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The bearing capacity of ice covers under static loads

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There is a growing interest in the use of ice covers to support loads for extended periods of time. Current practice for determining safe ice thicknesses for over-ice operations is essentially empirical, being based on experience and limited prototype testing. In this paper, the implications of a design criterion for the static load problem, which limits the maximum vertical deflection to the freeboard, are considered. An expression relating strain and deflection is developed. Field results for a static loading case were analyzed and indicated that the time dependent deflected shape of the ice cover could be represented using elastic theory and a time dependent Young's modulus. By combining the strain–deflection relationship with the deflection limit criterion it is shown that strains are small (*i.e.* less than 10^{-3} and in the primary stage of creep) for most cases of practical interest. A method, based on a correlation of observed deflection rates and associated initial elastic stresses from reported cases of static loading of ice covers, is proposed both as a design approach and a framework for analysis of field data.

Il se manifeste un intérêt croissant envers l'exploitation des champs de glace aux fins de porter des charges pendant des périodes prolongées. La pratique actuelle de détermination de l'épaisseur de glace qu'imposent des considérations de sécurité pour des opérations sur le dessus du champ de glace est essentiellement empirique et repose sur l'expérience acquise et les essais réalisés sur un nombre bien limité de prototypes. Cet article étudie les conséquences que contient implicitement un critère de calcul qui, pour le cas de la charge statique, limite la flèche maximale à la fraction émergente de la glace. Les auteurs présentent une expression reliant la dilatation (déformation relative) et la flèche. Ils ont analysé les résultats relevés en nature au cours d'un essai de chargement statique. Ces résultats montrent que la déformation du champ de glace fonction du temps peut être représentée au moyen de la théorie élastique moyennant la prise en compte d'un module d'Young fonction du temps. En combinant la relation dilatation–flèche et le critère relatif à la flèche limite, on démontre que les dilatations sont petites (*i.e.* moindres que 10^{-3} et dans la zone du fluage primaire) dans la majorité des cas d'intérêt pratique. En s'appuyant sur une corrélation établie entre les taux de déformation observés et les contraintes élastiques initiales correspondantes lors de chargements statiques de champs de glace, les auteurs proposent une technique utilisable à la fois comme critère pratique de calcul et comme outil d'analyse des données relevées en nature.

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Ice covers are being used increasingly for winter roads, airstrips, and the support of drill rigs and cranes. Field experience with the bearing capacity of ice covers has provided an essentially empirical basis for the determination of the ice thickness required for over-ice operations (Kingery *et al.* 1962; Gold 1971; Dykins 1969; Baudais *et al.* 1974; Rose and Masterson 1975). There is a need, however, to place the design, construction, and use of load supporting ice structures on a proper engineering basis to ensure an appropriate level of safety for men and equipment. Increased understanding of the properties and behavior of ice is making this possible. Examples of the application of ice engineering to bearing capacity problems have been reported by Assur (1956), Meneley (1974), Michel *et al.* (1974), and Kivisild *et al.* (1975).

A bearing capacity problem for which there is still not a proven method of solution is the determination of the thickness of ice required to support safely a static load for a given period of time. This problem requires the establishment of a performance criterion that will ensure that the ice cover will not fail (*e.g.* specification of the maximum allowable stress or time dependent deflection). One criterion that has been applied is that the maximum deflection must not exceed the freeboard (Panfilov 1972; Meneley 1974). The purpose of this paper is to explore the implications of this criterion, and to present a design method based on it and on field observations of the deflection of ice covers under static load.

Maximum Deflection Criterion for Design

For the static loads that are being considered in this paper, ice will creep and the deflection of the cover will increase continuously with time. There are sound practical reasons for limiting vertical deflection for this situation to the freeboard existing between the free water surface and the top of the cover, quite apart from its implications with respect to the stress and strain imposed on the ice. If the freeboard is exceeded, water can flood the surface through cracks or similar openings. This water may interfere with operations, damage stored material, and freeze around equipment or stores, making removal difficult. If freezing does not occur, the flood water must be considered as a load effectively neutralizing part of the buoyant force over the area it covers. Freezing into good quality ice, however, will add to the structural strength of the cover. Water on the surface will increase the temperature of the ice beneath it, possibly having a serious effect on strength properties, particularly for sea ice. Water on the surface also has a psychological effect on users, particularly those not familiar with the factors controlling the safe bearing capacity of ice covers. This was a major reason, for example, for stipulating that the load placed on the ice cover used for a parking lot during the 1970 Winter Games near Saskatoon, Saskatchewan, should never exceed the buoyancy of the cover (Meneley 1974).

There is little information on the creep or plastic behavior of ice plates. It is usually assumed that this behavior for the biaxial stress condition associated with the bearing capacity problem can be described by results from simple compression or tension tests. Meyerhof (1962) has analyzed the problem assuming ice behaves as an ideally plastic solid, but little progress has been made in either the validation of this approach, or in the development of approaches (such as Nevel's 1966), that take into consideration the creep or viscoelastic properties of ice. In discussions of the bearing capacity problem, the actual strain that an ice cover experiences when performing in a satisfactory manner has been almost totally ignored.

The basic differential equation describing the deflection of an infinite elastic plate on an elastic foundation due to a vertical load is:

$$\nabla^4 w = p/D \tag{1}$$

where w = the vertical deflection, p = the total load intensity due to buoyancy change and external load applied normal to ice surface, D = the plate stiffness = $Eh^3/12(1-\nu^2)$, h = the thickness of the ice cover, and E, ν = Young's modulus and Poisson's ratio, respectively.

The geometry of the problem is shown in Fig. 1. This is usually the starting point for both short term and static load problems.

Wyman (1950) has obtained solutions for the deflection, w, within and outside a loaded area. The deflection at distance r from the centre of a distributed load, q, applied uniformly over a circular area of radius, a, is given by:

$$w = \frac{q}{\rho g}\left[1 + \frac{a}{l}\left(\text{ker}'\frac{a}{l}\text{ber}\frac{r}{l} - \text{kei}'\frac{a}{l}\text{bei}\frac{r}{l}\right)\right] \tag{2}$$

for $r \leqslant a$, and

$$w = \frac{q}{\rho g}\frac{a}{l}\left(\text{ber}'\frac{a}{l}\text{ker}\frac{r}{l} - \text{bei}'\frac{a}{l}\text{kei}\frac{r}{l}\right) \tag{3}$$

for $r \geqslant a$, where ber, bei, ker, kei, are modified Bessel functions, g = gravitational acceleration, ρ = the density of the water supporting the ice cover, and l = a characteristic length:

$$l = \left(\frac{Eh^3}{12k(1-\nu^2)}\right)^{1/4} \tag{4}$$

$k = \rho g$ is the subgrade reaction. The maximum extreme fibre stress which occurs under the

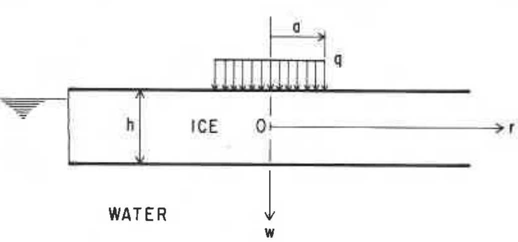

FIG. 1. Schematic of an infinite plate hydrostatically supported.

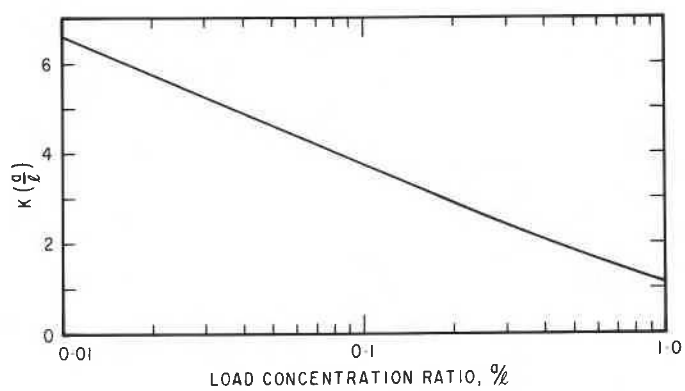

FIG. 2. Non-dimensional term relating deflection to strain.

centre of the loaded area is given by:

$$[5] \quad \sigma_{\max} = 3q\left(\frac{a}{h}\right)^2 \cdot \frac{l}{a} \cdot (1+v)\,\mathrm{kei}'\,\frac{a}{l}$$

The maximum strain for these two load conditions occurs at the top and bottom of the ice cover at the center of the loaded area. For relatively small vertical deflections (less than half the thickness) it is given by:

$$[6] \quad \epsilon = h/2R$$

where R = the radius curvature of the deflected ice surface and

$$[7] \quad \frac{1}{R} = \frac{\mathrm{d}^2 w}{\mathrm{d}r^2}$$

The relation between the deflection and the maximum strain at the center of the loaded area, from [2], [6], and [7], is

$$[8] \quad \epsilon = \frac{h^2}{4l^2}\,\frac{\frac{a}{l}\mathrm{kei}'\frac{a}{l}}{1+\frac{a}{l}\mathrm{ker}\frac{a}{l}}\,\frac{w}{h} = \frac{h^2}{4l^2}K\left(\frac{a}{l}\right)\frac{w}{h}$$

The effective value of the ratio a/l normally lies between a minimum of 0.015, for a concentrated load, and a maximum of 0.4, for broadly distributed loads such as material storage areas. In Fig. 2, the term $K(a/l)$ is plotted against a/l for $0.01 \leqslant a/l \leqslant 1.0$. It can be seen that in the range $0.015 < a/l < 0.40$, it decreases from a value of about 6.0 to one of about 2.0.

An analysis by the authors of deflections of ice covers due to concentrated static loads indicates that time dependent deflections less than the freeboard can be described in the vicinity of the load by [2] and [3] and a characteristic length, l, that decreases in a continuous manner with time. An example of the apparent time dependence of l calculated from the results of Kingery *et al.* (1962) for the deflection of a 1.1 m thick sea ice cover under a load of 360 kN (81 000 lb) is shown in Fig. 3. For a maximum deflection of 72 mm (0.235 ft), or about 6.5% of the thickness of the ice cover, the apparent value of l was about 7.3 m (24 ft); the initial elastic value was 15.5 m (51 ft). For this particular example, the radius of the loaded area, a, was 3.5 m (11.5 ft) giving 0.23 and 0.48 for the initial and final values of a/l, respectively. The term $K(a/l)$, therefore, varied from 2.8 to 1.8. Assuming a value of 2.3 for $K(a/l)$ and a deflection equal to the freeboard ($w/h = 0.08$) gives a value of about 0.001 for the maximum strain.

If the deflection is limited to the freeboard, the maximum value for w/h is 0.08, assuming

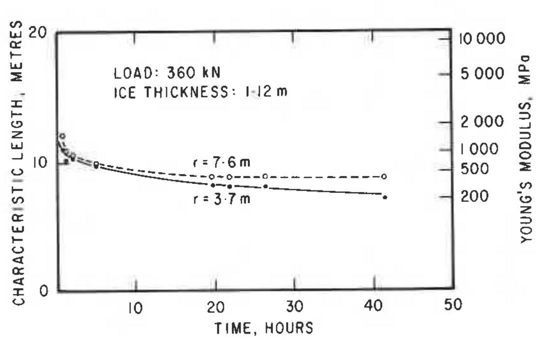

FIG. 3. Characteristic length from data of Kingery *et al.* (1962).

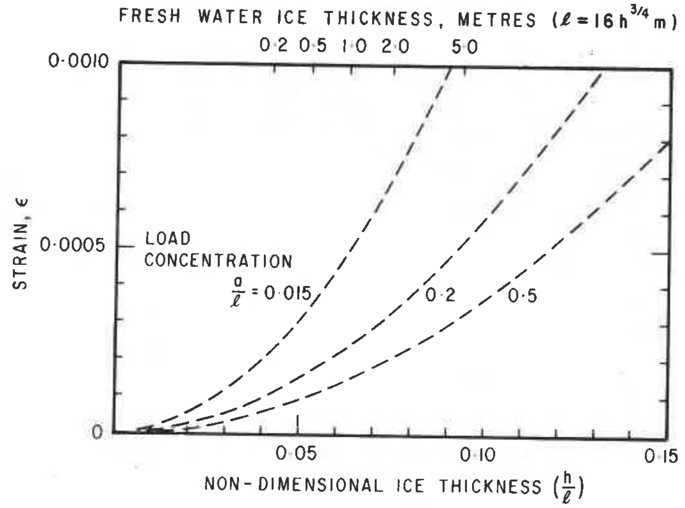

FIG. 4. Strain as a function of thickness for deformation $w/h = 0.08$.

the density of ice to be 920 kg/m^3 (57.4 lb/ft^3). Figure 4 shows the dependence of the maximum strain, ϵ, on the non-dimensional ice thickness h/l, for the deflection $= 0.08h$ and various values of load concentration a/l. The fresh water ice thicknesses corresponding to the value of l for elastic deflection (assuming $l = 16h^{3/4}$ m (Gold 1965)) is shown on the upper abscissa.

The State of Deformation of Ice

Figure 4 and the preceding example show that the extreme fibre strain developed in an ice cover during a deflection equal to its freeboard is quite small. It is, in fact, in the same range as that which can be attained anelastically with a stress of less than 1400 kPa (200 psi). Creep tests in uniaxial tension or compression show that for the range of stress associated with the static load problem, the secondary creep condition is only attained for ice after a strain of about 10^{-2}. The maximum strains that are induced, therefore, lie in the primary creep range. Analysis of the observations of Kingery *et al.* (1962) for an ice cover 1.12 m thick (44 in.) showed that even for a deflection corresponding to $w/h = 0.38$, the maximum strain was only 5×10^{-3}, *i.e.* still in the primary creep range.

Static loads are placed on ice covers for periods of about 1 to 100 days, or 10^5 to 10^7 s. If the maximum deflection allowed is equal to the freeboard (maximum strain of 10^{-3}), this would correspond to a range of average strain rate of about 10^{-8} to 10^{-10} s^{-1}. For ice subject to simple compression and tension, this range in strain rate would be associated with stress of less than 300 kPa (45 psi) (Gold 1965). Restricting deflection to the freeboard, therefore, requires that the ice be sufficiently thick to ensure that the maximum stress induced is low. There is very little information available on the primary creep behavior of ice for either the uniaxial or biaxial stress conditions for stresses corresponding to the strain rate range of 10^{-8} to 10^{-10} s^{-1}.

The stress distribution induced in the ice cover by a concentrated load is such that the radial and tangential stresses are equal beneath the load, and decrease and diverge with distance from the load. When the load is first applied, the elastic stress induced at and near the surfaces causes a viscous flow rate that exceeds that which would be calculated from the rate of deflection. These stresses will relax, therefore, but there will be a corresponding increase in stress in the interior of the cover. Observations by Gold (1965) have indicated that during primary creep, the creep rate is proportional to the stress raised to a power of about 1.5 to 2.0. If it is assumed that the strain rate has a linear dependence on distance from the neutral surface, the general shape of the stress distribution that would result is shown

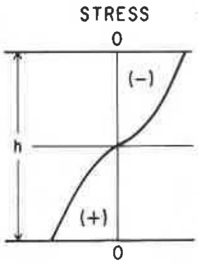

FIG. 5. General shape of viscoelastic stress distribution.

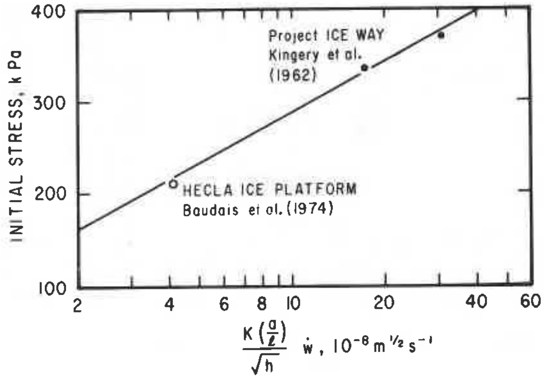

FIG. 6. Relation between initial stress and average strain rate derived from field data.

in Fig. 5. It does not deviate greatly from a linear distribution, which perhaps explains why it may be possible to describe the time dependent deflection of statically loaded ice covers using the elastic equations and a time dependent characteristic length, l, which, in fact, implies a time dependent elastic modulus.

Design for Static Loads

It is assumed that during primary creep, the dependence of the characteristic length, l, on the ice thickness still has the same form as for the elastic situation (*i.e.* $1 \propto h^{3/4}$) then from [8] a reasonable measure of the strain rate would be

[9] $$\dot{\epsilon} \propto \frac{K\left(\frac{a}{l}\right)}{\sqrt{h}}\dot{w}$$

In Fig. 6 the initial maximum elastic stress calculated using [5] is plotted against $(K(a/l)/\sqrt{h})\dot{w}$ for three cases of static loading described in the literature. The average deflection rate, $\dot{w}$, was determined by subtracting the calculated initial elastic deflection from the total deflection and dividing by the time required to attain that deflection. $K(a/l)$ was calculated using the characteristic length, l, associated with the maximum deflection. The correlation that is indicated in Fig. 6 could provide a basis for design.

It can be seen from Figs. 2 and 6 that the correlation between the initial elastic stress and the parameter $(K(a/l)/\sqrt{h})\dot{w}$ for ice of given thickness is relatively insensitive to changes in the characteristic length, l. It is known from field experience about what thickness of ice is required to support a given load (Gold 1971). Using this value as a first estimate, the elastic value for l associated with it, and the average allowable deflection rate determined from the required period of loading, an estimate can be made of $(K(a/l)/\sqrt{h})\dot{w}$. With this value, a first estimate of the initial maximum elastic stress can be obtained from Fig. 6. A second estimate of the ice thickness can then be determined from a knowledge of the load and its distribution using [5]. If the difference between the calculated thickness and the original estimate is sufficiently great, the procedure can be repeated, using a reduced value for l if considered appropriate.

Available field observations indicate that limiting the maximum deflection of a statically loaded ice cover to the freeboard results in a safe condition. Frankenstein (1963) conducted a number of short term loading tests (less than one hour) and found in general that failure occurred for $w/h = 0.5$ for concentrated loads and 0.9 for well distributed loads. These results are in agreement with what would be expected from the information presented in Fig. 4. In the work of Kingery *et al.* (1962), there was no indication of failure even for deflection ratios, w/h, larger than 0.25.

Conclusions

Analysis and field observations indicate that the use of the design criterion that the maximum deflection of a statically loaded good quality and continuous ice cover should not exceed the freeboard is both practical and safe. The maximum strain that would be developed under this condition is small and lies in the primary creep range of behavior. Strain rates and associated stress are also small and in the range for which there is little information, particularly for the biaxial stress condition existing in the ice cover.

Field observations indicate that for the range of deflection allowed by the criterion, the deflection near the load can be described by the equations derived for the elastic condition using a characteristic length, l, that decreases with time. This fact provides a basis for the correlation of the observed deflection rates of statically loaded ice covers to the maximum initial elastic stress. The ice thickness required for a given static load situation can be determined from this correlation and a knowledge of the load and its distribution. Further investigation of the creep behavior of ice plates is required to properly establish the validity of using a time dependent characteristic length for calculating the deflection of statically loaded ice covers.

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