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Designing ice bridges and ice platforms

Gold, L. W.

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DESIGNING ICE BRIDGES AND
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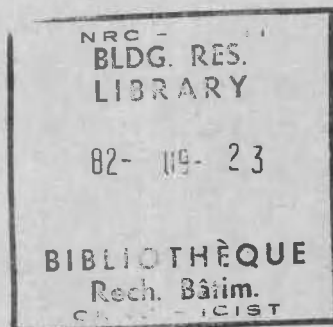
by L. W. Gold

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SOMMAIRE

Cette note aborde les calculs de base des plates-formes et des ponts de glace. Elle contient un résumé des observations qui montrent que le problème des charges en mouvement peut être traité analytiquement au moyen de la théorie des plaques minces élastiques sur support élastique. De plus, l'auteur exprime une opinion sur les données encore manquantes. Il aborde aussi le problème du calcul des plates-formes de glace, pour lesquelles on a pas encore trouvé de méthode de calcul analytique, et envisage les possibilités de progrès dus notamment aux mesures des déformations dans la couche de glace.

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DESIGNING ICE BRIDGES AND ICE PLATFORMS

L.W. Gold
Associate Director

Division of Building Research
National Research Council of Canada
Ottawa, Canada

Canada

The current basis for design of ice bridges and ice platforms is considered. Experience and performance observations showing that the moving load problem can be treated analytically using the theory of thin elastic plates on an elastic foundation are summarized, and an opinion is expressed as to the information still required. The basis for the design of ice platforms, for which a validated analytical method has not yet been established, is reviewed; and the possibility of progress on the problem, provided by the capability for measuring strains in ice covers, is pointed out.

One of the earliest descriptions of the construction and use of an ice bridge in Canada is contained in the records of the parish of Sainte-Marie-Madeleine [1].* In 1878 it was decided to construct a new church of stone that was to be obtained from a quarry on the opposite side of a nearby river. Winter was late and it was not until 19 March 1879 that a road was completed on consolidated ice. About 360 m^3 of stone was taken over the ice by horse and sleigh by the time melting began and water started to flow over the bridge. This work was completed without accident, although it was continued right up to the time of rapid deterioration of the ice. It probably is representative of the experience-based practice of the day.

In many northern countries ice covers are used extensively for storing logs in preparation for floating them to the mills in spring. By the 1950's loads in excess of 50 tonnes were being placed on covers routinely, with relatively few incidents [2,3]. The preparation of the ice for such loads was based primarily on experience; the individuals responsible for it generally had no engineering or technical education.

There has been a growing use of ice covers for non-routine purposes, particularly for construction and for transportation and the development of resources in remote areas. As a result, considerable attention has been given during the past few years to placing the design of ice bridges and platforms on a proper engineering basis.

Two approaches have been used in developing this basis. One is the determination of the allowable load for given ice thickness from records of experience and observations of performance. The second is the specification of the "failure" condition as a limit state. In the limit state approach it is necessary to develop a valid mathematical description of the behaviour of ice covers under load and of the strains and stresses induced in them. The "failure" condition must then be defined and the allowable load specified as some fraction of the failure load.

It is very difficult and perhaps impossible to specify all the failure conditions for an ice cover. There are several reasons for this. A principal one is that ice is normally at a temperature within 40 Celsius degrees of its melting point and, therefore, in a "high temperature" state. Its strength and deformation properties are temperature- and time-dependent in this range.

Ice in ice bridges and platforms is subjected to a wide variation in structure and quality, although careful control may have been exercised in construction. In addition, temperature changes cause cracks and water currents cause erosion and thin areas. Surveys of ice failures have shown that most accidents are due to imperfections in the ice cover or to effects that have not yet been properly accounted for in design

* The author wishes to thank Professor B. Michel for bringing this reference to his attention.

or use, rather than to the exceedence of the allowable load for the average thickness of ice present [2,4]. The existence of these imperfections and non-normal conditions must be expected and can only be taken into account through careful observations during use.

Because of lack of knowledge concerning the deformation behaviour of ice and because of the variability in factors that determine the strength of ice covers, field data are still the principal basis for the specification of allowable loads. The theoretical descriptions of behaviour now evolving with the development of the limit state approach are providing a rational mathematical framework for the analyses and presentation of this experience. Too often, however, experience is not recorded. In addition, it is not yet possible to measure in the field all the factors necessary to describe fully the response of an ice cover to load. It is relatively easy to measure deflection; but only recently have methods for measuring strain been successfully demonstrated [5,6]. There is as yet no satisfactory method for measuring stress. If the use of ice covers for supporting loads is to become more of an engineering science than an art it will be necessary to develop and demonstrate design methods and criteria that can ensure safe and satisfactory performance. This paper is a brief review of the current state of the development of this knowledge for both moving and static load problems.

Moving Loads

The starting point for the moving load problem has been the theory of a thin elastic plate on an elastic foundation [7,8,9]. Observations indicate that this theory should be sufficiently accurate for speeds in excess of 1 km/h as long as proper account is taken of the strain rate dependence of the elastic modulus [10]. At low speeds the shape of the deflected surface is essentially the same as that for the static elastic case [11]. If it is assumed that load, P , acts over an area of effective radius, a , then the expression relating P , ice thickness h , and maximum stress σ_m (which occurs under the load) is

$$P = \frac{\pi \sigma_m b h^2}{3(1+\nu)k e^{1/2} b} = B(b) \sigma_m h^2 \quad (2)$$

where

$$b = \frac{a}{\sqrt[4]{k}}$$

$$k = \left[\frac{E h^3}{12 \rho g (1 - \nu^2)} \right]^{1/4}$$

E = elastic modulus

ν = Poisson's ratio

ρ = density of water

$kei'(b)$ is the first derivative of one of the modified Bessel functions.

For the limit state, the maximum stress is assumed to be the tensile strength of ice. If this property of the ice and the modulus of elasticity are known, ice thickness required for given loads can be determined using superposition (and computer if necessary) for complex load geometries [8,12].

Observations give $\ell = 16 h^{\frac{3}{2}}$ m for fresh water ice [3]. This corresponds to an elastic modulus of 6.9×10^3 MPa, a relatively high value. Sinha [13] shows that this value is associated with a period of loading of about 20 s at -10°C . It would be expected that the modulus of elasticity could relax to about 50% of that value for very slowly moving loads. Such a decrease would cause a decrease in ℓ of about 16%. If the effective radius of the load area is 1.25 m, the corresponding increase in $B(b)$ is less than this amount.

If the effective radius of loading is 1.5 m, B has a value of about 0.75 for $h = 0.25$ m; 0.5 for $h = 1.0$ m; and 0.42 for $h = 2.0$ m. For the range of ice thickness and effective area of loading usual for vehicles travelling on ice its value is about 0.6.

In view of the uncertainty in the time dependence of $B(b)$ and in the value to be used for tensile strength, it is often assumed in the analyses of performance data that

$$\frac{P}{g} = P' = Ah^2 \quad (3)$$

where P' is in kg, h in m, and A is a constant.

Observations on the successful use of ice covers have shown that loads are usually in the range $P' = 3.5 \times 10^4 h^2$ to $17.5 \times 10^4 h^2$. The usual recommended upper limit for loads to be placed on ice of thickness h is about $P' = 7.0 \times 10^4 h^2$. $P' = 17.5 \times 10^4 h^2$ defines the approximate upper limit for situations for which risk is acceptable (e.g., tanks in wartime). During a survey of wood placed on ice covers, nine failures occurred during the placing of 42,500 truck loads for which loadings ranged from $P' = 0.7 \times 10^4 h^2$ to $10.5 \times 10^4 h^2$. The distribution in the number of loads when plotted against P'/h^2 was approximately normal, with the maximum at $P' = 4.2 \times 10^4 h^2$ [3].

Experience has shown that good quality ice covers not subject to thermal stress should support moving loads satisfactorily to loadings of $P' = 14 \times 10^4 h^2$. The ice should not be subjected to repetitive loads at this level and its use must be under the control of an individual knowledgeable about ice and the factors that determine the strength of ice covers. For uncontrolled situations experience indicates that loadings should be restricted to $P' = 3.5 \times 10^4 h^2$, but even then failures can be expected owing to imperfections in the cover and the effects of thermal stress.

If it is assumed that B has a value of 0.6, the range of maximum stress associated with $3.5 \times 10^4 < P^1/h^2 < 17.5 \times 10^4$ is about 0.5 to 3 MPa. The upper value exceeds the stress that would be expected to cause cracks to form at the under surface of the cover, but experience has shown that a cover can tolerate such crack formation for moving loads without failure. Experience also shows that the lower value is a safe, allowable, tensile stress for ice.

Although field observations have made it possible to delineate in a reasonable manner the elastic modulus and allowable maximum stress to be used for the design of ice bridges, much still has to be done. Relatively little attention has been given to the effect of temperature and salinity. Kerr and Palmer [14] have shown that for the elastic case the elastic or rigidity modulus can be replaced by an effective value for plate bending calculations, given by

$$D = \frac{1}{1-\nu^2} \int_{-Z_0}^{h-Z_0} Z E(Z) dZ \quad (4)$$

where Z_0 is the position of the neutral plane and $E(Z)$ is the value of the modulus at distance Z below the surface.

The deflection of the cover and, therefore, the linear strain variation through the ice cross-section is determined by the load and D . As $E(Z)$ varies through the section, the stress distribution is no longer linear. Since the bottom surface of the cover is always at the melting point, the maximum stress may, in fact, occur at some position in the interior. The elastic modulus of ice, however, becomes less temperature dependent with decreasing period of loading [13], and the value of 6.9×10 MPa is probably reasonable for fresh-water ice in most moving-load situations and the temperature range 0 to -20°C .

The effective value of ℓ and tensile strength will increase with increasing vehicle speed. Increasing ℓ causes B to decrease, and this partly offsets the effect of the increase in strength. Increasing the vehicle speed, however, causes another effect that must be taken into consideration.

When a vehicle travels on ice covers, a hydrodynamic wave is set up in the underlying water. This wave travels with a speed that depends on the depth of the water, thickness of ice cover, and modulus of elasticity of the ice. If the speed of the vehicle coincides with that of the hydrodynamic wave, the deflection due to load reinforces that associated with the wave. This problem has been considered by Assur [15], Nevel [16] and Eyre [11].

Figure 1 presents measurements of the ratio of actual maximum deflection to the elastic deflection at zero speed. The deflection is a maximum at a critical speed, u_c , that depends on the properties of the ice cover and the thickness of the ice

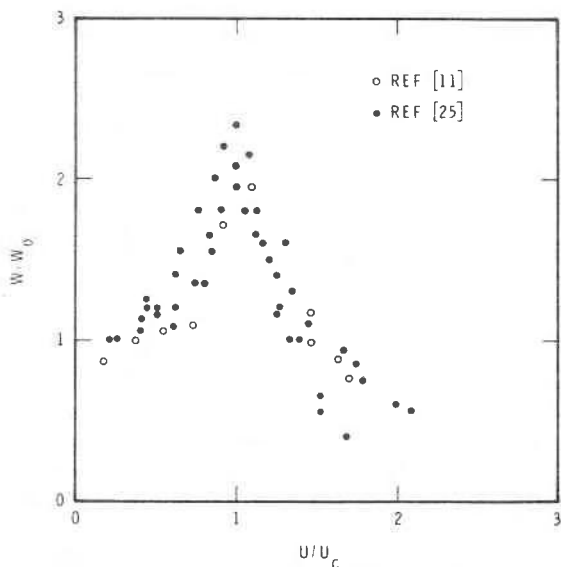


Figure 1
Dependence of the ratio of deflection, w , at speed u to the elastic deflection, w_0 , on the ratio of the speed to the critical speed, u_c

[15,16]. Plotting measurements in the non-dimensional form of Figure 1 shows that deflection at the critical speed is about two and one-half times that at low speeds. The theory presented by Nevel accounts for the features shown in Figure 1, except for those at the critical speed where the effect of dissipative processes that limit the maximum deflection have not yet been properly described. The dependence of maximum stress and tensile strength on vehicle speed has also still to be established. Reports of failure have indicated that speed has been a contributing factor in some cases. It is debatable whether vehicle speed is an important factor for loads of $P' = 3.5 \times 10^4 h^2$ and less.

More precise information is required on the dependence of ℓ and tensile strength on ice type, temperature distribution, and speed. It would be useful to carry out additional field experiments similar to those of Eyre [11] in which strains, deflections and acoustic emission to detect crack formation are measured as a function of vehicle load and speed. Such studies are needed, in particular, for sea ice for which present knowledge of elastic modulus and strength are appreciably less than for fresh-water ice.

Stationary Loads

A demonstrated limit state design method for determining the safe thickness for stationary loads has yet to be established. The reason for this is the lack of knowledge concerning the relations among load, deflection, deflection rate, strain, strain

rate, and stress. The mathematics of the problem are difficult because of the non-linear relation between strain rate and stress.

The appropriate criterion for the stationary load problem is probably one of performance based on allowable deflection or deflection rate rather than allowable stress, particularly for loads that will be in place for periods of more than one day. For shorter periods, maximum strain rates are in the range for which it may be necessary to limit the maximum stress as well.

Much attention has been given to the stages of failure for ice covers under stationary loads. The information provided by such experiments may be misleading because they involve deflections greater than the freeboard. Perhaps the only situations for which deflections of this magnitude can be tolerated routinely are those such as the storage on ice covers of wood that is to be floated to mills after spring thaw. In general, if the ice is to support material that must be retrieved or activity such as drilling, there are practical reasons for keeping the deflection less than the freeboard in addition to the limits that this places on stress and strain [17].

If deflection is limited to the freeboard, the maximum strain induced is less than 1% for normal ice thickness. This means that deformation is confined to the primary creep stage and, from the point of view of the deformation behaviour of ice, is one of small strain. The implications of this have not yet been fully appreciated or exploited. In dealing with the problem most investigators have assumed a linear viscous behaviour or a constitutive relation of the form

$$\dot{\epsilon} = \dot{\epsilon}_0 \left(\frac{\sigma}{\sigma_0} \right)^n \quad (5)$$

where $\dot{\epsilon}$ is the strain rate for constant stress σ , and $\dot{\epsilon}_0$, σ_0 and n are constants. The value for n has usually been taken to be that found for the secondary creep stage.

Work by Gold [18] demonstrated that for the uniaxial constant load condition the expression relating strain, stress and time has the form

$$\dot{\epsilon}(\sigma, t) = A(t) \sigma^{n(t)} \quad (6)$$

where A and n tend with strain to their constant secondary creep stage values. It was also found that n depended on the type of ice and on the strain history. For simple compression at -10°C and stress between 0.4 and 1.5 MN/m², n was independent of the stress at a given time and changed only slowly with time after about 150 min.

In determining n from creep tests, it is usual to use the linear region of the creep strain/time curve. This region occurs over a time period that depends on stress and temperature. The value of n to be used in the constitutive equation for bending problems, however, must be determined from strain rate values for the same time for

each stress and temperature since what is required for calculations is the strain rate dependence of the stress at a given time and temperature.

Murat [19] observed that the maximum deflection versus time for beams subjected to a constant four-point load and simply-supported plates subjected to a constant load at their centre became essentially linear after an initial transient phase. This constant rate of deflection developed while the maximum strain was still of the order of 0.1%. It required about 5 h to establish the constant deflection rate for the beams at the lowest load level; for the plates, it took over 40 h. Taking these results and the earlier work of Gold [18] and of Krausz [20] into consideration, it seems clear that the transient phase before the apparent steady-state behaviour must be associated with the transition from the initial elastic condition to the condition for which the stress distribution through the section depends primarily on strain rate. The strain to do this is in agreement with the time dependence of n observed by Gold.

Masterson et al. [5] state that deflection of the platform they observed varied with time raised to the power 0.47. When the deflection measurements presented in their Figure 6 are plotted on a log-log scale, however, this dependence is found only for time in excess of about 10 days. The ratio of their measured maximum strain to deflection, assuming that 0 deflection coincides with 0 measured strain, is also constant after that time. These observations suggest that for thick platforms designed to support loads for many days the initial transient phase extends over periods of days in contrast to minutes or hours in laboratory scale experiments.

These observations are consistent with analyses of Sinha [13,21] indicating that delayed elastic behaviour dominates immediately following the application of load. With time it becomes progressively less significant with respect to the component of the strain due to viscous flow. In fact, the evidence suggests that it is the neglect of the delayed elastic behaviour that is responsible for the apparent time (and perhaps stress) dependence of A and n in equation (6).

Murat [19] found from his experiments on beams and simply-supported plates that at a given time during the period of constant deflection rate

$$\frac{\dot{\delta}h}{L^2} \propto \left(\frac{P}{h^2} \right)^n \quad (7)$$

where P is the constant applied load and L the length of beam or diameter of plate. He found the average value of n to be 2.32 for beams subjected to four-point load and 2.70 for simply-supported plates. The tests were carried out at -10°C . The lower range of the maximum stress induced in these tests probably overlaps the upper range for satisfactory performance of ice platforms supporting a stationary load for more than one day. From the geometry of the tests it is possible to state (for them) that

$$\dot{\epsilon}_m \propto \frac{\dot{\Delta}h}{L^2} \propto \left(\frac{P}{h^2} \right)^n \quad (8)$$

If a general relation exists between $\dot{\epsilon}_m$ and $\dot{\Delta}h$ for ice platforms, it should be possible to develop at least an empirical relation of the form of equation (8) between maximum strain rate and load.

The measurements of Masterson et al. [5] on an ice platform about 6.5 m thick confirm that it is reasonable to assume that during deflections under stationary loads both strain and strain rate increase linearly through the cross-section of the cover to their maximum values in tension or compression. Observations have indicated also that for deflections less than the freeboard the shape of the deflected surface for a reasonable distance away from the load can probably be described by an equation of the same form as the initial elastic one. If this is correct, one can assume $\dot{\epsilon}_m \propto \frac{\dot{\Delta}h}{L^2}$, where L is a function of time and is the characteristic length that fixes the shape of the surface in the vicinity of the load. Because of the elastic foundation effect of water, the maximum strain rate is not directly proportional to the deflection rate.

With the capability that now exists for measuring strains directly in platforms, it should be possible to establish the expressions relating load, ice thickness, deflection rate, and maximum strain rate. If this can be achieved, the performance criterion can be specified in terms of allowable deflection, maximum strain rate, or stress. Under certain conditions it could be stated as allowable maximum (P/h^2) , as given by Murat [19]. If a stress criterion is used, it will be necessary to establish a relation of the form of equation (5), from which the stress through the cross-section can be calculated from the strain rates. The deflection rate to be used for the calculations would be determined from the length of time the load is to be on the ice, the maximum allowable deflection, and an apparent initial deflection that includes the delayed elastic component of the strain.

The performance criterion must be chosen so that, when satisfied, cracks will not propagate during the period when the platform is undergoing the allowed deflection. Calculations indicate that when deflection is limited to the freeboard for loads that will be in place for more than one day, the maximum strain rate and stress are sufficiently low for this to be the case. The limit state conditions, however, have still to be established through experiment and performance measurements. As the limit state values of the critical stress and strain rate are not known, the safety factor associated with current experience-based practice cannot be specified.

A good example of the current design approach for ice platforms is that used for platforms supporting off-shore drilling activity in the high Arctic [5,22,23]. Loads have been in the range of 500 to 1500 tonnes. The performance criterion applied has been that total deflection must not exceed the freeboard and that the initial elastic

maximum stress must not exceed 345 kPa. The basis for the specification of maximum stress was that cracks are not observed to form in fresh-water ice during the first 1% creep strain caused by a constant uniaxial compressive stress of less than that value [24].

Over time, the loads placed on these platforms have been increased. Platforms for the heavier loads have been designed on the basis of past experience, primarily by establishing from performance measurements the time dependence of the apparent value of ℓ . Ice thickness was chosen to ensure that the allowable amount of maximum deflection would not be exceeded during the period in which the load would be on the ice. The range of loads placed on the platforms is now sufficiently wide to make it possible to establish an empirical expression relating design load, P_1 , ice thickness, h_1 , and deflection, w_1 , of the form

$$w_1 = w_0 \left(\frac{h_0}{h_1} \right)^m \left(\frac{P_1}{P_0} \right)^n \quad (9)$$

where w_0 , h_0 , and P_0 , are reference deflection, ice thickness and load, and m and n are constants [22].

Stress levels within the platforms have been checked through computer calculations by making assumptions that allow strain rates to be specified and using a relation between stress and strain rate of the form of equation (5). It is difficult to generalize from this experience because the load distribution is not simple and the platforms are tapered rather than of constant thickness. Measurements now being made of the time-dependent strains through the cross-section of the platforms will provide a better basis for future use of the observations on performance in developing and confirming a more universal design method.

Conclusions

Traditionally, the use of ice bridges and platforms has been based on experience. Observations have shown that ice can be assumed to behave elastically for loads moving at a speed greater than about 1 km/h if proper account is taken of the time and temperature dependence of the elastic modulus. Values of allowable maximum tensile stress determined from field practice lie in the range of 0.5 to 3 MPa. The risk associated with moving loads that induce a maximum stress in this range is determined primarily by imperfections in the ice cover and uncontrollable factors such as temperature changes. Information is required on the time dependence of the maximum stress and tensile strength to establish the dependence of bearing capacity on vehicle speed.

The mathematical basis for the design of ice platforms for stationary loads has not yet been developed. The principal barrier is the non-linear relation between strain rate and stress and the difficulties this poses for analysis. Practically

all stationary load problems are associated with maximum strains less than about 1%. Observations indicate that after an initial transient period, probably associated with the transition of the initial elastic state to the fully viscous state, the stress has a power law dependence on the strain rate. The exponent at a given time appears to be essentially constant for stress greater than about 0.4 MPa; it is not known whether it remains constant for strain rates inducing stress below that value. If the expressions relating load, ice thickness, deflection, deflection rate, strain, and strain rate can be established, it should be possible to develop a general design method that takes into account the initial transient behaviour. The recently demonstrated ability to measure strains in situ should allow these relations to be found.

Limiting deflection of the ice cover to the freeboard is probably a practical performance criterion for load durations of more than one day. This criterion effectively ensures that the maximum strain rate stays below the critical value required for propagation of cracks. If deflections are allowed to exceed the freeboard that would normally be associated with an all-ice platform of given thickness, or if loads are to be stored for short periods only, the maximum allowable stress level and a valid method of calculating the maximum stress will have to be established.

This paper is a contribution from the Division of Building Research, National Research Council of Canada, and is published with the approval of the Director of the Division.

References

- [1] Loranger, M., La merveille d'un pont de glace. *Apostolet* 79, Oblats de Marie Immaculée, 50 (2), 1879, p. 7-8.
- [2] Gold, L.W., Field study on the load bearing capacity of ice covers. *Woodlands Rev., Pulp & Paper Mag. Canada*, 61, 1960, p. 153-154, 156-158.
- [3] Gold, L.W., Use of ice covers for transportation. *Can. Geotech. J.*, 8 (2), 1971, p. 170-181.
- [4] Sundberg-Falkenmark, M., Load bearing capacity of ice. *Swed. Inst. Meteor. and Hydrol., Series 1*, Stockholm, 1963.
- [5] Masterson, D.M., Anderson, K.G. and Strandberg, A.G., Strain measurements in floating ice platforms and their application to platform design. *Can. J. Civ. Eng.*, 6 (3), 1979, p. 394-405.
- [6] C-CORE., The measurement of subsurface strain on Roche 0-43 artificially thickened sea ice drilling platform. Contract Report No. 78-16, Memorial University, St. John's, Nfld., 1978.

- [7] Westergaard, H.M., New formulas for stresses and strains in concrete pavements of airfields. *Trans. Am. Soc. Civil Eng.*, 113, 1948, p. 425-444.
- [8] Wyman, M., Deflections of an infinite plate. *Can. J. Res. A*, 28, 1950, p.293-302.
- [9] Kerr, A.D., The bearing capacity of floating ice plates subjected to static or quasi-static loads. *J. Glaciol.*, 17 (76), 1976, p. 229-268.
- [10] Gold, L.W., Ice pressure and bearing capacity, *In Geotechnical Engineering for Cold Regions*, Chapt. 10. (editors Andersland, O.B. and Anderson, D.M.), McGraw Hill, 1978, p. 505-551.
- [11] Eyre, D., The flexural motions of a floating ice sheet induced by moving vehicles. *J. Glaciol.*, 19 (81), 1977, p. 555-569.
- [12] Nevel, D.E., Safe ice loads computed with a pocket calculator. *Proc. Workshop on Bearing Capacity of Ice Covers*. Assoc. Cttee. Geotech. Res., National Research Council of Canada, TM 123, 1979, p. 205-222.
- [13] Sinha, N.K., Rheology of columnar-grained ice. *Exper. Mechan.*, 18 (12), 1978, p. 464-470.
- [14] Kerr, A.D. and Palmer, W.T., The deformations and stresses in floating ice sheets. *Acta Mechanica*, 15, 1972, p. 57-72.
- [15] Assur, A., Traffic over frozen or crusted surfaces. *Mech. Soil Vehicle Systems*. *Proc. 1st Int. Conf. Mech. Soil Vehicle Systems*, Torino-Saint Vincent, 12-16 June 1961.
- [16] Nevel, D.E., Moving loads on a floating ice sheet. *Cold Regions Engineering Laboratory*, U.S. Corps of Engineers, Hanover, N.H., Research Report 261, 1970.
- [17] Frederking, R.M.W. and Gold, L.W., The bearing capacity of ice covers under static loads. *Can. J. Civil Eng.* 3 (2), 1976, p. 288-293.
- [18] Gold, L.W., The initial creep behaviour of columnar-grained ice, Part I: Observed Behaviour: Part II Analysis. *Can. J. Phys.*, 43, 1965, p. 1414-1434.
- [19] Murat, J.R., La capacité portante de la glace de mer. Ph.D. Thesis, École Polytechnique de Montréal, 1978.
- [20] Krausz, A.S., The creep of ice in benching. *Can. J. Phys.*, 41 (1), 1963, p. 167-177.
- [21] Sinha, N.K., Short term rheology of polycrystalline ice. *J. Glaciol.*, 21 (85), 1978, p. 457-473.
- [22] Masterson, D.M., Anderson, K.G. and Strandberg, A.G., Reply to Discussion: Strain measurements in floating ice platforms and their application to platform design. *Can. J. Civ. Eng.*, 7 (3), 1980, p. 565-568.
- [23] Beaudais, D.J., Watts, J.S. and Masterson, D.M., A system for offshore drilling in the Arctic Islands. *Offshore Tech. Conf.*, Houston, Texas, Paper No. TC 2622, 1976.
- [24] Gold, L.W., The process of failure of columnar-grained ice. *Phil. Mag.*, 26 (2), 1972, p. 311-328.

- [25] Beltaos, S., Field studies of the response of floating ice sheets to moving loads. Proc. Workshop on Bearing Capacity of Ice Covers, Assoc. Cttee, Geotech. Res., National Research Council of Canada, TM 123, 1979, p. 1-11.

IAHR DISCUSSION SHEET.

Author: Loren Gold

Paper: Invited IAHR Paper, "Designing Ice Bridges and Ice Platforms."

Discusser: Phil Johnson, P. E.

1045 Lakeview Terrace, Fairbanks, Alaska, 99706, USA

Discussion:

I am discussing the second Section of Gold's paper titled "Moving Loads."

Gold presents what can be called the "Critical Speed" reaction of a floating ice sheet to a moving load although he does not develop the theme completely or carry it to a logical conclusion. It is not a complete description of the reaction of an ice sheet to a moving load as I hope to show.

The principal point is that the moving load will generate a hydrodynamic wave that will travel through the water and deform the ice at the location of the wave. If the vehicle is traveling at the same speed as the wave (the critical speed), the deflection of the ice due to the wave is joined by that of the vehicle and the total deflection (depression of the ice sheet) under the vehicle is substantially increased. Nevel [16] (Gold's References) showed that theoretically the deflection would go to infinity but that observations showed that it merely reached a finite maximum. Gold combines the data of Eyre and Belthaus to show that the deflection increase does occur but that the maximum is around 2-1/2 times the static deflection on moderately thick fresh-water ice. This all seems confirmed by theory and field tests and I do not disagree.

Continuing further, efforts have been made for many years to find the tensile stress in a floating ice sheet. Precise solutions have been developed which depend upon calculating the the deflection of the ice sheet under the load. However, the total deflection at the critical speed depend upon the two factors which deform the ice sheet - the weight of the moving vehicle and the presence of the moving wave - so it is difficult to find the total stress in the ice as a function of deflection. It is almost certain that increased deflection indicates increased stress but the relationship is not known more precisely. Stress, particularly tensile stress in the bottom of an ice sheet, is important because an ice sheet will begin to fail when the tensile stress at that point reaches the tensile strength of the ice. Although Gold did not state this precisely, I agree with his implication that increased deflection under the moving vehicle at the critical speed also indicates increased stress in the ice.

Not addressed by Gold or most others who have published on the Critical Stress problem is the probability of encountering these increased stresses. While not

specifically stated, the impression is given that a vehicle traveling on the ice is very apt to encounter them and the results, also not discussed in detail, are apt to be unpleasant. Many authors advise that vehicle speeds on ice should be strictly controlled and kept low - usually at 10 mph or slower - ignoring the fact that the critical speed for an ice sheet on shallow water is at or near that speed.

Based on the discussion above, we can identify two conditions that must be met before a moving vehicle on an ice sheet will encounter critical speed conditions. The first is that there must be a hydrodynamic wave in the water while the second is that the vehicle must be traveling with this wave. Because of this second condition, it appears that the vehicle itself must generate the wave by traveling in a straight line at the critical speed and then travel in the same direction at the same speed to utilize the critical speed conditions that have been developed. Figure 1 of Gold's shows that the vehicle speed must be very close to the critical speed for the deflections to reach a maximum. It appears that if the vehicle speed varies somewhat, the full critical speeds would not be developed.

Nevel* addressed this matter from another angle and stated "...one should not travel at a constant speed near the critical velocity. When passing through the critical velocity, one should do so quickly in order to insure a transient rather than a steady state condition." Nevel's worry about the steady-state conditions and indifference to the transients makes sense and confirms the conclusions reached.

It appears that random changes ⁱⁿ speed and the direction of travel that would occur when a vehicle is traveling freely on an ice sheet would be adequate to prevent the steady-state conditions necessary to generate the critical speed conditions and that such a vehicle would have little chance of encountering them. However, ice travel is often constrained to a particular track by clearing an ice road. In this case, randomness in the track followed will be eliminated. Further, travel conditions on that track will tend to be uniform so that a vehicle would tend to find a comfortable speed and hold it. In such a case, a vehicle would be apt to encounter critical speed conditions. This could be avoided by posting the ice road with the critical speed and advising drivers to avoid them. This would completely avoid critical speed conditions except for a careless driver.

One further point remains. Research into ice reaction to vehicles traveling on an ice sheet has been confined to lakes where the existence of the critical speed phenomena has been demonstrated and quantified. Can the same effect be found on river ice where the water is moving with respect to the ice, where some of the flow might be turbulent, where water depth may change rapidly and the water may

*Nevel, Donald E., Bearing capacity of floating ice sheets, unpublished CRREL document, December 1968.

flow in a curved path? It does not appear reasonable and should be checked.

The above discussion shows that it would be unusual for a vehicle traveling somewhat randomly on an ice sheet to encounter critical speed conditions. The probability would be increased if the travel were restricted to a prepared ice road but could be eliminated by posting the road with the critical speed and advising travelers to avoid that speed. In addition, it may well be impossible to meet the critical speed conditions on river ice. All in all, it seems questionable whether the approach to the problem of moving loads should be based on the "critical speed" effect.

Author's Reply
by:

L.W. Gold, Division of Building Research, National Research Council,
Ottawa, Canada

Mr. Johnson has identified important questions that still remain to be answered concerning the dependence of the safe performance of ice covers on vehicle speed. It is difficult to measure the stress that is induced in the ice by moving loads. As vehicle speed increases, however, ice becomes more elastic in its behaviour and elastic theory should become more appropriate for describing deflections and stresses. It is very important that measurements be made in the field of the dependence on vehicle speed of the maximum strain rate induced in the cover. Observations should be made at the same time of cracks that are induced by the load, probably using sonic techniques. This information, along with laboratory measurements of the strain rate and temperature dependence of the elastic modulus, should provide the information that is required on the dependence of safe performance on vehicle weight and speed.