0$$

where

$$\rho = \frac{R}{\bar{R}}$$

$$\lambda_m = \int_0^1 \frac{2}{m} \left[\cos(mk) \mathrm{Si}(mk-mz) - \sin(mk) \mathrm{Ci}(mk-mz) + N_m \right] z(k-z) dz$$

and

$$\lambda(k) = \int_0^1 z(k-z) \log \left(\frac{k}{k-z} \right) dz$$

Let

$$\sum_m \frac{2}{m^2} \exp (-m^2 t) = T_1 (t)$$

and

$$\lambda(k) - \sum_m \lambda_m \exp (-m^2 t) = T_2(t)$$

then Eq. (11) reduces to

$$\rho + (1-\rho)\ T_1 - \frac{L_1 T_2}{L_2} \left(\frac{\partial^2 \rho}{\partial r^2} + \frac{1}{r} \frac{\partial \rho}{\partial r} \right) - 1 = 0$$

which has a solution in the form

$$\rho = 1 - \frac{K_0 \{ [\ L_2\ (1-T_1)\ r^2/L_1 T_2\ ]^{\frac{1}{2}} \}}{K_0 \{ [\ L_2\ (1-T_1)\ a^2/L_1 T_2\ ]^{\frac{1}{2}} \}} \quad . \ . \ . \quad (12)$$

where K { } is the modified Bessel Function of the second kind.

Note that the function expressing the dependence of R on time appears only in the argument of K { } and that as $r \longrightarrow \infty$ the second term on the right-hand side of Eq. (12) disappears. It is seen, hence, that the assumption made earlier is not only convenient but is also mathematically consistent.

Having determined R, the displacement field (1) may be formulated as

$$V_r = V_\theta = 0$$

$$V_z = -1 \left[\left[1 - \frac{K_0 \{ [L_2\ (1-T_1)/L_1 T_2]^{\frac{1}{2}} r \}}{K_0 \{ [L_2\ (1-T_1)/L_1 T_2]^{\frac{1}{2}} a \}} \right]\right] L_1 \left\{ \log \frac{k}{k-z} - \right.$$

$$\left. \sum_m \left[\left[\frac{2}{m} \left[-\sin(mk)\mathrm{Ci}(mk-mz) + \cos(mk)\mathrm{Si}(mk-mz) + N_m \right] \right]\right] \exp(-m^2 t) \right\} \quad . \ . \ . \quad (13)$$

Employing Eq. (9) and letting $r \longrightarrow a$, the vertical component of the shearing stress on pile surface is obtained.

$$(\tau_{rz}) \text{ at pile surface} = (k-z) \left\{ \frac{1-\sum_m \frac{2}{m^2} \exp(-m^2 t)}{\lambda(k)-\sum_m \lambda_m \exp(-m^2 t)} \; \frac{L_1}{L_2} \right\}^{\frac{1}{2}} \cdot$$

$$\cdot \frac{K_1 \{[L_2 (1-T_1)/L_1 T_2]^{\frac{1}{2}} a \}}{K_0 \{[L_2 (1-T_1)/L_1 T_2]^{\frac{1}{2}} a \}} \cdot$$

$$\cdot \left\{ \log \frac{k}{k-z} - \sum_m \frac{2}{m} [\cos(mk)Si(mk-mz) - \sin(mk)Ci(mk-mz) + N_m] \exp(-m^2 t) \right\} \quad . \; . \; (14)$$

DISCUSSION

Certain features of both Eqs. (13) and (14) are worthy of mention. For example, at time $t = 0$, $T_1(t) = 1$ and hence both R and $\frac{\partial R}{\partial r}$ tend to zero. This is to be expected as the time $t = 0$ signifies the very initiation of loading.

When $r \to a$ $V_z \to 0$ which implies that the soil is "stuck" to the pile surface and that the pile is rigid in comparison with the clay surrounding it. Both implications are plausible if the magnitude of the surface load p is "moderate" and if the pile surface is "rough".

Figure 1 shows the variation of F, the load supported by the pile of $a = .02$ resulting from skin friction, as a function of both time and position. Note that in particular at $t = 0$, $F = 0$ everywhere along the pile and that at $t \to \infty$;

$$F = \pi a \, \frac{K_1 \, \{[\, L_2/L_1\lambda(k)]^{\frac{1}{2}} \, . \, a\}}{K_0 \, \{[\, L_2/L_1\lambda(k)]^{\frac{1}{2}} \, . \, a\}} \cdot \left[\frac{L_1}{L_2\lambda(k)}\right]^{\frac{1}{2}} \cdot$$

$$\left[\, (k-1)^2 \, \left(\log \frac{k}{k-1} + \frac{1}{2}\right) - (k-z)^2 \, \left(\log \frac{k}{k-z} + \frac{1}{2}\right) \right] \cdot$$

The fact that for ($t>0$, $z = 1$) the curves in Figure 1 are not tangential to the z axis (although they have a very small gradient) is somewhat inconsistent with the boundary conditions but then the displacement field (1) has one of the simplest forms possible.

Johannessen and Bjerrum (1965) propose the equation

$$\tau_a = \sigma_v' \cdot K \tan \phi_a' \qquad \ldots \qquad (15)$$

to represent the variation of the skin friction τ_a. This is an empirical equation which is based on a few full-scale experiments on "driven piles" and in which σ_v' is the effective vertical stress and $K \tan \phi_a'$ is apparently a constant for any one time. The authors do not discuss the variation of this parameter with time in any length but indicate a value of 0.12 for about 13 months after installation and a value of 0.20 for times after a two-year period for the <u>particular experiments they have performed</u>.

Although there is a certain measure of agreement between Johannessen and Bjerrum's formulation and the formulation presented here (compare Figure 4 (c) of their paper with Figure (1) there appears to be a slight inconsistency involved in Eq. (15).

If the bedrock is rigid, then at (r = 0, z = 0) : $\frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r} = 0$

and hence both ϵ_{rz} and τ_a must vanish. If, on the other hand, the bedrock deforms due to the load of the pile, then at z = 0:

$$\frac{\partial V_z}{\partial r} > 0$$

in which case positive skin friction should develop at the pile tip. Equation (15) contradicts both requirements as it predicts a maximum value for the negative skin friction at tip. (Note that in their experiments the soil was free to drain at z = 0 and hence σ_z' assumed it maximum value here.)

CONCLUDING REMARKS

The concept presented in this paper is applicable to initial and boundary value problems in soil mechanics; the fact that it discusses the skin friction on a pile is somewhat incidental.

The basic concept is simple enough; it employs a kinematically admissible displacement field for the clay particles surrounding the pile to estimate the skin friction acting on it. The technique is not new although in soil mechanics it has almost invariably been used to obtain solutions to eigenvalue problems, e.g., slope stability.

The domain of the application of the concept presented here is potentially extensive although in each particular situation the boundary conditions and the configuration of the system to be analysed would decide whether such a technique is likely to yield fruitful results. As an immediate application it may be extended to treat the case of floating piles where both positive and negative skin friction develop along the pile and where an equation of the form in Eq. (15) would obviously not provide the correct answer.

REFERENCES

Hill, R. (1963). A General Method of Analysis for Metal-Working Processes. J. Mech. Phys. Solids, Vol. 11. Pergamon Press.

Johannessen and Bjerrum (1965). Measurement of the Compression of a Steel Pile to Rock due to Settlement of Surrounding Clay. Sixth Int. Conf. on Soil Mech. and Found. Eng. Montreal.

Le Lievre, Brian (1967). Ph.D. Thesis, University of Waterloo.

This paper is a joint contribution of the Department of Civil Engineering, University of Waterloo and the Division of Building Research, National Research Council.

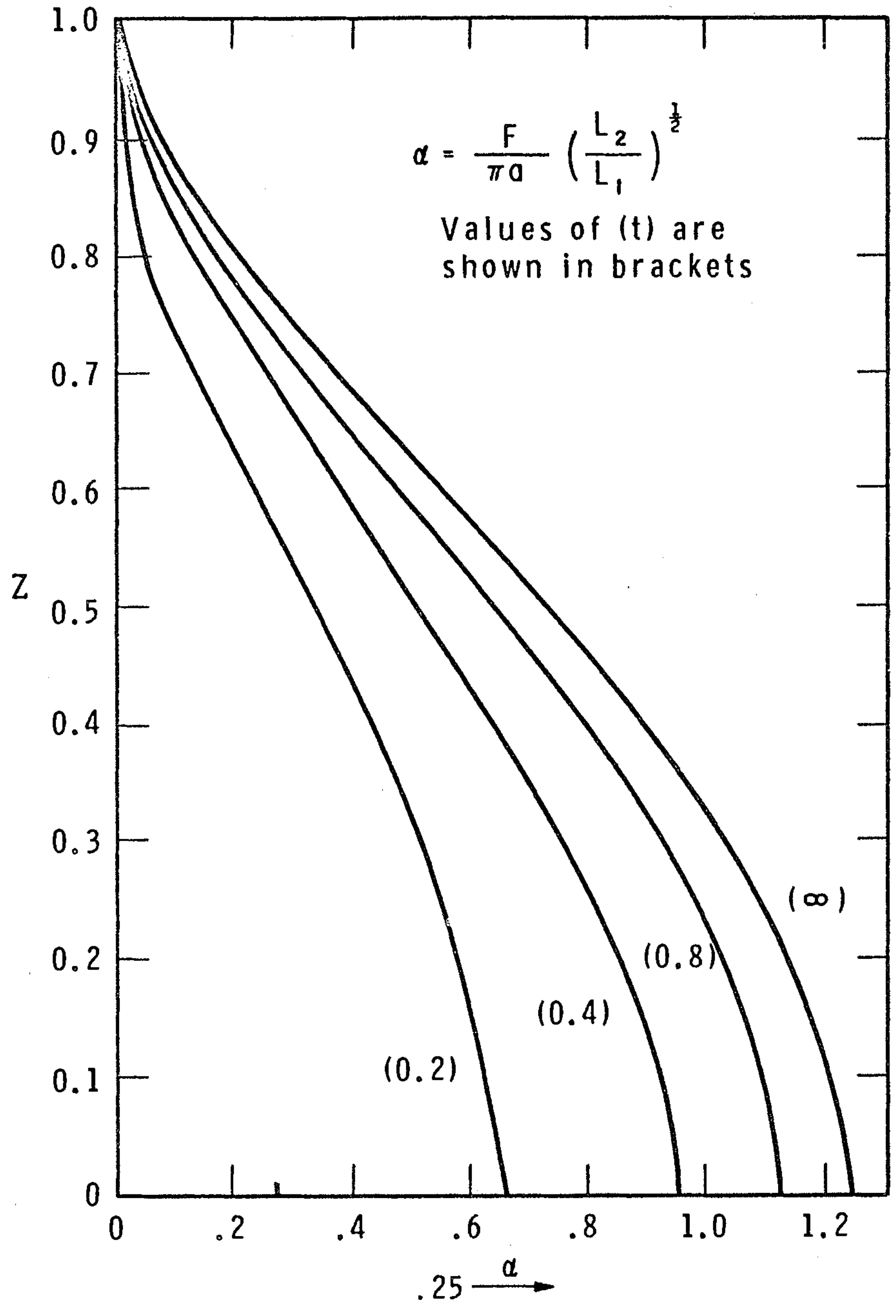

FIGURE 1

VARIATION OF LOAD SUPPORTED BY PILE OF UNIT LENGTH, k = 1.01

BR 3829