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PERFORMANCE OF DISCRETE ORDINATES METHOD IN A GAS TURBINE COMBUSTOR SIMULATOR

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ABSTRACT

Predictive accuracy of discrete ordinate method (DOM) was assessed by applying it to the prediction of incident radiative fluxes on the walls of a gas turbine combustor simulator(GTCS), and comparing its predictions with measurements. Input data utilized for the DOM were measured gas concentration and temperature profiles and inner wall temperatures of the GTCS which is a cylindrical enclosure containing a turbulent diffusion flame of propane and air. Effects of order of approximation (S_4 and S_6) and using uniform and non-uniform gas absorption coefficients for the non-homogeneous medium on the accuracy of the predicted heat fluxes were also investigated. Comparisons show that S_4 approximation is adequate for the prediction of incident wall heat fluxes and the use of an absorption coefficient profile based on measured gas concentrations and temperatures improves the accuracy significantly.

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INTRODUCTION

Over the past decade, S_n or Discrete Ordinates Method (DOM) has been the most widely used technique for obtaining numerical solutions to radiative transfer equation (RTE). The DOM has many important advantages that account for its popularity: Apart from the obvious ease with which the method can be incorporated into Computational Fluid Dynamics (CFD) calculations due to its compatibility with the solutions of differential equations for turbulent, reacting and radiating flows, it is computationally efficient [1] and relatively easy to code, and requires single formulation to invoke higher order approximation. This method has first been suggested by Chandrasekhar [2] for one- dimensional astrophysics problems. Carlson and Lathrop [3] have developed the DOM to solve multi-dimensional neutron transport problems. More recently, it was applied to radiative heat transfer problems successfully [4-16] since pioneering works of Truelove [4-6] and Fiveland [7-10]. Selçuk and Kayakol [14,15] investigated the solution accuracy of DOM by predicting the distributions of radiative flux density and source term of a rectangular enclosure problem and comparing the results with exact solutions produced previously for the same problem. The problem was based on data taken from a large-scale experimental furnace with steep temperature gradients typical of operating furnaces.

A significant number of industrial furnaces and combustors are cylindrical in shape. Therefore, in a previous paper[16], it is considered necessary to evaluate the DOM by applying it to the predictions of radiative flux and source term distributions of a cylindrical enclosure based on data reported previously on a pilot-scale experimental furnace with steep temperature gradients typically encountered in industrial furnaces and by comparing its predictions with the exact values. It was concluded that DOM predicts radiative transfer in absorbing-emitting medium with acceptable accuracy.

In this paper, the predictive accuracy of the DOM is assessed by applying it to the prediction of incident radiative heat fluxes at the walls of a gas turbine combustor simulator and comparing its predictions with the measurements. The GTCS, which is a cylindrical enclosure containing a turbulent diffusion flame of propane and air, was designed, constructed and operated within the framework of NATO-AGARD Project T51/PEP, with the objective of providing experimental data for testing the accuracy of a radiation code to be used in conjunction with a novel computational fluid dynamics code developed for the same GTCS [17]. Effect of order of approximation (S_4 and S_6) and the effect of using uniform and non-uniform gas absorption coefficients for the non-homogeneous medium on the accuracy of the predicted heat fluxes are also investigated.

DESCRIPTION OF THE METHOD

For the application of DOM, the GTCS is considered as a right cylindrical shaped enclosure containing an absorbing-emitting, non-scattering radiatively grey medium whose temperature and absorption coefficient are assumed to be available at all points within the enclosed medium. Surfaces bounding the medium are assumed to be grey with the known temperature profile.

For the problem under consideration, the discrete ordinates representation of RTE takes the following form[16]

$$\frac{\mu_m}{r}\frac{\partial\left(r I^m\right)}{\partial r}-\frac{1}{r}\frac{\partial\left(\eta_m I^m\right)}{\partial \phi}+\xi_m\frac{\partial I^m}{\partial z} = -k_a I^m + k_a I_b \tag{1}$$

where $I^m\ [\equiv I(r, z; \theta,\phi)]$ is the total radiation intensity at position (r, z) in the discrete direction $\Omega_m(\mu_m = \sin\theta\cos\phi,\ \eta_m = \sin\theta\sin\phi,\ \xi_m = \cos\theta)$ which is defined in terms of the polar angle θ between z axis and Ω_m, and the azimuthal angle ϕ between r and the projection of Ω_m in the x-y plane. I_b represents the total blackbody radiation intensity at the temperature of the medium and k_a is absorption coefficient of medium. Assuming the surface bounding the medium to be grey, Equation (1) is solved with following boundary conditions

$$\text{at } r = R\ :\quad I^m = \varepsilon_w I_{bw} + \frac{1-\varepsilon_w}{\pi}\sum_{m'} w_{m'}\mu_{m'} I^{m'} \qquad \mu_m < 0 \tag{2}$$

$$\text{at } r = 0\ :\quad I^m = I^{m'} \quad \mu_{m'} = -\mu_m \qquad \mu_m > 0 \tag{3}$$

$$\text{at } z = 0\ :\quad I^m = I_{bw} \qquad \xi_m > 0 \tag{4}$$

$$\text{at } z = L\ :\quad I^m = \varepsilon_w I_{bw} + \frac{1-\varepsilon_w}{\pi}\sum_{m'} w_{m'}\xi_{m'} I^{m'} \qquad \xi_m < 0 \tag{5}$$

where I_{bw} ($\sigma T_w^4/\pi$) is the black-body radiation intensity at the temperature of surface, T_w. The w_m is angular quadrature weight and ε_w is the wall emissivity. The centerline (r = 0) is treated as a fictitious, perfectly specular reflecting boundary. In Equations (2) through (5), the values m and m′ denote outgoing and incoming directions, respectively.

A control volume form of discrete ordinates equations can be obtained by multiplying Equation (1) by 2πrdrdz and integrating over a control volume in axisymmetric cylindrical geometry as follows

$$\mu_{m,\ell}\left(A_{re}I_{re}^{m,\ell} - A_{rr}I_{rr}^{m,\ell}\right) - \left(A_{re} - A_{rr}\right)\left(\gamma_{m,\ell+1/2}\, I_p^{m,\ell+1/2} - \gamma_{m,\ell-1/2}\, I_p^{m,\ell-1/2}\right)/w_{m,\ell} +$$

$$\xi_{m,\ell}\left(B_{ze}I_{ze}^{m,\ell} - B_{zr}I_{zr}^{m,\ell}\right) = -k_a v_p I_p^{m,\ell} + k_a v_p I_b \tag{6}$$

where $I_p^{m,\ell}, I_p^{m,\ell\pm1/2}$ are cell-center intensities along directions (m,ℓ), and $(m,\ell\pm1/2)$, respectively. $I_{re}^{m,\ell}, I_{rr}^{m,\ell}, I_{ze}^{m,\ell}, I_{zr}^{m,\ell}$ are reference (r) and exit (e) intensities at each face of control volumes along r- and z-directions, respectively. $I_{rr}^{m,\ell}, I_{zr}^{m,\ell}, I_p^{m,\ell-1/2}$ can be assumed known from boundary conditions or calculations in adjoining control volumes, but

$I_{re}^{m,\ell}$, $I_{ze}^{m,\ell}$, $I_p^{m,\ell+1/2}$, $I_p^{m,\ell}$ are unknown. Therefore, to solve Equation (6), auxiliary relations among these intensities are necessary.

$$I_p^{m,\ell} = \alpha I_{re}^{m,\ell} + \left(1-\alpha\right) I_{rr}^{m,\ell} \tag{7}$$

$$I_p^{m,\ell} = \alpha I_{ze}^{m,\ell} + \left(1-\alpha\right) I_{zr}^{m,\ell} \tag{8}$$

$$I_p^{m,\ell} = \alpha I_p^{m,\ell+1/2} + \left(1-\alpha\right) I_p^{m,\ell-1/2} \tag{9}$$

Finite-difference weighting factor, α assumes values in the range $0.5 \le \alpha \le 1$. Diamond Difference Scheme,(DDS), α=0.5, is based on the assumption of linear variation of the radiation intensity within each control volume. As is reported in the literature [18], it is unstable and gives oscillatory (positive-negative) solutions that propagate throughout the spatial domain. The size of control volume should be made small enough to avoid negative and oscillatory radiation intensity. However, even with a small control volume, this problem may occur if there is a significant difference between the radiation intensity values on the adjacent faces of control volume as in the case of an enclosure with one hot and three cold black walls. Therefore, it is necessary to employ negative intensity fix-up procedure (switch-to-step scheme (α=1.0), [18]) for DDS when negative intensity is encountered.

With DDS, solving Equation (6) for cell-centre intensity, $I_p^{m,\ell}$ may be evaluated as

$$I_p^{m,\ell} = \frac{\mu_{m,\ell} A I_{rr}^{m,\ell} + \beta_{m,\ell} I_p^{m,\ell-1/2} + \xi_{m,\ell} B I_{zr}^{m,\ell} + k_a v_p I_b}{\mu_{m,\ell} A + \beta_{m,\ell} + \xi_{m,\ell} B + k_a v_p} \tag{10}$$

where

$$A = A_{rr} \ . \ A_{re} = (2\pi r_r \Delta z) + (2\pi r_e \Delta z) \tag{11}$$

$$B = B_{ze} + B_{zr} = 2[\pi(r_e^2 - r_r^2)] \tag{12}$$

$$\beta_{m,\ell} = -(\gamma_{m,\ell+1/2} + \gamma_{m,\ell-1/2})(A_{re} - A_{rr}) / w_{m,\ell} \tag{13}$$

Every $I_p^{m,\ell}$ value on every cell-centre and faces of control volume can be computed by stepping from control volume to control volume. The direction of recursive evaluation is in accord with the direction of physical propagation of radiation beam as defined by (μ_m, η_m, ξ_m). Single index m is used for a discrete direction for the remaining part.

Once the intensity distribution is determined, the incident heat flux on a wall is calculated as

$$q^{-} = \int_{2\pi} \zeta \, I \, d\Omega = \sum_{m} w_m \, \zeta \, I^{m} \tag{14}$$

where ζ direction cosines μ, ξ in r, z directions, respectively.

GAS TURBINE COMBUSTOR SIMULATOR

GTCS is a cylindrical enclosure containing turbulent diffusion flame of propane with air. The combustion chamber, shown schematically in Fig. 1, was one of three main sections of the test rig (air/fuel inlet, flow conditioning section and combustion chamber). It is 420 mm in length (excluding exit cone length) and 101.6 mm in diameter. It was constructed as a confined turbulent diffusion flame, with bluff body flame stabilising. The air is injected into chamber around the bluff body while the fuel is injected through the centre of the disc. Experimental data for wall and gas temperatures, incident radiative heat fluxes and gas compositions were obtained for fuel and air flow rates of 0.0009 m^3/s and 0.0425 m^3/s respectively, corresponding to an equivalence ratio of 0.5.

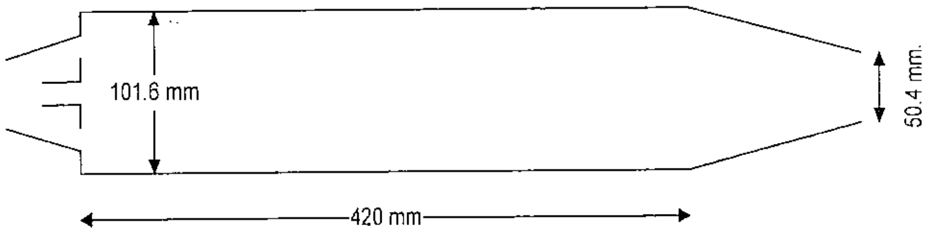

Figure 1. A schematic diagram of GTCS.

Wall and gas temperatures were measured by using K and S type thermocouples, respectively. Measurement of incident radiative heat flux on the wall was carried out by using a Medtherm 64TP-2-25 heat flux transducer of Schmidt-Boelter type (0-90852 W/m^2). It had a sapphire window attachment with 150^0 view angle and provisions for water cooling and nitrogen gas purging for the window.

Species concentrations in the gas were measured by using a water-cooled gas sampling probe, gas conditioning system and a Varian Star 3400 gas chromatograph.

The ranges of measurements for wall temperatures and incident radiative fluxes on the walls were 57.2-362.0 mm and 51.5-351.5 mm, respectively. Gas temperatures were measured in the ranges of 51.9-354.2 mm in axial and 0-46 mm in radial directions, respectively. Measured gas compositions fall within 21-291 mm in axial and 0-45 mm in radial directions, respectively.

APPLICATION OF DOM TO GTCS

Volume of the GTCS where measurements were available, was subdivided into 13 and 14 control volumes in radial and axial directions, respectively, so that coordinates of the points of gas temperature and composition measurements coincide with the centre of the control volumes which form the grid points of the numerical solution. This subdivision results in a shift of burner wall 6.9 mm away from the actual position and an increase of 1.2 mm in the actual radius of the combustor. Temperatures of the control volumes, not falling within the range of gas temperature measurements, such as those adjacent to the walls were obtained by interpolation. For the purpose of modelling radiative exchange, the converging exit of the GTCS was approximated by a circular disc of the same diameter as the main body of the combustion chamber, having an inner circular opening of the same diameter as that of the exit cone.

Figure 2 shows a schematic illustration of combustor dimensions, gas and wall temperature distributions and radiative properties of the surfaces and openings, used as input data for the radiation code.

To determine the effect of using a uniform or non-uniform absorption coefficient for non-homogeneous medium on the accuracy of the predicted heat fluxes, the code has been run twice. In the first run, an absorption coefficient, calculated from the averages of CO_2 and H_2O concentrations measured, the mean of measured gas temperatures and the mean beam length of the whole enclosure was used. The absorption coefficient was found to be 0.53 m^{-1} at the mean combustor temperatures of 1633K.

The second run was carried out by using an absorption coefficient profile for the non-homogeneous medium. The absorption coefficient for each control volume was calculated by using

$$k_a = -(1/L_m)\ln(1-\varepsilon_g) \tag{15}$$

where L_m and ε_g represent the mean beam length and gas emissivity for each control volume, respectively. Gas emissivity was determined by employing Leckner's total emissivity correlations[19] together with measured values for partial pressures of CO_2 and H_2O and temperature and the calculated mean beam length for each control volume. For control volumes for which $p_g L_m < 0.03$ or for which temperature < 1000 K , k_a is calculated from Planck mean absorption coefficient, $k_{a,P}$.

$$k_{a,P} = \frac{\int_0^\infty I_{b,\lambda} k_\lambda \, d\lambda}{\int_0^\infty I_{b,\lambda} \, d\lambda} \tag{16}$$

The values of $k_{a,P}$ plotted as function of temperature for CO_2 and H_2O [20] are a used for this purpose. The resulting profile for absorption coefficient is shown both in graphical form (Figure 3) and tabular form (Table 1).

T= 300 K ε=1.0 | T= 1058 K ε=0.8

L_z= 420 mm

1840	1827	1812	1776	1743	1690	1640	1609	1530	1494	1423	1376	1217	T= 1058 K
1856	1844	1827	1794	1760	1711	1660	1617	1547	1497	1426	1367	1216	T= 1065 K
1872	1862	1843	1812	1777	1733	1680	1625	1565	1500	1429	1358	1218	T= 1078 K
1889	1880	1853	1830	1794	1755	1700	1633	1582	1503	1432	1349	1211	T= 1073 K
1907	1900	1881	1852	1815	1769	1717	1650	1582	1512	1429	1352	1203	T= 1053 K
1909	1904	1892	1870	1833	1788	1729	1664	1594	1513	1428	1349	1189	T=1029 K
1936	1928	1912	1888	1852	1807	1753	1680	1603	1517	1428	1342	1167	T= 992 K
1940	1934	1920	1900	1863	1818	1766	1694	1610	1528	1422	1331	1135	ε=0.8 T= 938 K
1937	1933	1926	1908	1883	1846	1791	1724	1643	1543	1427	1316	1091	T= 867 K
1919	1919	1913	1904	1885	1855	1818	1764	1686	1580	1444	1295	1038	T= 781 K
1908	1907	1900	1893	1882	1867	1840	1806	1750	1652	1491	1263	974	T= 686 K
1650	1727	1802	1849	1866	1861	1837	1812	1781	1690	1499	1059	827	T= 594 K
628	808	1061	1263	1440	1592	1688	1713	1708	1663	1518	1203	858	T= 513 K
814	905	1031	1131	1220	1296	1344	1356	1354	1331	1359	1101	929	T= 448 K

T= 1000 K ε=1.0

L_r= 52 mm

Figure 2 Dimensions and gas temperature distribution for GTCS

For both runs, DDS and S_n[3] are used as spatial differencing and angular quadrature schemes, respectively.

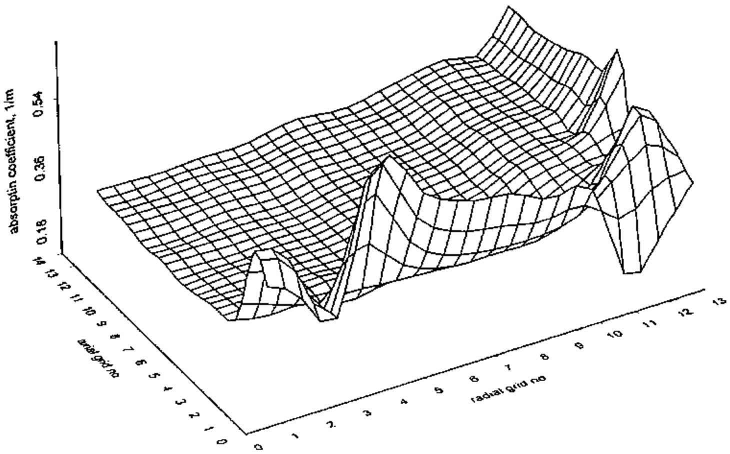

Figure 3. Variation of absorption coefficient for GTCS

Table 1. Non-uniform absorption coefficients calculated by using mean beam length based on dimensions of control volume under consideration for GTCS

Control Volume		z direction													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
r direction	1	0.44	0.53	0.31	0.24	0.24	0.23	0.22	0.22	0.23	0.22	0.23	0.24	0.24	0.25
	2	0.37	0.44	0.28	0.24	0.24	0.23	0.23	0.22	0.23	0.22	0.23	0.24	0.24	0.25
	3	0.31	0.29	0.26	0.24	0.24	0.23	0.23	0.22	0.23	0.23	0.24	0.24	0.25	0.25
	4	0.70	0.55	0.25	0.24	0.24	0.23	0.23	0.23	0.23	0.23	0.24	0.25	0.25	0.26
	5	0.61	0.44	0.25	0.24	0.23	0.23	0.23	0.23	0.23	0.23	0.24	0.25	0.25	0.26
	6	0.53	0.34	0.25	0.24	0.24	0.24	0.25	0.25	0.25	0.25	0.26	0.27	0.27	0.28
	7	0.50	0.31	0.25	0.25	0.24	0.25	0.25	0.25	0.26	0.26	0.26	0.27	0.28	0.29
	8	0.49	0.30	0.26	0.24	0.25	0.26	0.26	0.27	0.27	0.27	0.27	0.28	0.28	0.28
	9	0.44	0.30	0.25	0.24	0.25	0.27	0.27	0.28	0.28	0.28	0.28	0.29	0.29	0.30
	10	0.46	0.32	0.29	0.28	0.30	0.32	0.31	0.32	0.32	0.31	0.32	0.32	0.32	0.32
	11	0.18	0.39	0.34	0.33	0.33	0.34	0.34	0.34	0.34	0.33	0.33	0.33	0.34	0.34
	12	0.23	0.48	0.54	0.38	0.35	0.37	0.33	0.35	0.34	0.34	0.34	0.33	0.33	0.32
	13	0.37	0.41	0.43	0.35	0.31	0.56	0.49	0.48	0.47	0.45	0.44	0.43	0.44	0.44

RESULTS AND DISCUSSION

Figure 4 illustrates the comparison between the incident heat fluxes predicted by S_4 with uniform and non-uniform absorption coefficient and the measurements. As can be seen from the figure, the use of non-uniform absorption coefficient leads to considerable improvement in the accuracy of the predicted fluxes. In order to show the effect of using higher order approximation on accuracy, incident fluxes determined by S_6 for non-uniform absorption coefficient have also been plotted on the same diagram. When the profiles of S_4 and S_6 predictions for a uniform absorption coefficient are compared with the measured values, the effect of higher order approximation on the accuracy of the predicted fluxes seems insignificant.

A condensed comparison between S_4 and S_6 of incident heat fluxes with the measured values is contained in Table 2. Absolute and maximum percentage errors reported in Table 2 are based on measured data. As can be seen from the table, increasing the order of approximation from S_4 to S_6 does not lead to significant improvement in the predictive accuracy. However, the use of a profile for absorption coefficient rather than a uniform value for the whole enclosure decreases the absolute error by nearly 50 %.

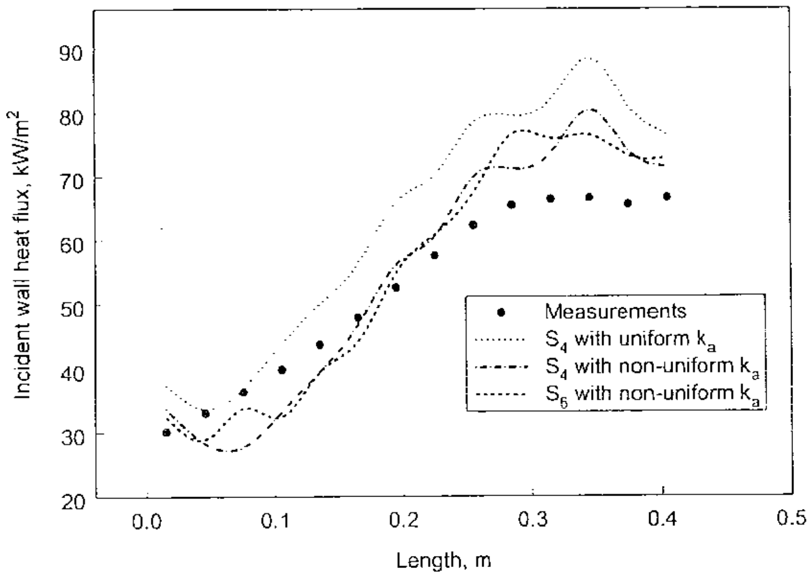

Figure 4. Comparison between S_4 and S_6 predictions of incident heat fluxes along the side wall of GTCS and measurements.

Table 2. Comparison between S_4 and S_6 predictions of incident heat fluxes and measured data for GTCS

Designation		Average absolute percentage error		Maximum percentage error	
		S_4	S_6	S_4	S_6
Absorption	Uniform	18.32	19.13	-32.82	-29.49
Coefficient, k_a	Non-uniform	11.57	10. 53	23.56	18.63

CONCLUSIONS

S_4 and S_6 approximations with uniform and non-uniform absorption coefficient are applied to a gas turbine combustor simulator with the objective of developing an accurate and efficient radiation code to be used in conjunction with a novel CFD code under development for the same system. Accuracy of the radiation code was evaluated by comparing predicted values of radiative heat fluxes incident on the walls with the measurements. Comparisons show that S_4 approximation with non-uniform absorption coefficient produces predictions in reasonably good agreement with the measured data. Absorption coefficient profile representative of the non-homogeneous medium is the most effective parameter in improving the accuracy. Effect of increasing the order of approximation from S_4 to S_6 on the accuracy of predicted results is insignificant for the system under consideration.

NOMENCLATURE

A, B	control volume face areas in r, z direction, m^2
I	radiant intensity, $W\,m^{-2}\,sr^{-1}$
k_a	gas absorption coefficient, m^{-1}
I_{max}	maximum value of I
L_m	mean beam length, m
m	index for a direction
q	incident wall heat flux, $W\,m^{-2}$
T	temperature, K
V_p	volume of pth control volume, m^3
w_m	weight function in a direction m, m'
r, z	co-ordinate axes in cylindrical geometry
φ	azimuthal angle
Ω	direction vector of intensity

Subscripts

bw	black wall
e	exit-face
g	gas
m, m'	outgoing and incoming discrete direction
p	control volume centre
r	reference-face
w	wall

Greek symbols

α	finite-difference weighting factor
ε	emissivity
γ	angular differencing coefficient
θ	polar angle
ζ	direction cosines μ, ξ in r, z directions, respectively

Abbreviations

CFD	computational fluid dynamics
DDS	diamond differencing scheme
DOM	discrete ordinates method
RTE	radiative transfer equation

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