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Publisher's version / Version de l'éditeur:
https://doi.org/10.4224/20331542
Technical Translation (National Research Council of Canada), 1947-10-01

COPY NO.

NATIONAL RESEARCH COUNCIL OF CANADA

DIVISION OF MECHANICAL ENGINEERING

TECHNICAL TRANSLATION NO. TT - 37

MEASUREMENTS OF DENSITY IN FLOWING AIR BY MEANS OF THE DOUBLE-SLIT INTERFEROMETER

(DICHTEMESSUNG MIT HILFE DES DOPPELSPALT-INTERFEROMETERS)

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1 OCTOBER, 1947

NATIONAL RESEARCH LABORATORIES

Ottawa, Canada

TECHNICAL TRANSLATION

Division of Mechanical Engineering

Pages - 17
Fig. - 3

Tech. Trans. TT-37
Date- 1 October, 1947
File- 12-R4-22

Title: Dichtemessung mit Hilfe des Doppelspalt-Interferometers.

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Reference: ZWB/FB/1285. Report of the Kaiser Wilhelm Institute for Flow Research, Göttingen, September 28, 1940.

Subject: Measurements of density in flowing air by means of the double-slit interferometer.

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SYNOPSIS

The present report begins with a description of the double-slit interferometer and a derivation of the interferometer formula. Following that the application of the interferometer to measurements of density in flows is discussed and the formula is modified for this particular purpose. A calibration of the interferometer is essential. The possible sources of error are particularly stressed. The formulae given in this report are applicable to all interferometers. In conclusion measurements of density in a Laval nozzle are discussed. These measurements permit an insight into the accuracy of the measuring method.

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I OPERATION OF THE INTERFEROMETER.

Interference of light is generally produced by light from a luminous point arriving at a point, say a screen or a miscroscope, by two paths, the lengths of which differ by a few wave-lengths of the light. An interference image results, and points in the image plane appear either bright or dark, depending on whether the lengths of the two paths by which the light travels to the image plane differ by an even or odd multiple of half the wave-length of the light. It is important here that light of uniform phase, i.e., so-called coherent light, travels from the luminous point in the direction of the two light paths. The interference apparatus are generally distinguished by the manner in which separation of the light rays which are to cause interference is produced.

The problem is probably solved most easily in a double-slit interferometer (Fig. 1).

The light from an arc light B is projected on a slightly opened precision slit PS by means of a condenser K. The precision slit produces a beam of coherent light due to diffraction. The precision slit is imaged in the microscope M by means of the lens L. At the lens L a double slit DS is introduced into the light path. Both openings of the double slit are parallel to the precision slit. The light can enter the microscope only by two paths and produces there a system of strips parallel to the precision slit. If white light is used, i.e., light of different wave-lengths, then the points of greatest light intensity for different wave-lengths, in general, do not coincide. Only at the point to which the light travels on paths of equal length, i.e., the maximum of light intensity of zero order, does no interference occur, hence, an absolutely white strip. This is bounded by two dark strips with varicoloured edges. The subsequent maxima and minima of light intensity are more varied in colour and less marked and can be easily distinguished from the principal maximum, by these facts alone.

The double slit may be replaced by two holes, the connecting line of which is normal to the precision slit. The openings of the double slit depend on the one hand only on the shape given to the light ray, which is sent through the air to be investigated and, on the other hand, on the quantity of light required in the microscope. The lens L can also be replaced by a suitable concave mirror. However, it is not recommended that the microscope be replaced by a photographic plate.

The mounting of the apparatus is extremely simple. The double slit is removed temporarily and care is taken that a clear image of the precision slit results in the microscope. Only then is the double slit inserted. The correct opening of the precision slit produces, thereupon, an interference image. The microscope must be protected from disturbing stray light as far as possible.

For the equality of the two light paths it is not, of course, geometrical length which is decisive, but the time required by the light to travel from the precision slit to the image point. The time, however, depends on the speed of light, i.e., on the refractivity of the media traversed by the light. If the two light rays are sent through two chambers of different refractivity then displacement of the interference image will result. Since the relation between optical density and mass density of a body is known, the possibility of a measurement of density from the displacement of the interference image is given.

Next we shall deal with the distance of the individual maxima of light intensity in the interference image, i.e., the so-called width of strip. The microscope is assumed to be replaced by a simple screen.

The separation of the gaps of the double slit is designated by d (Fig. 2), the distance of the double slit from the image plane by ℓ, the width of strip by σ and the mean wave-length of the light by λ. Furthermore let $s^2 = \ell^2 + \frac{d^2}{4}$.

With the assumption (which can always be made) that $d \ll \ell$, we may write:

$$(s + \frac{\lambda}{2})^2 = (\frac{d}{2} + \sigma)^2 + \ell^2$$

$$(s - \frac{\lambda}{2})^2 = (\frac{d}{2} - \sigma)^2 + \ell^2$$

from which it follows that

$$\sigma = \frac{s}{d}\lambda \approx \frac{\ell}{d}\lambda \qquad (1)$$

If the refractivity of a medium relative to a vacuum is designated by N_1, the speed of light in this medium by c_1 and the mass density by ρ_1, then the following equations, which are known from the theory of optics, hold true:

$$c_1 : c_2 = N_2 : N_1 = \lambda_1 : \lambda_2 \tag{2}$$

$$\rho_1 : \rho_2 = (N_1 - 1) : (N_2 - 1) \tag{3}$$

Let there be two chambers, 1 and 2, of the length b in the immediate vicinity of the double slit. One light ray is assumed to pass through each of the two chambers. If the same density $\rho_1 = \rho_2$ prevails in both chambers, then the maximum of light intensity should be in the image plane at the point $\delta = 0$. If $\rho_1 \neq \rho_2$, then the maximum of light intensity should be displaced by the length δ. The light must arrive at the maximum light intensity through both slits within the same time interval. If $c_1 < c_2$, then the following equation is **true:**

$$\frac{b}{c_1} + \frac{\sqrt{\ell^2 - (\frac{d}{2} - \delta)^2} - b}{c_o} = \frac{b}{c_2} + \frac{\sqrt{\ell^2 - (\frac{d}{2} + \delta)^2} - b}{c_o}$$

c_o is here the speed of light in the air of the laboratory. Because $d \ll 1$ and hence, also $\delta \ll 1$ we obtain

$$\delta = (N_2 - N_1) \frac{\ell b}{d} \frac{1}{N_o}$$

With sufficient accuracy we may equate N_o to unity in this equation and obtain, then, by means of equation (3)

$$\delta = (\rho_2 - \rho_1) \frac{N_o - 1}{\rho_o} \frac{\ell b}{d} = (\rho_2 - \rho_1) \frac{N_o - 1}{\rho_o} \frac{\sigma}{\lambda} b \tag{4}$$

The deviation from the zero position is hence proportional to the absolute density variation in the two chambers. It is also proportional to the length of chamber b and to the ratio $\frac{\ell}{d}$.

In order to obtain data on ρ_2, the density ρ_1 must be known. If the density in chamber 2 is not uniform throughout, only the mean density can be measured, and the density distribution must be known in order to give the density at each individual point. The sensitivity of the process increases with the length of chamber b. It also increases with the increase of the ratio $\frac{\ell}{d}$, but not to the same extent, since it must be taken into account that the width of strip σ increases at the same time, which is detrimental to the accuracy of reading. Nevertheless, one should attempt to make the ratio $\frac{\ell}{d}$ as large as possible; hence, to keep d to a minimum and to remove the image plane from the double slit as far as possible, i.e., to use a lens of long focal length, since with lenses of short focal length and large image distance the light intensity of the image is too small.

II MEASUREMENTS OF DENSITY IN NOZZLES AND TUNNELS.

In order to measure the density in nozzles and tunnels, the interferometer must be mounted in such a manner that the flow in figure 2 is visualized as normal to the plane of the drawing. Two sidewalls of the tunnel must be of glass, at least at the points where the light ray traverses the tunnel. Crystal glass free from striae is entirely adequate for this purpose. In this manner the mean density is measured on a straight line in a cross-sectional plane of the tunnel.

The limiting effect of the distance between the two light rays proves to be the sole difficulty in using the double-slit interferometer. However, this limiting effect is not too severe, as will be shown when discussing the accuracy of measurement (conclusion of Section IV b). In general, what is of interest are the density fields in the vicinity of any body introduced into the flow, e.g., density fields in the vicinity of an aerofoil, boundary layers along plates, etc. In all these cases the test bodies can be used simultaneously as comparison chambers by constructing them hollow and by passing the light ray 1 through them.

In order not to let the deviation δ increase too much, it is recommended that the anticipated approximate density where the measurement is to be taken be reproduced in the comparison chamber and the deviation from this anticipated density only be measured by means of the interferometer.

Turning now to the derivation of the density formula for the application of the interferometer and keeping in mind that in the reading by the microscope a magnification factor occurs in addition to the quantity δ, equation (4) can be written as follows:

$$\delta = A(\rho_2 - \rho_1) \qquad (4a)$$

The factor A is to be subsequently determined by calibration.

When beginning to work with the interferometer, it is most likely that the densities in the range of measurement and in the comparison chamber are not equal. The comparison density must be determined by measurement of the pressure p and the absolute temperature T. If, with air at rest in the tunnel, all quantities are designated by the subscript o, then there is a displacement of strip in the microscope even before introducing the flow

$$\delta_o = A(\rho_{2o} - \rho_{1o}) = A\,\frac{m}{R}\left(\frac{p_{2o}}{T_{2o}} - \frac{p_{1o}}{T_{1o}}\right)$$

Here, R is the absolute gas constant and m the molecular weight of the air.

If in the comparison chamber the pressure to be anticipated is now established (which can be calculated from the determination of the pressure p_1 and the temperature T_1 in the comparison chamber) and if the flow in the tunnel is started then a deflection is obtained in the microscope:

$$\delta = A\left(\rho_2 - \frac{m}{R}\,\frac{p_1}{T_1}\right)$$

Only the difference $\delta - \delta_o$ can be read in the microscope. From the two equations above

$$\rho_2 = \frac{m}{R}\left[\frac{p_1}{T_1} - \frac{p_{1o}}{T_{1o}} + \frac{p_{2o}}{T_{2o}}\right] + \frac{1}{A}(\delta - \delta_o) \qquad (5)$$

If the variables denoting the state of the air in the laboratory are denoted by the subscript o alone the formula (5a) is obtained for the frequently involved value $\frac{\rho_2}{\rho_o}$,

$$\frac{\rho_2}{\rho_o} = \frac{p_1}{p_o}\,\frac{T_o}{T_1} - \frac{p_{1o}}{p_o}\,\frac{T_o}{T_{1o}} + \frac{p_{2o}}{p_o}\,\frac{T_o}{T_{2o}} + \frac{1}{\rho_o}\,\frac{1}{A}\,(\delta - \delta_o) \qquad (5a)$$

This formula is generally simplified by the fact that some of the quantities occurring are equal. Above all, in most cases $p_{1o} = p_{2o} = p_o$.

The calibration of the interferometer can be carried out in such a manner that the values at the second point of measurement are left unchanged and various densities are produced in the comparison chamber. The calibration values are denoted by the superscript E. Then:

$$\frac{1}{\rho_o}\,\frac{1}{A} = \frac{\frac{p^E_{1o}}{p_o}\,\frac{T_o}{T^E_{1o}} - \frac{p^E_1}{p_o}\,\frac{T_o}{T^E_1}}{\delta^E - \delta^E_o} \qquad (6)$$

If care is taken that the deflections $\delta - \delta_o$ do not become too large in the measurements, then the last term of equation (5) is small as compared with the first term on the right hand side of the equation, so that, when calibrating it is sufficiently accurate to write

$$p^E_{1o} = p_o \;; \qquad T_o = T^E_1 = T^E_{1o}$$

Thus:

$$\frac{1}{\rho_o}\,\frac{1}{A} = \frac{1 - \frac{p^E_1}{p_o}}{\delta^E - \delta^E_o} \qquad (6a)$$

It is quite sufficient if the value $\frac{1}{\rho_o}\,\frac{1}{A}$ is obtained as a mean value of about five calibration values. It is of course most favourable for calibrating to work with deflections $\delta^E - \delta^E_o$ as large as possible.

III SOURCES OF ERROR AND CORRECTIONS.

a) Deflection of the Laboratory Floor.

If a microscope of 50 diameters magnification is used it is still possible to observe displacement of strips of 0.002 centimetre reasonably well. Since, as a rule, the apparatus will have a length of several metres, vibrations or elastic deflections of the floor may easily falsify the measurements. Perceptible vibrations of the floor generally make measurements impossible. Care must be taken that all persons who are near the interferometer during a measurement remain in one place from the time of reading of δ_o until the reading of δ.

b) <u>Density Variations of the Air in the Laboratory.</u>

The interferometer is of course very sensitive to these variations. In general, density variations easily arise from thermal convection. The mere presence of a person in the vicinity of the double slit can cause oscillations of the interference image. It is therefore advisable, even though not absolutely necessary, to conduct the light rays through a tube, particularly where they are at some distance from each other.

c) <u>Influence of the Boundary Layer Along the Walls of the Tunnel.</u>

Boundary layers which may falsify the measurements of density originate along the glass walls bounding the flow. The order of magnitude of the resultant error will be estimated.

Let T_2 be the temperature of the flow. It is sufficient for the estimation to assume that the glass walls have the stagnation temperature T_2 of the flow. Let the density of the flow be ρ_2 and along the glass walls $\rho_2' = \rho_2 \frac{T_2}{T_2'}$. If δ^* is a measure for the thickness of the boundary layer and if linear increase of the air density in the boundary layer is assumed, then the mean density of the boundary layer is $\frac{1}{2}(\rho_2' + \rho_2)$. The mean density of the air ~~flowing~~ along the light ray is then

$$\overline{\rho_2} = \frac{1}{b}\left[\rho_2(b-2\delta^*) + \rho_2\left(\frac{T_2}{T_2'} + 1\right)\delta^*\right] = \rho_2\left[1 - \left(1 - \frac{T_2}{T_2'}\right)\right]\frac{\delta^*}{b} \quad 1)$$

Hence the error in the measurement of density caused by the boundary layer is of the order of magnitude:

$$\left(1 - \frac{T_2}{T_2'}\right)\frac{\delta^*}{b} = \frac{\frac{K-1}{2}M^2}{1 + \frac{K-1}{2}M^2}\frac{\delta^*}{b} \qquad (7)$$

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1) Translator's Note: The latter portion of this equation appears incorrectly in the original document and should read

$$= \rho_2\left[1 - \left(1 - \frac{T_2}{T_2'}\right)\frac{\delta^*}{b}\right]$$

where M, as usual, represents the Mach number. In order to be able to estimate the error, it is essential to obtain by some means the thickness of boundary layer δ^*. The error will be of importance only in long narrow tunnels and with high Mach numbers.

d) <u>Change in Optical Density of the Glass Due to Cooling of the Flow.</u>

The glass walls undergo a change in temperature due to the air flowing past them. The change in temperature results in a change of mass density and consequently of the optical density of the glass. The error thus resulting is given below without calculation for a case which occurs frequently.

It is assumed that the temperature of the glass remains unchanged at the point of passage of light ray 1. At the point of passage of light ray 2 a linear temperature gradient is assumed to arise due to the flow. The difference in temperature between the outside and inside of the glass is assumed to be ten per cent of the difference of temperature ΔT between the outer wall of the glass and that of the flow. Let the ratio of the thickness of the two glasses to the length of the light path in the flowing air, i.e., to the width of the tunnel, be 1:5.
Let

the refractive index of the glass be $N = 1.50$,
the temperature coefficient of linear expansion of the glass $\beta = 0.8 \cdot 10^{-5}$ c.g.s. units,

and
the error in the density ratio, $\frac{\rho_2}{\rho_0}$, caused by the expansion of the glass be $\Delta\frac{\rho_2}{\rho_0}$.

Then

ΔT	9°	18°	32°	49°
$\Delta \frac{\rho_2}{\rho_0}$	0.006	0.012	0.022	0.033

It is proved thus that the error can be too high to be permissible for an exact calculation. It is proportional to the ratio of the thickness of glass to the width of the tunnel and to the coefficient of expansion of the glass as well as to the difference of temperature ΔT. This error can be reduced by selecting a glass as thin as possible and with a low temperature coefficient of expansion. On account of the complicated conditions of heat conduction in the glass, it will be preferable in the individual case to determine experimentally the error described above by comparing the results obtained after various durations of flow. A duration of flow so brief that steady conditions cannot develop in the glass must be used as a start. Another possibility for determining the error is to observe the deflection δ_o for various coolings of the glass.

The error can also be avoided if considerable cooling of the glass is permitted. The reading of δ_o must in that case be carried out immediately after the reading of δ, hence with cooled glass walls. In this manner the error was successfully eliminated in the measurements given below. It is important in this case to determine what temperature T_{2o} can be ascribed to the air in the tunnel immediately after the cessation of the flow.

e) Dependence of the Optical Density of Air on Water Vapour Content and Water Content.

Regarding the effect of water vapour, reference is made to the formula in Kohlrausch's Lehrbuch der Praktischen Physik, 16th Edition, page 269, which in the present notation is expressed as

$$N_2 - 1 = N_L - 1 - \frac{273}{T_2} 5.5 \cdot 10^{-8} e \tag{8}$$

in which N_L is the refractive index of dry air, N_2 the refractive index of the air containing water vapour which flows in the tunnel and e the partial pressure of the water vapour.

The difference in the value of $N_2 - 1$ (which is decisive for the measurement of density) between dry air and that saturated with water vapour at 20°C and 760 mm. Hg pressure is $9.0 \cdot 10^{-7}$. Accordingly, for a value of N_L = 1.000273 this difference is 0.003 of the value of $N_2 - 1$.

The water vapour content of the air affects accurate density measurements only at temperatures above those of normal room temperature and at high humidity of the air. In this case the water vapour content of the comparison chamber would have to be taken into account.

In order to estimate the influence of water droplets existing in the air on the optical density of the air, it would be important to know the size of the water droplets. However, as a rule, it will be unknown. Therefore two extreme cases will be estimated. In the first case it is assumed that the diameter of the water droplet is small as compared with the wave-length of the light. In the second case it is assumed that all the water is concentrated in one layer. The velocities in water in the two cases are assumed to be equal and likewise the velocities in air. For the first case the Lorentz-Lorenz law, by which the specific refraction of a molecular mixture equals the sum of the specific refractions of the individual mixture components, is used. If the mass is designated by G, with W as subscript for water and L for air, then the specific refraction according to Lorentz-Lorenz is $\frac{N^2 - 1}{N^2 + 2} \cdot \frac{G}{\rho}$ and then

$$\frac{N_2^2 - 1}{N_2^2 + 2} \cdot \frac{G_W + G_L}{\rho_2} = \frac{N_L^2 - 1}{N_L^2 + 2} \cdot \frac{G_L}{\rho_L} + \frac{N_W^2 - 1}{N_W^2 + 2} \cdot \frac{G_W}{\rho_W}$$

From this equation after some transformations and simplifications

$$N_2 - 1 = (N_L - 1)(1 + \frac{3}{2} \frac{N_W + 1}{N_W^2 + 2} \cdot \frac{N_W - 1}{N_L - 1} \frac{G_W}{G_L} \cdot \frac{\rho_L}{\rho_W})$$

The second estimation will now be dealt with. Let the volume of the air be $V_L = \frac{G_L}{\rho_L}$, the volume of the water $V_W = \frac{G_W}{\rho_W}$ and the volume of the mixture $V_2 = V_L + V_W$. If the length of the light path in water is denoted by ℓ_L and ℓ_W, then $\ell_2 = \ell_L + \ell_W$ and, associating a mean velocity c_2 with the light when passing through the water and air layers, the following can be written

$$\frac{\ell_2}{c_2} = \frac{\ell_L}{c_L} + \frac{\ell_W}{c_W}$$

However, since $\ell_2 : \ell_L : \ell_W = V_2 : V_L : V_W$, according to equation (2)

$$N_2 \; V_2 = N_L \; V_L + N_W \; V_W$$

Then after some transformations and simplifications,

$$N_2 - 1 = (N_L - 1)\left(1 + \frac{N_W - 1}{N_L - 1} \frac{G_W}{G_L} \cdot \frac{\rho_L}{\rho_W}\right)$$

The term representing the error caused by the presence of water differs in the two equations merely by the factor $\frac{3}{2} \frac{N_W + 1}{N_W^2 + 2} = 0.927$, for a refractive index of water $N_W = 1.33$. With a given mean error in measurement this factor is without significance. Since practically the same result was obtained for both extreme cases, it may be assumed that the error in the density measurement, caused by the presence of water, is, irrespective of the size of the water particle,

$$\frac{N_W - 1}{\rho_W} \frac{\rho_L}{N_L - 1} \frac{G_W}{G_L} = 1.3 \cdot \frac{G_W}{G_L} \tag{9}$$

Assuming that all the water vapour of saturated air at 20°C and 760 mm. Hg atmospheric pressure is condensed by adiabatic expansion, an error of almost two per cent is obtained for $G_W/G_L = 0.014$.

f) Refraction of Light Rays.

If there is a considerable density gradient in the flow, then the light ray 2 will undergo a change of direction due to refraction. This change can be easily calculated; however, there is generally an additional change of direction due to a gradient of the optical density in the glass. This change of direction is in most cases opposite and cannot be calculated on account of the complicated conditions of heat conduction in the glass. The experiment, however, shows that the refraction even with a density gradient of

$$\frac{\partial \rho_2}{\partial x} = 0.3 \cdot 10^{-3} \text{ c.g.s. units}$$

is still so slight that the interference image is impaired by it but not destroyed.

IV MEASUREMENTS OF DENSITY IN A LAVAL NOZZLE.

a) Description of the Procedure in Making a Measurement.

The author used the double-slit interferometer for measurements of density in a Laval nozzle. The cross-section of the nozzle is shown in figure 3 in full scale[1]. The width of the nozzle was b = 5 cm. throughout. The arrow indicates the direction of the flow. The laboratory air was drawn through the nozzle into a vacuum tank. Upstream from the nozzle there were devices for the production of air of different humidities which will not be described here. Downstream from the nozzle there was a quick-closing stopcock. One half of a symmetrical nozzle was used and its curved wall was adjustable. The comparison chamber is denoted by 1. It was heat-insulated against rapid changes in temperature from the outside by a layer of hard rubber. By means of a pressure hole it was possible to produce various densities in the comparison chamber and to measure the pressure prevailing there. A thermocouple was introduced into the chamber for measuring the temperature. A second pressure hole made it possible to measure the static pressure at point 3. A second thermocouple was introduced near this point into the curved "cheek of the nozzle" (Düsenbacke) in order to determine at this point the temperature existing in the cheek. The arrangement described made it possible to measure pressure and density at any point of the nozzle by means of one comparison chamber and one pressure hole by shifting the curved cheek of the nozzle. Reading by means of a vernier permitted the determination of displacements of the cheek of 0.01 centimetre. The double slit consisted of two square holes with sides approximately 0.18 centimetre. The centres of the two holes were approximately 0.9 centimetre apart. The double slit is shown in the figure by dotted lines. Care was taken that the line joining hole 2 of the double slit with the hole for the static pressure was exactly normal to the plane cheek of the nozzle. However, this by no means guarantees measurements of density and pressure at the same point. On the contrary, it shows that for exact measurements in such a short nozzle, constant values of density and pressure over the cross-section cannot be assumed. For the measurements a curvature correction, based on the assumption that the streamlines decreased linearly from the curved to the plane cheek of the nozzle, was introduced for the determination of the mean density and mean pressure. This calculation will not be carried out here since

1) Translator's Note: The figure as shown has been enlarged from microfilm and redrawn and probably differs in size from the original referred to.

it is not a case of interferometer correction but rather one of the error resulting from the application of the one-dimensional nozzle theory to short Laval nozzles.

An arc light of 20 amperes at 220 volts was used as light source. Its light was projected on a precision slit by means of a condenser. A lens of two metres focal length produced an image of the precision slit in a microscope of 50 diameters magnification. The distance between double slit and microscope was approximately six metres. A scale having 50 divisions was in the ocular of the microscope; one division thereby corresponding to 1/50 millimetre. The distance between two maxima of light intensity amounted to approximately 19 divisions, as can be easily calculated from formula (1). With this large width of strip it would not be easy to read with accuracy the centre of a maximum of light intensity. The edges of the first two maxima of light intensity, however, showed a marked transition from a definite red to a definite blue which was particularly characteristic. At this point the readings were taken. This can be done directly, since, with a change in density, the entire system of strips shows a uniform displacement.

It was possible to read one division with accuracy. The error which appeared in the measurement due to the reading of δ and δ_o therefore amounted to not more than one division, i.e., approximately $\frac{\sigma}{20}$. If $\lambda = 5.7 \cdot 10^{-5}$ cm. and $N_o - 1 = 2.9 \cdot 10^{-4}$, this results according to formula (4) in an inaccuracy of the density values

$$\frac{\rho}{\rho_o} \text{ of } \pm 0.002$$

By comparison with this, the temperature and pressure readings may be considered completely accurate. It is obvious that the percentage error in the determination of $\frac{\rho}{\rho_o}$ increases for smaller density ratios.

The comparison chamber was on the one hand connected with a comparatively large glass sphere and on the other hand with the outer air by a three-way cock. The volume of the glass sphere was large compared with that of the chamber. A mercury manometer was connected to the glass sphere. By means of a water-jet vacuum pump the density desired in the chamber could be produced in the sphere. This density underwent only a slight change when the chamber was connected with the glass sphere.

During the reading of δ_o the chamber was connected with the outside air. Hence, $p_{1o} = p_o = p_{2o}$ was to be substituted in equation (5a). Furthermore, it became evident that with the flow brought to a standstill, the air between the cheeks of the nozzle assumed the temperature of these within one second. It was possible to demonstrate this by momentarily switching the flow on and off. In this way air of the temperature T_o came between the cheeks. A small deflection, which became evident in the microscope, disappeared within one second. The temperature T_{2o}, therefore, was to be equated to the temperature measured in the cheek of the nozzle at point 3.

Measurement was carried out in the following manner. Each day on which measurements were taken, the interferometer was, first of all, calibrated by adjustment of various densities in the comparison chamber and by reading the appropriate deflections. There was no difficulty in obtaining sufficient accuracy when calibrating.

For the measurement itself two persons were always required, one for reading the deflections in the microscope and the other one for operating the manometers and reading the galvanometers of the two thermocouples. First, the arc light was switched on and the curved cheek of the nozzle was screwed to the point desired. The air density anticipated during the measurement was obtained in the glass sphere by evacuation. Then everyone got to the place assigned to him and was not permitted to leave it before the conclusion of the measurement. In the beginning the comparison chamber was connected to the outside air in order to make sure of the quality of the interference image in the microscope. Following that, the flow was switched on by means of the quick-acting cock and the comparison chamber at the same time connected to the glass sphere. In the microscope the interference image was generally displaced by several divisions and was rather indistinct. During the first seconds another displacement of the image by several divisions occurred due to cooling of the glass. After the interference image had become stationary, the displacement δ was read in the microscope and, at the same time, the mercury manometers were blocked by means of glass cocks. Immediately after that, the flow was brought to a standstill by closing the quick-acting cock and the comparison chamber was at the same time connected to the outside air. Whilst one of the attendants speedily read the deflection δ_o in the microscope, the other one watched the deflections of the two galvanometers. After recording these, it was possible to read the blocked manometers.

With some practice this manner of reading was easily carried out. Importance must be attached to quick succession of the two readings δ and δ_o in order to prevent too large changes in the optical density of the glass. Speedy reading of the galvanometers is less important, since their deflections do not change as rapidly and the errors caused by them are smaller.

If the cooling of the glass had not been taken into account in the measurement under consideration, the error at Mach number $M = 1.6$ would have amounted to 3 per cent of the value of $\frac{\rho}{\rho_o}$.

b) <u>Results and Accuracy of Measurement.</u>

The results of these measurements are given below. Pressure and density measurements were carried out at a temperature of 17°C in the laboratory with relative humidities of 0, 40, 75 and 90 per cent. With dry adiabatic expansion of the air, the density values are now well suited for the analysis of the quality of the interferometric test procedure, since density values, which are obtained by calculation from the static pressure measurements by assuming a dry adiabatic expansion, can be used for comparison. In a report to be published shortly the author will show that moist air also expands in a dry adiabatic manner in short nozzles at $M < 1$. For $M > 1$, on the other hand, this can generally not be claimed. In this case condensation phenomena are encountered, and density measurements of moist air in this region are therefore no longer of interest here, since there are no possibilities for comparisons to permit an analysis of their quality. The density values for moist air in the region $M > 1$, measured by interferometer, are not included in the table below. In the region $M < 1$, there is no difference in the results of density measurements of dry and moist air (within measuring accuracy).

The figures in the first column of the table indicate different positions of the curved cheek of the nozzle in millimetres. The figures in the second column represent the values for $\left(\frac{\rho}{\rho_o}\right)_f$ mathematically calculated from the nozzle dimensions by assuming dry adiabatic expansion without boundary layers. The density values $\left(\frac{\rho}{\rho_o}\right)_{ad}$, obtained from static pressure measurements by assuming dry adiabatic expansion, are compiled in column 3. The difference between these two values is attributed to the presence of a boundary layer. Columns 4-7 give the values for various relative densities as obtained by interferometer. Their mean value $\frac{\bar{\rho}}{\rho_o}$ is given in column 8. The mean error for each individual nozzle position $\Delta\frac{\rho}{\rho_o}$ is shown in column 9 and the mean error for the first seven nozzle positions in column 10. Finally the appropriate Mach numbers are given in column 11. Each of the data given was obtained by one individual measurement, taking into account the curvature correction mentioned above. It was shown in section 3 (a and d) how the errors a and d can be avoided. According to the formulas given above, the other errors are smaller than the inaccuracy of the present results due to uncertainties in the microscope reading.

1	2	3	4	5	6	7	8	9	10	11
x	$(\frac{\rho}{\rho_o})_f$	$(\frac{\rho}{\rho_o})_{ad}$	$(\frac{\rho}{\rho_o})_o$	$(\frac{\rho}{\rho_o})_{0.40}$	$(\frac{\rho}{\rho_o})_{0.75}$	$(\frac{\rho}{\rho_o})_{0.90}$	$\frac{\bar{\rho}}{\rho_o}$	$\Delta\frac{\rho}{\rho_o}$		M
70		0.937	0.939	0.940	0.938	0.935	0.938	0.0019		0.387
65	0.884	0.894	0.896	0.901	0.893	0.895	0.896	0.0030		0.490
60	0.833	0.846	0.846	0.845	0.847	0.848	0.846	0.0012		0.592
55	0.783	0.796	0.794	0.798	0.794	0.797	0.796	0.0018	0.0018	0.689
50	0.731	0.743	0.741	0.744	0.741	0.744	0.742	0.0015		0.806
45	0.679	0.686	0.687	0.687	0.687	0.687	0.687	0.0010		0.911
40	0.626	0.632	0.631	0.633	0.633	0.634	0.633	0.0011		1.015
35	0.578	0.583	0.581							1.105
30	0.532	0.535	0.536							1.200
25	0.486	0.489	0.491							1.292
20	0.438	0.441	0.441							1.389
15	0.390	0.393	0.395							1.493
10	0.341	0.339	0.335							1.620

The error in measurement is thus definitely within the limits which were to be expected in accordance with the estimation in Section IVa.

The Laval nozzle, employed in the measurements, is favourable for the application of the interferometer inasmuch as it was possible to keep the double slit distance d quite small. On the other hand, the Laval nozzle is extremely unfavourable as to length of the light ray b = 5 cm. in the air to be investigated. It is therefore possible to operate with the same accuracy as here in a tunnel of 50 centimetres width with a distance of light ray d = 10 cm. In somewhat less accurate measurements the temperature measurements may, as a rule, be omitted, particularly when operating with Mach numbers $M < 1$.

The double-slit interferometer is hence a relatively simple and accurate aid for the determination of density in flow.

V SUMMARY

In the double-slit interferometer the density is measured from the displacements of an interference image. This image originates from interference of two light rays conducted parallel to each other; one of which is conducted through the air to be investigated and the other one through air with density known. The accuracy of the measuring procedure depends on the distance between the two light rays. The error in measurement is generally very slight. In the example of measurements of density in a Laval nozzle, quoted in this report, the mean error amounts to less than two per thousand.

The instrument is particularly suitable for point-by-point measurements of two-dimensional density fields in the vicinity of test pieces. It is worth mentioning that the double-slit interferometer can be obtained at low cost and is easy to mount.

/MJ

FIG 1, 2, 3
TECHTRANS TT-37

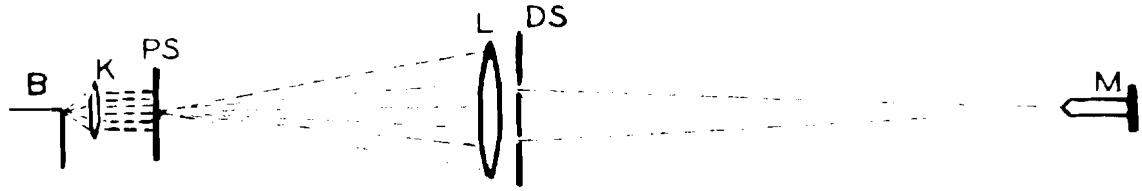

Fig. 1: Double-slit interferometer.

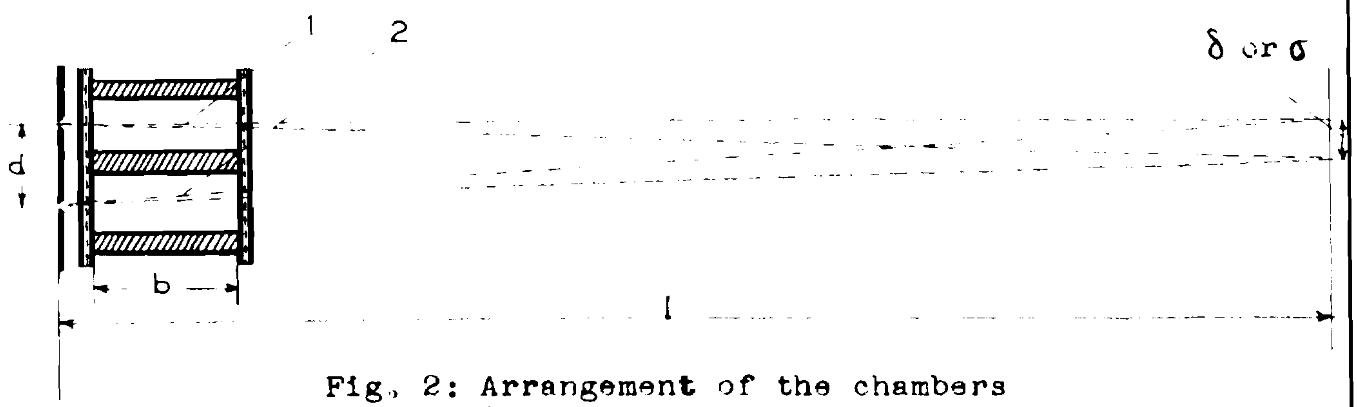

Fig. 2: Arrangement of the chambers in the interferometer.

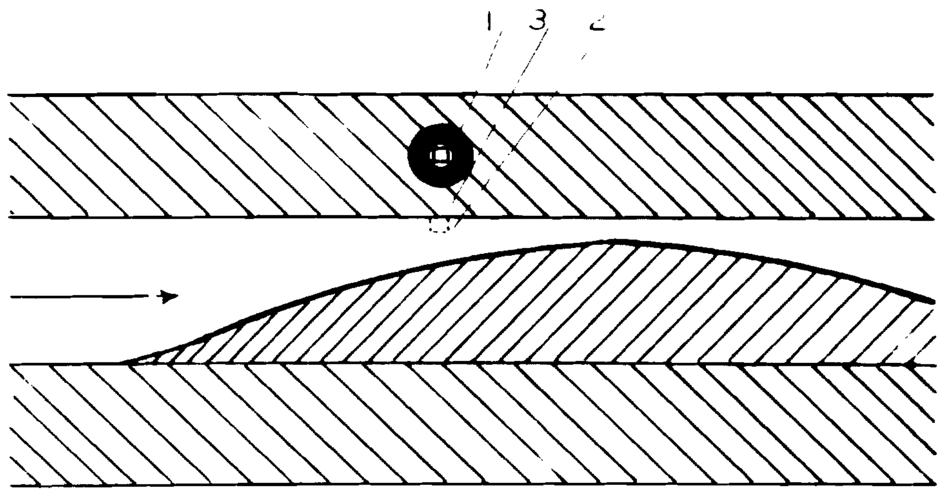

Fig. 3: Longitudinal section through nozzle and chamber.