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RÉSUMÉ

Examen et comparaison des méthodes de calcul des forces d'encrêtement de la glace. Les auteurs donnent un aperçu d'un nouveau modèle d'encrêtement qui prévoit les hauteur et profondeur limitantes des amas de fragments. Ce modèle utilise une équation de diffusion pour décrire la répartition normale des contraintes dans la masse de fragments. Les résultats des méthodes de calcul sont comparés à ceux des essais en laboratoire et à une situation de terrain typique. Les modèles de milieux continus semblent prévoir de façon assez précise les forces d'encrêtement, tandis que la méthode de l'énergie potentielle les sous-estime grandement.

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ON MODELLING OF ICE RIDGE FORMATION

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Abstract

A review and comparison of calculation methods of ice ridging forces is presented. An overview of a new ridging model that predicts the limiting rubble height and depth is included. This model uses a diffusion equation to describe the normal stress spread in the bulk rubble. Results of calculation methods are compared to those of laboratory tests and a typical field situation. Continuum models appear to predict reasonable ridging forces while the potential energy method greatly underestimates the forces.

Introduction

Knowledge of the forces associated with ice ridging and dimensions of the sail and keel are needed to address several practical problems. These include the prediction of ice forces on wide structures, the use of grounded rubble for protection of structures and for construction purposes, and the estimation of the forces acting on pack ice or large multi-year ice floes. The process of ridging remains poorly understood in spite of the attention it has attracted. The simple calculation method used in practice appears to be inadequate in many situations.

This paper presents a discussion of analytical models, including the results of a new model, that relate ice forces during ridging to rubble geometry. Emphasis is restricted to a comparison of the methods and the constraints on their validity.

Review of Ridging Models

Early observations of ice ridging were reported by Zubov (1945). He described the geometry of many ice rubble features and suggested equating the loss of the kinetic energy of an ice floe to the increase in the potential energy of the rubble in order to calculate ridge-building force. Parmerter and Coon (1973) developed a numerical kinematical model that predicted ridge geometries that were in excellent agreement with field observations. An ice sheet was assumed to progress horizontally in steps until it broke in bending. Broken ice blocks were placed in the keel and the sail at each step. After a number of steps, the limiting height and depth of the ridge were reached. Since no direct estimate of the force in the ice sheet could be made, Parmerter and Coon (1973) suggested that a lower bound to the force can be calculated by equating the work done by the ice sheet to the increase in the potential energy of the ridge. The resulting values were in the range of 100 to 10^4 N/m, which was considered to be in agreement with expected average forces in an ice cover several hundred kilometers long.

The lower bound method was a minor part of Parmerter and Coon's study, which was concerned with large-scale movements of ice covers. Still, because of its simplicity and the need to predict ice forces, several authors used that method to derive a simple equation for ridge-building force. That equation is frequently (and erroneously) attributed to

Parmerter and Coon (1973). A review by Kovacs and Sodhi (1980) described the literature on this "energy balance" method (see also Sodhi and Kovacs, 1984).

Forces observed in ice covers at length scales relevant to wide structures (10 to 100 m) are much higher than the values predicted by the above method. Kovacs and Sodhi (1980) tried to overcome this discrepancy by adding a frictional force. They considered the case of an ice sheet sliding over a rubble accumulation to calculate the friction force on the surface of that sheet. The result is restricted to the assumed mode of ice behavior and does not apply when the bulk rubble is undergoing deformation. Visual accounts of ridges and rubble features (Zubov, 1945) indicate that complex modes of rubble deformation may occur.

Guidance to the formulation of more accurate models came from laboratory experiments on ice rubble (e.g., Tatinclaux and Cheng, 1978; Prodanovic, 1979; and Hellmann, 1984). These investigations showed that deforming bulk rubble approximately follows a Mohr-Coulomb yield condition. Mellor (1980) realized that a continuum approach can be used to model brash ice and examined some cases related to ice ridging.

The present authors (Sayed and Frederking, 1984) developed a continuum model by considering the bulk rubble to be rigid-plastic obeying the Mohr-Coulomb yield criterion. The ridge was idealized as a wedge at the passive critical state. The resulting forces in the ice sheet were higher than those calculated from the potential energy method. This plasticity analysis is probably suitable for the early stages of ridge building, when most of the rubble is deforming (at yield). As a ridge approaches its limiting height and depth a different situation may develop, with only parts of the rubble deforming.

Stress Distribution in Bulk Rubble

Although laboratory experiments suggest that deforming bulk rubble may follow the Mohr-Coulomb criterion, substantial effort is still needed to fully characterize the material behavior. Numerical models can be constructed once the appropriate constitutive equations are established.

A simpler alternative approach is one used to perform stress calculations in particulate media. Considering that forces are transmitted through random contacts between particles, the stress applied at a boundary has been shown to spread according to the one-dimensional diffusion equation. Smoltczyk (1967), Harr (1977), and Chikwendu and Alimba (1979) developed this theory. Detailed derivations can be found in these references; only a simple case, given by Harr, is mentioned below as a justification and to illustrate this approach. Equilibrium of the blocks shown in Fig. 1 gives

$$\sigma_x (x + \Delta x, y) = \tfrac{1}{2}[\sigma_x (x, y-\Delta y) + \sigma_x (x, y+\Delta y)] \qquad (1)$$

where x and y are Cartesian coordinates, and σ_x is the normal stress in the x direction. Expanding σ_x using Taylor's theorem and assuming that Δx and Δy tend to zero gives,

$$\frac{\partial \sigma_x}{\partial x} = D \frac{\partial^2 \sigma_x}{\partial y^2} \qquad (2)$$

where

$$D = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \tfrac{1}{2} \frac{(\Delta x)^2}{\Delta y} \qquad (3)$$

Based on physical and dimensional arguments, the diffusivity was chosen as

$$D = \nu x \qquad (4)$$

The value of ν given by Harr (1977) is $\frac{\pi}{8}$, while Chikwendu and Alimba (1979) calculated 0.273 for spherical particles. Ice rubble would correspond to high value for ν, possibly close to $\frac{\pi}{8}$, owing to the angularity of typical ice blocks.

It should be noted that Equation (2) can be derived using a probabilistic approach that is essentially different from the above simple deterministic method. The aim of this section is not to present a rigorous proof but rather to propose the use of Equation (2) to calculate stresses in bulk rubble.

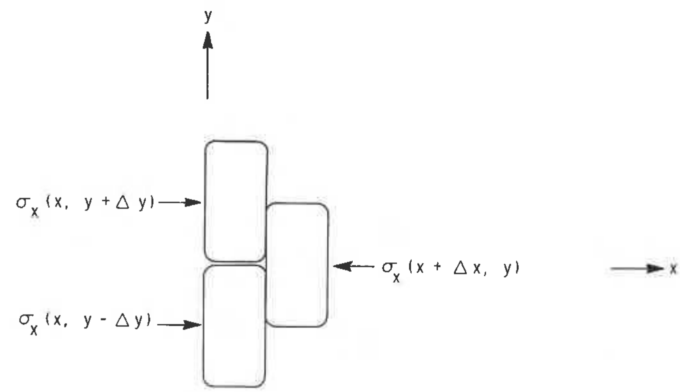

FIGURE 1
NORMAL STRESS DISTRIBUTION BETWEEN ICE BLOCKS

Overview of a Ridge Limiting-Height Model

A new model of ridge formation as it reaches the limiting height and depth is briefly described. The analysis is too long to be adequately presented here. Therefore, the discussion is restricted to an outline of the approach and part of the results.

It is assumed that half a two-dimensional symmetrical wedge (as shown in Fig. 2) represents a sail or a keel. In this idealized model, the advancing ice sheet applies a horizontal force, F, at the edge of a rubble pile-up of height H. A vertical body force, γ, acting downwards, is taken as the bulk unit weight of the sail or buoyancy of the keel.

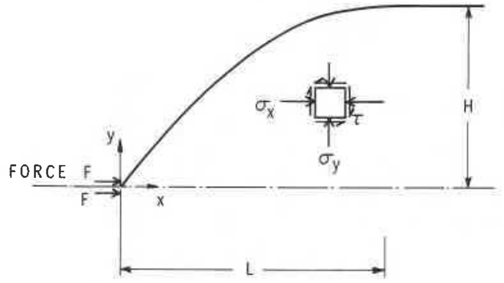

FIGURE 2
DEFINITION SKETCH OF ICE RIDGING MODEL

Assuming that deformation is two-dimensional and neglecting inertia terms, the equilibrium equations become

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau}{\partial y} = 0 \tag{5}$$

$$\frac{\partial \tau}{\partial x} + \frac{\partial \sigma_y}{\partial y} = \gamma \tag{6}$$

where σ_x, σ_y, and τ are the normal and shear stress components, and γ is the unit weight of the ice rubble in the sail or keel. Equations (2), (5), and (6), with the appropriate initial and boundary conditions can determine the stress distribution. Details of the solution of these equations are too lengthy to be included here. Our results show that different stress distributions can be obtained by changing the ratio $\frac{F}{\gamma H^2}$.

In the early stages of the development of a ridge, the horizontal force applied by the ice sheet will cause failure of the rubble, which can then move upwards. The height ceases to increase when it approaches a value sufficient to inhibit further failure of the rubble. Thus, the limiting height is assumed to occur when stresses reach the critical state over part of a vertical plane. The Mohr-Coulomb criterion is used to determine failure zones in the rubble. This failure condition is

$$\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} \geq \left(\frac{\sigma_x + \sigma_y}{2}\right) \sin \phi \tag{7}$$

where ϕ is the angle of internal friction. A typical failure zone in the rubble at the limiting height is shown in Fig. 3.

A relationship between the force F and the limiting sail height or keel depth H was found using the above approach. For the values $\nu = 0.4$ and $\phi = 50°$, which are likely to represent the properties of ice rubble, the result is

$$F = 0.76\ \gamma H^2 \tag{8}$$

Predicted slopes of the ridge, as can be seen from Fig. 3, are close to

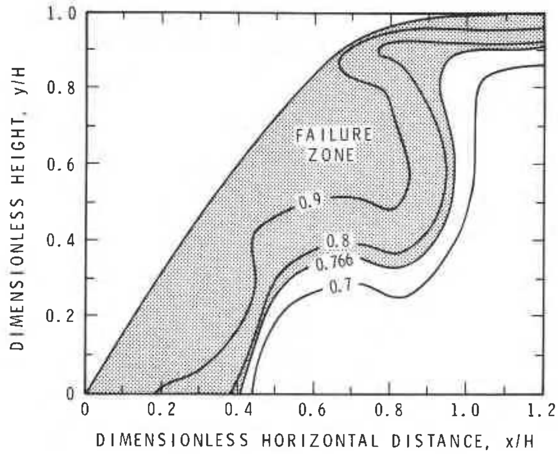

FIGURE 3

CONTOURS OF THE RATIO OF MAXIMUM SHEAR STRESS TO AVERAGE NORMAL STRESS SHOWING FAILURE ZONE IN A RUBBLE PILE-UP AT THE CRITICAL HEIGHT ($\phi = 50°$, $\nu = 0.4$)

45°. Details of this analysis will be presented elsewhere (M. Sayed and R.M.W. Frederking. A model of ice rubble pile-up. Manuscript in preparation.).

Comparison of Calculation Methods

The force predicted from the limiting-height model (Equation (8)) is proportional to the square of the height. A similar relationship was obtained from the plasticity analysis (Sayed and Frederking, 1984)

$$F = C\gamma H^2 \tag{9}$$

where C depends on the side slopes of the rubble. A wedge as shown in Fig. 2 with one side at a 45° slope, and the other side horizontal corresponds to C = 1.08.

The force calculated using the potential energy method (Kovacs and Sodhi, 1980) would be

$$F = \frac{1}{2} \gamma_i t \ H \tag{10}$$

where γ_i and t are the unit weight and thickness of ice respectively.

Friction along an ice sheet sliding over the slope is

$$F_f = \mu\gamma_i tH \cot \beta \tag{11}$$

where μ is a coefficient of friction and β is the slope. This would increase the force given by Equation (10) by approximately 20% for the case of wedge with slope $\beta = 45°$ and using $\mu = 0.1$.

For neutrally buoyant ridges, the ratio of keel depth to sail height depends on the unit weight and buoyancy of rubble. It is simple to show that the limiting-height and plasticity models (Equations (8) and (9)) give

$$\frac{F_s}{F_k} = \frac{\gamma_k}{\gamma_s} \tag{12}$$

where F_s, F_k, γ_s, and γ_k are the forces associated with sail and keel formation, the unit weight of the sail, and the buoyancy of the keel respectively. The potential energy method predicts forces linearly proportional to rubble height; consequently forces associated with the sail and keel would be equal.

Forces predicted by the various method for a typical ridge (ice concentration = 0.75, ice specific gravity = 0.9 and t = 1.5 m) are compared in Fig. 4. Although no field measurements directly relating forces to ridge geometries are available, consider as an example observed ridges in the southern Beaufort Sea. Geometry of floating ridges and rubble accumulations against structures can be uniform over lengths of the order of 100 m. Sail heights can reach 5 m and keel depths do not exceed 20 m. Forces in ice covers of commensurate lengths (see Johnson et al., 1985; and Sayed et al., 1986) are of the order of 100 kN/m. It can be seen from Fig. 4 that the limiting height and plasticity models apparently give values of the correct order, whereas the potential energy method grossly underestimates ice forces.

Calculated forces are compared to the experimental results of Timco and Sayed (1986) in Fig. 5. The experiments produced two-dimensional floating ridges using model ice. Calculations are made using ice

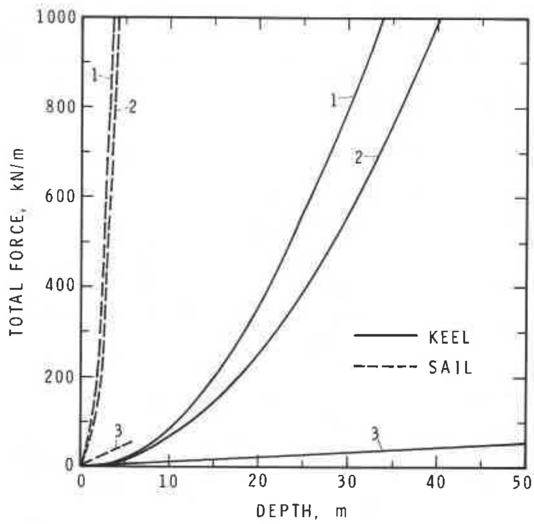

FIGURE 4

PREDICTED TOTAL ICE FORCES AND THE CORRESPONDING KEEL AND SAIL DEPTHS CALCULATIONS: 1 - PLASTICITY METHOD
2 - LIMITING HEIGHT AND DEPTH METHOD
3 - POTENTIAL ENERGY METHOD, FRICTION ADDED

concentration of 0.75, ice thickness of 0.05 m, and ice specific gravity of 0.95. Forces corresponding to both keel and sail formation are added to give the total force in the ice sheet. The limiting-height and plasticity models give forces of the same order though lower than the experimental results. The potential energy method predicts very low forces.

It appears from the above examples that continuum models predict reasonable ice ridging forces. The plasticity model gives forces higher than the limiting-height model by approximately 40%. This is expected because the plasticity analysis considers all rubble to be yielding, which may be suitable for the early stage of ridge development. A similar trend was observed in model tests by Timco and Sayed (1986).

The potential energy method greatly underestimates ridging forces. It seems to be more suitable for situations where an ice sheet slides over a shore. For example, the forces calculated by Kovacs and Sodhi (1980) approached reasonable values for cases where ice would slide for

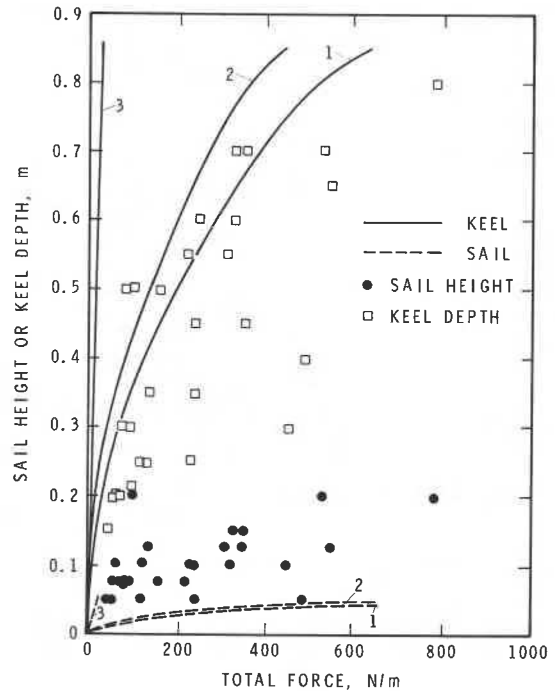

FIGURE 5

COMPARISON OF PREDICTED TOTAL FORCE - RUBBLE HEIGHT AND DEPTH WITH THE EXPERIMENTAL RESULTS OF TIMCO AND SAYED (1986)
CALCULATIONS: 1 - PLASTICITY METHOD
2 - LIMITING HEIGHT AND DEPTH METHOD
3 - POTENTIAL ENERGY METHOD, FRICTION ADDED

relatively long horizontal distances (20 m to 90 m) before rubble pile-ups 3 m to 6 m high were formed.

Summary

A review and comparison of calculation methods of ice ridging forces was presented. A diffusion equation was proposed as a representation of normal stress spread in bulk rubble. A model of ridges at their limiting height and depth that uses that approach was outlined.

Calculations indicate that the plasticity and limiting-height models predict forces of the order expected when deformation is uniform. This corresponds to ridges and floating rubble pile-up against or near wide

structures. The potential energy method greatly under-estimates ice forces in these cases. It seems more suited for situations where an ice sheet slides over a shore for relatively long distances.

Observed ridge keel depth and sail heights, and measured ice pressures in the southern Beaufort Sea are best explained by the plasticity and limiting height models that predict an H^2 dependence of F rather than the linear H dependence of the energy models.

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