Some observations on the dependence of strain on stress for ice
Gold, L. W.

Publisher's version / Version de l'éditeur:
https://doi.org/10.1139/p58-131
Canadian Journal of Physics, 36, 10, pp. 1265-1275, 1958-10

NATIONAL RESEARCH COUNCIL
CANADA
DIVISION OF BUILDING RESEARCH

SOME OBSERVATIONS ON THE DEPENDENCE OF STRAIN ON STRESS FOR ICE

BY

L. W. GOLD

Reprinted from
CANADIAN JOURNAL OF PHYSICS
VOL. **36**, NO. **10**, OCTOBER 1958, p. 1265

RESEARCH PAPER NO. 65
OF THE
DIVISION OF BUILDING RESEARCH

NRC 4880 OTTAWA PRICE 25 cents

This publication is being distributed by the Division of Building Research of the National Research Council as a contribution towards better building in Canada. It should not be reproduced in whole or in part, without permission of the original publisher. The Division would be glad to be of assistance in obtaining such permission.

Publications of the Division of Building Research may be obtained by mailing the appropriate remittance (a Bank, Express, or Post Office Money Order or a cheque made payable at par in Ottawa, to the Receiver General of Canada, credit National Research Council) to the National Research Council, Ottawa. Stamps are not acceptable.

A coupon system has been introduced to make payments for publications relatively simple. Coupons are available in denominations of 5, 25, and 50 cents, and may be obtained by making a remittance as indicated above. These coupons may be used for the purchase of all National Research Council publications including specifications of the Canadian Government Specifications Board.

SOME OBSERVATIONS ON THE DEPENDENCE OF STRAIN ON STRESS FOR ICE[1]

L. W. GOLD

ABSTRACT

Observations were made on the longitudinal and transverse strain of rectangular ice samples stressed at the rate of 2 kg/cm^2/sec to a maximum stress of 10 kg/cm^2. The duration of the stress was kept less than 10 seconds. Under these conditions the ice behaved elastically. The observations indicated that two deformation processes contributed to the measured strain. It is concluded that the two sources of strain are deformation of the grains and slip at the grain boundaries.

INTRODUCTION

Ice is a crystalline solid which, under natural conditions, exists in an environment at a temperature near that at which it melts. With respect to mechanical stress, it exhibits in its normal behavior characteristics peculiar to this thermal state which are not completely understood at present. It is not surprising, therefore, to encounter in the literature a wide spread in the values of the elastic moduli of ice.

Papers on the elasticity of ice have been discussed at some length by the Snow, Ice and Permafrost Research Establishment (U.S. Army Corps of Engineers 1951), Dorsey (1940), Hess (1940), and Weinberg (1936). Values for Young's modulus measured by bending, extension, and compression methods range from 0.3×10^{10} to 11.0×10^{10} dynes/cm^2. Dorsey (1940) in his review states that it is questionable whether useful values can be obtained by the usual static methods. The temperature range in which most of the reported static values were measured (0° C to -10° C) is too small to indicate any significant dependence on temperature.

The elastic moduli have been found to be more consistent when measured by sonic and ultrasonic methods. When Boyle and Sproule (1931) measured the velocity of longitudinal waves in ice they found that this velocity was temperature dependent. Ewing, Crary, and Thorne (1934) measured the longitudinal and the extensional wave velocity; their value for the longitudinal wave velocity agrees well with the results of Boyle and Sproule. Northwood (1947) measured the longitudinal, extensional, and Rayleigh wave velocity. His longitudinal and extensional wave velocities were higher than those reported previously. He also found a dependence on temperature which was less than that reported by Boyle and Sproule. Sonic observations on multigrained ice have not shown any conclusive evidence that the wave velocity depends on the direction of propagation through the ice.

[1]Manuscript received April 28, 1958.
Contribution from the Division of Building Research, National Research Council, Ottawa, Canada, and presented with the approval of the Director of the Division.
Issued as N.R.C. No. 4880.

The single crystal of ice exhibits hexagonal symmetry and five constants are required to describe its elastic behavior under stress. Jona and Scherrer (1951) measured these five constants ultrasonically. Green and MacKinnon (1956) carried out observations on two of the constants and their results are in reasonable agreement with those of Jona and Scherrer.

Voigt (1928), assuming random orientation of the optic axis of the constituent crystals, developed equations giving the isotropic values for Young's modulus, Poisson's ratio, the rigidity modulus, and the bulk modulus from the stiffness moduli for the single crystal. Reuss (1929) developed similar equations using the compliance moduli. Hill (1952) showed that measured values should lie between the values calculated from the Voigt and Reuss equations. Table I gives reported sonic values for the Young's modulus, rigidity modulus, bulk modulus, and Poisson's ratio for ice at a temperature of $-5°$ C. The value attributed to Boyle and Sproule was obtained by fitting a straight line by least squares to their results for propagation perpendicular to the direction of growth of the ice. Also given in Table I are the values calculated from the results of Jona and Scherrer using the Voigt and Reuss relationships. These were corrected to $-5°$ C using the temperature dependence as found by Northwood and assuming Poisson's ratio independent of temperature. The value for the bulk modulus found by Richards and Speyers (1914) using a piezometer is also given. Table I shows that though there is still some discrepancy, the results of measurements of the elastic moduli of ice by sonic methods are in reasonable agreement.

DEPENDENCE OF STRAIN ON STRESS

Studies on metals carried out at elevated temperatures have shown that the magnitude of the strain in phase with an applied stress depends on the rate at which the stress is applied and the absolute temperature of the specimen (Zener 1948). Kê (1948) discusses the energy loss associated with this phenomenon and the resulting dependence of the rigidity modulus on temperature and frequency of applied stress and attributes the observed results for metals, in part, to grain boundary slip. It is found that the energy loss has a maximum value which depends on the frequency of the applied stress for a given temperature, and upon the temperature for a given frequency of stressing. As the frequency of stressing is lowered, the peak in the energy loss versus temperature curve is displaced toward lower temperatures. It is also found that the rigidity modulus decreases from an upper value, called the unrelaxed modulus, to a lower value called the relaxed modulus; the maximum rate of change occurs when the energy loss is at a maximum. Crawford (1957) has observed the same phenomena in an organic glass. He found also that, corresponding to the rapid decrease in the rigidity modulus with increasing temperature, there was a rapid increase in Poisson's ratio. The increase in Poisson's ratio shifted to lower temperatures as the frequency of stressing was lowered.

Observations such as those by Kê and Crawford show that measured values for the elastic properties will depend on the duration of the stress at a given temperature. If the duration of the stress is long enough, the relaxed value for the elastic moduli will be observed. If the duration is short enough, these observations will not be distorted by creep and the resulting permanent deformation. Ice creeps readily under stress. This fact likely explains the large differences in the elastic moduli of ice measured by static methods since in many of the experiments the duration of the stress was long enough to allow creep to occur. The observations reported in this paper on the dependence of strain on stress for ice in the temperature range $-3°$ C to $-40°$ C were made under conditions of low stress and low rate of stressing. The duration of the stress was short enough that any permanent deformation which resulted was much smaller than the measured strain.

SYMMETRY CONSIDERATIONS FOR MULTIGRAIN ICE

Under normal conditions of growth with a plane ice–water interface, multigrain ice is composed of long, candlelike, single crystals. A single grain may extend from the upper surface of the ice to the ice–water interface, the long axis of the grain coinciding with the direction of freezing. The direction of the optic axis in each grain is determined by the nucleating conditions, but it is usually assumed, with some justification, that the optic axis will have a preferred orientation in the direction of freezing. Wilson, Zumberge, and Marshall (1954) have carried out observations on natural lake ice and their report contains a discussion on the preferred orientation of the optic axis of the grains as well as a number of photographs showing the typical candlelike texture.

Because of the candlelike structure of multigrain ice, it should exhibit a symmetry described by Love (1952) as transverse isotropic. A material with this symmetry requires five constants to describe its elastic behavior under stress. If we let z be the direction of freezing of a very large multigrain ice sheet so that the freezing plane is parallel to the x and y axes and the long axis of the grains lies in the z direction, the relationships between stress and strain can be expressed by:

$$e_{xx} = (T_{xx}/E_x) - \sigma_x(T_{yy}/E_y) - \sigma_z(T_{zz}/E_z), \tag{1}$$

$$e_{yy} = (T_{yy}/E_y) - \sigma_x(T_{xx}/E_x) - \sigma_z(T_{zz}/E_z), \tag{2}$$

$$e_{zz} = \{(T_{zz}/E_z) - (\sigma_x/E_x)\}(T_{xx} + T_{yy}), \tag{3}$$

$$e_{xy} = T_{xy}/G_{xy}, \tag{4}$$

$$e_{yz} = T_{yz}/G_{yz}, \tag{5}$$

$$e_{zx} = T_{zx}/G_{zx}, \tag{6}$$

where e_{xx}, e_{yy} . . . are the strains and T_{xx}, T_{yy} . . . are the stresses.

$E_x = E_y$ is the Young's modulus for compression in a direction parallel to the ice surface;

E_z is the Young's modulus for compression in a direction perpendicular to the ice surface;

G_{xy} is the elastic shear modulus for stresses acting in a direction parallel to the ice surface in planes perpendicular to that surface;

$G_{zx} = G_{yz}$ is the elastic shear modulus for stresses acting in a direction parallel to the ice surface in planes parallel to that surface;

σ_z is Poisson's ratio relating strains in the x and z directions and in the y and z directions;

σ_x is the Poisson's ratio relating strains in the x and y directions.

$$E_x = 2G_{xy}(1+\sigma_x). \tag{7}$$

In the experiments to be reported, ice samples were loaded in compression in the x direction (perpendicular to the long axis of the grains for multigrain ice) so that

$$T_{yy} = T_{zz} = T_{xy} = T_{yz} = T_{zx} = 0.$$

EXPERIMENTAL PROCEDURE

The observations were carried out on rectangular ice specimens 5 cm × 10 cm and 20 cm long prepared at −10° C with a standard metal cutting lathe and a wood planer. A small, manually driven testing machine with a capacity of 2000 kg (Hounsfield tensometer) was used for applying the loads. The test pieces were mounted between 5 cm × 10 cm rectangular steel plates 1.25 cm thick backed up by steel cones 5 cm in diameter and about 1 cm thick. The load was applied through balls which seated in the cones.

An extensometer with a 2-in. gauge length and a strain magnification of 3300 was used for measuring strains. The extensometer was frozen to the test piece during each load cycle. Load and strain were recorded on a Baldwin X-Y recorder, Model MD-2. The stress on the test piece was kept below 10 kg/cm^2 (140 p.s.i.) at all times.

The test pieces were prepared from multigrain ice and from large single crystals. Large single crystals of ice were prepared by seeding de-aerated tapwater and extracting heat vertically as shown in the sketch of the apparatus in Fig. 1. The water level was always maintained at the same height as the freezing interface by allowing water to drip from tap A. The cross section

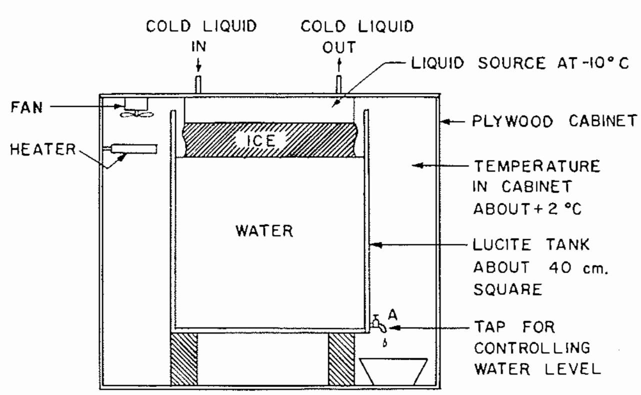

FIG. 1. Cabinet for preparing single crystals of ice.

of the crystal could be enlarged or decreased by adjusting this rate of flow. The single-crystal specimens were cut so that the stress was applied perpendicular to the direction of the optic axis, which coincided, in this case, with the direction of growth. The direction of the optic axis was determined optically with a Leitz polarizing microscope.

For the multigrain test pieces the average grain diameter was varied. Since it was found that the measured strains behaved erratically from one load cycle to the next when the average grain diameter was approximately equal to the width of the test piece, the largest grain diameter was kept less than the width by a factor of 4. Very fine-grained ice (average grain diameter approximately 2 mm) was prepared by seeding the water with snow. The test pieces were always cut so that the grain boundaries were perpendicular to the 10-×20-cm face. The observations were carried out in a cold room, the temperature of which could be varied over the range 0° C to −40° C and held at any temperature in this range to ±0.5° C. Because of the method of controlling the temperature, the humidity in the room is low and any exposed ice surface will evaporate. Between experiments, therefore, the test pieces were stored in kerosene as this did not appear to affect their elastic behavior even after continual immersion for 6 months. The temperature of the test pieces was taken as that of the kerosene in which they were stored, measured with a standard laboratory grade thermometer accurate to ±0.25° C. This temperature was very close to the average temperature of the room in which the tests were carried out. Temperatures at which the observations were made were chosen randomly with time. The test pieces were allowed to stand 24 hours at a new temperature before measurements were made.

It was observed that usually the strain varied across the sample as shown in Fig. 2 although this was not always true. Since it was necessary, therefore,

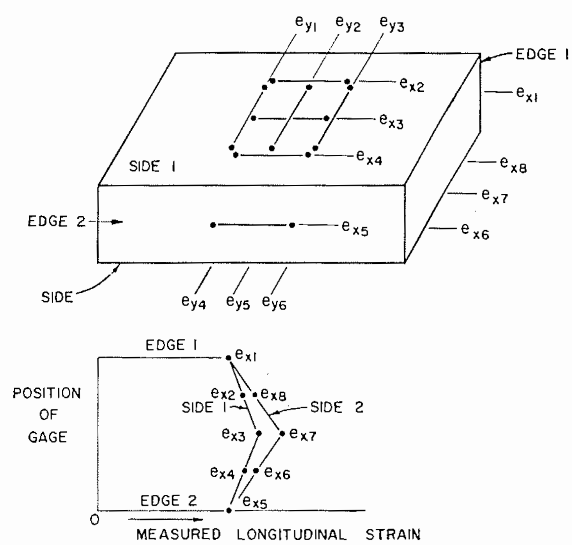

FIG. 2. Location of strain measurement positions on test piece and example of variation of measured longitudinal strain with position.

to measure the strain pattern set up during stressing, the following procedure was adopted. The test piece was mounted in the tensometer and centered as closely as possible by eye. A slight load was placed on the test piece and this load was always maintained between load cycles so that the position of the test piece in the testing machine was not changed. The strains were then measured at the positions shown in Fig. 2, as it was possible to rotate the test piece in the testing machine without disturbing its orientation with the applied stress. A complete load cycle was required to obtain the strain in each position. The strains e_{x3} and e_{x7} were measured twice; thus, the test piece was subject to 16 load cycles during each experiment. The order of measuring the strains was random with regard to location.

It was assumed that the elastic moduli for each test piece were constant at a given temperature and on this basis the average strain $\bar{e}_{xx}$ corresponding to the average stress T_{xx} given by the applied load divided by the cross-sectional area of the sample was

$$\bar{e}_{xx} = \frac{1}{8}\sum_{i=1}^{8} e_{xi}, \tag{8}$$

$$E_x = T_{xx}/\bar{e}_{xx}.$$

The transverse strain was measured over the central part only as shown in Fig. 2. An average transverse and corresponding longitudinal strain were calculated using the following equations:

$$\bar{e}_l = \{(e_{x2}+e_{x4}+e_{x6}+e_{x8})/8\}+\{(e_{x3}+e_{x7})/4\},$$

$$\bar{e}_t = \{(e_{y1}+e_{y3}+e_{y4}+e_{y6})/8\}+\{(e_{y2}+e_{y5})/4\},$$

in which e_{x3}, e_{x7}, e_{y2}, and e_{y5} are given a weight of 2

$$\sigma_x = \bar{e}_t/\bar{e}_l. \tag{9}$$

The method of averaging the strains corrected for any bending moment introduced into the test piece by not having the specimen correctly centered between the loading heads.

Observations on the elastic moduli of fine-grained ice were also made by measuring the velocity of propagation of an ultrasonic pulse. Barium titanate crystals, cut to excite the transverse vibration, were used to induce the vibrations in the ice and to detect the transmitted wave. The velocity of extensional and transverse waves propagated perpendicular to the grain boundaries were made using apparatus essentially the same as described by Leslie (1950). The samples used for the ultrasonic observations were protected from evaporation inside a special lucite box. The temperature in this enclosure was measured with a copper–constantan thermocouple.

EXPERIMENTAL RESULTS

Testing Machine

Preliminary observations showed that if the applied stress was kept low and if the length of time taken to complete a load cycle was of the order of 10 seconds, the relationship between the stress and the resulting strain was linear and there was little or no permanent deformation.

A dependence of E_x on temperature was found for multigrain ice whereas E_x for single crystals was found to be independent of temperature within the accuracy of the observations. This is shown in Fig. 3. It can also be seen in Fig. 3 that E_x for multigrain ice depends on the average grain diameter; the smaller the average grain diameter, the smaller is E_x for any fixed temperature.

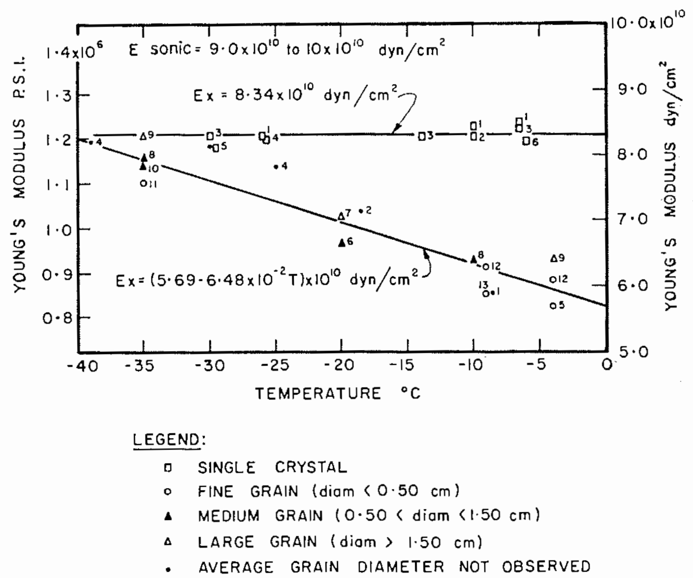

FIG. 3. Dependence of static values of Young's modulus on temperature.

In Fig. 3, the number at the upper right of each symbol specifies the test piece used. Some test pieces were used for more than one set of observations.

The equation for the straight line fitted by least squares to the multigrain ice data in Fig. 3 is

$$E_x = (5.69 - 6.48 \times 10^{-2} T) \times 10^{10} \text{ dynes/cm}^2,$$

where T is the temperature in °C. The coefficient of correlation between E_x and T was found to be 0.82.

The calculated values of $\bar{e}_l$ for multigrain ice were adjusted for an average stress of 5 kg/cm² acting over the central area on which the observations for the transverse strain were made. The calculated values for $\bar{e}_t$ were also adjusted to correspond with this average stress. In Fig. 4 are plotted the adjusted values of $\bar{e}_l$ and $\bar{e}_t$. The equation of the least squares fit line is

$$\bar{e}_t = -28.8 \times 10^{-6} + 0.88 \bar{e}_l.$$

The coefficient of correlation is 0.92. It can be seen from Fig. 4 that there is no obvious dependence on grain size.

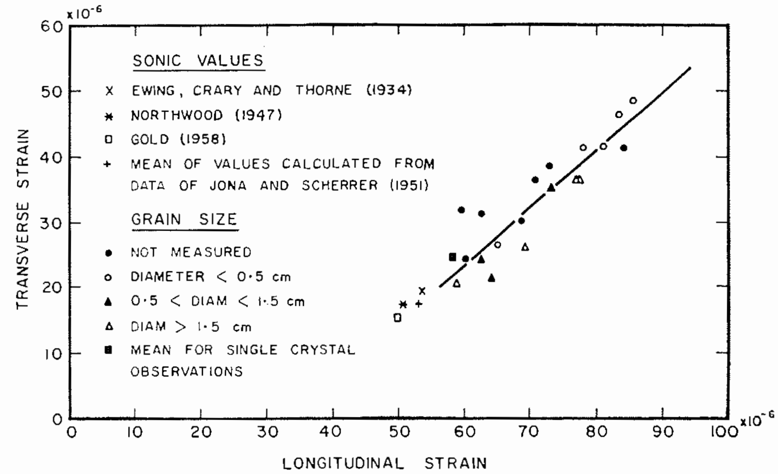

FIG. 4. Longitudinal strain and transverse strain for

stress $= 4.9 \times 10^6$ dynes/cm^2,
$= 5$ kg/cm^2.

The results of the measurements on single crystals were unusual in that $\bar{e}_l$ for an average stress of 5 kg/cm^2 was found to be equal to $(58.5 \pm 1.5) \times 10^{-6}$ for all tests whereas $\bar{e}_t$ varied from 19.5×10^{-6} to 33.5×10^{-6}. It was found that $\bar{e}_{xx} - \bar{e}_l$, a measure of the variation in strain across the sample, correlated with $\bar{e}_t$. In Fig. 4, the value of $\bar{e}_t$ for $\bar{e}_{xx} - \bar{e}_l = 0$ is plotted. For $\bar{e}_t > 25.0 \times 10^{-6}$, $\bar{e}_{xx} - \bar{e}_l$ was negative. The value of $\bar{e}_{xx} - \bar{e}_l$ for multigrain ice did not correlate well with $\bar{e}_t$.

Sonic Observations

In Fig. 5 the velocity of the extensional wave and the distortional wave

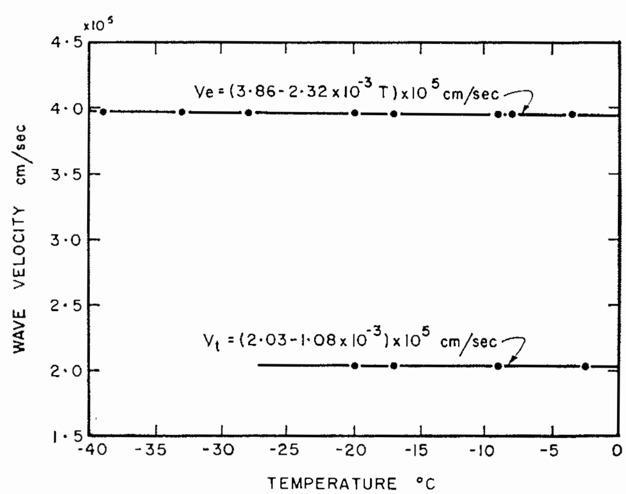

FIG. 5. Velocity of the extensional and distortional wave plotted against temperature.

measured ultrasonically in a multigrain specimen are plotted against temperature. A third wave, assumed to be the Rayleigh wave, was also observed. Because of the uncertainty in the path length associated with this wave, its velocity was not used in the calculations of the elastic moduli. The estimated velocity of the assumed Rayleigh wave was almost equal to the velocity of the shear wave.

The greatest source of error in calculating the velocities of the various waves through the ice was in estimating the time of arrival of the transmitted pulse and in measuring the length of the specimen. The circuit used for measuring time was calibrated against an accurate crystal-controlled oscillator. The extensional velocity is considered accurate to $\pm 1\%$ but the error in the velocity of the distortional wave may be greater than this. Least square fit o the results yield for the velocity of the extensional wave,

$$v_e = (3.86 - 2.32 \times 10^{-3} T) \times 10^5 \text{ cm/sec}$$

and for the velocity of the distortional wave

$$v_t = (2.03 - 1.08 \times 10^{-3} T) \times 10^5 \text{ cm/sec.}$$

T is again the temperature in °C. Values for G, E, and σ were calculated for a temperature of $-5°$ C and are given in Table I.

TABLE I

SONIC VALUES FOR YOUNG'S MODULUS, E; RIGIDITY MODULUS, G; POISSON'S RATIO, σ; AND BULK MODULUS, K, FOR MULTIGRAIN ICE AT A TEMPERATURE OF $-5°$ C

Source		E dynes/cm² $\times 10^{10}$	E p.s.i. $\times 10^6$	G dynes/cm² $\times 10^{10}$	G p.s.i. $\times 10^6$	σ	K dynes/cm² $\times 10^{10}$	K p.s.i. $\times 10^6$
Boyle and Sproule	(1931)	8.95	1.30					
Ewing, Crary, and Thorne	(1934)	9.17	1.33	3.36*	0.49	0.365	11.3*	1.64
Northwood	(1947)	9.80	1.42	3.68*	0.53	0.33	9.61*	1.39
Jona and Scherrer	(1951)							
Using Voigt relationships		9.38	1.36	3.52*	0.51	0.33	8.81*	1.28
Using Reuss relationships		9.18	1.33	3.45*	0.50	0.33	8.92*	1.29
Gold	(1957)	9.94	1.44	3.80	0.55	0.31*	8.72*	1.26
Richards and Speyers (static method, temp. $-7°$ C)	(1914)						8.34	1.21

*Calculate from relationships between E, G, σ, and K for isotropic materials.

The values given for E and σ in Table I were used to calculate longitudinal and transverse strains for a stress of 5 kg/cm². These strains are plotted in Fig. 4.

DISCUSSION

The results shown in Fig. 3 indicate that there is a contribution to the longitudinal strain in multigrain ice which does not occur in the single crystal specimens and, therefore, this contribution is probably associated with the grain boundaries. The observed qualitative dependence of Young's modulus on grain size is in agreement with the observation of Kê (1948) on multigrain metals. Figure 4 gives further confirmation that grain boundary slip is responsible for the dependence of the elastic moduli of multigrain ice on temperature. It shows that two deformation processes appear to contribute to the longitudinal and transverse strains. One process contributes an amount about the same as would be calculated using the elastic moduli measured sonically. The second process, which does not appear to operate in single crystals, involves a linear relationship between stress and strain as does the first, but it is much more temperature-dependent. If this contribution involves a relative movement between grains with no change in volume, then, because of the symmetry, the increase in the transverse strain would equal the increase in the longitudinal strain. In the present study, the increase in the transverse strain was found to be 0.88 times the increase in the longitudinal.

It can also be seen from Figs. 3 and 4 that because the two deformation processes involve different relationships between the longitudinal and transverse strains, the observed value of Poisson's ratio increases with increasing temperature. Since two processes appear to contribute to the strain, caution should be exercised in applying the usual relationships of classical elasticity, particularly the one given by equation (7),

$$E_x = 2G_{xy}(1+\sigma_x).$$

It is not possible at this time to draw conclusions on the relaxation time associated with the assumed grain boundary slip for ice. It was found during the experiments, however, that doubling and halving the rate of stressing did not change the observed value for Young's modulus. Therefore, the time associated with the stress cycle must have been reasonably larger than the relaxation time associated with the grain boundary slip. It is concluded, therefore, that Fig. 3 gives the relaxed value for the Young's modulus of multigrain ice as a function of temperature. This implies that not only does the grain boundary behave in a viscous manner but also that the allowable amount of slip which can occur before the grains become locked in new positions is a function of both the temperature and the stress.

CONCLUSIONS

The following conclusions have been drawn from the observations:

(1) For ice with a temperature in the range $-3°$ C to $-40°$ C, strain is linearly related to the applied stress if the duration of the stress is short enough and the maximum stress is kept to a low value. A maximum stress of 10 kg/cm^2 and a rate of stressing of 2 kg/cm^2/sec with the duration of the stress less than 10 seconds were found sufficient to satisfy the conditions.

(2) For multigrain ice two deformation processes, each involving a linear dependence of strain on stress, contribute to the measured strain at low rates of stressing. One process is probably dependent on temperature only to the degree exhibited by moduli measured sonically, whereas the second is markedly temperature-dependent.

(3) Since at the low rate of stressing the Young's modulus of multigrain

ice was temperature-dependent whereas for the single crystals it was temperature-independent, grain boundary slip is associated with the markedly temperature-dependent deformation.

(4) Since the observed values for Young's modulus did not change when the rate of stressing was double or half the rate normally used, these values correspond to conditions of maximum allowable relaxation of shear stresses at the grain boundaries.

(5) The amount of slippage that can occur at the grain boundaries depends on the temperature and the magnitude of the applied stress.

ACKNOWLEDGMENTS

The author wishes to acknowledge the assistance of Mr. L. Jones and Mr. D. Matz in obtaining the results herein presented.

REFERENCES

Boyle, R. W. and Sproule, D. O. 1931. Can. J. Research, **5**, 601.
Crawford, S. M. 1957. Proc. Phys. Soc. (London) B, **70**, 1045.
Dorsey, N. E. 1940. Properties of ordinary water-substance (Reinhold Publishing Corp., New York).
Ewing, M., Crary, A. P., and Thorne, A. M. 1934. Physics, **5**, 165.
Green, R. E. and MacKinnon, L. 1956. J. Acoust. Soc. Am. **28**, 1922.
Hess, H. 1940. Z. Gletscherkunde, **27**, 1. (SIPRE Translation No. 4, U.S. Army Corps of Engineers, 1950).
Hill, R. 1952. Proc. Phys. Soc. (London) A, **65**, 349.
Jona, F. and Scherrer, P. 1952. Helv. Phys. Acta, **25**, 35.
Kê, T-S. 1948. Trans. Am. Inst. Mining Met. Engrs. **176**, 448.
Leslie, J. R. 1950. Proc. Am. Soc. Testing Materials, **50**, 1314.
Love, A. E. H. 1952. A treatise on the mathematical theory of elasticity, 4th ed. (Cambridge University Press, London).
Northwood, T. D. 1947. Can. J. Research, A, **25**, 88.
Reuss, A. 1929. Z. angew. Math. Mech. **9**, 49.
Richards, T. W. and Speyers, C. L. 1914. J. Am. Chem. Soc. **36**, 491.
U.S. Army Corps of Engineers, 1951. SIPRE Rept. No. 4.
Voigt, W. 1928. Lehrbuch der Kristallphysik (Leipzig: Teubner).
Weinberg, B. 1936. Intern. Union Geodesy Geophysics. Intern. Assoc. Sci. Hydrol. Bull. **23**, 509.
Wilson, J. T., Zumberge, J. H., and Marshall, E. W. 1954. U.S. Army Corps of Engineers, SIPRE Rept. No. 5.
Zener, C. 1948. Elasticity and anelasticity of metals (University of Chicago Press, Chicago).

A list of all publications of the Division of Building Research is available and may be obtained from the Publications Section, Division of Building Research, National Research Council, Ottawa, Canada.