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SIMPLIFIED ANALYSIS OF DYNAMIC STRUCTURE–GROUND INTERACTION

BY

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Simplified Analysis of Dynamic Structure–Ground Interaction[1]

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A simplified method of analysis is presented for the determination of dynamic properties of single-story structures founded on flexible foundations. The general equations for natural frequency, mode shapes, and modal damping are applied to structures founded on an elastic half-space and on piles. The results of parameter studies, including the effects of hysteretic soil material damping, are presented for these two cases.

L'auteur expose une méthode simplifiée pour la détermination des propriétés dynamiques des structures à un seul étage reposant sur des fondations souples. Il applique ensuite à deux types de structures les expressions générales fournissant la fréquence propre, les modes et l'amortissement caractérisant chaque mode: les structures fondées sur un demi-espace élastique, et les structures fondées sur des pieux. L'article présente pour ces deux cas les résultats d'études paramétriques incluant l'influence de l'amortissement d'un sol à hystérèse.

[Traduit par la Revue]

Introduction

In the design of structures to resist the effects of dynamic loads such as earthquakes or wind, the assumption is generally made that the structure is supported by a rigid foundation. Since these dynamic loads and the dynamic response of structures also impose loads on the foundations, and since all real materials deform under applied loads, it may readily be appreciated that the rigid-base assumption represents an approximation to the real conditions. Whereas for most conventional structures it is sufficiently accurate and adequate to assume a rigid foundation for purposes of design to resist dynamic loads, it occasionally becomes necessary to consider the effects of a flexible, or compliant, foundation. This is particularly the case when massive structures such as nuclear power plants or dams or tall structures such as free-standing towers or high-rise buildings are founded on relatively soft materials. This gives rise to a phenomenon generally called dynamic structure–ground interaction.

Despite the availability of versatile and accurate methods of analysis to solve such problems—particularly the finite element method—simplified methods of analysis are useful for the following reasons:

(a) They can be used as first-order approximations to the more refined complex problem.

(b) They often permit the user to appreciate the essential features of the problem more readily than the solution to the complex problem would permit.

(c) They permit the isolation of the important parameters that govern the behavior of the system more readily than would be possible by numerical solutions such as the finite element method.

The simplified method of analysis presented here deals with the determination of the natural frequencies, mode shapes, and modal damping ratios of structure–ground interaction systems under dynamic loads. Once these quantities are known, the structural response and forces induced by seismic loads or other dynamic disturbances can be determined conveniently by response spectrum techniques.

General Method of Analysis

Before considering specific types of foundations, such as circular footings on an elastic half-space, or pile foundations, general relationships will be formulated which are valid for all types of elastic foundations. The major assumptions and simplifications incorporated in the analysis are the following.

1. The mathematical model of a given structure is simplified to a single-story structure on a foundation whose load–deformation char-

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acteristics can be described by Hooke's law. This model is represented in Fig. 1; it contains the essential features present in structure–ground interaction, namely interstory damping, frequency-dependent foundation properties, foundation damping, and foundation mass. This model has been used in a number of previous investigations by Jennings and Bielak (1973), Parmelee (1967), Rainer (1971), and Veletsos and Nair (1974). Cross-coupling between translational and rotational base motion is neglected. Multistory structures can be reduced to this simple model by methods outlined by Jennings and Bielak (1973).

2. The modal damping ratio is determined from energy principles in which the properties of the uncoupled mode shapes are employed. This can be expected to give reasonable results when the modal damping ratio is relatively small, say less than 10% of critical.

3. The seismic response is determined from a modal solution employing response spectra of ground motions. A similar approach can be used to evaluate the effects of wind loading. This requires as basic quantities the natural frequencies, mode shapes, and modal damping ratios of the dynamic system.

The present approach is thought to be simpler and more general than previously available solutions, which dealt specifically with the foundations on an elastic half-space (for example, Bielak 1975; Jennings and Bielak 1973; Rainer 1971; Veletsos and Nair 1974). Bielak (1975) also considers shallow buried foundations. Results for pile foundations have been presented using discretized mathematical models (Penzien 1970; Ohta *et al.* 1973).

An iteration procedure is used here to solve for the modal frequencies and mode shapes of a structure–ground interaction system. Iteration can be considered as an approximate method, but since the problem converges rapidly the answers can be obtained to any desired degree of accuracy. The use of the iteration approach leads to the derivation of simple relationships for the fundamental frequency, mode shapes, and the modal damping ratio of the system, as will be demonstrated subsequently.

Natural Frequencies

The mathematical model of the structure under investigation consists of a base mass m_0 resting on an elastic half-space and a top mass

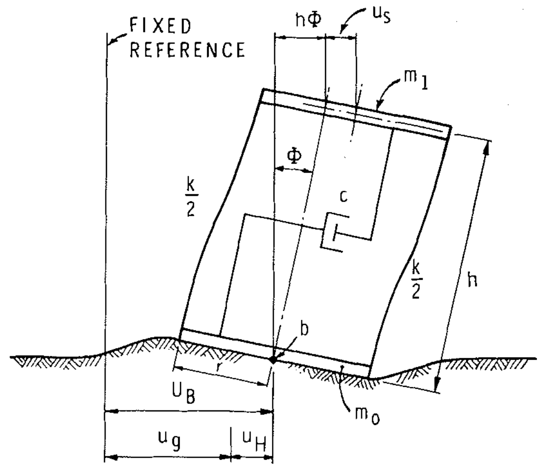

FIG. 1. Model of structure–ground interaction system.

m_1, as shown in Fig. 1. The horizontal base stiffness and rocking stiffness can be generalized to include the stiffness contributions from side layer soil and pile foundations. The equations of motion and derivation of the transfer functions involving real and imaginary terms are presented by Jennings and Bielak (1973), Parmelee (1967), Rainer (1971), and in a forthcoming paper.[2]

If only the real parts of the transfer function are retained and base excitation is zero (*i.e.* undamped free vibration is assumed) the following equation is obtained for the relative base displacement u_H, interstory displacement u_s and rocking displacement $h\phi$

$$[1]\quad \begin{bmatrix} 1-\dfrac{\Omega_H^2}{p^2} & 1 & 1 \\ 1 & 1-\dfrac{\omega_0^2}{p^2} & 1 \\ 1 & 1 & 1-\dfrac{\Omega_\Phi^2}{p^2} \end{bmatrix} \begin{bmatrix} u_H \\ u_s \\ h\Phi \end{bmatrix} = 0$$

where

$$[2]\quad \Omega_H^2 = K_H/m_1 - \alpha p^2 = \omega_H^2 - \alpha p^2$$

$$[3]\quad \omega_0^2 = k/m_1 = \text{square of the fixed-base frequency of the structure}$$

$$[4]\quad \Omega_\Phi^2 = K_\Phi/m_1 h^2 - \beta p^2 = \omega_\Phi^2 - \beta p^2$$

and $\alpha = m_0/m_1$, $\beta = I/I_1$, K_H and K_Φ are hor-

[2]J. H. Rainer. Paper in process.

izontal and rotational stiffnesses of the foundation on the ground, and p is the frequency of the interaction system. I is the mass moment of inertia of m_1 and m_0 about their own axes of rotation, and $I_1 = m_1h^2$ is the geometric mass moment of inertia of the structure. Other terms are defined in Fig. 1. The frequency equation obtained from Eq. [1] is then:

$$[5] \qquad \frac{1}{p^2} = \frac{1}{\omega_0^2} + \frac{1}{\Omega_H^2} + \frac{1}{\Omega_\Phi^2}$$

or

$$[6] \qquad \frac{p^2}{\omega_0^2} = 1/\left(1 + \frac{1}{\Omega_H^2/\omega_0^2} + \frac{1}{\Omega_\Phi^2/\omega_0^2}\right)$$

Since p^2 is also contained in Ω_H^2 and Ω_Φ^2, successive approximations are required for the evaluation of p^2.

If the products of p^2 with α and β are neglected, Eq. [6] reduces to the well-known Southwell-Dunkerley approximation (Jacobsen and Ayre 1958):

$$[7] \qquad \frac{1}{p^2} \simeq \frac{1}{\omega_0^2} + \frac{1}{\omega_H^2} + \frac{1}{\omega_\Phi^2}$$

Equation [7] also gives the first approximation for p^2 of the fundamental mode. Successively improved values of p^2, up to any desired degree of accuracy, are obtained with further cycles of iteration, using Eqs. [5] or [6]. These equations are equally valid for the second and third mode of the mathematical model in Fig. 1. However, the numerical computations are quite sensitive. If these frequencies are required it is probably better to use other methods of evaluating the eigenvalues.

Determination of Mode Shapes

Once the eigenvalues have been determined, the corresponding mode shapes can be found by substituting in Eq. [1] and solving for the displacement components. If the expression for p^2 in Eq. [6] is substituted into Eq. [1], the following relationships for the modal amplitude ratios δ, ξ, and γ are obtained:

$$[8] \qquad \delta = \frac{u_s}{u_t} = \frac{p^2}{\omega_0^2} = \frac{1}{1 + \dfrac{\omega_0^2}{\Omega_\Phi^2} + \dfrac{\omega_0^2}{\Omega_H^2}}$$

$$[9] \qquad \xi = \frac{u_H}{u_t} = \frac{p^2}{\Omega_H^2} = \frac{1}{1 + \dfrac{\Omega_H^2}{\omega_0^2} + \dfrac{\Omega_H^2}{\Omega_\Phi^2}}$$

$$[10] \qquad \gamma = \frac{h\Phi}{u_t} = \frac{p^2}{\Omega_\Psi^2} = \frac{1}{1 + \dfrac{\Omega_\Phi^2}{\omega_0^2} + \dfrac{\Omega_\Phi^2}{\omega_H^2}}$$

where $u_t = u_s + u_H + h\Phi$.

The modal amplitude ratios will be accurate if the resonant frequency p^2 is accurate since the relationships Eqs. [8] to [10] do not involve any additional approximations.

Modal Damping Ratio

A modal damping ratio λ_E can be obtained from energy considerations as presented by Novak (1974*a*, *b*):

$$[11] \qquad \lambda_E = \frac{C_s u_s^2 + C_H u_H^2 + C_\Phi \Phi^2}{2p(m_0 u_H^2 + m_1(u_s + u_H + h\Phi)^2 + I\Phi^2)}$$

where $C_s = 2\lambda_0\sqrt{km_1} = 2\lambda_0\omega_0 m_1$ is the interstory damping constant in units of force per velocity, and C_H and C_Φ are the damping constants in the horizontal and rotational direction, respectively, of the foundation. By evaluating the various damping terms C and substituting the modal amplitudes u_s, u_H, and $h\Phi$, the modal damping ratio λ_E for any mode can be evaluated. Some numerical comparisons between an equivalent modal damping ratio λ_{eq} and λ_E have been presented by Rainer (1975).

By expressing the displacement amplitudes in the form of modal amplitude ratios of Eqs. [8], [9], and [10], Eq. [11] becomes:

$$[12] \qquad \lambda_E = \frac{1}{(1 + \alpha\xi^2 + \beta\gamma^2)}[\lambda_0\delta^{3/2} + \Lambda_H\xi^2 + \Lambda_\Phi\gamma^2]$$

where $\Lambda_H = C_H/2pm_1$ and $\Lambda_\Phi = C_\Phi/2pm_1h^2$. The damping constants C_H *and* C_Φ can include the contributions from various sources of energy dissipation such as radiation damping, material damping, partial burial, and pile foundations.

Structures with Foundations on Elastic Half-space

Although the above relationships for natural frequency and damping ratios are valid for any geometric configuration of the base, as well as for shallow buried foundations or pile foundations, specific solutions will now be obtained for structures with circular foundations resting on an elastic half-space. This resembles a common configuration of nuclear power reactors when the influence of shallow burial can be neglected.

Natural Frequency

As can be seen from Eq. [5], the natural frequency of the interaction system depends on the frequencies ω_0^2, Ω_h^2, and Ω_θ^2, as defined by Eqs. [2] to [4], except that the subscripts h and θ apply to the half-space solution and replace the more general subscripts H and Φ, respectively. This subscript h should not be confused with h, the height of the structure.

With appropriate substitution for the properties of circular foundations and simplification, Eq. [7] becomes:

$$[13] \quad \frac{\omega_0^2}{p^2} = 1 + \frac{k}{Gr} \times \left(\frac{1}{e(1 - \alpha p^2/\omega_h^2)} + \frac{h^2/r^2}{d(1 - \beta p^2/\omega_\theta^2)}\right)$$

where

$$e = \frac{32(1 - \nu)k_h}{7 - 8\nu}, \quad d = \frac{8k_\theta}{3(1 - \nu)},$$

$$\omega_h^2 = \frac{Gre}{m_1}, \quad \text{and} \quad \omega_\theta^2 = \frac{Gr^3 d}{I_1}$$

G = shear modulus of the ground, and k_h and k_θ are the frequency-dependent horizontal and rotational stiffness coefficients for the circular footing on the half-space, and ν is Poisson's ratio. An approximation for the resonance frequency of a structure on an elastic half-space that is analogous to the completely general case given by Eq. [7] is obtained if the terms $\alpha p^2/\omega_h^2$ and $\beta p^2/\omega_\theta^2$ are neglected.

The primary parameters that affect the frequency reduction for a structure on an elastic half-space are:

(a) k/Gr, the ratio of the stiffness of the structure to that of the foundation resting on the ground, and

(b) h^2/r^2, the square of the aspect ratio of the structure.

Secondary influences on the natural frequency reduction are Poisson's ratio of the elastic half-space, and the translational and rocking frequencies ω_h^2 and ω_θ^2 as defined after Eq. [13]. The former is the frequency of the top mass on the elastic half-space in the horizontal direction, the latter is the rocking frequency considering only the geometric mass moment of inertia of the top mass about the base.

It should be noted that the shear wave velocity of the ground, $V_s = (G/\rho)^{1/2}$, is a significant parameter in the sense that it is a function of the shear modulus G, and thus a convenient parameter designating soil stiffness (ρ is the mass density of the ground). V_s also plays a minor role in the determination of the frequency-dependent foundation stiffness coefficients k_h and k_θ since these are generally functions of $a = pr/V_s$. However, V_s should not be used by itself to establish criteria for assessing the importance of ground–structure interaction effects, since the other important parameters, namely foundation stiffness (as given by Gr), structural stiffness (as given by k), and the aspect ratio (h/r) are not incorporated in V_s.

Modal Damping Ratio for Elastic Half-space

The modal damping ratio is given by Eq. [12] where, for the elastic half-space,

$$[14] \quad \Lambda_H = \Lambda_h = C_h/2pm_1 = \frac{K_h c_h a}{2m_1 p^2} = \frac{\omega_h^2}{p^2}\left(c_h \cdot \frac{a}{2}\right)$$

and

$$[15] \quad \Lambda_\Phi = \Lambda_\theta = C_\theta/2pm_1h^2 = \frac{K_\theta c_\theta a}{2m_1 h^2 p^2} = \frac{\omega_\theta^2}{p^2}\left(c_\theta \cdot \frac{a}{2}\right)$$

Soil material damping D can be incorporated as $\mathbf{c} = c(a/2) + D$ for both the h and θ subscripts, as shown by Veletsos and Verbic (1973), Rainer (1975), and Bielak (1975). The damping coefficients c_h and c_θ are those applicable to a footing on an elastic half-space, as computed for example by Veletsos and Verbic (1974).

For the fundamental mode, Eq. [12] can be simplified by using the approximations[3]

$$\Omega_h^2 \simeq \frac{K_h}{m_1} = \omega_h^2$$

$$\Omega_\theta^2 \simeq \frac{K_\theta}{m_1 h^2} = \omega_\theta^2$$

[3]This approximation can be shown to be reasonable for structures founded on an elastic half-space but may not apply to other foundation types, such as pile foundations.

and

$$(1 + \alpha\xi^2 + \beta\gamma^2) \simeq 1.0$$

With the aid of Eqs. [8] to [10],

$$[16] \quad \lambda_E' = \lambda_0(\delta^{3/2}) + c_h'(\xi) + c_\theta'(\gamma)$$

where

$$c_h' = c_h \cdot (a/2) \quad \text{and} \quad c_\theta' = c_\theta \cdot (a/2)$$

The important parameters that affect the modal damping ratio γ_E are readily identified from Eq. [16]. These are:

(a) the structural damping ratio λ_0 and the foundation damping coefficients c_h and c_θ;

(b) the modal amplitude ratios δ, ξ, and γ; and

(c) the nondimensional frequency **a** for the footing on the half-space.

The following implications for the modal damping ratio can be seen from Eq. [16].

Since $\delta^{3/2} = (p/\omega_0)^3$ from Eq. [8], the contribution of the structural damping term is seen to vary as the cube of the frequency reduction ratio. This dependence has also been established by Veletsos and Nair (1974) and Bielak (1975). As long as the structural modal amplitude ratio δ is large compared with the base amplitude ξ and rocking amplitude γ, the contribution from foundation damping will be negligible. As δ decreases relative to ξ and γ, the contribution of structural damping towards the system damping ratio diminishes rapidly and the system damping ratio is then dominated by foundation damping. It also follows that the contribution of the structural damping will become negligible if the frequency ratio (p/ω_0) becomes significantly less than 1.

Modal Damping for Higher Modes

Since p^2/ω_h^2 is significant relative to 1 for the two higher modes of this single-story model, the approximations in Eq. [16] are not acceptable and Eq. [12] has to be used. For the type of structure investigated here, numerical results as presented by Jennings and Bielak (1973) and Rainer[2] show that the second and third modes are highly damped as a result of foundation radiation damping. Therefore, for an estimate of seismic response of this simplified model only the contributions of the fundamental mode need be considered.

Parameter Study

The characteristics of the mathematical model shown in Fig. 1 and described by the equations presented by Parmelee (1967) and Rainer (1971) were investigated to show the influence of the main parameters governing the dynamic behavior. Because of the mathematical model chosen, the results are strictly applicable to structures with circular foundations on an elastic half-space. They can, however, be adapted to rectangular footings by deriving an equivalent radius (Richart *et al.* 1970).

In order to reduce the number of variables to manageable proportions, only the dominant parameters were varied. Fixed parameters are: Poisson's ratio of ground, $\nu = 0.333$; mass ratio $\alpha = 1.03$; inertia ratio $\beta = 0.226$. The values for α and β chosen are representative of some nuclear reactor structures.

For the description of the frequency-dependent foundation properties, the algebraic expressions derived by Veletsos and Verbic (1974) were used. Although the frequency-dependent stiffness was employed for the rocking motion, this had negligible effects on the results compared with using constant values. The frequency ratio and the modal amplitude ratios were iterated four times throughout the set of parameters employed.

The variation of the frequency reduction ratio p/ω_0 is shown in Fig. 2, and the damping ratio λ_E is plotted as a function of the primary parameters k/Gr, and h/r in Figs. 3 and 4. Since the modal damping ratio in Eq. [16] is a function of **a** and since $a^2 = (k/Gr)(p^2/\omega_0^2)(1/b_1)$, the mass density ratio $b_1 = m_1/\rho r^3$ also becomes a plotting parameter for modal damping ratio. Figure 3 shows that for small values of aspect ratio h/r the modal damping ratio increases rapidly for increasing stiffness ratios k/Gr. For slender structures, *i.e.*, large values of h/r, the modal damping ratio becomes relatively small.

For large values of the stiffness ratio k/Gr, magnitudes of modal damping ratios are plotted as λ_E *versus* h/r in Figs. 4a and b, for soil material damping ratios $D = 0$ and 0.05, respectively. The results can be utilized as follows: for λ_0 less than about 10%, the parameters for which the modal damping ratio will be smaller than the structural interstory damping ratio are those that lie below the

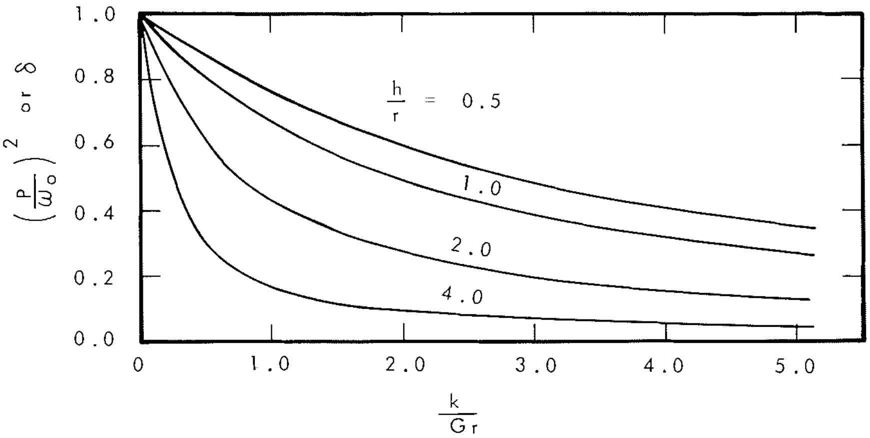

FIG. 2. Frequency reduction ratio for interaction structure on elastic half-space.

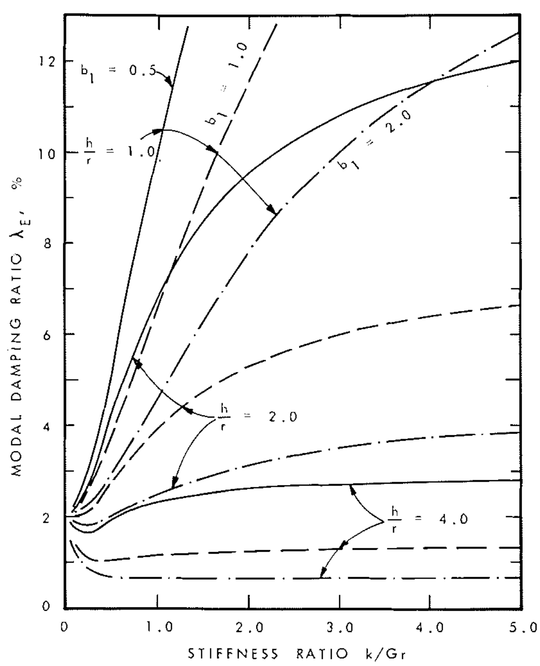

FIG. 3. Modal damping ratios for structures on elastic half-space. $\lambda_0 = 2\%$ ($\gamma = 0.333$, $\alpha = 1.03$, $\beta = 0.226$, $D = 0.0$).

ordinate of the applicable structural damping ratio.

The following general observations are made:

(i) For small values of interstory damping, structures with large aspect ratios h/r and structures of large values of mass ratio b_1 will produce small modal damping ratios.

(ii) For $k/Gr > 2.0$, the modal damping ratio is relatively insensitive to the stiffness ratio k/Gr, but depends primarily on the aspect ratio h/r and the mass density ratio b_1.

(iii) For large structural damping ratios λ_0, the modal damping can be smaller than λ_0 for a wide range of commonly encountered values of aspect ratio h/r and mass density ratio b_1.

(iv) For $k/Gr > 2.0$, the increase in the system damping ratio λ_E for slender structures becomes nearly equal to the increase in the soil material damping ratio D. This is evident by comparing corresponding ordinates for values of h/r greater than about 1.5 in Figs. 3 and 4 for $D = 0.00$ and 0.05, respectively.

This latter observation agrees with the results from the approximate relation, Eq. [16]. Since for soft foundations $\xi + \gamma$ is nearly equal to 1.0, and δ is small, the system damping ratio depends directly on the foundation damping ratios and soil material damping ratios as follows:

$$\lambda_E'' = c_h \frac{a}{2} \xi + c_\theta \frac{a}{2} \gamma + D(\xi + \gamma)$$

These results can have important consequences in the design of structures with flexible foundations. The assumption of high values of system damping may not be justified if a flexible foundation condition is present, particularly in tall structures. However, material damping in the foundation soil will contribute to increases of the modal damping ratio. The results presented point to the importance of establishing realistic levels of soil material

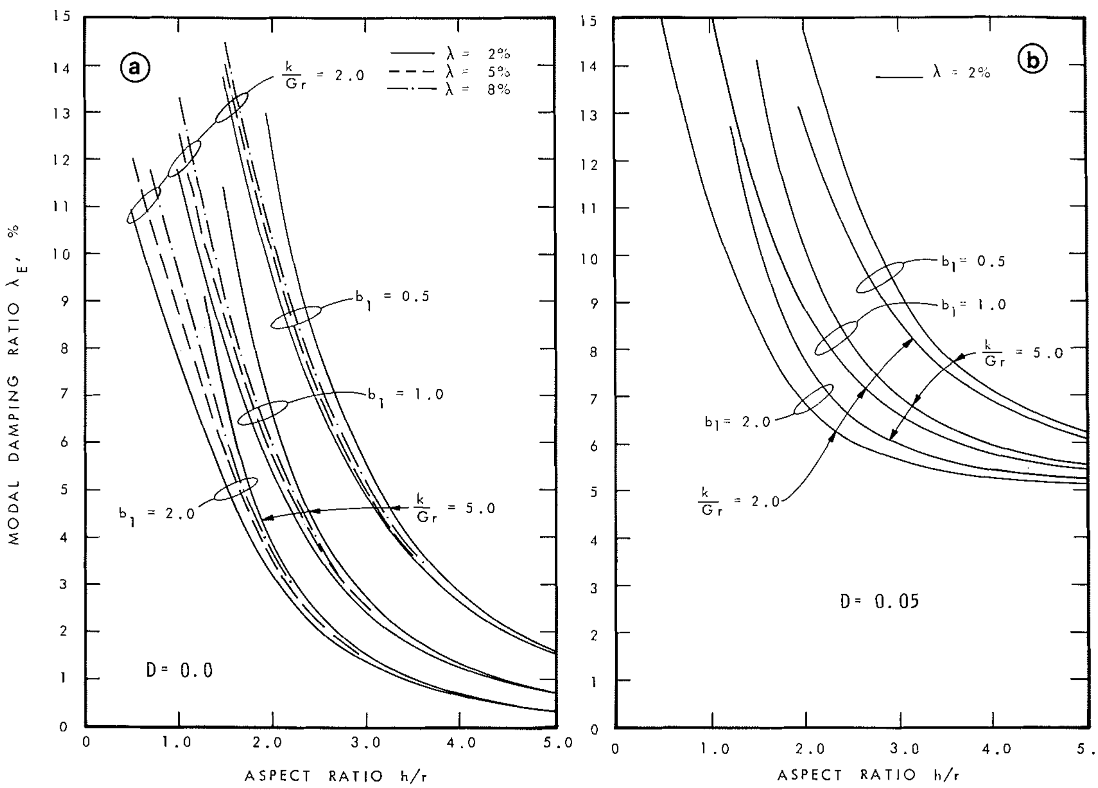

FIG. 4. (a) Modal damping ratio for structure on elastic half-space, for $k/Gr \geqslant 2.0$ ($\gamma = 0.333$, $\alpha = 1.02$, $\beta = 0.226$, $D = 0.0$). (b) Modal damping ratio for structure on elastic half-space, for $k/Gr \geqslant 2.0$ ($\gamma = 0.333$, $\alpha = 1.03$, $\beta = 0.226$, $D = 0.05$).

damping for the strain levels that are expected to occur.

Structures with Pile Foundations

For structures founded on piles the general relations for frequency reductions (Eq. [5]) and for modal damping ratio (Eq. [12]) are also applicable. However, the evaluation of the various stiffness and damping terms has to proceed differently than for the elastic half-space. For completeness, the contributions of lateral soil layer are also included here, but no numerical results are presented.

A number of assumptions have to be made in order to permit the simplified solution of pile foundations:

(a) The pile group efficiency factor for the foundation is based on static consideration and is assumed known.

(b) The efficiency factor applied to stiffness is assumed to be applicable also to geometric and material damping.

(c) The rocking stiffness of the pile foundation arises from the axial stiffness of the piles and the in-plane rotational stiffness of the pile tops. The fraction of total rocking stiffness due to axial pile stiffness and pile top rotation depends on the pile properties and the geometric layout of the pile foundation.

(d) For the results to be applicable to seismic disturbances, the assumption has to be made that the horizontal motion of the pile foundation without the structure represents the free-field motion of the ground.

The horizontal and rotational stiffnesses K_{H} and K_{Φ} are evaluated by summing the contributions of mutually independent sources of stiffness:

$$K_{\text{H}} = K_{\text{h}} + K_{x} + K_{u} \text{ and } K_{\Phi} = K_{\Theta} + K_{z\phi} + K_{\phi\phi} + K_{\psi}$$

Similarly the damping contributions to C_{H} and C_{Φ} are summed as

$$C_{\mathrm{H}} = C_{\mathrm{h}} + C_{\mathrm{x}} + C_{\mathrm{u}} \text{ and } C_{\Phi} = C_{\theta} + C_{z\phi} + C_{\phi\phi} + C_{\psi}$$

where the subscripts have the following meaning, in the horizontal and rotational directions, respectively:

H, Φ: total quantity
h, θ: half-space contribution
x, ϕ: pile-foundation contribution
u, ψ: side layer contribution

($z\phi$ and $\phi\phi$ refer to rocking contributions from axial motion and in-plane rotation of the piles, respectively).

The frequency is then obtained from Eq. [5] by making use of the total stiffness K_{H} and K_{Φ}. It should be noted that the stiffnesses of the pile foundation, K_{x} and K_{ϕ}, are the net stiffnesses of the entire pile foundation, obtained by multiplying the individual pile stiffness by the total number of piles and by the group efficiency factor.

The modal damping ratio λ_{E} can be evaluated from Eq. [12] once Λ_{H} and Λ_{Φ} are known. The various contributions to C_{H} and C_{Φ} are the damping coefficients obtained from Eqs. [14], [15] for the elastic half-space, from Novak (1974*b*) for the side layer, and Novak (1974*a*) for the pile foundation. For the latter, for example,

$$C_{\mathrm{x}}p = K_{\mathrm{x}}a_{\mathrm{x}}\frac{f_{11,2}}{f_{11,1}}.$$

Substitution of the above relationship and summing the other contributions to C_{H} and C_{Φ} results in:

$$[17] \quad \Lambda_{\mathrm{H}} = \frac{1}{m_1 p^2}\left[K_{\mathrm{h}}\left(\frac{a}{2}c_{\mathrm{h}} + D_{\mathrm{h}}\right) + K_{\mathrm{x}}\left(\frac{a_{\mathrm{x}}}{2}\frac{f_{11,2}}{f_{11,1}} + D_{\mathrm{x}}\right) + K_{\mathrm{u}}\left(\frac{a_{\mathrm{u}}}{2}\frac{\bar{S}_{\mathrm{u}2}}{S_{\mathrm{u}1}} + D_{\mathrm{u}}\right)\right]$$

$$[18] \quad \Lambda_{\Phi} = \frac{1}{m_1 h^2 p^2}\left[K_{\theta}\left(\frac{a}{2}c_{\theta} + D_{\theta}\right) + K_{z\phi}\left(\frac{a_{\mathrm{x}}}{2}\frac{f_{18,2}}{f_{18,1}} + D_{z\phi}\right) + K_{\phi\phi}\left(\frac{a_{\mathrm{x}}}{2}\frac{f_{7,2}}{f_{7,1}} + D_{\phi\phi}\right) + K_{\psi}\left(\frac{a_{\mathrm{u}}}{2}\frac{\bar{S}_{\psi2}}{S_{\psi1}} + D_{\psi}\right)\right]$$

The terms $f_{7,2}$, $f_{11,2}$, and $f_{18,2}$ are geometric damping parameters and $f_{11,1}$, and $f_{18,1}$ stiffness parameters as evaluated by Novak (1974*b*). Subscripts 7, 11, and 18 pertain to pile top bending, horizontal and vertical displacement of piles, respectively; $\bar{S}_{\mathrm{u}2}$, $S_{\mathrm{u}1}$, $\bar{S}_{\psi2}$, and $S_{\psi1}$ are damping and stiffness terms for side layer reaction as evaluated and tabulated by Novak (1974*a*). The nondimensional frequencies **a** pertain to the respective foundation element and the adjacent soil at the resonance frequency of the interaction structure. Similarly the hysteretic material damping D is that applicable to the soil adjacent to the deforming foundation element. For pile foundations the material damping has been treated analogously with the half-space solution (Veletsos and Verbic 1973; Rainer 1975) and with the side layer solution (Bielak 1975).

The following substitutions are made:

$$(K_{\mathrm{h}} + K_{\mathrm{x}} + K_{\mathrm{u}})/K_{\mathrm{H}} = r_{\mathrm{h}} + r_{\mathrm{x}} + r_{\mathrm{u}} = 1.0$$

$$(K_{\theta} + K_{z\phi} + K_{\phi\phi} + K_{\psi})/K_{\Phi} = r_{\theta} + r_{z\phi} + r_{\phi\phi} + r_{\psi} = 1.0$$

and

$$K_{\mathrm{H}}/m_1 = \omega_{\mathrm{H}}^2, \; K_{\Phi}/m_1 h^2 = \omega_{\Phi}^2$$

Equations [17] and [18] then become

$$[19] \quad \Lambda_{\mathrm{H}} = \left(\frac{\omega_{\mathrm{H}}^2}{p^2}\right)\left[r_{\mathrm{h}}\left(\frac{a}{2}c_{\mathrm{h}} + D_{\mathrm{h}}\right) + r_{\mathrm{x}}\left(\frac{a_{\mathrm{x}}}{2}\frac{f_{11,2}}{f_{11,1}} + D_{\mathrm{x}}\right) + r_{\mathrm{u}}\left(\frac{a_{\mathrm{u}}}{2}\frac{\bar{S}_{\mathrm{u}2}}{S_{\mathrm{u}1}} + D_{\mathrm{u}}\right)\right]$$

$$[20] \quad \Lambda_{\Phi} = \left(\frac{\omega_{\Phi}^2}{p^2}\right)\left[r_{\theta}\left(\frac{a}{2}c_{\theta} + D_{\theta}\right) + r_{z\phi}\left(\frac{a_{\mathrm{x}}}{2}\frac{f_{18,2}}{f_{18,1}} + D_{z\phi}\right) + r_{\phi\phi}\left(\frac{a_{\mathrm{x}}}{2}\frac{f_{7,2}}{f_{7,1}} + D_{\phi\phi}\right) + r_{\psi}\left(\frac{a_{\mathrm{u}}}{2}\frac{\bar{S}_{\psi2}}{S_{\psi1}} + D_{\psi}\right)\right]$$

It may be observed that the contributions of the various sources of damping are scaled in proportion to their respective stiffness ratios, r.

Whereas in principle all the terms required for calculating the damping ratio are known, judgment is needed in assessing pile group action

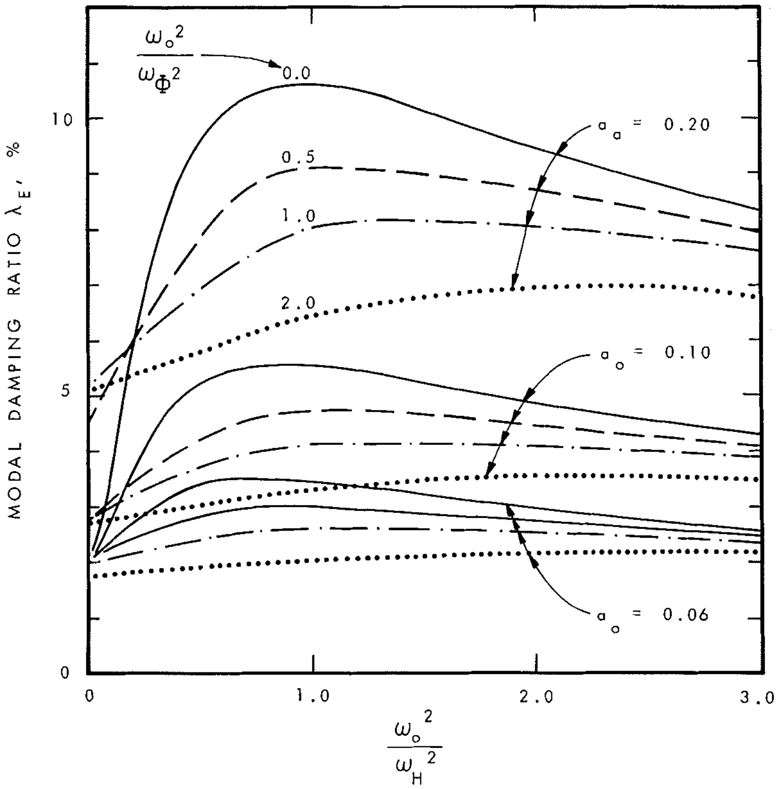

FIG. 5. Modal damping ratios for structures on pile foundations. ($\alpha = 1.0$, $\beta = 0.226$, $\lambda_0 = 2\%$, $D = 0.0$, $f_{18,2}/f_{18,1} = 1.60$, $f_{7,2}/f_{7,1} = 0.70$, $r_{z\phi} = 0.44$, $r_{\phi\phi} = 0.56$).

and material damping. Furthermore, iterative procedures are needed when material properties are strain dependent.

For ease of numerical evaluation, it may be advantageous to express Λ_H and Λ_Φ in terms of $a_0 = a(p/\omega_0)$, the nondimensional frequency relative to the frequency of the fixed-based structure. Also, parametric approximations for the various damping terms are possible. These and other topics are treated in greater detail by Rainer.[2]

Parameter Study of Pile Foundations

Since the horizontal and the rotational stiffness of the entire pile foundation can be varied somewhat independently, it is advantageous to retain $\omega_H^2/\omega_0^2 = K_H/k$ and $\omega_\Phi^2/\omega_0^2 = K_\Phi/kh^2$ as independent variables in the parameter study.

Figure 5 presents modal damping ratios of the pile system as a function of ω_0^2/ω_H^2, ω_0^2/ω_Φ^2 and the nondimensional frequency a_0. The following parameters are used: mass ratio $\alpha = 1.0$, $\beta = 0.226$, interstory damping ratio $\lambda_0 = 2\%$, and soil material damping $D = 0$. From Novak (1974*a*, *b*), $f_{18,2}/f_{18,1} = 1.6$; $f_{11,2}/f_{11,1} = 2.38$ and $f_{7,2}/f_{7,1} = 0.70$ for concrete piles. Rotational stiffness ratios are $r_{z\phi} = 0.44$ and $r_{\phi\phi} = 0.56$.

The results in Fig. 5 show that major changes in the modal damping ratio occur mainly at low values of ω_0^2/ω_H^2, *i.e.*, for relatively stiff foundations. Considerable damping arises from the rocking motion as is evident from the substantial values of λ_E near $\omega_0^2/\omega_H^2 = 0$. This contrasts with structures founded on an elastic half-space, where for relatively stiff foundations rocking contributes very little to the modal damping ratio. Modal damping also increases substantially with increasing values of a_0. This implies that with increasing pile diameter, and maintaining constant rocking and horizontal pile group stiffness, as well as constant soil stiffness and pile slenderness ratio, greater modal damping values are obtained. It can also be ascertained from specific examples

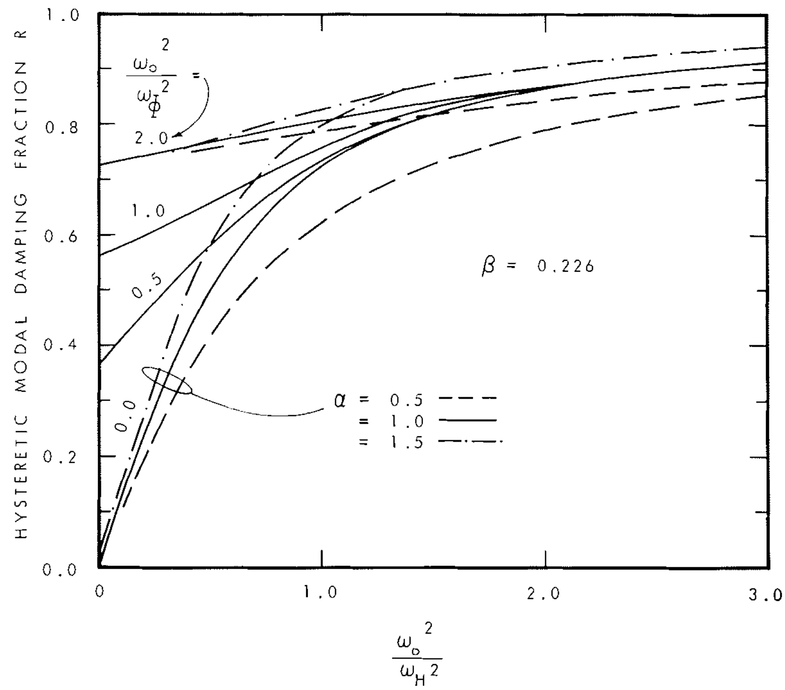

FIG. 6. Hysteretic modal damping fraction for structures on pile foundations.

that increases in structural stiffness, as is reflected by larger values of ω_0^2 and a_0, result in increasing modal damping values. Similar results are obtained for $\lambda_0 = 5\%$, except that near $\omega_0^2/\omega_H^2 = 0$ the modal damping values are larger than those in Fig. 5 for $\lambda_0 = 2\%$ as shown by Rainer.[2]

Equation [12], with Eqs. [19] and [20], can be rearranged so that all terms containing D are collected; thus the total modal damping ratio λ_E becomes

$$\lambda_E = \lambda_{\bar{E}} + RD$$

where R is called the 'hysteretic modal damping fraction'. Plots for R are presented in Fig. 6 for relevant parameters shown there. D is assumed the same for horizontal translation and axial motion of the pile. Figure 6 shows that for stiff foundations, little of the material damping contributes to the modal damping ratio. As the foundation stiffness decreases relative to the structure, an increasing proportion of material damping becomes effective in the total modal damping ratio. Only for very soft foundations is nearly the entire amount of material damping ratio effectively additive to the modal damping ratio that arises from structural and geometric foundation damping.

Summary and Conclusions

The treatment of dynamic structure–ground interaction as presented in this paper can be summarized as follows:

1. The natural frequency of the fundamental mode and the corresponding modal amplitude ratios can be found by a simple analytical expression. Iteration is required for high degrees of accuracy.

2. The modal damping ratio λ_E can be evaluated from an expression derived from Novak's damping relationship (Novak 1974*a*). This involves primarily the modal amplitude ratios and damping coefficients for the structure and the foundation soil. Relationships for foundations on an elastic half-space and more general formulations including pile foundations and lateral soil layer restraint on the footing are presented.

3. This procedure facilitates the isolation and identification of the important parameters that govern dynamic structure–ground inter-

action and enables one to perform wide-ranging parameter studies.

4. The dynamic properties of structures on pile foundations can be determined similarly as for structures on elastic half-space subject to certain simplifying assumptions. The influence on modal damping ratio of elastic energy propagation into the ground and of hysteretic material loss in the soil has been presented for some specific structural parameters.

The following conclusions have been reached:

1. The natural frequency of the fundamental mode of a structure–foundation system is primarily dependent on the stiffness ratio of structure to ground and the aspect ratio of height to width of foundation.

2. The system damping ratio for the fundamental mode is a linear combination of the products of the damping coefficients of the ground and the corresponding squares of the modal amplitude ratios of the structure, and the interstory damping ratio of the structure times the interstory modal amplitude ratio to the 3/2 power.

3. The variation of the modal amplitude ratios shows a rapid decrease of relative displacement and a similar increase of rocking displacement with increasing aspect ratios and stiffness ratios. This points to the predominant influence that rocking has on structure–ground interaction effects of moderately slender or very slender structures founded on an elastic half-space.

4. The contribution of structural interstory damping to the modal damping ratio decreases rapidly with increasing frequency reduction ratios. Alternatively it may be stated that with decreasing ratios of structure stiffness to foundation stiffness the contribution of the interstory damping becomes insignificant and the system damping will be dominated by foundation damping.

5. For stiffness ratios k/Gr greater than about 2, changes in soil material damping ratios are reflected in almost identical increases in system damping ratios for structures founded on an elastic half-space.

6. For a wide range of parameters, system damping ratios for structures on flexible soils can be smaller than the fixed based structural damping values, particularly for large aspect ratios and large mass density ratios. Consideration of soil material damping increases the system damping ratio and thereby reduces the range over which such reduced damping ratios can occur.

7. For structures on pile foundations substantial levels of modal damping can be achieved with large diameter piles even when soil material damping is neglected. The contributions of soil material damping to the modal damping ratio is most efficient for soft foundations; for stiff foundations only a small fraction of material damping contributes effectively to the modal damping ratio.

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Notation

a	nondimensional frequency $= pr/V_s$
b_1	mass density ratio $= m_1/\rho r^3$
c	damping coefficients (dimensionless)
C	damping constant (force per unit velocity)
D	soil material damping ratio (with subscripts)
$f_{7,1}, f_{11,1}, f_{18,1}$	stiffness parameter for vertical pile in in-plane bending, lateral, and axial direction, respectively.
$f_{7,2}, f_{11,2}, f_{18,2}$	damping parameter for pile in in-plane bending, lateral, and axial direction, respectively.
G	shear modulus of ground
h	height of single-story structure
I	sum of mass moment of inertia about own axes of rotation
I_1	geometric mass moment of inertia of structure $= m_1 h^2$
k	structural stiffness; frequency-dependent foundation stiffness coefficient (with appropriate subscript)
K	foundation stiffness (with subscript)
m_0	base mass
m_1	top mass
p	resonant frequency of interaction structure
r	radius of base mass or pile; stiffness ratio of foundation (with subscript)
$\bar{S}_{u2}, \bar{S}_{\psi 2}$	side layer damping parameters for horizontal and rotational motion, respectively.
$S_{u1}, S_{\psi 1}$	side layer stiffness parameters for horizontal and rotational motion, respectively.
u	modal amplitudes
v_c	wave velocity in pile
V_s	shear wave velocity of ground $= (G/\rho)^{1/2}$
α	mass ratio $= m_0/m_1$
β	inertia ratio $= I/I_1$
γ, δ, ξ	modal amplitude ratios of rotational displacement, interstory displacement, and base displacement, respectively.
λ_0	structural damping ratio (fraction of critical)
λ_E	modal damping ratio for interaction structure
Λ	damping ratio for foundation element (with appropriate subscript)
Φ	angle of rocking rotation for interaction structure
ν	Poisson's ratio
ρ	mass density of soil
ω_0	angular resonance frequency of fixed based structure
ω	angular component resonance frequency without correction terms (with subscripts)
Ω	angular component resonance frequency with correction terms (with subscripts)

Subscripts for horizontal translation and rotation, respectively:

H, Φ	total quantity
h, θ	half-space contribution
x, ϕ	pile-foundation contribution
u, ψ	side layer contribution

(zϕ and $\phi\phi$ refer to rocking contributions from axial motion of piles and in-plane rotation of pile tops, respectively).