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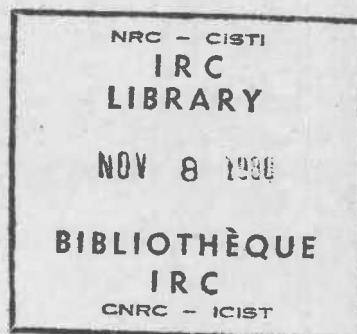
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## ***Experimental Determination of the Z-Transfer Function Coefficients for Houses***

by S.A. Barakat

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## RÉSUMÉ

Des expériences conçues spécialement en vue de déterminer les coefficients de fonction de transfert  $\alpha$  charge calorifique dus aux apports solaires, ainsi que les coefficients de fonction de transfert air de pièce, ont été réalisées sur un site d'essai. Trois séries de coefficients ont été obtenus pour différents niveaux de masse de maisons à ossature de bois. Ces niveaux, qui couvrent une plage inférieure à celle du ASHRAE Handbook, sont particulièrement utiles pour les calculs énergétiques visant les maisons. Les niveaux de masse examinés sont basés sur 46, 130 et 535 kg/m<sup>2</sup> (9,4, 26,6 et 110 lb/pi<sup>2</sup>) d'aire de plancher.

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# EXPERIMENTAL DETERMINATION OF THE Z-TRANSFER FUNCTION COEFFICIENTS FOR HOUSES

S.A. Barakat, Ph.D., P.E.

## ABSTRACT

Specially designed experiments were performed at an outdoor test facility to determine the cooling-load z-transfer function coefficients due to solar gain input, as well as the room-air transfer function coefficients. Three sets of coefficients were obtained for different mass levels in wood-frame houses. These levels, which cover a range lower than those covered in the ASHRAE Handbook, are particularly useful for energy calculations for houses. The mass levels examined are based on 46, 130, and 535 kg/m<sup>2</sup> (9.4, 26.6, and 110 lb/ft<sup>2</sup>) of floor area.

## INTRODUCTION

The z-transfer function technique (also called the thermal response factor or the weighting factor method), introduced by Stephenson and Mitalas (1967; Mitalas 1972, 1978; ASHRAE 1985), is a well-known method for energy analysis of buildings. With this technique, hourly thermal loads can be calculated based on the physical description of the building and the ambient weather condition. These loads are then used, together with information about the heating/cooling system, to calculate the space air temperature and heat addition/extraction rates. To perform these calculations, the method utilizes sets of precalculated transfer function coefficients. These are given in the ASHRAE procedures (ASHRAE 1975) for three room mass levels; namely, light, medium, and heavy, characterized respectively by 146, 341, and 635 kg/m<sup>2</sup> (30, 70, and 130 lb/ft<sup>2</sup>) of floor area.

An examination of conventional wood-frame houses yields a room specific mass value as low as 50-60 kg/m<sup>2</sup> (10-12 lb/ft<sup>2</sup>), which is much lower than ASHRAE values for lightweight buildings. The precalculated response factors given in ASHRAE (1975) are, therefore, not appropriate for most houses. This is confirmed by the lack of agreement between measured and predicted energy consumption when ASHRAE values were used in an energy analysis program (Konrad and Larsen 1978) to simulate the performance of a wood-frame structure. Figure 1 shows the results of one such calculation. This lack of agreement indicates that the simulated performance consistently implies a much heavier building.

In this study, experiments were carried out to determine the cooling-load transfer function coefficients for solar heat gain input and the room-air transfer function coefficients for three different mass levels of houses corresponding to 46, 130, and 535 kg/m<sup>2</sup> (9.4, 26.6, and 110 lb/ft<sup>2</sup>) of floor area.

## THE Z-TRANSFER FUNCTION METHOD

For a dynamic heat transfer system, dependent and independent variables can be related by differential equations. The solution of these equations may take several forms. In this case, the input and output z-transforms (i.e., transforms of independent and dependent variables) are related by a z-transfer function as follows:

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$$O(z) = I(z)G(z)$$

where  $O(z)$  = z-transform of the output  
 $I(z)$  = z-transform of the input  
 $G(z)$  = system transfer function

The z-transformation of a time-dependent function (e.g., solar radiation, outside air temperature) is a sequence of values of the function at equal time intervals (usually one hour); that is, it is a time series representation of the function.

The most important characteristic of the z-transfer function method, as compared with other calculation methods, is that the inputs and the outputs are sequences of values equally spaced in time. Thus, the weather records of outside air temperature and solar radiation, which are given on an hourly basis, can be used as input with little or no preprocessing. The main limitation of the z-transfer method is that the system under consideration must be constant with time and linear; (in other words, the thermal properties and heat transfer coefficients used in the system must be constant, and independent of temperature).

The z-transfer function method for energy calculations is explained in Chapter 26 of the 1985 edition of ASHRAE Fundamentals (ASHRAE 1985). The calculation consists of two steps. In the first step, the space air temperature is assumed to be constant at some reference value and the instantaneous heat gains/losses for the room are calculated. These include solar and internal heat gains and envelope conduction and infiltration losses. A cooling/heating load for the room (defined as the rate at which heat must be removed/added to maintain the air temperature in the room at the constant reference value) is calculated for each component of instantaneous heat gain/loss.

The z-transfer function that relates the z-transforms of the room heat gain,  $I(z)$ , to the room cooling<sup>+</sup> load,  $Q(z)$ , may be expressed as

$$\frac{Q(z)}{I(z)} = \frac{v_0 + v_1 z^{-1}}{1 + w_1 z^{-1}} \quad (1)$$

where  $v_0$ ,  $v_1$ , and  $w_1$  are the z-transfer function coefficients. While  $v_0$  and  $v_1$  are different for each type of heat gain,  $w_1$  is the same for all types and depends only on the thermal storage characteristics of the room. In the time domain, the room cooling load is given by

$$Q_t = v_0 I_t + v_1 I_{t-\Delta} - w_1 Q_{t-\Delta} \quad (2)$$

where  $t$  is the time and  $\Delta$  is the time interval between successive values, usually one hour.

At the end of the first step, the cooling loads from various heat gains are added to get the total cooling load for the room.

In the second step, the net heat input to the room (total cooling load less any heat extraction or addition provided by the HVAC system) is used to calculate the deviation of the space air temperature from the reference value set during the cooling load calculation.

The z-transfer function that relates the z-transforms of the space temperature deviation from the reference,  $\theta(z)$ , to the net heat input to the space,  $QN(z)$ , is termed the room-air (or air-temperature) transfer function and is given by

$$\frac{QN(z)}{\theta(z)} = \frac{g_0 + g_1 z^{-1} + g_2 z^{-2}}{1 + p_1 z^{-1}} \quad (3)$$

where  $g_0$ ,  $g_1$ ,  $g_2$  and  $p_1$  are the transfer function coefficients. The value of the coefficient  $p_1$  is equal to  $w_1$  for the same room mass. The net heat input and space temperature are related by

$$QN_t = g_0 \theta_t + g_1 \theta_{t-\Delta} + g_2 \theta_{t-2\Delta} - p_1 QN_{t-\Delta} \quad (4)$$

or

$$\theta_t = \frac{1}{g_0} [(QN_t + p_1 QN_{t-\Delta}) - g_1 \theta_{t-\Delta} - g_2 \theta_{t-2\Delta}] \quad (5)$$

<sup>+</sup>To avoid repetition, references to heating loads will be eliminated; these loads can always be considered as negative cooling loads.

Some interesting properties of the transfer function coefficients can be obtained by applying the steady-state conditions. For the cooling-load transfer function coefficients, it can be shown that

$$\frac{v_0 + v_1}{1 + w_1} = f \quad (6)$$

where  $f$  represents the fraction of the steady-state heat gain that appears as a cooling load. The coefficients given in ASHRAE Fundamentals are such that  $f = 1$ . An estimate of  $f$  can be made using the procedure in Chapter 26 of the Handbook. The  $v$  coefficients can be adjusted accordingly.

Similarly, under steady-state conditions, Equation 4 can be rearranged to give

$$\frac{g_0 + g_1 + g_2}{1 + p_1} = K_T \quad (7)$$

where  $K_T$  is the thermal conductance of the room, including all heat flow paths.

In ASHRAE literature, the air-temperature coefficients are presented as normalized factors to remove the effect of variations in room thermal conductance, and the effect of room size. The normalized transfer function coefficients are calculated as

$$\begin{aligned} g_0^* &= (g_0 - K_T) / A_f \\ g_1^* &= (g_1 - p_1 K_T) / A_f \\ g_2^* &= g_2 / A_f \end{aligned} \quad (8)$$

where  $A_f$  is the floor area. Equation 7 becomes

$$g_0^* + g_1^* + g_2^* = 0 \quad (9)$$

Another property inherent in the air temperature coefficients is a measure of the heat capacity of the room. It can be shown that the heat capacity,  $C$ , is given by

$$C = A_f \frac{2g_0^* + g_1^*}{1 + p_1} \quad (10)$$

## THE OUTDOOR TEST FACILITY

The test facility, described in detail in Barakat (1984), consists of three one-story insulated wood-frame buildings with basements. They are divided into four two-room units and four single-room units. Three of the four two-room units (Units 1 to 3) were used for the experiments. Construction details and thermal resistance values of the walls and ceilings are given in Tables 1 and 2.

Each of the two-room units consists of a south and a north room with a connecting door. Each south room has a south-facing window with a net glass area of  $2.6 \text{ m}^2$ ; each north room has a  $1 \text{ m}^2$  window facing north. The three units differ only in the type and amount of thermal mass used as interior finish of the walls, as given in Table 2. All rooms in the facility are heated to  $20^\circ\text{C}$  with an electric baseboard heater controlled by a precision controller to within  $0.1^\circ\text{C}$ . To eliminate any heating load due to outdoor air infiltration, a slight positive pressure is maintained in all rooms, using air preheated to the same temperature as the air in the test room.

Measurements were taken of the following: the average indoor air temperature at mid-height of each room (1.2 m above the floor); the temperature of the attic air and the corridor air; the solar radiation incident on a horizontal surface, and on a south- and a north-facing vertical surface; the direct normal component of the solar radiation; the electric space-heating energy consumption of each room; and the wind speed and direction.

A data logger was used to scan and record the data on magnetic tape. Wind data, radiation and temperature values were scanned every minute, averaged over a specific period, and recorded. Energy consumption values were accumulated and recorded for the same period.

## EXPERIMENTAL PROCEDURE

In principle, the experiments were based on measuring the room response (heating-system power output or room-air temperature) to a known input (solar gain or perturbation in the system input to the room). Application of the transfer function relationships of Equation 2 or 4 using the measured input and output data would then yield the values of the coefficients. Since the test units are located outdoors, the overall measured output contained a second significant component, which is the overall response of the room to the outdoor weather parameters. This component had to be subtracted before the transfer function could be obtained.

To eliminate the dynamic effect of solar gain through windows and the associated overheating of the space on sunny days, both windows of each unit were shaded from solar radiation by an external, vertical shade about 40 cm in front of the window.

For the solar transfer function experiments, the shade on the south window was totally or partially removed on a sunny day to allow solar gain through the window for 1 hour. This created a 1-hour solar input pulse on the room. The magnitude of the pulse (size of opening) was chosen so that the room-air temperature would not exceed the set point (reference) temperature. All the parameters, including the electric heating power consumption for the room, were measured every five minutes. Mid-hour values were then used for the analysis.

For the room-air transfer function experiments, the window remained fully shaded and an additional electric heater, placed in the center of the unit, was started at a prescribed time during the night, and kept on for three or four hours, causing the room to overheat. The temperature rise above the reference value (setpoint) is the output response for Equation 5 (or input for Equation 4).

Figures 2 and 3 represent examples of the measured parameters and outdoor weather conditions during a solar pulse experiment, and during a heat-addition pulse experiment, respectively. Further details of both types of experiments are given in a companion publication (Barakat 1985).

## DATA ANALYSIS

In both pulse experiments, the total output, that is, the change in power consumption for the solar gain experiments, and the rise in room temperature for the heat addition experiments, was the result of two simultaneous inputs: the intentionally generated perturbation and the outdoor weather parameters (ambient temperature, wind and solar radiation on the building surfaces). To separate the net output due to the pulse, the component due to outdoor weather variation was first subtracted from the total output. A transfer function relationship was then established between the input pulse and the corresponding net output, and the coefficients were determined. The procedure is described in detail in the following paragraphs.

### Step 1. Measuring Room Response to Outdoor Weather Parameters

A number of periods were chosen during which no pulses were applied and the room temperature remained at a set point. The measured power consumption, therefore, represented the response to the outdoor weather parameters. Using multiple linear regression, a transfer function was obtained linking the input parameters and the room power consumption,  $E_a$ . This function has the form

$$\frac{E_a(z)}{\Delta T^*(z)} = \frac{a_0 + a_1 z^{-1}}{1 + b_1 z^{-1}} \quad (11)$$

In the time domain this translates into

$$E_a(t) = a_0 \Delta T^*(t) + a_1 \Delta T^*(t-1) - b_1 E_a(t-1) \quad (12)$$

where  $\Delta T^*$  is a weighted temperature difference defined as

$$\Delta T^* = \frac{\sum_{i=1}^n (UA)_i \Delta T_i}{\sum_{i=1}^n (UA)_i}$$

and  $UA$  = overall heat transfer coefficient for a particular wall, window or ceiling

$\Delta T = T_s - T_r$ , for outside walls

$= T_a - T_r$ , for partition walls and ceiling

$= T_o - T_r$ , for windows

$T_s$  = sol-air temperature of the outside surface

$T_a$  = air temperature of the adjacent room or attic

$T_o$  = outdoor air temperature

$T_r$  = room temperature (set at 20°C).

The overall heat transfer coefficients,  $UA$ , were calculated based on the construction details. Sol-air temperatures were calculated using the following data: surface absorptance; solar radiation incident on the surface (measured for south and north walls and calculated from horizontal measurements for east and west walls [Barakat 1980]); and the outdoor film heat transfer coefficient calculated hourly as a function of wind speed (ASHRAE 1985).

The coefficients ( $a_0$ ,  $a_1$ , and  $b_1$ ) of Equation 12 were determined using multiple linear regression. The adequacy of these coefficients was checked by using them to predict the power consumption during other periods. Figure 4a shows measured power consumption for a four-day period in January 1983; it also shows the predicted power using the transfer function coefficients calculated for the same data, with the first value as an initial condition. Figure 4b shows the measured power values for a different period in March 1983 compared with the values predicted using the coefficients produced for the data in Figure 4a. The prediction, which has an rms error of less than 25 W (compared with an average power of 380 W over the period), is good.

### Step 2. Generating an Input Pulse

For a period with an input pulse, the coefficients generated in Step 1 were used to predict the room response due to ambient weather,  $E_a(t)$ , using Equation 12. The net output response due to the pulse,  $E_p(t)$  was then calculated as

$$E_p(t) = E(t) - E_a(t) \quad (13)$$

where  $E_p(t)$  represents  $Q_t$  for the solar gain experiments (Equation 2) and  $QN_t$  for the room-air temperature experiments (Equation 4), and  $E(t)$  is the total power consumption of the room (including additional heater input in the room-air temperature experiments). Step 2 is illustrated in Figure 5 for a solar pulse at noon in the south room of Unit 2 and in Figure 6 for a heat-addition pulse in Unit 1. In these figures and all those that follow, the hourly discrete data points are joined by straight lines for both measured and predicted results. Data used to produce the response factor start only at the pulse time.

For the solar pulse, the input is calculated as

$$P = FAI \quad (14)$$

where  $I$  is the average solar radiation intensity over the 1-hour pulse period,  $F$  is the solar gain factor for double glazing calculated for that hour (Barakat 1980); and  $A$  is the net glazing area exposed during the period.

### Step 3. Calculating the Transfer Function Coefficients

Multiple linear regression analysis was used to produce a first approximation to the transfer function coefficients in Equations 2 and 4. These coefficients were then used as initial assumptions for a sophisticated prediction program to generate more accurate values of the coefficients. A prediction program was used (DFMFP 1968), which calculates the local minimum of a function (the error value) of several variables (the coefficients) using the



method of Fletcher and Powell (1963). This is the same method commonly used for the design of recursive digital filters (Steiglitz 1970). The minimized function was the square of the difference between the measured and predicted responses. The method substitutes the predicted value for the previous hour in Equation 2 or 4 to calculate the value and error for the present hour, rather than the measured value for the previous hour, as in regression calculations.

## RESULTS

### Solar Response Factors

A total of 11 experiments were performed during the 1980/81 heating season in the south rooms of Units 1, 2, and 3 of the test facility. Table 3 gives the average calculated values of the transfer function coefficients for all experiments and the average for each room (light, medium, heavy). The table also gives values of the rms difference between the measured and predicted responses. In addition, Figure 7a to c shows examples of both the measured and predicted cooling load response to a solar pulse for each of the light, medium and heavy rooms.

As with the response factors in the ASHRAE Handbook, the coefficients presented in Table 3 are normalized, as explained earlier in this Note, to give an  $f$  value (in Equation 6) of 1. Before normalization, average calculated values of  $f$  were 0.7, 0.89, and 0.86 for the three room mass levels, respectively.

### Room-Air Response Factors

Experiments were performed in two heating seasons, 1980/81 and 1982/83. Between the two periods, an additional external layer of insulation was applied to the walls and ceilings of all test units. The change in thermal characteristics is reflected in the data in Table 1. In 1980/81, experiments were performed in all three 2-room units for both rooms simultaneously (partition door open). In 1982/83, the medium-weight unit was not available for further experiments. Details of the experiments performed during both heating seasons (19 in all) are given in Table 4. Figure 8a to c shows examples of the measured and predicted net heat input to the room (output) corresponding to the measured variation in room temperature (input) for each of the three room mass values. The resulting average response factors for all the experiments are listed in Table 4. Results of individual tests are also given by Barakat (1985).

Due to measurement errors, the transfer function coefficients obtained did not exactly satisfy the steady-state condition given by Equation 7. To satisfy this condition, the  $g_2$  coefficient was calculated from this equation. The overall thermal conductance of the unit,  $K_T$ , was obtained by adding the corridor-wall conductance to the indoor-outdoor UA value of the unit obtained in a previous investigation (Barakat 1984). The maximum change in any  $g_2$  value was less than 10%.

In addition, to be consistent with the ASHRAE presentation, the room-air response factors were normalized using Equation 8. These adjusted and normalized factors are presented in Table 5 for the three mass levels.

As indicated earlier, the thermal capacity of a building can be determined using Equation 10. Table 5 gives the resulting values for the three units, as well as the values calculated from the construction details, the building elements, and the specific heat of each element (taken from Table 2). The data in the table show good agreement between the two sets of thermal capacity values.

## DISCUSSION

The response factors obtained in this study, together with the values in the ASHRAE Handbook, are summarized in Table 6. Examination of the decay coefficients ( $w_1$  and  $p_1$ ) shows that the new values are consistent with the mass levels of the building. The new response factor values, therefore, fill the gap that existed for wood-frame houses. Additional work is needed, however, to produce the remaining sets of cooling load response factors (conduction, lights, etc.).

Work is underway to calculate the z-transfer function coefficients analytically, for the same mass levels reported in this study, using the procedure developed by Mitalas (1983). Preliminary results of these calculations (Sander 1985) are reported in Table 7. Calculated values for the solar cooling load and the room-air transfer function coefficients show fairly good agreement with experimental values. The large values of the new g coefficients of the room-air transfer function are probably due to the locations of heat input and temperature measurement. As mentioned previously, temperature was measured as the average (of 12 locations) at room mid-height. However, if the measured average temperature is less than the true mixed average assumed in any calculation, this will result in higher values of the g coefficients compared with those calculated analytically. In addition, such differences may account for the discrepancy between the assumed linear indoor convective film coefficients used in the calculation, and the actual convective coefficients for the walls of the test units.

The response factors in Table 7 were used in the ENCORE Program (Konrad and Larsen 1978) to simulate the performance of Unit 1 of the test facility for the same period shown in Figure 1. Room temperature and power consumption are compared with measured values in Figure 9. The new response factor values gave results that corresponded much better to actual monitored performance for such lightweight buildings. Further improvement in the simulation results, particularly for heavyweight buildings, can be expected if more coefficients are used to express the z-transfer function.

## SUMMARY

New sets of z-transfer function coefficients were generated experimentally for the solar cooling load and the room-air temperature. The coefficients were obtained for wood-frame houses with three different specific mass levels: 46, 130, and 535 kg/m<sup>2</sup> of floor area. These coefficients improved the thermal performance prediction capability of energy analysis programs based on the response factor principle. The response factors reported in this Note complement those in the ASHRAE Handbook and increase the scope of the response factor method.

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TABLE 1  
Characteristics of Test Units

	1980/81	1981/82
Wall thermal resistance ( $\text{m}^2 \text{ } ^\circ\text{C/W}$ )	2.1	2.2
Ceiling thermal resistance ( $\text{m}^2 \text{ } ^\circ\text{C/W}$ )	3.5	5.6
Floor thermal resistance ( $\text{m}^2 \text{ } ^\circ\text{C/W}$ )	7.0	7.0
South-window glass area ( $\text{m}^2$ )	2.6	2.6
North-window glass area ( $\text{m}^2$ )	1.0	1.0
Window thermal resistance ( $\text{m}^2 \text{ } ^\circ\text{C/W}$ )	0.35	0.35
Floor area per room ( $\text{m}^2$ )	13.9	13.9
Room height (m)	2.4	2.4

TABLE 2  
Thermal Storage Characteristics of Test Units

Test unit	Thermal capacity <sup>a</sup> (MJ/K)	Construction
1	1.96	<u>Light</u> - Standard wood-frame, 12.7 mm gypsum board finish on walls and ceilings, carpet over wooden floor.
2	4.21	<u>Medium</u> - As above, but 50.8 mm gypsum board finish on walls and 25.4 mm finish on ceiling.
3	11.62	<u>Heavy</u> - Interior wall finish of 101.6 mm brick, 12.7 mm gypsum board finish on ceiling, carpet over wooden floor.

<sup>a</sup>Includes framing in walls and ceiling

TABLE 3  
Solar Transfer Function Coefficients

Weight	$v_0$	$v_1$	$w_1$	RMS fit error (W)
Light	0.635	-0.240	-0.605	11.7
Medium	0.400	-0.140	-0.740	9.2
Heavy	0.335	-0.205	-0.870	9.2

TABLE 4  
Room-Air Response Factors

Date	$g_0$	$g_1$	$g_2$	$p_1$	RMS fit error (W)	Number of tests
<u>Light</u>						
Average 1981	334.4	-398.8	91.2	-0.601	33.5	3
Corrected $g_2$	334.4	-398.8	82.8	-0.601		
Average 1983	368.9	-508.4	153.7	-0.693	25.9	6
Corrected $g_2$	368.9	-508.4	151.0	-0.693		
<u>Medium</u>						
Average 1981	446.3	-547.7	120.2	-0.724	34.0	3
Corrected $g_2$	446.3	-547.7	113.5	-0.724		
<u>Heavy</u>						
Average 1981	453.7	-529.2	81.5	-0.901	30.2	3
Corrected $g_2$	453.7	-529.2	80.1	-0.901		
Average 1983	400.3	-452.2	51.4	-0.895	24.0	4
Corrected $g_2$	400.3	-452.2	56.3	-0.895		

TABLE 5  
Normalized Room-Air Response Factors

Unit	Data	$g_0^*$	$g_1^*$	$g_2^*$	$p_1$	Thermal capacity (J/K)
Light	1981	10.40	-13.37	2.97	-0.601	
	1983	11.92	-17.35	5.43	-0.693	
	Average	11.16	-15.36	4.20	-0.647	1.98(1.96) <sup>a</sup>
Medium	1981	14.50	-18.58	4.08	-0.724	3.78(4.21) <sup>a</sup>
Heavy	1981	14.67	-17.55	2.88	-0.901	
	1983	13.00	-15.02	2.02	-0.895	
	Average	13.84	-16.29	2.45	-0.898	11.18(11.62) <sup>a</sup>

<sup>a</sup>Values in brackets are from Table 2

TABLE 6  
Experimental and ASHRAE Response Factors

	Exptl. light	Exptl. medium	ASHRAE light	ASHRAE medium	Exptl. heavy	ASHRAE heavy
Specific mass (kg/m <sup>2</sup> ) of floor area	46	130	146	341	535	635
<u>Solar</u>						
v <sub>0</sub>	0.635	0.400	0.224	0.197	0.335	0.187
v <sub>1</sub>	-0.240	-0.140	-0.044	-0.067	-0.205	-0.097
w <sub>1</sub>	-0.605	-0.740	-0.820	-0.870	-0.870	-0.93
<u>Room-air</u>						
g <sub>0</sub> * (W/m <sup>2</sup> K)	11.16	14.50	9.54	10.28	13.84	10.50
g <sub>1</sub> * (W/m <sup>2</sup> K)	-15.36	-18.58	-9.82	-10.73	-16.29	-11.07
g <sub>2</sub> * (W/m <sup>2</sup> K)	4.20	4.08	0.28	0.45	2.45	0.57
p <sub>1</sub>	-0.647	-0.724	-0.82	-0.87	-0.898	-0.93

TABLE 7  
Calculated Response Factors

	Light	Medium	Heavy
Specific mass (kg/m <sup>2</sup> ) of floor area	46	130	535
<u>Solar</u>			
v <sub>0</sub>	0.48	0.38	0.37
v <sub>1</sub>	-0.06	-0.14	-0.27
w <sub>1</sub>	-0.58	-0.76	-0.90
<u>Conduction</u>			
v <sub>0</sub>	0.77	0.71	0.66
v <sub>1</sub>	-0.35	-0.47	-0.56
w <sub>1</sub>	-0.58	-0.76	-0.90
<u>Lighting (incandescent)</u>			
v <sub>0</sub>	0.50	0.50	0.50
v <sub>1</sub>	-0.08	-0.26	-0.240
w <sub>1</sub>	-0.58	-0.76	-0.90
<u>Room-air temperature</u>			
g <sub>0</sub> * (W/m <sup>2</sup> K)	6.87	9.08	10.86
g <sub>1</sub> * (W/m <sup>2</sup> K)	-7.21	-9.31	-11.39
g <sub>2</sub> * (W/m <sup>2</sup> K)	0.34	0.23	0.53
p <sub>1</sub>	-0.58	-0.76	-0.90

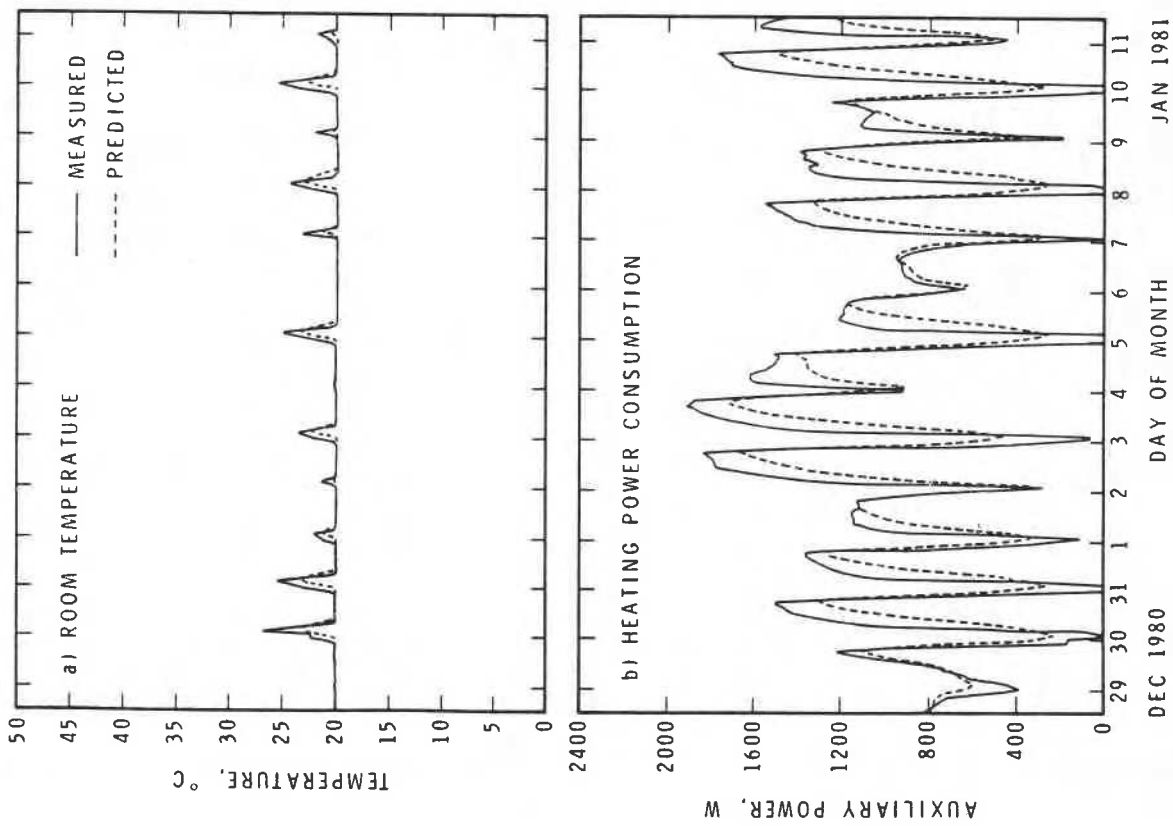


Figure 1. ENCORE simulation results for light unit using ASHRAE response factors

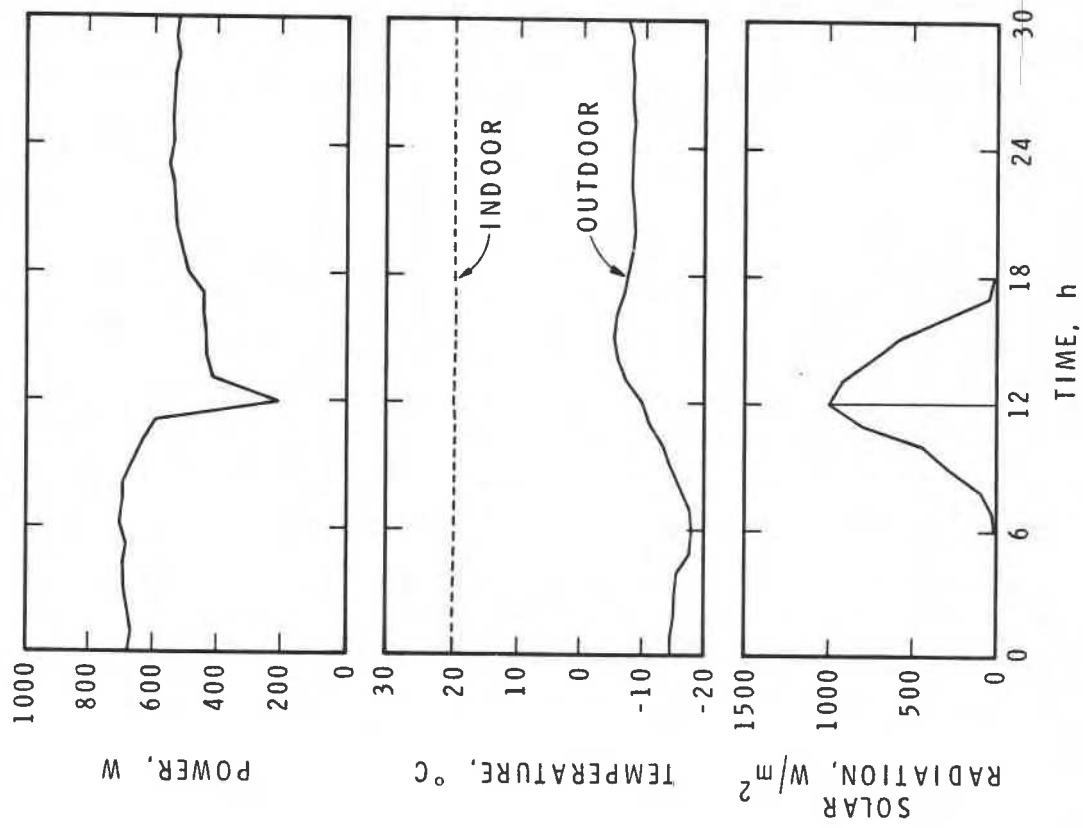


Figure 2. Example of measured values during a solar pulse input

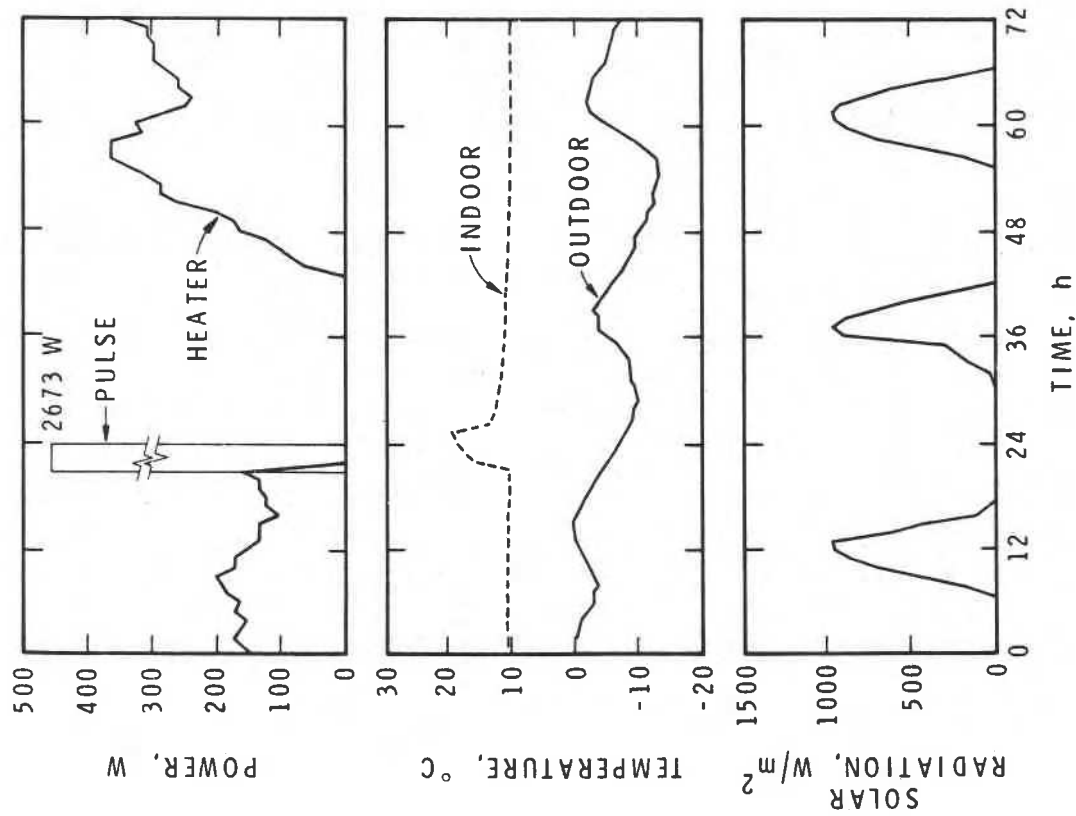


Figure 3. Example of measured values during a room temperature pulse input

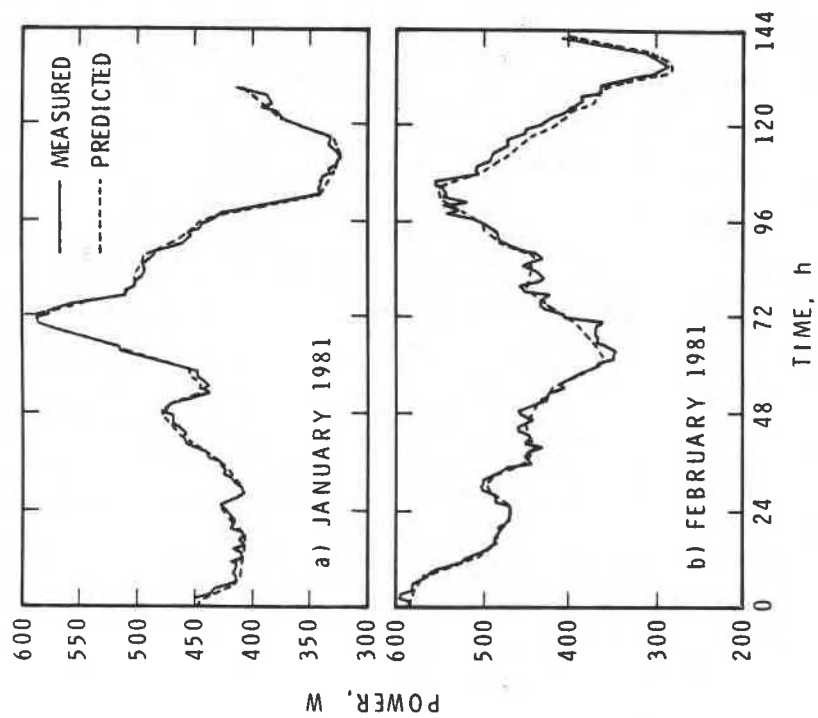


Figure 4. Measured and predicted power  $E_a$  for heavy unit

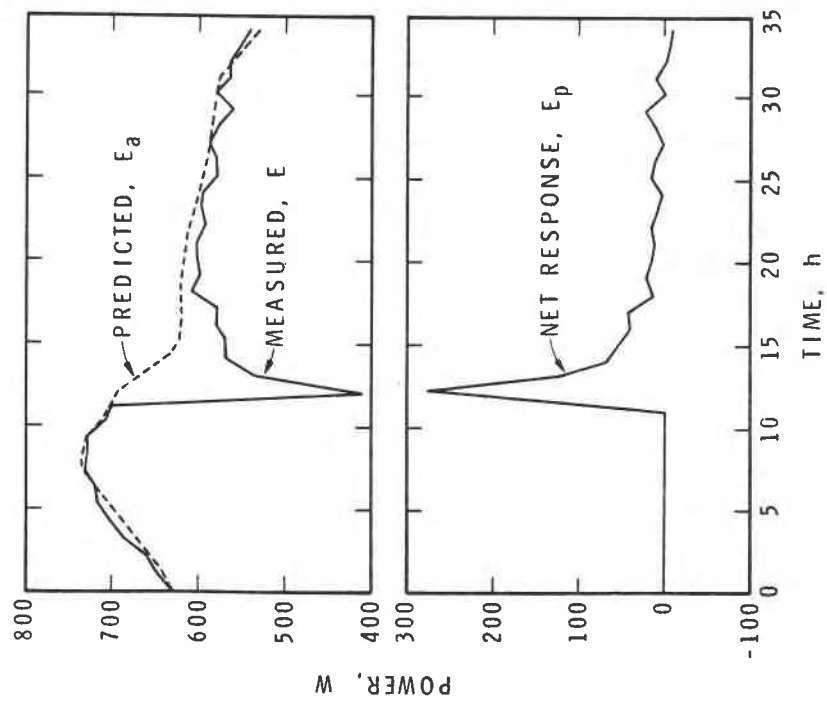


Figure 5. Net response to a solar gain pulse

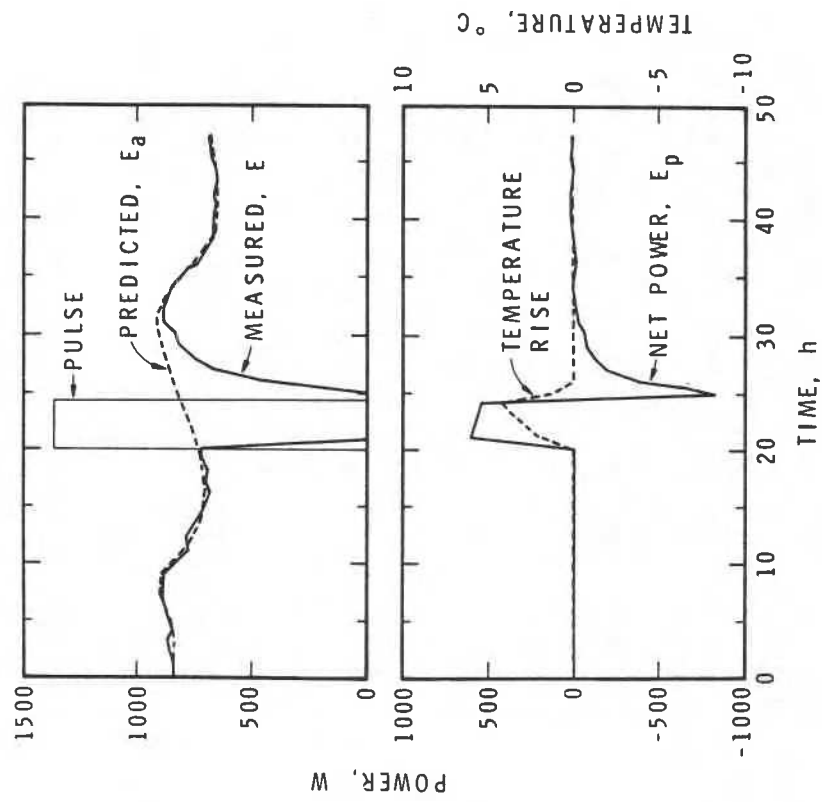


Figure 6. Net input and output for a room-temperature pulse



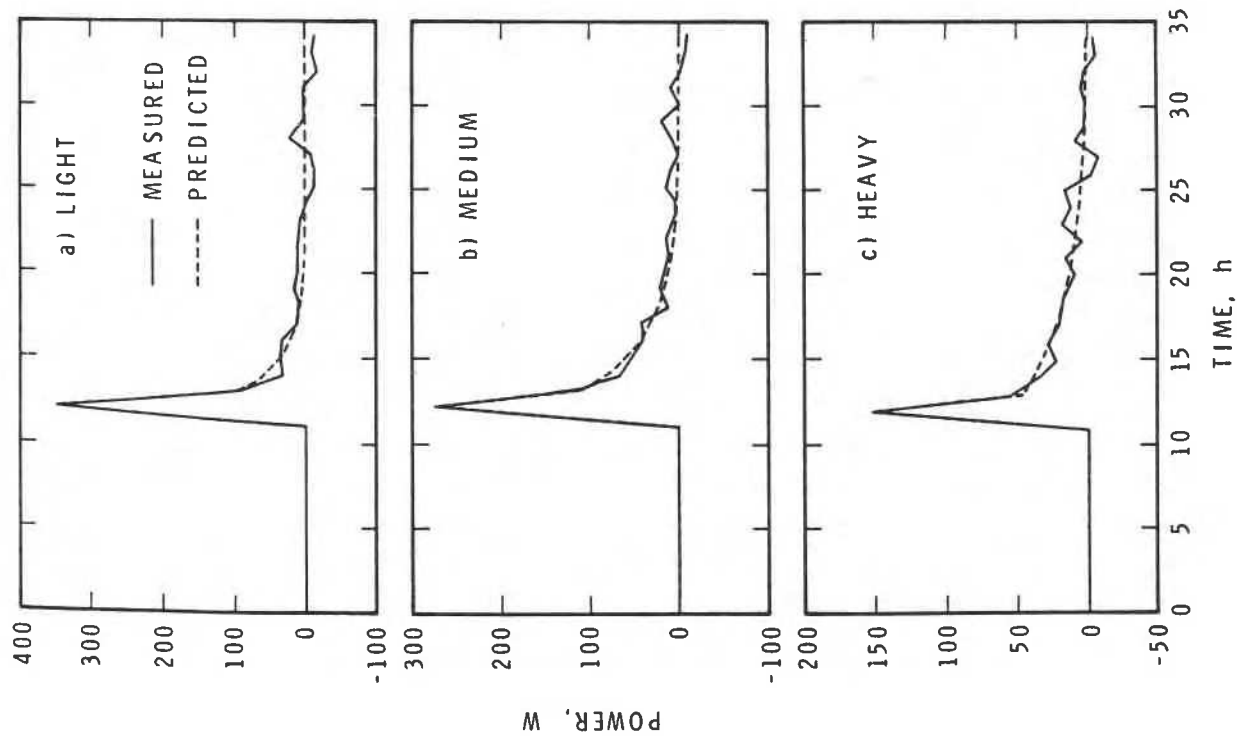


Figure 7. Measured and calculated response to a solar gain pulse

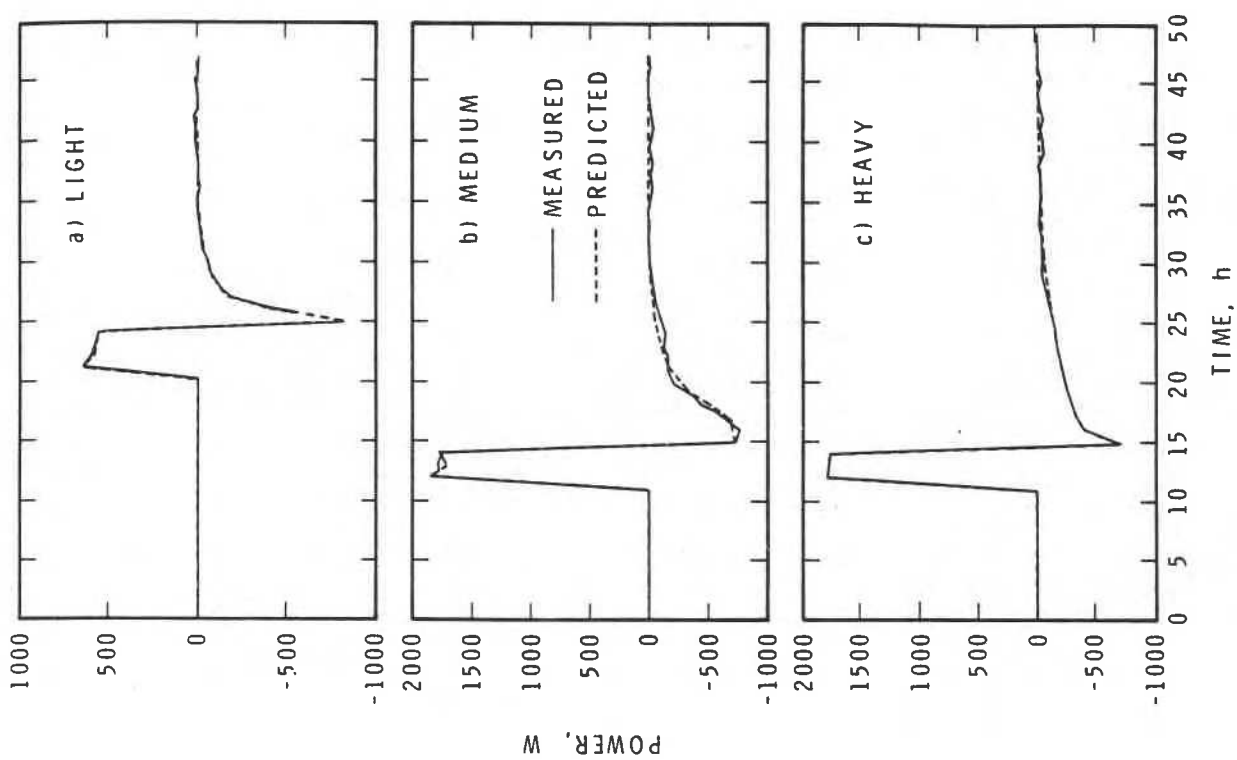


Figure 8. Measured and calculated response to a room-temperature pulse

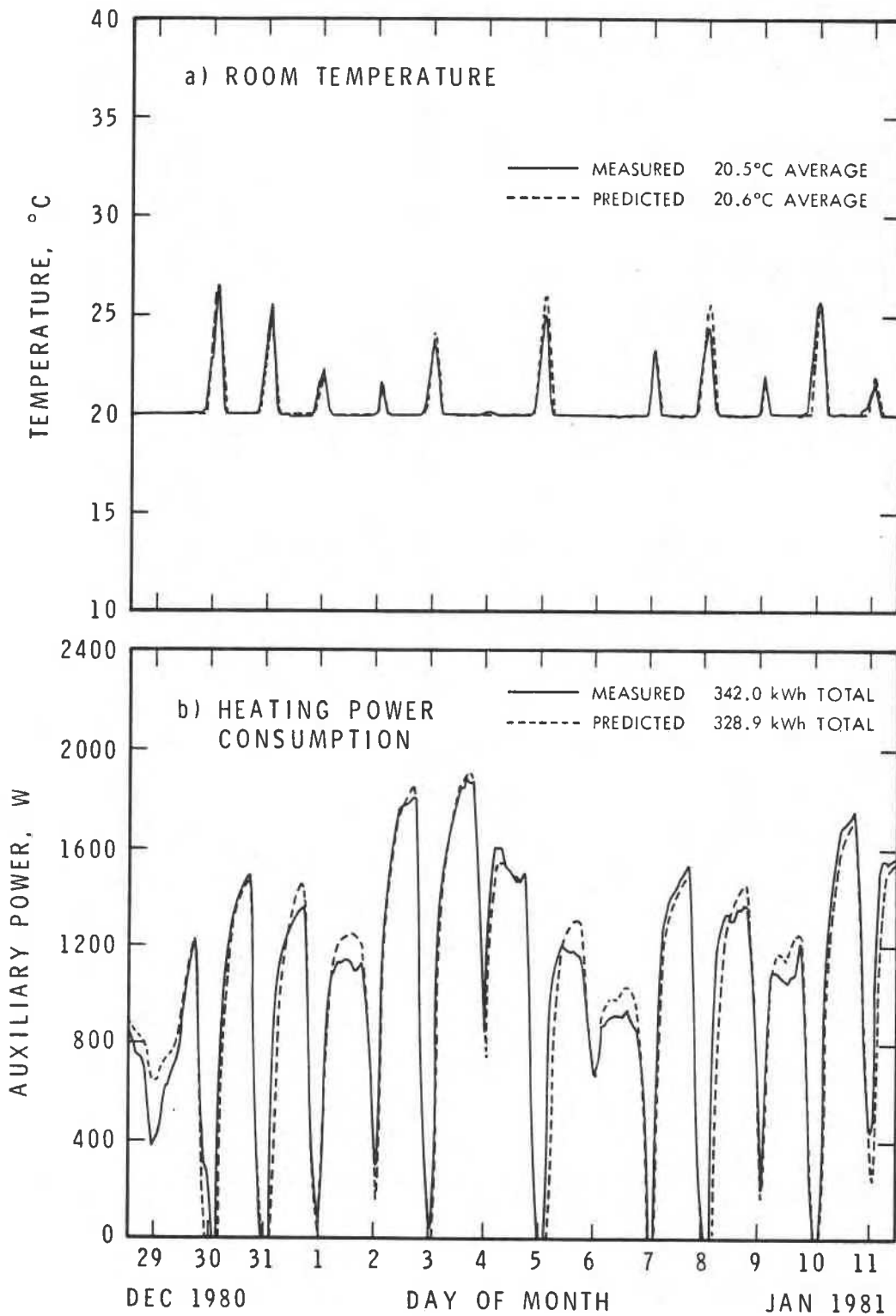


Figure 9. ENCORE simulation results for light units using new response factors (Table 7)

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