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RÉSUMÉ

On présente ici un modèle de crêtes de pression en glace flottante. Un coin symétrique à deux dimensions est censé représenter la voile (ou la quille) d'une crête. Les auteurs se servent du modèle stochastique de transfert des contraintes dans les milieux particulaires et des équations d'équilibre pour déterminer la répartition des contraintes. Ils ne tiennent pas compte de la cinématique du problème dans leur analyse. Ils utilisent le critère de Mohr-Coulomb pour déterminer les zones de rupture et, en conséquence, les hauteur et profondeur limites de la crête. Le modèle prévoit la poussée exercée sur la couverture de glace en rapport avec la hauteur et la profondeur limites de la crête, et il fournit des informations concernant la géométrie de celle-ci. Les résultats sont comparés à ceux obtenus par d'autres méthodes de calcul. Les poussées calculées semblent être du même ordre que celle [illegible] vrages.

Model of Ice Rubble Pileup

By M. Sayed[1] and R. M. W. Frederking[2]

Abstract: A model of floating ice pressure ridges is developed. A two-dimensional symmetric wedge is assumed to represent the sail (or keel) of a ridge. The stochastic model of stress transfer in particulate media and the equilibrium equations are used to determine the stress distribution. The kinematics of the problem are not considered in the analysis. The Mohr-Coulomb criterion is used to determine failure zones and consequently the limiting height and depth of the ridge. The model predicts forces in the ice cover associated with limiting ridge height and depth and gives some information about ridge geometry. The results are compared to other computation methods. Force predictions appear to be of the same order observed when floating ice impinges on wide structures.

Introduction

When floating sea ice sheets are driven by environmental forces such as wind or current against shores or man-made structures, they break into small blocks. These blocks, which in the period immediately after their formation behave as a granular material with little or no cohesion, may accumulate to form a variety of features. Unlike flat slopes where an ice sheet may slide or "ride up," a steep obstacle will cause ice rubble to "pile up." The height and depth of floating rubble continue to increase as the ice sheet advances and breaks against previously formed rubble. After reaching a certain limiting height and depth, the rubble starts to grow horizontally. Similar rubble features, called pressure ridges, also occur when ice floes press against each other. In shallow water areas (≤ 25 m), keel depth can extend to the sea floor, thereby grounding the rubble.

Interest in ice rubble pileup has increased because of recent oil explorations in the Beaufort Sea. Estimates of the associated stresses and the limiting height and depth are important for designing drilling structures and marine pipelines. Floating ridges can also cause extreme loading conditions.

A comprehensive description of many ice rubble features was given by Zubov (1945). A recent review of available literature was given by Kovacs and Sodhi (1980) and Sodhi and Kovacs (1984). Details of the morphology of pressure ridges may be found in papers by Weeks and Kovacs (1970) and Tucker and Govoni (1981). Most field studies of rubble interaction with structures, however, remain proprietary. Published data include that of Strilchuk (1977) and Frederking and Wright (1982).

Analytical treatment of this subject is still very limited. A kinematic

[1]Assoc. Res. Ofcr., Geotech. Section, Inst. for Res. in Construction, Natl. Res. Council of Canada, Ottawa, Ontario, Canada K1A OR6.

[2]Sr. Res. Ofcr., Geotech. Section, Inst. for Res. in Construction, Natl. Res. Council of Canada, Ottawa, Ontario, Canada K1A OR6.

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numerical model of pressure ridges was developed by Parmerter and Coon (1972). Although predicted ridge geometries appeared reasonable, the forces in the ice sheet associated with a certain ridge height could not be calculated directly. Parmerter and Coon suggested that a lower bound could be obtained by equating the work done by the ice sheet to the increase in the potential energy of the ridge. Their values ranged from 10^2 to 10^4 N/m. Several authors (e.g., Kovacs and Sodhi 1980) used this potential energy method to derive a simple equation that gives the force in an ice sheet associated with rubble pileup. This equation gives forces of the order expected in an ice cover with length scales of several kilometers. In such a case, ridging or piling occurs only over part of the length at any instant. This sporadic behavior gives average forces lower than local forces. Ice rubble deforms more uniformly over smaller length scales (10 to 100 m), a situation relevant to most engineering applications. Indeed, measured ice forces on obstacles of these dimensions where rubble pileup occurs are of the order of a few hundred kilo-Newtons per meter (Strilchuk 1977; Frederking et al. 1985). The potential energy method grossly underestimates the forces in this case.

Kovacs and Sodhi (1980) added a frictional component by considering a simple case of a single layer of ice sliding over a rubble pileup or a shore. Visual accounts of rubble pileups (Zubov 1945; Kovacs and Sodhi 1980) suggest that more complex modes of deformation take place. Forces calculated by Kovacs and Sodhi approached reasonable values only for cases where the ice sheet slides for a long distance over a shore.

Simple shear tests by Prodanovic (1979) on model ice rubble show that bulk rubble obeyed the Mohr-Coulomb yield condition. Other laboratory experiments by Keinonen and Nyman (1978) and Tatinclaux and Cheng (1978) produced similar results. Those experiments gave angles of internal friction close to 50° with little or no cohesion. Consequently, Mellor (1980) treated the rubble as Mohr-Coulomb material to study some cases related to pileup. The writers (Sayed and Frederking 1984) developed a continuum model of ridges and pileups by considering the bulk rubble to be rigid-plastic obeying the Mohr-Coulomb yield criterion. Stresses in a wedge at the passive critical state were obtained.

This paper is aimed at predicting the limiting height and depth of a rubble pileup and the associated stresses. At this limiting state, parts of the rubble may be at a preyield condition. The plastic analysis (Sayed and Frederking 1984) is more suitable for the earlier stages of rubble pileup.

Model

Consider an ice sheet advancing against a steep obstacle. Broken blocks form a buoyant wedge that grows horizontally after reaching the limiting height and depth. The weight of the sail would be equal to the buoyancy of the keel. It is assumed that the rubble has a two-dimensional symmetrical geometry, as shown in Fig. 1, which represents a sail or a keel. In this idealized model, the advancing ice sheet applies a horizontal load at the apex. A vertical body force acting downwards is taken as the bulk unit weight of the sail (or buoyancy of the keel). The model implies that an ice block may remain either in the sail or in the keel. The actual process might

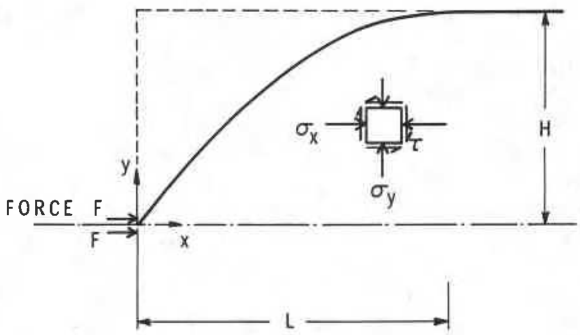

FIG. 1. Definition Sketch of Ice Rubble Pileup Model

be more complex than that, with an exchange of blocks between sail and keel.

Because available experimental data on broken ice do not give a stress-strain relationship, preyield behavior, which is important in the current case, cannot be defined. A stochastic model of normal stress distribution is used here to overcome this problem. The particulate nature of rubble and the relatively large size of blocks suggest that stresses are transmitted through random contacts.

Smoltczyk (1967) and Harr (1977) developed the theory for stochastic stress distribution in particulate media. A review of their theory is beyond the scope of this paper. They showed, however, that normal stresses applied at a boundary are distributed inside the bulk material according to the one-dimensional diffusion equation. The normal stress below a concentrated load acting on a half-space would be Gaussian and, unlike linear elasticity, tensile stresses could be eliminated. The stochastic model was further developed by Chikwendu and Alimba (1979). Recently Golden (1985) presented a general treatment of this subject.

The two-dimensional equilibrium equations of the stresses and a one-dimensional diffusion equation for the distribution of the horizontal normal stress as predicted by particulate media models are used herein. In this connection, mass balance and strain-displacements relations are not used. The Mohr-Coulomb criterion is used to determine failure zones.

Governing Equations

Deformation is assumed to be two-dimensional and relatively slow such that inertia of the rubble is negligible. The equilibrium equations in rectangular Cartesian coordinates x and y (Fig. 1) are as follows:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau}{\partial y} = 0 \quad (1)$$

$$\frac{\partial \tau}{\partial x} + \frac{\partial \sigma_y}{\partial y} = -\gamma \quad (2)$$

where σ_x, σ_y, and τ are the normal and shear stress components (compressive normal stresses are considered positive); and γ = the unit weight of bulk rubble (or buoyancy of the keel).

The normal horizontal stress distribution σ_x is assumed to follow the one-dimensional diffusion equation with x playing the role of the time variable (Chikwendu and Alimba 1979; Golden 1985; Harr 1977; Smoltczyk 1967). The diffusion coefficient has a dimension of length; it depends in general on the voids ratio and the shape and size distribution of the blocks. It was assumed in the references just cited that the diffusion coefficient increases linearly from the boundary on which external forces act. This eliminated tensile stresses, which cannot be sustained by cohesionless materials, and agreed more closely with experimental observations. Golden (1985) formally discussed the admissible functions describing the diffusion coefficient. The linear dependence on the distance x is the simplest choice permitted by his theory. Thus, the horizontal normal stress distribution may be given by

$$\frac{\partial \sigma_x}{\partial x} = \nu x \frac{\partial^2 \sigma_x}{\partial y^2} \quad \ldots\ldots (3)$$

where ν is a dimensionless coefficient. Chikwendu and Alimba (1979) calculated a value of 0.273 for ν in the case of spherical particles with cubic (the loosest) packing. Intuitively, this value should increase for angular particles. Harr (1977) used a value that gives stress under a concentrated line load acting on a half-space equal to that of the Boussinesq elastic solution. This would result in a value of $\pi/8$ for ν. There are no experimental data to determine the appropriate value for ice rubble. Still, the angularity and flakiness of typical blocks in ice rubble suggest a high value, possibly close to $\pi/8$. Failure or rapid deformation of the bulk rubble is assumed to be governed by the linear Mohr-Coulomb criterion. Thus, failure occurs when

$$\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} \geq \left(\frac{\sigma_x + \sigma_y}{2}\right) \sin \phi \quad \ldots\ldots (4)$$

where ϕ is the angle of internal friction. Unlike a perfect plasticity hypothesis, shear stresses are allowed to exceed the failure condition in part of the rubble. Other parts may not reach failure.

Stress Distribution

The normal horizontal stress σ_x is governed by the parabolic partial differential equation (Eq. 3). Boundary conditions should be specified at the centerline and the stress-free surface. The initial condition at the apex is

$$\sigma_x(0, y) = 2F\delta(y) \quad \ldots\ldots (5)$$

where $\delta(y)$ is the delta function and 2F is the line force. (A factor of two is used so that the force, F, acts on half the wedge.) The other two components of the stress tensor, σ_y and τ, are obtained from the equilibrium equations (Eqs. 1 and 2). Integrating Eq. 1 with respect to y and using Eq. 3 gives

$$\tau = -\nu x \frac{\partial \sigma_x}{\partial y} \quad \ldots \quad (6)$$

The shear stress τ and normal stress gradient $\partial\sigma_x/\partial y$ should vanish at the centerline. Therefore the integration constant was taken as zero. Integrating Eq. 2 with respect to y and using Eq. 3 gives

$$\sigma_y = C - \gamma y + \nu\sigma_x + \nu x \frac{\partial \sigma_x}{\partial x} \quad \ldots \quad (7)$$

where C is a function of x.

The shear stress should be zero at the centerline ($y = 0$) and the top of the rubble [$y = y_0(x)$]. For large values of x the boundary elevation becomes $y_{0_{x\to\infty}}(x) = \mathrm{H} =$ constant. The solution of Eq. 3 is simplified by considering the boundary to be at $y_0(x) = \mathrm{H}$ for all values of x. Thus

$$\frac{\partial \sigma_x}{\partial y}(x, \mathrm{H}) = \frac{\partial \sigma_x}{\partial y}(x, 0) = 0 \quad \ldots \quad (8)$$

From observing the solution (presented later), it was felt that changing the boundary position for small values of x would not affect the solution appreciably (the decay of σ_x for small x). This approximation, albeit in the absence of mathematical justification, appears adequate to examine the validity of the current modeling approach. A more rigorous treatment would require an evolution equation for y_0 to be derived from the mass conservation (continuity) condition. Strains and displacements, however, are not included in the analysis because of the uncertainty regarding the appropriate constitutive equations. Such a method would increase the complexity of the solution without improving the accuracy.

Eq. 3 can be solved, subjected to the boundary conditions expressed in Eqs. 5 and 8, by using the method of images (Crank 1975). The stress distribution due to a line force 2F acting on a half-space ($x \geq 0$) is given by

$$\sigma_x = \frac{2\mathrm{F}}{\sqrt{2\nu\pi x}} \exp\left(-\frac{y^2}{2\nu x^2}\right) 8 \quad \ldots \quad (9)$$

Adding image forces at $y = \pm 2\mathrm{H}, \pm 4\mathrm{H}, \pm 6\mathrm{H}, \ldots$ to satisfy the boundary conditions in Eq. 8, superposition gives

$$\sigma_x = \frac{2\mathrm{F}}{\sqrt{2\nu\pi x}} \sum_{n=-\infty}^{\infty} \exp\left[-\frac{(y + 2n\mathrm{H})^2}{2\nu x^2}\right] \quad \ldots \quad (10)$$

Substituting Eq. 10 in Eq. 6 gives the shear stress

$$\tau = \frac{2\mathrm{F}}{\sqrt{2\nu\pi x^2}} \sum_{n=-\infty}^{\infty} (y + 2n\mathrm{H}) \exp\left[-\frac{(y + 2n\mathrm{H})^2}{2\nu x^2}\right] \quad \ldots \quad (11)$$

Considering that the normal stress σ_y should vanish at the boundary $y = y_0$, from Eqs. 7 and 10

$$\sigma_y = \gamma(y_0 - y) + \frac{2F}{\sqrt{2\nu\pi x^3}} \sum_{n=-\infty}^{\infty} \left\{ (y + 2nH)^2 \exp\left[-\frac{(y+2nH)^2}{2\nu x^2}\right] - (y_0 + 2nH)^2 \exp\left[-\frac{(y_0+2nH)^2}{2\nu x^2}\right] \right\} \quad \text{(12)}$$

The boundary height y_0 increases from zero at the origin to reach the limiting height H at $x = L$ as shown in Fig. 1. In the range $0 \leq x \leq L$, y_0 is limited by the vertical spread of the stresses σ_x and τ. The determination of y_0 is discussed later.

Limiting Height

A horizontal force applied by the ice sheet will initially cause failure of the bulk rubble, which, as a result, can move upwards. The height ceases to increase when it approaches a value sufficient to prevent failure (upward movement) in the rubble. It is assumed that the limiting height H corresponding to a certain force is reached when stresses are at the critical passive state (given by Eq. 4 with $\sigma_x \geq \sigma_y$) along a vertical plane. Any further increase in H would reduce shear stresses to a prefailure state. Because of the choice of the boundary conditions, σ_x has a nonzero value but τ and σ_y are zero at $y = H$. Consequently, failure will always occur in a layer near the top of the rubble. Since a layer of thickness less than a block size has no physical significance, any failure occurring in the top 10% of the rubble pileup height in the limiting case was ignored. This assumption is appropriate for most cases of interest.

Rubble height y_0 in the sloping part of the pileup ($x \leq L$) should correspond to low stresses (σ_x) such that an increase in height would cause blocks to roll freely down the slope. The value of y_0 at a certain x may be calculated by trial and error. An approximation that seems to give reasonable results is to choose y_0 as the height at which σ_x (from Eq. 10) is reduced to a small fraction (taken as 0.1) of its maximum value (at $y = 0$).

Analysis of Results

Results obtained for a stress diffusion coefficient $\nu = 0.4$ and an angle of internal friction $\phi = 50°$ are presented later. Eqs. 10–12 were solved (using $y_0 = H$) to give the stress components σ_x, τ, and σ_y. The value of H was incremented in steps for a given F. At each step, the failure condition was tested by substituting the calculated stresses in Eq. 4. This procedure was repeated for increasing x. The appropriate values of H and L are reached when failure occurs only in the top 10% of the wedge. Contours of the ratio

$$\frac{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}}{\left(\frac{\sigma_x + \sigma_y}{2}\right)}$$

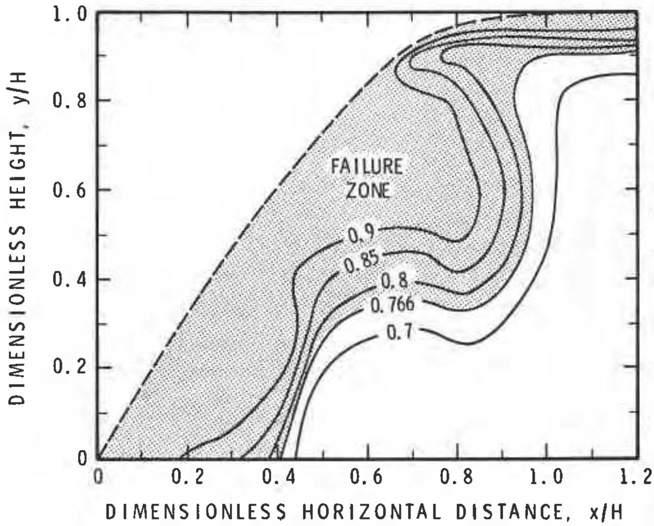

FIG. 2. Contours of Ratio of Maximum Shear Stress to Average Normal Stress Showing Failure Zone in Rubble Pileup at Critical Height ($\phi = 50°$, $\nu = 0.4$)

are shown in Fig. 2. Failure occurs when this ratio exceeds sin ϕ (0.766 for $\phi = 50°$). The resulting relationship between line force and limiting height for half a wedge is

$$F = 0.76\gamma H^2 \qquad (13)$$

The corresponding normal and shear stress distributions are shown in Figs. 3 and 4. The slope of the wedge in Fig. 2, which is approximately 45°, is greater than the average values reported in the literature. Comparison with field observations is difficult, however, since measurements are always done after the pileup process is completed. Also, the current analysis assumed F is steady; a time-dependent F could change the slope.

These values of parameters are likely to represent the properties of ice rubble as discussed earlier. Similar computations were carried out, however, for a range of ν from 0.27 to 0.45, and of ϕ from 45 to 55°, in order to examine sensitivity of the results to each parameter. Wedge geometry and stress distributions followed similar patterns as in Figs. 2–4. The relationship between force and limiting height for this range of parameter values is shown in Fig. 5.

The force given by Eq. 13 is proportional to the square of the height of a sail or a keel. Since the pileup is neutrally buoyant, the ratio of sail to keel height depends on their weight and buoyancy, respectively. Thus

$$\frac{F_s}{F_k} = \frac{\gamma_k}{\gamma_s} \qquad (14)$$

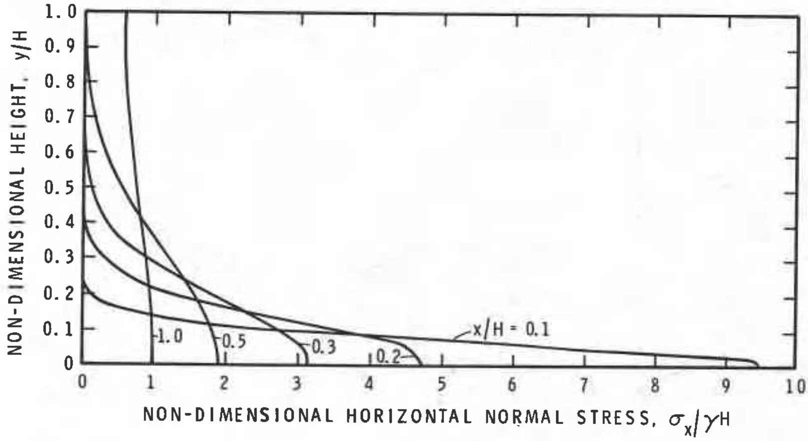

FIG. 3. Horizontal Normal Stress Distribution

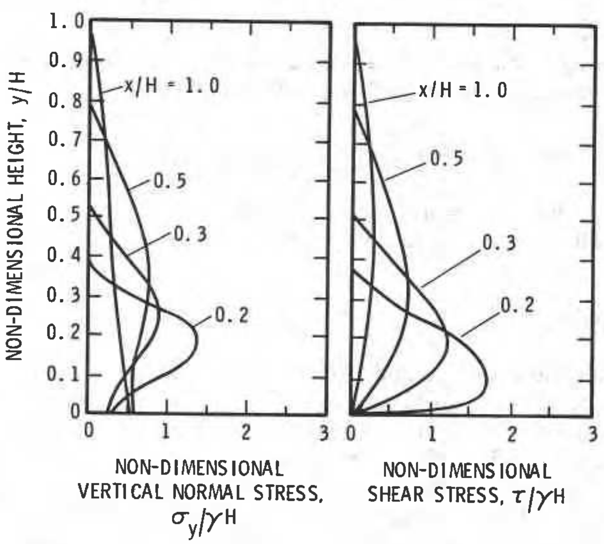

FIG. 4. Vertical Normal Stress and Shear Stress Distribution

where F_s, F_k, γ_s, and γ_k are the forces associated with the sail and the keel, the unit weight of the sail, and the buoyancy of the keel, respectively.

Comparison of Predicted Force with Other Methods

Forces calculated from Eq. 13 are compared with those predicted by other methods in Fig. 6. A bulk weight of 6,600 N/m^3 is used, which is typical of a pileup sail. As expected, plasticity analysis (Sayed and Frederking 1984) gives higher forces because all of the rubble is assumed to be at the critical state. That assumption would be appropriate for the early stages of deformation before the limiting height is reached.

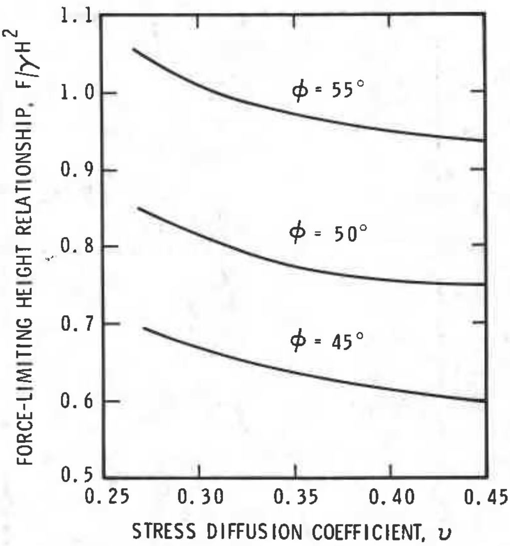

FIG. 5. Dependence of Force-Limiting Height Relationship on Parameters ν and ϕ

The potential energy method gives

$$F = \frac{1}{2}\,\gamma_i H t \quad (15)$$

where γ_i and t are the unit weight and thickness of the ice sheet, respectively. This corresponds to a single layer of an ice sheet sliding over a wedge of height H. Adding friction as suggested by Kovacs and Sodhi (1980) increases the force by 20% for the wedge considered here, which is still much lower than that predicted by the current analysis. Eq. 15 appears to be more suited to cases in which an ice sheet slides over a gentle slope for a relatively large distance.

Although no published data are available to verify the model directly, observed stresses and rubble heights (Frederking 1985; Strilchuk 1977) appear to agree with the current results.

Conclusion

A model of floating ice rubble pileup has been developed. The stochastic model of stress transfer in particulate media was used to obtain the stress distribution in an idealized sail or keel of a pileup. The diffusion equation was solved using an approximate boundary condition (rectangular boundary). A more elaborate solution is still required to accurately solve the free boundary problem. Kinematics were not included in the analysis. A Mohr-Coulomb criterion was used to determine failure zones in the rubble and the limiting rubble height. Values of the stress diffusion coefficient were chosen according to plausible physical arguments. Parametric study showed that results can vary only within a relatively small range. Block

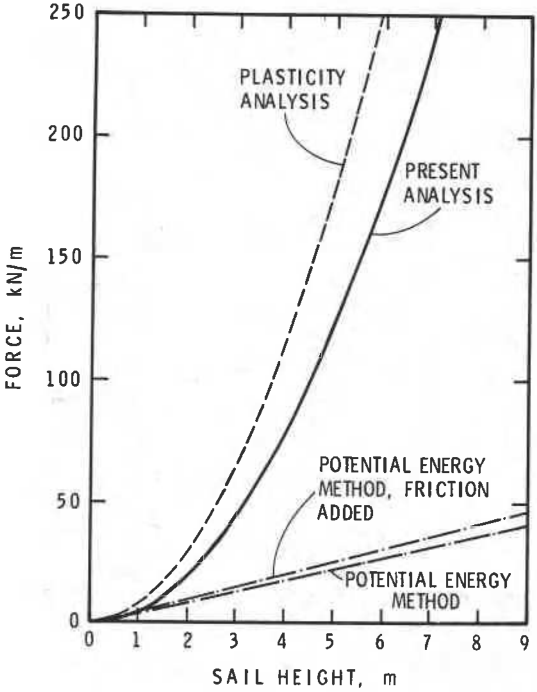

FIG. 6. Comparison of Current Analysis with Other Methods ($\gamma = 6{,}600$ N/m^3, $\phi = 50°$, and $\nu = 0.4$)

size does not affect the parameters ν and ϕ and does not appear explicitly in the analysis.

Force predictions appear to be of the order observed when floating ice impinges on wide structures. The current values are lower than those calculated using a plasticity analysis and substantially exceed those obtained from the potential energy method.

There is a need for field data concerning the geometry of a pileup during its formation, the limiting pileup height, and the associated force. Further experiments to determine the constitutive equations for bulk ice rubble are also needed.

Acknowledgment

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Appendix II. Notation

The following symbols are used in this paper:

C = function of x in Eq. 6;
F = line force acting on half wedge;
H = limiting height (or depth) of rubble;
H_k = limiting depth of keel;

H_s = limiting height of sail;
t = thickness of ice sheet;
x = horizontal coordinate;
y = vertical coordinate;
γ = unit weight of bulk rubble;
γ_i = unit weight of ice;
δ = Delta function;
ν = dimensionless stress diffusion coefficient;
σ_x = horizontal normal stress;
σ_y = vertical normal stress;
τ = shear stress; and
ϕ = angle of internal friction.

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