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RÉSUMÉ

La présente note évalue quatre méthodes de calcul du rayonnement solaire incident sur les surfaces inclinées à partir du rayonnement solaire horizontal total. Le rayonnement solaire horaire calculé sur des surfaces verticales orientées au sud et au nord est comparé aux valeurs mesurées. Enfin, des hypothèses sur la distribution du rayonnement solaire diffus sont étudiées en comparant des modèles isotropes et des modèles anisotropes.

COMPARISON OF MODELS FOR CALCULATING SOLAR RADIATION ON TILTED SURFACES

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ABSTRACT - Four methods of calculating solar radiation incident on tilted surfaces from global horizontal solar radiation are assessed. The calculated hourly solar radiation on south- and north-oriented vertical surfaces is compared with actual measured values. In addition, diffuse solar radiation distribution assumptions are examined by comparing the isotropic and anisotropic distribution models.

INTRODUCTION

Solar heat gain is an important component of building energy balance. It is, therefore, essential to be able to calculate accurately the solar radiation incident on building surfaces at different orientations and tilt angles from total horizontal radiation measurements, which are more readily available. Among the models developed to predict the solar radiation incident on a tilted surface, the basic difference is the correlation each uses for calculating the diffuse component of the global horizontal radiation. The correlations are usually based on data from a limited number of locations. In a recent study Spencer (1) compared diffuse radiation predicted by four models with measured values for five Australian locations. The models were those of Bugler (2), Liu and Jordan (3), Boes et al. (4), and Orgill and Hollands (5). Spencer found that the Orgill and Hollands model, with modified coefficients, appeared to give the best prediction of diffuse solar energy. The other three gave predictions that were close to each other.

The present study assesses the ability of four radiation models to predict solar radiation on tilted surfaces for a Canadian location. Three of the models are the same as Spencer's (i.e., those of Liu and Jordan (3), Boes et al. (4), and Orgill and Hollands (5)); the fourth was developed by Hay (6), based on data from four Canadian measuring stations. Each model is used to calculate solar radiation incident on vertical south-facing and vertical north-facing surfaces based on hourly solar radiation data measured on a horizontal surface at the National Research Council Canada (NRCC) passive solar test facility in Ottawa. The calculated values were then compared with the corresponding measured values on an hourly basis for seven one-week periods.

A common assumption in estimating solar radiation on tilted surfaces is that the diffuse component of the solar radiation (sky diffuse radiation) has an isotropic distribution over the hemispherical sky (7). It has been shown, however, that sky diffuse radiation is anisotropic in many instances (8,9) and that the isotropic assumption may introduce errors in calculations for tilted surfaces. The isotropic assumption and two anistropic distribution models were compared and the effect of the assumed distribution on calculated radiation for tilted surfaces was examined.

INCIDENT SOLAR RADIATION ON A TILTED SURFACE

The total solar radiation incident on a tilted surface, H_T, comprises three components: direct or beam component, H_{bT}, sky diffuse component, H_{ds}, and reflected diffuse component, H_{dr}. Thus,

$$H_T = H_{bT} + H_{ds} + H_{dr} \tag{1}$$

If the sky diffuse radiation is assumed to be isotropic, equation (1) can be expressed as

$$H_T = H_b \frac{\cos \Theta_i}{\cos \Theta_z} + H_d \left(\frac{1+\cos s}{2} \right) + (H_b+H_d) \left(\frac{1-\cos s}{2} \right) \rho \tag{2}$$

where H_b and H_d are the beam and diffuse components of the global horizontal radiation, H, Θ_i is the incident angle, Θ_z is the zenith angle of the sun, s is the angle between the surface and the horizontal (slope), and ρ is the ground reflectance.

Diffuse Radiation Correlations

A brief description of how each of the models arrives at H_b and H_d from H is given below.

1. Liu and Jordan model (3)

This method relates the ratio of daily diffuse to total horizontal radiation, H_d / H, to the ratio, K_T, of daily horizontal to extraterrestrial radiation, by the correlation

$$H_d / H = 1.0045 + 0.04349\, K_T - 3.5227\, K_T^2 + 2.6313\, K_T^3 \qquad (3)$$

Although the correlation was based on daily rather than hourly values, it has often been used to estimate hourly diffuse radiation. The correlation is based on data from Blue Hill, Mass., U.S.A. (Lat 42°13'N).

2. Orgill and Hollands model (5)

The correlation between H_d / H and K_T is based on hourly data for Toronto, Canada (Lat 43°48'N), and is expressed by the following relations:

for

$$0 < K_T < 0.35 \qquad H_d / H = 1.0 - 0.249\, K_T$$

$$0.35 < K_T < 0.75 \qquad H_d / H = 1.557 - 1.84\, K_T \qquad (4)$$

$$K_T > 0.75 \qquad H_d / H = 0.1769$$

3. Boes et al. model (4)

This model is based on hourly data from three U.S. stations: Albuquerque (Lat 35°3'N), Omaha (Lat 41°23'N) and Blue Hill. It correlates the direct normal solar radiation, DN, in KW/m^2 to the ratio, K_T, by the following relations:

for

$$K_T > 0.5 \text{ and } \theta_z < 10° \qquad DN = 0.4$$

$$K_T < 0.3 \qquad DN = 0 \qquad (5)$$

$$0.3 < K_T < 0.84 \qquad DN = -0.52 + 1.8\, K_T$$

$$K_T > 0.84 \qquad DN = 1.0$$

The beam component can then be derived from the direct normal using the relation

$$H_b = DN \cos \theta_z \qquad (6)$$

4. Hay model (6)

This model is based on hourly measured radiation data at four Canadian locations: Toronto, Montreal (Lat 45°30'N), Goose Bay (Lat 53°18'N) and Resolute (Lat 74°43'N). It incorporates multiple reflections of short-wave radiation between the earth's surface and the atmosphere. It therefore requires hourly ground reflectance and cloud cover value. The model correlates the ratio of the diffuse component, H_d', to global radiation, H' (where the prime denotes values before multiple reflections), to a modified K_T ratio by

$$H_d' / H' = 0.9702 + 1.6688\, K_T' - 21.303\, K_T'^2 + 51.288\, K_T'^3 - 50.081\, K'^4 + 17.551\, K'^5 \qquad (7)$$

in which

$$K'_T = \frac{H'}{H_o} = \frac{H}{H_o}(1 - \rho\beta) \qquad (8)$$

and

$$H_d' / H' = \frac{H_d - H\rho\beta}{H(1-\rho\beta)} \qquad (9)$$

where ρ is ground reflectance and β is atmospheric back scatteration, a function of cloud cover.

Sky Diffuse Radiation Distribution Models

A method of calculating diffuse radiation from the sky, H_{ds}, allowing for varying degrees of anisotropy, is given by Hay (9):

$$H_{ds} = H_d \left[\frac{DN}{I_o} \frac{\cos \theta_i}{\cos \theta_z} + \left(1.0 - \frac{DN}{I_o}\right)\left(\frac{1 + \cos s}{2}\right) \right] \qquad (10)$$

where I_o is the solar constant.

The method developed by Klucher (8) multiplies the isotropic sky diffuse component by two correction factors to give the sky diffuse component, H_{ds}, that is,

$$H_{ds} = H_d \left(\frac{1 + \cos s}{2}\right)\left(1 + F \sin^3 \frac{s}{2}\right)\left(1 + F \cos^2 \theta_i \sin^3 \theta_z\right) \qquad (11)$$

where $F = 1 - (H_d / H)$

CALCULATION PROCEDURE

At the NRCC passive solar test facility, global solar radiation values on a horizontal surface and on vertical south-facing and vertical north-facing surfaces are measured every minute, then averaged and recorded on tape every 15 min. Data gathered during seven weeks in 1980/81 were chosen to represent periods with different sky conditions. The conditions for the seven weeks are given in Table I. Data were checked for continuity and consistency before the 15-min data were added to produce hourly radiation values. Solar position, extraterrestrial radiation, and incident angle were calculated following the procedure described in Ref. 10. The hour angle was taken as the apparent solar time equivalent to the mid-point of the hour.

Values of solar radiation incident on vertical south-facing and vertical north-facing surfaces were calculated from the measured horizontal radiation values, using each of the models. Ground reflectance was assumed to have values of 0.7 and 0.2 for completely snow-covered and grass-covered ground, respectively; for partial snow cover a value between the two was chosen that would best represent the actual conditions.

RESULTS AND DISCUSSION

For building energy calculation, solar radiation data are required as hourly, monthly, or seasonal integrated values. Hourly values are required for detailed computer simulations, while monthly or seasonally integrated totals are used with simplified energy calculation methods. The following comparison attempts to address the two applications separately.

Hourly Solar Radiation Values

To compare the four diffuse radiation models, values of the solar radiation incident on vertical south-facing and north-facing surfaces were calculated using each of the models, assuming isotropic distribution of the sky diffuse radiation. The absolute value of the difference between calculated and measured values was determined for every hour between sunrise and sunset each day, and was averaged over the daytime hours for every week. These average absolute error values are given in Table II(a) for a vertical south-facing surface, and in Table II(b) for a vertical north-facing surface.

To aid in assessing the models the accumulated number of hours during which the absolute error was less than or equal to a chosen threshold value are given in Tables III(a and b) for south-facing and north-facing surfaces, respectively. Following the study by Spencer (1), these thresholds were chosen as 0.1 and 0.2 MJ/m^2 for the south-facing surface; lower values of 0.05 and 0.1 MJ/m^2 were chosen for the north-facing surface. The lower threshold values are approximately equal to 10 and 20% of the average hourly solar radiation incident on the south or north surfaces over the seven-week period.

For solar radiation incident on a vertical south surface, both Hay's model and that of Orgill and Hollands provided the best prediction (that is, the least absolute error and the highest percentage of time with a small error); the Liu and Jordan model provided the worst. For predicting radiation on a vertical north surface, however, the order was reversed and the Liu and Jordan model was best. The difference in the predictions for the north-facing surface, however, is less than that for the vertical south surface.

To examine the effect of assuming an anisotropic sky diffuse radiation distribution, values of solar radiation incident on the south and north surfaces were calculated using each of the four models in combination with the two anisotropic sky diffuse distributions represented by equations (10) and (11). The absolute error values averaged over the seven-week period for every combination are given in Tables IV(a and b) for vertical south and north surfaces, respectively. It may be seen that neither of the anisotropic assumptions improved the vertical south predictions; on the contrary, the error increased in all cases, with the minimum effect occurring with Hay's two models. Similarly, for

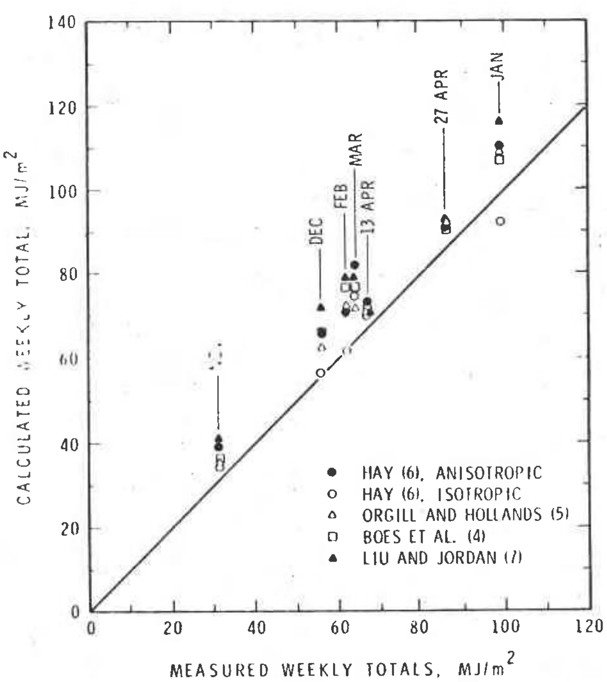

Figure 1

Comparison of weekly total radiation on a vertical south surface

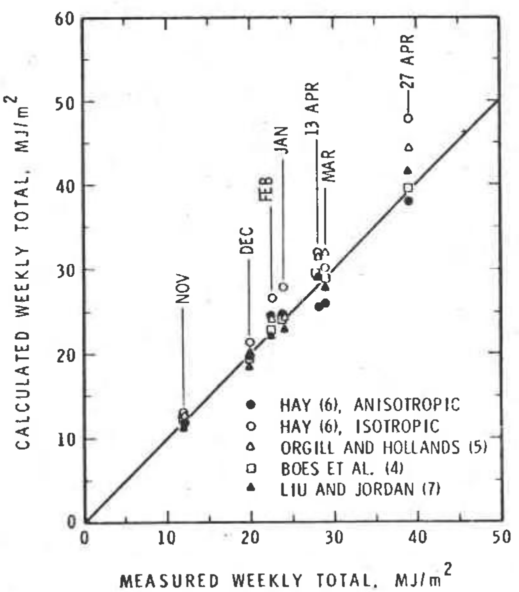

Figure 2

Comparison of weekly total radiation on a vertical north surface the vertical north surface the error increased for all cases except when both Hay's models were used. In this case the error decreased to a value aproximatly that of the other three models, using the isotropic assumption.

Weekly Solar Radiation Values

The calculated and measured weekly values of solar radiation incident on vertical south-facing and north-facing surfaces are compared in Figures 1 and 2, and the percentage differences between them are given in Table V(a and b). For these comparisons calculations were performed using the isotropic sky diffuse assumption with all four diffuse models. In addition, the Hay anisotropic assumption was used with the Hay diffuse model because it was the only combination that provided an improvement over the isotropic assumption.

As shown in Figure 1, all the models over-predict the total amount of radiation on a vertical south surface, but to varying degrees. The Hay model with the isotropic assumption gives the smallest error, while the Liu and Jordan model shows the largest error. All models except the Hay isotropic combination predict the total radiation on the vertical north surface quite well. Although the Orgill and Hollands model produced low absolute errors on an hourly basis, it seems to over-predict the hourly radiation values, quite consistently, leading to error in the integrated value, which is therefore significantly larger than that resulting from the models of Boes et al. and Liu and Jordan.

CONCLUSION

Solar radiation values measured on vertical south-facing and vertical north-facing surfaces have been compared with values calculated for the same surfaces using measured total horizontal radiation and four different diffuse radiation correlations in combination with three models for sky diffuse radiation distribution. The diffuse radiation correlations of Hay (6), Orgill and Hollands (5), and of Boes et al. led to good predictions of both hourly and weekly total values of solar radiation on vertical south and vertical north surfaces. The Liu and Jordan model, on the other hand, produced large errors in the prediction of radiation incident on a vertical south surface. The use of an anisotropic assumption for diffuse sky radiation resulted in greater errors in almost all cases. Owing to the limited data on which the conclusions are based, further study of the anisotropic assumptions is recommended.

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Table I - Details of Weekly Periods

Week	Daytime Hours	Sunshine Hours	Ground Reflect.
24 Nov	69	11.6	0.5
22 Dec	69	17.1	0.7
5 Jan	67	37.5	0.7
1 Feb	65	18.1	0.7
9 Mar	91	27.3	0.3
13 Apr	100	47.5	0.2
27 Apr	105	62.2	0.2

Table II(a) - Comparison of Hourly Radiation on Vertical South Surface

Week	Average Absolute Error, MJ/m² Hay	Orgill & Hollands	Liu and Jordon	Boes
24 Nov	0.1	0.08	0.14	0.1
22 Dec	0.12	0.14	0.24	0.17
5 Jan	0.22	0.32	0.36	0.29
4 Feb	0.13	0.18	0.28	0.24
9 Mar	0.15	0.12	0.17	0.16
13 Apr	0.06	0.06	0.07	0.07
27 Apr	0.07	0.08	0.08	0.08
Av.	0.12	0.13	0.18	0.15

Table II(b) - Comparison of Hourly Radiation on Vertical North Surface

Week	Average Absolute Error, MJ/m² Hay	Orgill & Hollands	Liu and Jordan	Boes
24 Nov	0.027	0.02	0.019	0.021
22 Dec	0.046	0.029	0.041	0.038
5 Jan	0.071	0.045	0.048	0.048
1 Feb	0.078	0.032	0.034	0.04
9 Mar	0.052	0.081	0.06	0.064
13 Apr	0.075	0.065	0.054	0.059
27 Apr	0.11	0.086	0.078	0.072
Av.	0.068	0.055	0.050	0.051

Table III(a) - Comparison of Hourly Radiation on Vertical South Surface

	Percentage Time when Error Less than Threshold							
	Hay		Orgill & Hollands		Liu and Jordan		Boes	
Week	0.1*	0.2	0.1	0.2	0.1	0.2	0.1	0.2
24 Nov	71.0	85.5	82.6	89.9	56.5	75.4	71.0	81.2
22 Dec	63.8	78.3	63.8	72.5	36.2	44.9	47.8	66.7
5 Jan	35.8	56.7	35.8	49.2	26.9	41.8	35.8	53.7
1 Feb	55.4	70.8	49.2	58.5	33.8	43.1	44.6	52.3
9 Mar	47.2	68.1	52.7	76.9	45.1	62.6	45.1	70.3
13 Apr	78.0	96.0	75.0	100.0	69.0	95.0	68.0	96.0
27 Apr	67.6	96.2	65.7	95.2	63.8	95.2	63.8	95.2
Av.	61.0	80.6	61.7	80.0	49.7	69.1	55.0	76.3

* Threshold values, MJ/m²

Table III(b) - Comparison of Hourly Radiation on Vertical North Surface

	Percentage Time when Error Less than Threshold							
	Hay		Orgill & Hollands		Liu and Jordan		Boes	
Week	0.05*	0.1	0.05	0.1	0.05	0.1	0.05	0.1
24 Nov	86.9	91.3	88.4	95.7	89.9	98.6	89.9	97.1
22 Dec	72.5	85.5	82.6	91.3	65.2	94.2	71.0	89.9
5 Jan	50.7	68.7	64.2	82.1	68.7	85.1	65.7	79.1
1 Feb	64.6	70.8	76.9	93.8	73.8	92.3	72.3	86.2
9 Mar	64.8	83.5	47.2	67.0	60.4	84.6	51.6	78.0
13 Apr	55.0	72.0	49.0	69.0	54.0	82.0	52.0	77.0
27 Apr	37.1	60.0	39.0	67.6	47.6	77.1	44.8	69.5
Av.	59.9	75.1	60.8	78.8	63.6	86.6	61.5	81.1

* Threshold values, MJ/m²

Table IV(a) - Comparison of Sky Diffuse Distribution Models for Vertical South Surface

Model	7-Week Average Absolute Error, MJ/m² Hay	Orgill & Hollands	Boes	Liu and Jordan
Isotropic model	0.12	0.13	0.15	0.18
Hay anisotropic model	0.14	0.18	0.28	0.34
Klucher anisotropic model	0.15	0.18	0.25	0.31

Table IV(b) - Comparison of Sky Diffuse Distribution Models for Vertical North Surface

Model	7-Week Average Absolute Error, MJ/m² Hay	Orgill & Hollands	Boes	Liu and Jordan
Isotropic model	0.068	0.055	0.051	0.050
Hay anisotropic model	0.053	0.062	0.073	0.074
Klucher anisotropic model	0.099	0.074	0.063	0.055

Table V(a) - Comparison of Weekly Total Radiation on Vertical South Surface

Week	Percent Error					Meas-ured Value MJ/m^2
	Hay Aniso-tropic	Hay Iso-tropic	Orgill & Hollands Iso-tropic	Boes et al. Iso-tropic	Liu & Jordan Iso-tropic	
24 Nov	26.8	9.9	13.0	15.7	31.4	31.3
22 Dec	17.7	1.1	13.6	18.7	28.9	56.1
5 Jan	11.1	-6.5	9.7	7.9	17.5	99.3
1 Feb	15.2	0.2	18.3	23.8	28.2	62.2
9 Mar	27.6	17.5	13.1	19.9	22.6	64.3
13 Apr	8.6	6.7	7.9	8.1	8.2	67.3
27 Apr	5.3	6.3	7.3	6.5	6.4	86.2
Av.	14.3	4.0	11.3	13.2	18.5	466.7

Table V(b) - Comparison of Weekly Total Radiation on Vertical North Surface

Week	Percent Error					Meas-ured Value MJ/m^2
	Hay Aniso-tropic	Hay Iso-tropic	Orgill & Hollands Iso-tropic	Boes et al. Iso-tropic	Liu & Jordan Iso-tropic	
24 Nov	1.9	1.1	9.3	5.9	-0.7	12.0
22 Dec	-1.5	7.5	0.1	-3.5	-7.9	20.0
5 Jan	0.6	15.5	0.5	0.5	-5.3	24.3
1 Feb	9.3	21.5	6.1	1.4	-1.2	22.7
9 Mar	-11.4	4.4	11.1	0.2	-3.3	29.0
13 Apr	-10.0	13.8	12.6	3.9	2.3	28.4
27 Apr	-5.0	20.3	11.4	-1.2	4.0	40.0
Av.	-3.4	14.0	7.9	0.6	-1.1	176.4