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Gold, L. W.

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BY

L. W. GOLD

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CRACK FORMATION IN ICE PLATES BY THERMAL SHOCK

L. W. Gold

N.R.C. No. 7548

CRACK FORMATION IN ICE PLATES BY THERMAL SHOCK*

L. W. GOLD
Snow and Ice Section, Division of Building Research, National Research Council, Ottawa, Canada
Received December 28, 1963

ABSTRACT

Cracks were formed in the surface of smooth ice blocks by bringing them into contact with a second colder ice block. The temperature change within each block and the associated thermal stress were calculated. The ultimate strength of a smooth ice surface was found to be between 30 and 40 kg/cm^2. The surface temperature shock necessary to produce this stress was about 6° C. There was a marked preference for the cracks to form parallel to the basal and prismatic planes.

Maximum depth of crack penetration was obtained with ice blocks made up of only one or two crystals with their C axis perpendicular to the surface. The minimum observed value for the strain energy release rate at crack arrest was calculated to be between 150 and 200 ergs/cm^2 for each cm^2 of new crack surface, indicating that for the crystallographic orientation and stress distribution of those experiments there was relatively little plastic deformation at the crack tip. The associated crack edge stress intensity factor was between 3.0×10^6 and 3.5×10^6 dynes cm$^{-3/2}$. It was observed that the calculated strain energy release rate at crack arrest increased with decreasing average size of segments formed by the cracks. It is considered that the calculated strain energy release rates for these cases may exceed the true values because of reduction in the stress between the surface and the bottom of the crack prior to crack arrest.

During an investigation on crack formation in ice during creep under a constant compressive load the question arose as to whether there was a dependence of crack formation on crystallographic orientation (Gold 1960). Since that time further observations have shown that during creep the cracks tend to form either parallel or perpendicular to the basal plane. Experiments were designed to show whether this dependence still occurs if the stress required to cause cracks to form is developed quickly. This was accomplished by bringing an ice plate at $-10°$ C into intimate contact with a brass plate cooled with dry ice. The cracks that resulted formed within 1 second of bringing the two plates together. With plates cut from single crystals in such a way that the crystallographic axis of hexagonal symmetry was either perpendicular or parallel to the plate surface, it was observed that the cracks preferred to form in the basal or prismatic planes (Gold 1961). Under this condition of stressing, ice does exhibit cleavage planes.

If two plates made from the same material, but conditioned to different temperatures, are brought into contact, the temperature at their interface immediately assumes a value midway between the temperatures of the plates. This temperature shock is well defined. If the plates are reasonably thick, the dependence of the temperature on time and distance from the interface can be readily calculated. In addition, an approximate solution for the resulting thermal stress distribution can be obtained. It is possible to determine, there-

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fore, the minimum temperature difference necessary to cause cracks to form and so to calculate the associated surface stress. With a knowledge of the crack geometry and of the thermal stress at the time the cracks formed, conclusions can be drawn as to the strain energy released during their formation. This paper reports such experimental observations made on ice, and gives, as well, additional evidence on the dependence of crack formation on crystallographic orientation observed with multigrained ice plates.

PREPARATION OF ICE

Ice was made by unidirectional freezing of water in a galvanized steel tank about 60 cm in diameter. The water was taken from the normal drinking supply; no attempt was made to purify it further; it was de-aerated with an aspirator before being placed in the tank. When the surface of the water had cooled to 0° C, it was seeded with ice grains about 1 mm or less in diameter. Because ice crystals grow more readily in the direction normal to the axis of hexagonal symmetry than parallel to it, a bias soon developed in the crystallographic orientation of the grains such that the symmetry axis tended to be perpendicular to the direction of freezing. The ice that formed was columnar, some grains extending through the full thickness of the ice (10 or more cm). The ice had an average grain diameter, as determined by the linear intercept method, between 0.15 and 0.60 cm. If the water was allowed to freeze without seeding, very large grains were obtained, some of average grain size of 15 to 20 cm.

Blocks about 15 by 18 cm and 10 cm thick were cut from the ice so that the long axis of the grains was perpendicular to the 15×18-cm face. This face was then machined with a special milling machine with a spiral cutter about 15 cm long. The final thickness of the blocks was 9 cm; they were immediately stored in kerosene to prevent deterioration of the surface by evaporation.

EXPERIMENTAL PROCEDURE

The technique used for applying a thermal shock was as follows. One block was kept in kerosene in a cold room at a temperature of about −10° C. A second block was placed in kerosene in a portable insulated box and cooled to some temperature lower than −10° C in a refrigerated compartment. The temperature of the blocks was maintained constant to ±0.1° C for at least 24 hours before a test. Just before a test the top of the insulated box in the refrigerated compartment was removed and a tap opened which allowed the kerosene level to lower until it was just below the machined surface of the block. The top of the box was immediately replaced and the box taken into the cold room. The block in the cold room was removed from its kerosene and quickly wiped with tissue at the same temperature as the block. The top of the insulated box was then removed and the two blocks brought immediately into contact. Just before removing the insulated box from the refrigerated compartment, and the block in the cold room from its kerosene, the temperature of the kerosene in each case was measured with a thermocouple to ±0.05° C.

Crack formation was observed visually and, once initiated, was completed within a fraction of a second. The time to crack initiation after the blocks were brought into contact was estimated to the nearest second by counting. The blocks were separated immediately after cracks formed. There was no visual or audible evidence of crack deepening prior or subsequent to separation of the blocks once the cracks had formed. In the earlier experiments in which the thermal shock was induced with a brass plate cooled with dry ice, crack deepening after the initial formation of the crack pattern was clearly audible. If a crack did not form in the cooled block within 10 seconds of bringing them together, the blocks were separated and returned to their respective containers. The temperature of the cold block was then lowered, and after at least 48 hours the test was repeated.

SOLUTION OF THE THERMAL PROBLEM

If two perfectly flat plates at different but constant temperatures are brought together, ideally the temperature at the interface will immediately assume a value midway between the two. In practice, surfaces are not perfectly flat and consequently ideal contact is not achieved. As a result, the temperature of the surface of the warmer plate will be higher than the midpoint temperature and that of the colder plate lower. With increasing time, the two surfaces gradually attain this temperature. The time-dependence of the temperature in each plate is determined, in part, by the nature of the contact.

In the present experiments a film of kerosene was intentionally left on each surface so as to ensure uniform thermal contact and to establish over the contact area a film of known thermal properties. Normally the contact between blocks was such that the kerosene wetted each surface completely. Occasionally, air bubbles were trapped between the two surfaces; this was considered a poor contact condition.

It was observed that when the contact was good a strip of metal 1.25×10^{-3} cm thick could be slid between the two blocks, but that one 3.8×10^{-3} cm thick could not. It was therefore assumed that the separation between the blocks was 2.5×10^{-3} cm. Similarly it was found that the separation between two surfaces in poor contact was about 5.0×10^{-3} cm.

Consider a semi-infinite medium of thermal conductivity k_i and diffusivity λ initially at a constant temperature T_i. Let there be a film of thickness z_0 and conductivity k_f on its surface. Carslaw and Jaeger (1959) have shown that if the temperature of the surface of the film is suddenly changed to the value T_f, the time-dependence and distribution of temperature in the solid are given by a solution of

$$\partial T/\partial t = \lambda \cdot \partial^2 T/\partial z^2$$

subject to certain conditions at the interface between the film and the medium ($z = 0$). For the experiment under discussion these conditions require that the temperature gradient in the ice at its surface be

$$(\partial T/\partial t)_{z=0} = -h(T_f - T_s),$$

where $h = k_f/k_i Z_0$,
Z_0 = half the thickness of the kerosene layer separating the two ice blocks,
T_s = temperature at the ice–kerosene interface,
T_f = temperature at the midplane of the kerosene layer.

The solution can be expressed in the following convenient form:

$$(1)\qquad \frac{T(z,t)}{T_f - T_i} = \frac{T(z,t)}{\Delta T} = \operatorname{erfc}\left[\frac{z}{2\sqrt{\lambda t}}\right] - \exp(hz + h^2\lambda t)\operatorname{erfc}\left[\frac{z}{2\sqrt{\lambda t}} + h\sqrt{\lambda t}\right],$$

where $T(z, t)$ = the change in the temperature at time t and depth z,
and erfc = the complementary error function.
The following values were used for the constants:
$k_i = 56.0\times10^{-4}$ gm-cal/cm sec °C -15° C (Jakob 1949),
$\lambda = 0.011$ cm²/sec (Dorsey 1940),
$k_f = 3.80\times10^{-4}$ gm-cal/cm sec °C at -15° C (extrapolated value using Smithsonian table),
$Z_0 = 1.25\times10^{-3}$ cm (good contact),
$= 2.50\times10^{-3}$ cm (poor contact).
The values obtained for h were
good contact: 54.3 cm^{-1},
poor contact: 27.1 cm^{-1}.

Using equation (1) it can be calculated that when t = 10 seconds the thermal wave will have propagated a distance of less than 1.5 cm in from the interface. This shows that for the purpose of temperature calculations an ice block 9 cm thick can be assumed to behave as a semi-infinite medium for the first 10 seconds after the application of a temperature shock to its surface.

SOLUTION OF THE THERMAL STRESS PROBLEM

Consider a plate of thickness d with its surface in the x–y plane, the positive z direction being into the plate. Suppose that the plate undergoes a temperature change $T(z, t)$ that depends only on the time and the distance z from the surface. Timoshenko and Goodier (1951) give the following solution for the thermal stress developed in the plate at time t and depth Z well away from a free edge:

$$(2)\qquad S_{xx} = S_{yy} = -BT(z,t) + \frac{B}{d}\int_0^d T(z,t)dz + \frac{12(z-d/2)}{d^3}B\int_0^d T(z,t)(z-d/2)dz,$$

where $B = E\alpha/(1-\sigma)$,
E = Young's modulus,
σ = Poisson's ratio,
α = coefficient of linear expansion.

The leading term on the right-hand side is the thermal stress that would be present if the temperature change $T(z, t)$ occurred in a semi-infinite medium. The second term is the adjustment due to the boundary condition of zero normal stress at the edges of the plate of thickness d. Similarly, the third term is the adjustment due to the boundary condition of zero bending moment at the edges.

JUSTIFICATION FOR APPLYING THE PLATE THEORY

Consider the problem shown in Fig. 1. A constant pressure P is applied over a strip of width a at the top of a very long slot of depth b. Lachenbruch (1961) has tabulated approximate values for P_{yy}/P at various distances y/b from the edge of the slot for various values of a/b, where P_{yy} is the stress at the surface and normal to the slot. Using his numerical results, it is possible to estimate by superposition the stress P_{yy} at the surface due to an arbitrary stress $P(z, t)$ applied over the face of the slot.

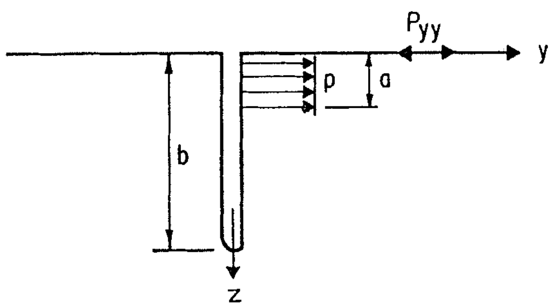

FIG. 1. Constant pressure "p" applied to edge of a long slot.

Consider now a semi-infinite solid with a temperature distribution $T(z, t)$ given by equation (1). The stress that is induced by this temperature distribution is

$$S_{xx} = S_{yy} = -BT(z, t). \tag{3}$$

Suppose now that it were possible to cut a square into the surface of the solid by making four cuts normal to the surface and to a depth b. The stress normal to the surface of the cuts must be zero. In order to satisfy this boundary condition, it is sufficient to add to the stress for the semi-infinite case the stresses P_{yy} and P_{xx} due to a force $P(z, t) = BT(z, t)$ applied to the surface of the cuts. Using the results of Lachenbruch's analysis, it is possible to calculate the stress at the surface of the square due to the four edge loads. This calculation should be reasonably accurate as long as it is not carried too close to the corners. The stress over the surface of the square is then obtained by adding the stress due to the supposed edge loads to that given in equation (3).

This calculation was made for ice for cuts 15 cm long and 9 cm deep and time $t = 2$ seconds. It was found that for a cut 9 cm deep or greater, the contribution of the edge load to the surface stress was independent of the depth of the cut b, that is, the block could be considered as a very long column. It was observed also that for distances away from the edge greater than 3.5 cm the stress was everywhere less than 5% lower than that calculated using the finite plate equation, the stress being maximum at the center. Equation (2) was therefore assumed valid and was used to calculate the stress in each experiment and the associated strain energy density.

Substituting equation (1) into (2), putting $d = 9$ cm, and neglecting terms of magnitude less than 1% of $BT(z, t)$, the following expression was obtained:

$$(4) \qquad \frac{S}{B\Delta T} = -\left[\operatorname{erfc}\frac{z}{2\sqrt{\lambda t}} - \exp(hz+h^2\lambda t)\operatorname{erfc}\left(\frac{z}{2\sqrt{\lambda t}}+h\sqrt{\lambda t}\right)\right] +0.125\sqrt{\lambda t}+1.65\times10^{-2}(z-4.5)(\lambda t-5.08\sqrt{\lambda t}).$$

This equation is plotted in Fig. 2 as a function of the time t for $z = 0$ and $h = 54.3$ cm^{-1} and 27.1 cm^{-1}. From Fig. 2 it may be seen that the stress at the plate surface rises to a maximum within about 2 seconds of contact for good contact, and within 5 seconds for poor. The maximum calculated stress is 10 to 20% lower than $B\Delta T$, i.e. that which would develop at the surface of a semi-infinite solid. The actual stress is probably within 5% of this value.

OBSERVATIONS

Thirty-four tests were conducted; details are given in Table I. The blank spaces in the time column indicate that no cracks formed during the test. An asterisk indicates that the cracks formed when the blocks were being separated,

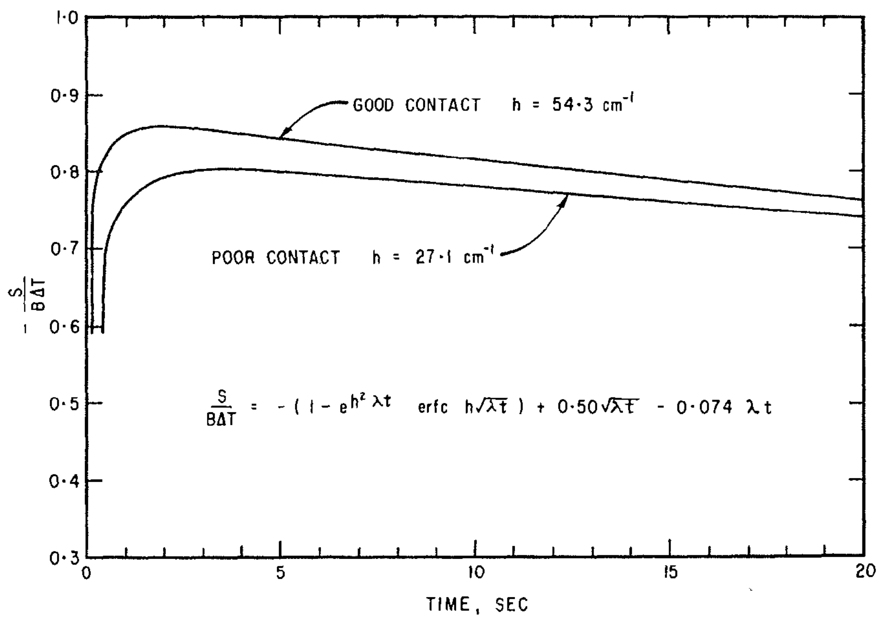

FIG. 2. Dependence of surface stress on time.

TABLE I

Thermal shock, ΔT, applied to ice surface and other relevant information for each test

Test No.	ΔT, °C	Contact	Time to formation of crack, sec	Average grain size, cm	Calculated maximum stress, kg/cm^2	Crack depth, cm
1	3.50	Good	*	$C\perp$†	21.6	
2	4.80	Good		$C\perp$	29.7	
3	4.90	Poor	*	$C\perp$	28.2	
4	5.35	Good	*	$C\perp$	33.0	
5	5.85	Good	2	$C\perp$	36.1	0.84
6	6.30	Good	2	$C\perp$	38.8	0.70
7	6.50	Poor	‡		37.5	
8	6.40	Good		$C\|$	39.6	
9	6.25	Good	2–3	0.35	38.5	0.29
10	6.45	Good	*	0.39	39.9	
11	6.90	Good	2–3	0.53	42.6	0.33
12	6.85	Good	2	$C\|$	42.3	0.45
13	6.90	Poor		$C\perp$	39.8	
14	6.85	Poor		0.15	39.5	
15	6.75	Good	1	2.32, $C\|$ and $\perp$	41.2	0.39
16	6.85	Good	4–5	$C\|$	41.6	0.61
17	6.85	Good	4–5	$C\perp$	41.6	0.90
18	7.30	Good	2–3	0.15	45.0	0.25
19	7.45	Good	2–3	1.80, $C\|$ and $\perp$	46.0	0.26
20	7.40	Poor	6–7	$C\perp$	42.1	0.55
21	6.25	Good	3–4	0.30	38.2	0.28
22	6.30	Good	3–4	2.48	38.5	0.29
23	6.30	Good	3–4	0.15	38.5	0.24
24	5.30	Poor		0.52	30.6	
25	5.30	Poor		0.34	30.6	
26	5.35	Poor	3–4	2.54	30.7	0.40
27	5.40	Good		0.52	33.3	
28	6.40	Good	3–4	0.52	39.1	0.34
29	6.40	Poor		0.34	36.9	
30	6.50	Poor	*	0.30	37.5	
31	6.30	Good		0.25	38.8	
32	6.30	Good		0.50	38.8	
33	6.40	Poor		0.34	36.9	
34	6.90	Good	2–3	0.34	42.6	0.37

*Indicates crack formed while separating blocks.
†$C\perp$ indicates symmetry axis perpendicular to shocked surface, and $C\|$, parallel.
‡A blank space in time-to-formation column indicates that no cracks formed.

after being in contact for at least 10 seconds. It was often difficult to separate the blocks, and normally it was necessary to twist them in such a way as to allow air to enter the interface at one edge. If cracks formed during separation the test was counted as no cracks forming due to thermal stress. In the column "average grain size," $C\perp$ indicates that the block was either a single crystal or perhaps two or three very large crystals with the C axis of symmetry perpendicular to the surface for each crystal. Similarly $C\|$ (in the same column) indicates single or large crystals with the C axis parallel to the stressed surface. Tests 15 and 19 were conducted on the two faces of a block made up of large grains with the direction of the symmetry axis of each grain either parallel or perpendicular to the surface. When only the average grain size is given there is a bias in the crystallographic orientation, as described earlier.

If cracks did not form in a surface during a test, this surface was tested again at a later date and at a different ΔT. Tests with the same average grain size, such as numbers 25, 29, 33 or 24, 27, 28, indicate repeated thermal shocking of the same surface.

The calculation of the surface stress raises a question concerning the choice of the value for Young's Modulus E and Poisson's ratio σ. Earlier work by Gold (1958) had shown that for uniaxial loading normal to the long axis of the grains in columnar grained ice E and σ are temperature-dependent. This dependence is probably associated with the relaxation phenomenon.

For a single crystal with the load applied perpendicular to the symmetry axis, the values of E and σ were found to have negligible temperature-dependence and were equal to those for multigrained ice at $-40°$ C. For the present problem, because the stresses S_{xx} and S_{yy} parallel to the surface of the plate were almost equal for distances away from the edge greater than 3.5 cm, it was considered that little relaxation of the stress would occur. This was considered reasonable, because the crystallographic symmetry axis of each grain was always almost parallel or perpendicular to the ice surface and the long direction of the columnar grains parallel to the z direction. Consequently, prior to crack formation, there was very little or no shear stress on planes upon which stress relaxation could take place during the period of stressing. E and σ were therefore assumed to be equal to the values obtained for single crystals and are given below along with the value used for the coefficient of linear expansion α:

$$
\begin{aligned}
E &= 8.34\times10^{10}\ \text{dynes/cm}^2 && \text{(Gold 1958),}\\
\sigma &= 0.40 && \text{(Gold 1958),}\\
\alpha &= (T = -15°\ \text{C})\ 50.9\times10^{-6}/°\ \text{C} && \text{(Butkovitch 1957),}\\
B &= E\alpha/(1-\sigma) = 7.07\times10^{6}\ \text{dynes/cm}^2\ °\ \text{C}.
\end{aligned}
$$

The value for E is about 6% lower than that obtained by sonic methods.

The surface stress at the time of the formation of the cracks was calculated with equation (4). If cracks did not form it was considered that the ice strength must exceed the maximum stress that developed. The calculated stresses are given in Table I. In Table II the stresses are grouped in steps of 5 kg/cm²

TABLE II

Number of tests in which cracks formed or did not form in the given stress interval

	Stress range, kg/cm²:					
	20–25	25–30	30–35	35–40	40–45	45–50
Number tested	1	2	5	17	7	2
Number that cracked			1	7	7	2
Number that did not crack	1	2	4	10		

and the number of tests for which cracks did or did not form is given. Although the number of tests is not sufficient to warrant a statistical analysis of the observations, they do show that for the conditions of the experiments, the tensile strength of a smooth ice surface in contact with kerosene was quite well defined and was between 30 to 40 kg/cm².

The observations show the statistical nature of failure. This was further demonstrated by the crack patterns that generally developed in the multi-grained ice. An example of this pattern is shown in Fig. 3, where the branching character is quite evident. It was generally possible to trace back along the cracks to a site where failure appeared to have started. Miles and Clarke (1961) observed the same behavior with cracks formed in crystals of magnesium oxide by thermal shock. The site for the specimen in Fig. 3 is located with an arrow. Crack branching appeared to occur primarily at grain boundaries.

DEPENDENCE OF CRACK FORMATION ON CRYSTALLOGRAPHIC ORIENTATION

To determine whether there was a dependence of crack formation on crystallographic orientation, etch pits were developed on the surface using the technique of Higuchi (1958). The sides of these pits are usually parallel to the trace of the basal and prismatic planes at the surface. The shape of the pit is rectangular, trapezoidal, or triangular, depending on the angle that the symmetry axis makes with the surface. Sometimes the shape is further complicated by etching on higher-order planes. In many cases it is possible to tell which edge of the pit is parallel to the basal plane and, therefore, to locate the plane containing the C axis. The shape of the pit can be used also to estimate the angle that the C axis makes with the surface.

A slot about 0.5 cm wide and of length exceeding 15 cm was cut in a piece of paper and laid over the etched surface. Each crack in the area exposed by the slot was observed with a microscope and its direction relative to the edges of the etch pits on the surface of grains that it traversed was noted.

For large-grained ice specimens (grain size 2 cm or larger) the dependence of crack formation on crystallographic orientation was obvious. Figure 4 shows the cracks formed in a thin plate cut from ice prepared in the same way as that for tests Nos. 15 and 19, in which the symmetry axis for each grain was either parallel or perpendicular to the surface. The plate was photographed with normal light to show the crack pattern and with polarized light to show the grains. It was observed that in those grains where the C axis was almost perpendicular to the surface the crack pattern was triangular; in those grains where the C axis was almost parallel to the surface the pattern was rectangular. The cracks often change direction abruptly on intersecting a boundary in order to be parallel or perpendicular to the basal plane.

For fine-grained ice it was observed that a strong dependence of crack formation on crystallographic orientation still existed even for the average smallest grain size tested, 0.15 cm. A total of 487 cracks was observed. Nineteen per cent were at grain boundaries, 67% were either parallel or perpendicular to the basal plane, and 14% had no obvious dependence on crystallographic orientation. It was often difficult to determine which edge of the etch pits was parallel to the basal plane and thus to determine what percentage of the cracks was parallel and what percentage perpendicular to it. In cases that were unambiguous, there was no marked preference for one orientation to predominate. Examples of the observed dependence of crack formation on

PLATE I

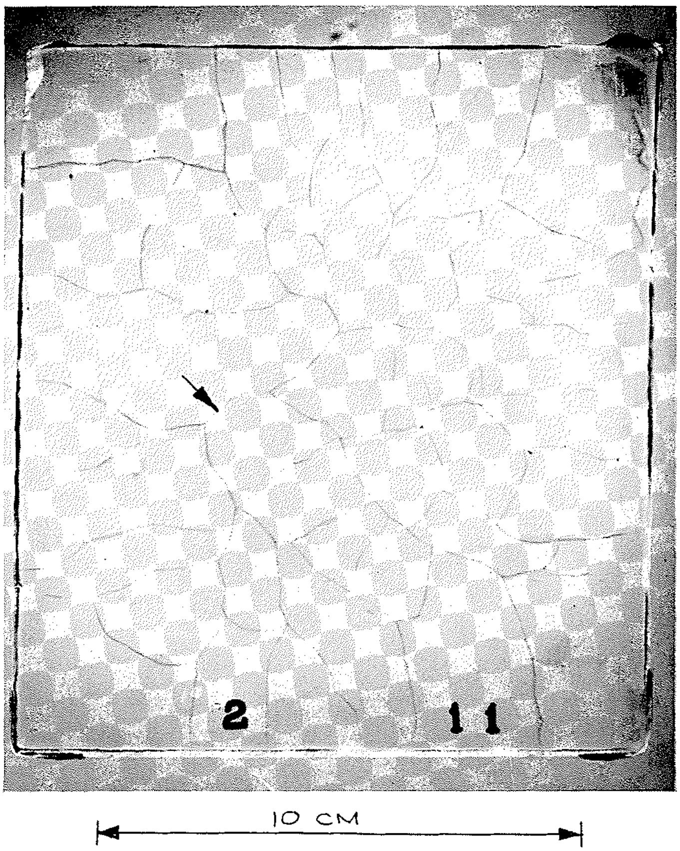

FIG. 3. Typical crack pattern. Arrow locates probable site at which crack nucleated.

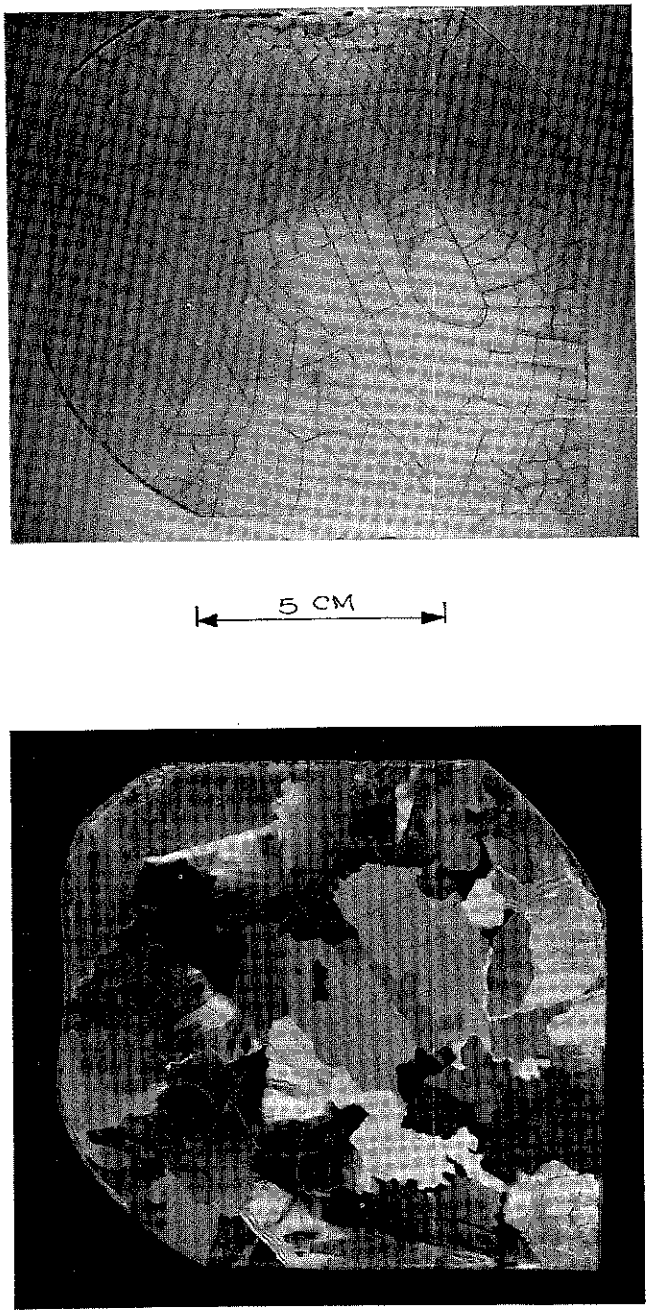

FIG. 4. Crack pattern in large-grained ice plate viewed with normal and polarized light. Symmetry axis of grains with rectangular pattern is almost parallel to surface, with triangular pattern almost perpendicular.

(a)

(b)

FIG. 5. Crack formation in multigrained ice. (a) Crack in lower left grain, parallel to basal plane, strikes grain boundary and branches into two cracks, one parallel and the other perpendicular to basal plane. (b) Cracks curve at grain boundaries to be parallel or perpendicular to basal plane in each grain.

crystallographic orientation are shown in Figs. 5(*a*) and (*b*). In Fig. 5(*a*) a crack in the basal plane of the grain in the lower left-hand corner strikes the grain boundary and branches into two cracks, one parallel and the other perpendicular to the basal plane. In Fig. 5(*b*) two cracks are present. The one running from bottom to top is parallel to one edge of the rectangular pits in the bottom grain. It changes direction very slightly at the grain boundary so as to be parallel to one edge of the triangular pits in the left center grain. This edge is probably parallel to one of the prismatic planes. On striking the boundary of the grain in the upper left-hand corner the crack curves so as to be in the basal plane. The crack coming from the right is parallel to one edge of the rectangular pits of the center grain, and, on striking the grain boundary, curves in the center left grain into the first crack.

Cracks that were parallel to the basal or prismatic planes were usually straight. Those that were not in these planes or at grain boundaries were usually curved. The surfaces of some of the straight cracks were examined. They were found to be very smooth, particularly those in the basal plane, often with the striated features associated with the surface of cleavage cracks.

CRACK DEPTH AND CRACK AREA

The crack patterns that developed in each test were photographed. The actual total length of crack L that formed in a center square portion of area A equal to about 100 cm^2 was determined from the photographs. The segments into which the cracks divided this area were counted and the average area per segment, $a = A/n$, determined.

The blocks were sectioned perpendicular to the cracked surface and the depth of the cracks d measured with a microscope. In Fig. 6 the average depth for each test is plotted against $\sqrt{(A/n)}$. For the multigrained ice,

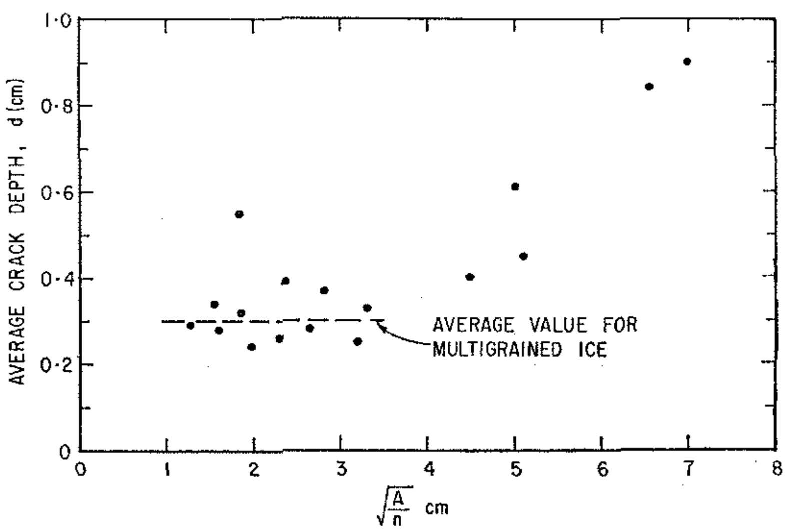

FIG. 6. The dependence of the average crack depth on the size of the segments formed by the cracks; n is the number of segments in area A of surface.

$\sqrt{(A/n)}$ was less than 4 cm. If the blocks were composed of one or two crystals it was greater than 4 cm except for one case when it was equal to 1.83 cm. For the multigrained ice, the crack depth was between 0.24 and 0.39 cm, with an average of 0.30. Relatively few cracks formed in the single crystals or in blocks made up of only two or three large grains. In these cases the cracks were usually straight and associated with basal or prismatic planes. They often traversed the full width of the plate. Such cracks would penetrate deeper into the ice, as shown in Fig. 6. The anomalous case at $\sqrt{(A/n)} = 1.83$ was in fact a block made up of a large crystal with its C axis perpendicular to the surface. Extensive crack branching occurred in this block, but the depth was greater than for the multigrained cases. The measured average crack depths are given in Table I.

The total measured crack length L for each test was multiplied by the average crack depth d and divided by the area A to obtain the average crack area per unit area of surface, D. For $\sqrt{(A/n)}$ greater than 3.5 cm, D was approximately constant with a value about 0.1 cm^2/cm^2. For $\sqrt{(A/n)}$ less than about 3 cm, D increased with decreasing $\sqrt{(A/n)}$ to a value of about 0.3 cm^2/cm^2 for $\sqrt{(A/n)}$ equal to 1.25 cm. It should be kept in mind that the amount of new crack surface that is formed per unit area is twice D.

ENERGY RELEASE RATE DURING CRACK PROPAGATION

It is often assumed in the initiation or arrest stage of a slowly propagating crack that the sum of the external work done on the solid in which the crack is forming and the change in elastic strain energy is equal to a constant times the increase of crack surface. Griffith (1921), in applying this hypothesis to glass, assumed the constant of proportionality to be equal to the surface energy. He obtained good agreement between theory and experiment. It was recognized that if plastic deformation occurred during crack propagation the proportionality constant would exceed the surface energy. Orowan (1955) and Irwin (1958*a*) extended the ideas of Griffith to such conditions. They, as well as others, have shown experimentally that if plastic deformation occurs the proportionality constant can exceed the surface energy of the material by a factor as large as 1000.

Sneddon (1946) has shown for an elastic solid that very near the crack tip the stresses in the plane perpendicular to the crack edge are given by

$$S = \frac{K}{\sqrt{2r}} f(\theta), \tag{5}$$

where r is the distance in the plane from the crack edge,
θ is the angle between the radius vector r and the plane containing the crack,
$f(\theta)$ is a known function of θ,
K is a constant and has been called "the crack edge stress intensity factor".

For conditions of plane strain and no external work Irwin (1958*a*) has shown that for a slowly propagating crack

$$G = \frac{(1-\sigma^2)}{E}\pi K^2, \tag{6}$$

where G is the strain energy released for unit elongation of a crack of unit depth.

Theoretical considerations and experimental observations indicate that this expression is still valid, even if there is plastic deformation at the tip of the crack, as long as the plastic strains are confined to distances from the crack edge that are small compared to the depth of the crack. This is equivalent to saying that the contribution to the strain energy change of the plastically deformed region is small compared to the contribution of the remaining part of the solid.

Expressions relating K to the applied stress and crack geometry have been obtained for a number of situations. The one of interest for the present observations on ice is that for a long crack normal to a plane surface and subject to a stress $P(z, t)$ perpendicular to its surface. This problem is not amenable to solution, but an approximate value for K can be obtained using the results of the analysis by Lachenbruch (1961).

As mentioned earlier, when the ice blocks were single crystals or composed of very large grains, the cracks were long and straight, usually extending from one boundary to another. With single crystals, only one or two cracks formed but if there were two or more large crystals, crack branching was observed, the branching sometimes occurring at the grain boundaries. The cracks in the multigrained ice were more irregular, usually straight within a grain and changing direction at or near the grain boundary. It was assumed that the stress distribution well away from the crack was that given by equation (4) and the crack edge stress intensity factor calculated using Lachenbruch's computed results. It was found that the strain energy release rate, G, increased with decreasing $\sqrt{(A/n)}$, being about 300 ergs/cm^2 for the largest value and 3500 ergs/cm^2 for the smallest. The corresponding values for the crack edge stress intensity factor, K, were about 3.0×10^6 and 10.5×10^6 dynes cm$^{-3/2}$ respectively. The block giving the lowest value of G (285 ergs/-cm^2) was a single crystal with the c axis perpendicular to the surface (No. 5 in Table I). Two parallel cracks a little over 3 cm apart formed. These cracks extended the full width of the block. The two cases of a single crystal with C axis parallel to the surface gave values for G of 815 and 1210 ergs/cm^2.

From Table I it can be seen that for the multigrained ice, including the two cases where the grains had their C axis either parallel or perpendicular to the surface, the average crack depth was about 0.30 cm and the change in temperature, ΔT, about 6.7° C. The value for K obtained with Lachenbruch's results for cracks 0.30 cm deep formed 2 seconds after the application of a temperature shock of 6.7° C, was about 9.0×10^6 dynes cm$^{-3/2}$, and for G, 2560 ergs/cm^2.

The strain energy release rate associated with 1 cm^2 of new surface at crack arrest is given by $G/2$. The calculated values of $G/2$ are plotted against $\sqrt{(A/n)}$ in Fig. 7.

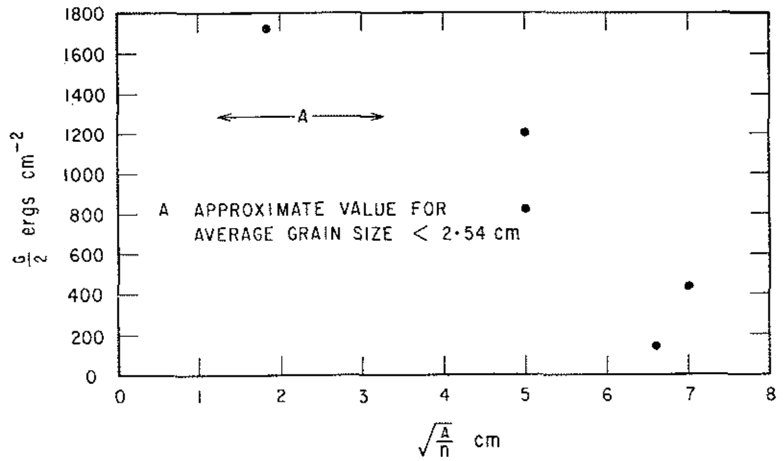

FIG. 7. Dependence on $\surd(A/n)$ of strain energy released at crack arrest for each cm² of new surface. A calculated assuming crack depth = 0.30 cm. Thermal shock = 5.85° C, cracks formed 2 seconds after contact.

For test No. 6, one crack formed about two seconds after the blocks were brought into contact. When the blocks were separated, a more extensive crack pattern developed. The calculation of the ultimate strength for this case was based on the formation of the first crack. The strain energy release rate per cm² of new surface calculated for the second event was about 1500 ergs/cm² and the value for $\surd(A/n)$ was 2.90 cm. This observation is not plotted in Fig. 7.

DISCUSSION

The thermal shock applied in the experiments established a stress distribution in which the principal stresses parallel to the shocked surface were equal over the central region of the surface, and the third principal stress was zero. The basal plane, which appears to be the only effective slip plane for ice at the temperature used in the tests, was always parallel or perpendicular to the shocked surface for grain sizes greater than 1.80 cm. There was a strong bias for it to be perpendicular to the surface for the ice with grain size less than 1 cm and for that used in tests 22 and 26 (Table I). Because of the orientation of the stress with respect to the basal plane, there was little or no shear stress developed on it prior to crack formation, and therefore it was considered that little or no creep and associated relaxation of the stress occurred. The results in Tables I and II show that for the ice used in the experiments the calculated stress required to cause cracks to form lay in a quite narrow range, and that it had no obvious dependence on grain size for grain size greater than 0.15 cm. There was a strong preference for cracks to form parallel to the basal and prismatic planes.

The observations indicate that when the segments that were formed by the cracks were large, the strain energy release rate, G, was about 300 ergs/cm². The strain energy release rate associated with unit area of new surface is one-half of G or about 150 ergs/cm^2. Skapski *et al.* (1957) give for the value of the surface energy of ice in contact with air 120 ± 10 ergs/cm^2. In the present experiments, the ice was in contact with kerosene, but it is debatable that it could maintain this contact continuously while cracks were propagating. The surface tension of the kerosene was measured by the ring method and was found to be 27 ergs/cm^2. Because kerosene wets ice, the minimum value for the surface energy of the new surface formed by the cracks would be about 93 ergs/cm^2. It would appear, therefore, that when the segments were large, the strain energy release rate at crack arrest was very nearly equal to the rate of increase of new surface times the surface energy, indicating that there was little plastic deformation at the crack edge.

With decreasing size of the segments formed by the cracks, the average crack depth decreased and the calculated strain energy release rate increased. Although this increase in the strain energy release rate at crack arrest could be attributed to increased plastic deformation at the crack edge as described by Orowan (1955), it is considered that this may not be the explanation in the present experiments for the following reason.

Consider a specimen made from a single crystal that is divided into two segments by a crack that forms in the basal or a prismatic plane. If the segments are big enough, the stress away from the new crack will not be much different from what it was prior to crack formation. Equation (4) would therefore be a reasonable approximation of the stress to use in calculating the crack edge stress intensity factor at crack arrest. If now, instead of a single crack, a multiple crack pattern develops, the disruption to the stress will be much more serious, particularly if grain boundaries are present. Shear stresses will be induced on the grain boundaries and basal planes initiating creep. With reduction in the size of the segments formed by the cracks, there will be an overlap in the perturbations in the stress caused by adjacent cracks. These factors would cause the stress near the surface to be lower than that given by equation (4). Consequently, the strain energy release rate calculated using the stress given by equation (4) would be greater than the true value. For the case of a crack 0.30 cm deep formed 2 seconds after the application of a temperature shock of 6.7° C, if the stress between the surface and the bottom of the crack is reduced by one-half, the value for K at crack arrest is about 4.5×10^6 dynes cm$^{-3/2}$ and for G 638 ergs/cm^2. If the stress is reduced to one-quarter of its initial value, the value for K is about 2.15×10^6 dynes cm$^{-3/2}$ and for G 145 ergs/cm^2. It is possible that such a reduction in stress prior to crack arrest due to deformation processes associated with crack formation could have occurred in the present experiments and been responsible for the decrease in average crack depth with decreasing $\sqrt{(A/n)}$.

One of the observed features of the cracks supports the above reasoning. The multigrained ice used in the experiments had a columnar grain structure, the long axis of the grains being perpendicular to the surface to which the thermal shock was applied. At crack arrest, the direction of propagation of the crack would be almost perpendicular to the surface, that is, in the direction of the long axis of the grains. Because the cracks preferred to form in the basal and prismatic planes, the stress distribution at the crack tip should not be much different relative to the crystallographic orientation of the grain, from what it was for the single crystal cases. Therefore, it would be expected that the amount of plastic deformation in the immediate vicinity of the crack edge would be about the same for all tests. Because the crack is propagating parallel to the long axis of the grains, it is not intercepting new grain boundaries or new grains with different orientation as in a granular material, and so an energy dissipation on this account would not be expected.

The foregoing reasoning suggests that two limiting conditions were encountered in the experiments. One was when the segments formed by the cracks were large enough and the cracks were so oriented that the stress well away from them was not much different from what it was prior to crack formation. In this case, the cracks propagated to the maximum depth of about 0.9 cm below the surface. The second was when the segments formed by the cracks were small enough that deformation associated with the crack formation process may have caused the stress in the segments to be significantly reduced prior to crack arrest. Under this condition, the depth of the crack would be reduced, the minimum for the conditions of the present experiments being about 0.30 cm.

In order to calculate the stress and strain energy associated with the thermal shock, a number of assumptions had to be made. These included the values of the elastic and thermal constants for ice and kerosene and the validity of equation (4). In addition, the experiment was not designed to measure the time as accurately as would have been desirable for the calculations. Two seconds after a thermal shock of 5.85° C is applied under good contact conditions, the value for K 0.9 cm below the surface is increasing at the rate of about 0.8×10^{6} dynes $\mathrm{cm}^{-3/2}$ per second. These possible sources of error should be kept in mind when considering the foregoing discussion. Values for the ultimate strength of ice obtained by Weeks (1962) using the ring tensile test, however, cover the same range as determined in the present experiments. Because the value of G for ice in contact with air is only about 25% greater than for ice in contact with kerosene, the value for K would be only about 10% larger. Therefore it would be expected that the ultimate strengths determined by both techniques should be almost equal if the formulae used for their calculations are valid.

The value of about 300 ergs/cm² obtained for the minimum value of G for ice is small in comparison with values determined for other materials. For example, Irwin (1957) quotes the following approximate values: lantern slide glass in moist air, 7×10^{3} ergs/cm²; ship plate steel, 15×10^{6} ergs/cm²; aluminum 24ST, 10^{8} ergs/cm². Kaplan (1961) found, for concrete, values ranging between 7×10^{3} and 2×10^{4} ergs/cm².

It may be useful to point out that the special case of thermal stressing used in the present observations is just one example of a family of problems in which the dimensional change of a solid is due to a physical process described by the diffusion equation. Another example is the absorption of water vapor by some solids. In fact, the absorption of water by some photoelastic materials, which obeys the diffusion theory as can be seen from observations presented by Mantle (1959), may provide a useful model for studying thermal stresses in the vicinity of cracks under various boundary conditions.

The present experiments demonstrate the important effect that surface characteristics and crystallographic orientation can have on the failure behavior of ice. Ice, normally considered a plastic material, can be loaded in such a way that very little or no plastic deformation occurs and the deformation behavior is that associated with a brittle solid. In the present experiments, ice surfaces maintained for an appreciable period of time without cracks forming a strain energy density considerably greater than that necessary for crack propagation. This is probably due to the small width and depth of surface imperfections that were present. Because ice is slightly soluble in kerosene, considerable smoothing of the surface takes place during storage for 48 hours or more at $-10°$ C. There would be little opportunity for surface imperfections present to grow to a critical size during the period of stressing because dislocations were not free to move appreciably under the stress distribution applied. Using the results of the present experiments in an equation obtained by Irwin (1958*b*) indicated that the maximum depth of surface imperfections was about 6.3×10^{-3} cm, a not unreasonable value for the surfaces used in the experiments. The strain energy release rate at the time of crack initiation was about 2000 ergs/cm^2.

A brittle behavior induced in ice by stress distribution is probably not confined to laboratory experiments. Ice on lakes and rivers is normally subjected to temperature changes that produce a stress distribution where the principal stresses parallel to the surface are equal and that perpendicular to it is zero. In addition, there is often a preferred crystallographic orientation with the basal planes of the ice grains parallel or perpendicular to the surface. Such a stress system would suppress plastic deformation and associated stress relaxation, and would influence the load-bearing capability of the ice cover.

CONCLUSIONS

Observations on crack formation in ice plates by thermal shock show that there is a marked dependence of the direction of crack formation on crystallographic orientation. For the condition of the surface obtained in the experiments, the ultimate strength was found to be between 30 and 40 kg/cm^2. The temperature shock at the surface necessary to produce this stress was about 6° C.

The calculated strain energy release rate at crack arrest was found to increase with decrease in the size of the segments formed by the cracks. The minimum value for the strain energy release rate was between 250 and 400 ergs/cm^2, indicating that there was little plastic deformation at the crack tip. The corresponding value for the crack edge stress intensity factor was between 2.8×10^6 and 3.5×10^6 dynes cm$^{-3/2}$. The maximum calculated values for the strain energy release rate were associated with the smaller size of crack segments and were between 2000 and 4000 ergs/cm^2. It is considered that with decreasing segment size the deformation associated with crack formation may have caused the stress between the surface and the bottom of the crack to be reduced prior to crack arrest. Consequently, the strain energy release rates at crack arrest given for these situations in this paper would exceed the true values.

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