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PREDICTING OPTICAL AND THERMAL CHARACTERISTICS OF TRANSPARENT SINGLE-GLAZED DOMED SKYLIGHTS

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ABSTRACT

Optical and thermal characteristics of domed skylights are important to solve the trade-off between daylighting and thermal design. However, there is a lack of daylighting and thermal design tools for domed skylights. Optical and thermal characteristics of transparent single-glazed hemispherical domed skylights under sun and sky light are evaluated based on an optical model for domed skylights. The optical model is based on tracking the beam and diffuse radiation transmission through the dome surface. A simple method is proposed to replace single-glazed hemispherical domed skylights by optically and thermally equivalent single-glazed planar skylights to accommodate limitations of energy computer programs. Under sunlight, single-glazed hemispherical domed skylights yield slightly lower equivalent solar transmittance and solar heat gain coefficient (SHGC) at near normal zenith angles than those of single-glazed planar skylights. However, single-glazed hemispherical domed skylights yield substantially higher equivalent solar transmittance and SHGC at high zenith angles and around the horizon. Under isotropic skylight, single-glazed hemispherical domed skylights yield slightly lower equivalent solar transmittance and SHGC than those of single-glazed planar skylights. Daily solar heat gains of single-glazed hemispherical domed skylights are higher than those of single-glazed horizontal planar skylights in both winter and summer season. In summer, the solar heat gain of single-glazed hemispherical domed skylights can reach 3% to 9% higher than those of horizontal single-glazed planar skylights for latitudes varying between 0° and 55° (north/south). In winter, however, the solar heat gains of single-glazed hemispherical domed skylights increase significantly with the increase of the site latitude, and can reach 232% higher than those of horizontal single-glazed planar skylights, particularly for high latitude countries.

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INTRODUCTION

Domed skylights have increasingly been incorporated into modern building design and in retrofitted buildings as new elements for daylighting and aesthetic purpose. They admit abundant natural light into buildings and can simulate the outdoors in many buildings such as atria and sport arenas. Their potential in reducing electrical lighting and heating/cooling energy costs of buildings is well recognized (AAMA 1987; AAMA 1981; Treado et al 1983; Jensen 1983). Domed skylights have also been associated with high energy costs, especially during warm seasons. Optical and thermal characteristics of domed skylights are very important to solve the trade-off between daylighting and thermal design. The shape of the skylight geometry is a crucial factor for the amount of daylighting contribution, solar heat gains and thermal heat gains/losses. The optimum design can only be accomplished by an accurate determination of the optical and thermal characteristics of domed skylights.

Extensive theoretical and experimental investigations have been conducted to predict the optical and thermal performance of planar skylights and windows. However, there is a lack of tools to predict the optical and thermal performance of domed skylights, mostly because of the difficulty to simulate their geometry. Theoretical models to predict the optical and thermal performance of domed skylights are very limited. Wilkinson (1992) considered translucent (diffuse transmitter) domed skylights, and developed models to predict the Daylight Factor inside the dome based on horizontal illuminance formulation. Diffuse radiation from isotropic and CIE overcast skies, and beam sun radiation were considered. However, beam solar radiation was treated as diffuse radiation. For daylighting calculations, IESNA (1993) suggested a mathematical procedure to calculate the visible transmittance of single and double glazed domed skylights. The procedure does not account for the dome shape, and it has not been validated against measurement.

Recently, Laouadi and Atif (1998) have developed an optical model to predict the transmittance, absorptance and reflectance of transparent, multi-glazed, hemispherical, domed skylights under sun and sky light. The model is based on tracking the beam and diffuse radiation transmission through the dome shape. Experimental testing studies of real domed skylights are also very limited, due to their complex geometry and size (Enermodal 1994). Atif et al (1997) calculated the visible transmittance of an atrium pyramidal skylight based on on-site horizontal illuminance measurements outside and inside the skylight. There was a significant difference between predicted and measured data. Laboratory testing of domed skylights using physical scale models in artificial skies has been conducted using illuminance measurements to calculate the Daylight Factor. These studies were restricted to the conditions of the simulation and have not been validated against real fact data (Navvb 1990). ASHRAE's procedure for thermal calculation assumes domed skylights as tilted glazings to calculate the U-value of the structure, and the extent of error of such an assumption has never been tested (ASHRAE 1997). Fenestration rating computer programs such as VISION (CANMET 1995) and WINDOW (LBL 1992) deal with only planar glazings. Building energy-simulation computer programs accommodate the dome geometry by dividing it into a number of inclined surfaces (Clarke 1985; ESRU 1996).

OBJECTIVES

The specific objectives of the paper are:

1. To predict the transmittance, absorptance, reflectance and solar heat gain coefficient of transparent single-glazed hemispherical domed skylights.
2. To develop a simple method where single-glazed hemispherical domed skylights are replaced by optically and thermally equivalent single-glazed planar skylights for energy calculations.
3. To compare the thermal characteristics between single-glazed hemispherical domed and horizontal planar skylights.

The prediction of the optical and thermal characteristics of domed skylights is important not only for energy calculation but also for solving the trade-off between daylighting and solar heat gains.

MATHEMATICAL FORMULATION

A domed surface is defined by its truncation angle (σ_0) and its radius (R). A dome shape is a representative form for any curved surface. The family of shapes covered range from a fully hemispherical surface to a planar surface. Solar irradiance incident on a domed surface depends on its geometry, orientation with respect to the south, and inclination with respect to the horizontal. Thermal and optical properties of transparent surfaces - U-value, solar heat gain coefficient (SHGC), transmittance, absorptance, and reflectance - are available only for planar surfaces (ASHRAE 1997; IESNA 1993). Thermal properties of domed surfaces are more complicated to predict than those of planar surfaces.

Optical properties of domed surfaces can be evaluated based on the optical properties of their counterparts planar surfaces and their geometry. A domed surface can be divided into a number of infinitesimal inclined planar surfaces. Irradiance incident on the domed surface may, thus, be readily calculated by summing up all irradiances incident on the inclined infinitesimal surfaces. Transmitted irradiance through the domed surface may be calculated by summing up all transmitted irradiances through the inclined infinitesimal surfaces that reach the dome base surface. Transmittance of the domed surface may, thus, be readily obtained. Other optical properties may be obtained in a similar manner. The mathematical model has been developed to predict the optical properties of transparent domed surfaces (Laouadi and Atif 1998). The underlying assumptions of this model are:

1. The light transmittance, absorptance and reflectance at any point on the dome surface are equal to those of a flat surface at the same incidence angle.
2. The amount of light reflection from the interior space under the dome back to the dome interior surface is not accounted for.

Optical Properties of Domed Surfaces

Domed surfaces receive beam solar radiation as well as sky and ground-reflected diffuse solar radiation. The amount of solar irradiance transmitted, absorbed, or reflected by a domed surface depends on the dome geometry and the beam and diffuse solar radiation.

Beam Radiation Transmission Process Through a Domed Surface. Figure 1 shows a schematic description of the beam radiation transmission process through a horizontal domed surface in a system of coordinates (x, y, z) moving with the sun. In tracking the solar radiation transmission, the dome surface is split into two portions: portion (A_1), which corresponds to the directly-transmitted component that reaches the dome base surface; and portion (A_2), which corresponds to the transmitted-reflected component. Only the first reflected radiation from the dome interior surface is assumed to reach the dome base surface. The extent of the maximum error of this assumption is less than 5% at an incidence angle 85° and less than 3% at an incidence angle 80°.

The incident beam irradiance is given by:

$$I_{b,dome} = \int_{A_1+A_2} I_b \cos\theta ds \qquad \textbf{(1)}$$

The transmitted, absorbed and reflected beam irradiances are given by:

$$IT_{b,dome} = \int_{A_1} I_b \tau(\theta) \cos\theta ds + \int_{A_2} I_b \tau(\theta)\rho(\theta) \cos\theta ds \qquad \textbf{(2)}$$

$$IA_{b,dome} = \int_{A_1+A_2} I_b \alpha(\theta) \cos\theta ds + \int_{A_2} I_b \tau(\theta)\alpha(\theta) \cos\theta ds \qquad \textbf{(3)}$$

$$IR_{b,dome} = \int_{A_1+A_2} I_b \rho(\theta) \cos\theta ds + \int_{A_2} I_b \tau^2(\theta) \cos\theta ds \qquad \textbf{(4)}$$

where:

A_1, A_2 : the dome surface portions for the directly-transmitted and transmitted-reflected radiation, respectively;

ds : the area of an elementary surface associated with the point *P*;

I_b : the beam solar radiation (W/m^2);

P : a point that moves in a plane perpendicular to the sun's rays plane and inclined with an angle σ with respect to the dome base surface plane;

τ,α,ρ : the transmittance, absorptance and reflectance of a planar surface, respectively;

θ : the incidence angle on the elementary surface *ds*.

Figure 2 shows the coordinates of the elementary surface *ds*.
The incidence angle on the elementary surface (θ) is given by:

$$\cos\theta = \sin\xi\cos\theta_z + \cos\xi\sin\theta_z\sin\varphi' \quad \textbf{(5)}$$

where:

ξ : the elevation angle of the point P with respect to the dome base surface plane;

φ' : the relative azimuth angle of the point P;

θ_z : the sun zenith angle

The sun zenith angle θ_z and the elevation angle ξ are expressed as:

$$\cos\theta_z = \cos L\cos\delta\cos\omega + \sin L\sin\delta \quad \textbf{(6)}$$

$$\sin\xi = \sin\varphi\sin\sigma \quad \textbf{(7)}$$

with:

L : the site latitude angle;

δ : the sun declination angle;

ω : the hour angle;

σ : the inclination angle of the plane of the point P with respect to the dome base surface plane;

φ : the equivalent angle to φ' in the inclined plane of the point P, given by:

$$\cos\varphi = \cos\xi\cos\varphi' \quad \textbf{(8)}$$

<u>Diffuse Radiation Transmission Process Through a domed Surface</u>. The total diffuse irradiance incident on a domed surface is expressed as follows:

$$I_{d,dome} = \int_{A_{dome}} I_{d,t}\,ds \quad \textbf{(9)}$$

The transmitted, absorbed and reflected diffuse irradiances are calculated as follows (Laouadi and Atif 1998):

$$IT_{d,dome} = I_{d,dome}\left\{\frac{F_{12}\tau_d}{1-\rho_d F_{11}}\right\} \quad \textbf{(10)}$$

$$IA_{d,dome} = I_{d,dome}\left\{\alpha_d + \frac{F_{11}\tau_d\alpha_d}{1-\rho_d F_{11}}\right\} \quad \textbf{(11)}$$

$$IR_{d,dome} = I_{d,dome}\left\{\rho_d + \frac{F_{11}\tau_d^2}{1-\rho_d F_{11}}\right\} \quad \textbf{(12)}$$

where:

A_{dome} : the dome surface area;

F_{11} : the view factor of the dome interior surface to itself;
F_{12} : the view factor of the dome interior surface to its base surface;
$I_{d,t}$: the total diffuse sky and ground-reflected radiation on an inclined surface (W/m^2);
τ_d, α_d, ρ_d : the transmittance, absorptance and reflectance of a planar surface for diffuse radiation;

The view factors F_{11} and F_{12} are expressed as:

$$F_{11} = 1 - F_{12} \text{ ; and } \quad F_{12} = A_h / A_{dome} = (1 + \sin\sigma_0)/2 \tag{13}$$

with A_h the area of the dome base surface.
By definition, the dome transmittance is the ratio of the transmitted irradiance to the incident irradiance. Similarly, the dome absorptance or reflectance is the ratio of the absorbed or reflected irradiance to the incident irradiance. These are expressed as:
For beam radiation,

$$\tau_{dome} = \frac{IT_{b,dome}}{I_{b,dome}}; \quad \alpha_{dome} = \frac{IA_{b,dome}}{I_{b,dome}}; \quad \rho_{dome} = \frac{IR_{b,dome}}{I_{b,dome}} \tag{14}$$

For diffuse radiation,

$$\tau_{d,dome} = \frac{F_{12}\tau_d}{1 - \rho_d F_{11}} \text{ ; } \alpha_{d,dome} = \alpha_d + \frac{F_{11}\tau_d \alpha_d}{1 - \rho_d F_{11}}; \ \rho_{d,dome} = \rho_d + \frac{F_{11}\tau_d^2}{1 - \rho_d F_{11}} \tag{15}$$

<u>Solar Heat Gain Coefficient And U-Value</u>

By definition, the solar heat gain coefficient (SHGC) is the fraction of the incident irradiance that enters the glazing and becomes heat gain. It includes both the directly-transmitted portion and the fraction of the absorbed portion, which is redirected to the indoor space by convection and radiation. The SHGC for diffuse radiation is calculated assuming a uniform radiance distribution (ASHRAE 1997). The U-value of the dome structure is calculated based on one-dimensional heat flow. These parameters are expressed as follows:
For beam radiation,

$$SHGC_{dome} = \tau_{dome} + N_i \ \alpha_{dome} \tag{16}$$

For diffuse radiation,

$$SHGC_{d,dome} = \tau_{d,dome} + N_i \ \alpha_{d,dome} \tag{17}$$

$$U_{dome} = \frac{1}{1/h_0 + 1/h_i + R_c} \tag{18}$$

where:
h_0 : the outdoor combined convection and radiation coefficient;
h_i : the indoor combined convection and radiation coefficient;
R_c : the glazing conduction resistance.
N_i : the fraction of the absorbed irradiance that is inwardly redirected to the indoor space under the dome, and is expressed as follows (ASHRAE 1997):

$$N_i = U_{dome} / h_0 \tag{19}$$

Computing the optical and thermal properties of a dome is not straightforward before the evaluation of the double integral in Equations (2), (3), (4) and (9) for each time of the day. An alternative approach is to compute the optical and thermal properties of a planar surface that is optically and thermally equivalent to a domed surface. This is particularly important to building energy-simulation and fenestration-rating computer programs. This approach has the following advantages:

1. It eliminates the need for the input of complex geometrical data of domed surfaces. The user only needs to input the optical properties of the dome-equivalent planar surface.
2. It allows daylighting and thermal performance to be compared as between domed and planar surfaces.
3. It can treat inclined domed surfaces by simply reducing them to inclined dome-equivalent planar surfaces.
4. It facilitates the prediction of the optical properties of domed surfaces on the basis of measurements conducted for the dome-equivalent planar surface. Horizontal illuminance measurement inside and outside the dome may be used to measure the equivalent transmittance.

OPTICAL AND THERMAL CHARACTERISTICS OF A DOME-EQUIVALENT PLANAR SURFACE

A simple method is proposed to calculate the optical and thermal characteristics of a planar surface that is optically and thermally equivalent to a domed surface. The dome-equivalent planar surface would have the same aperture, the same construction materials, the same orientation and inclination angles, and would produce similar amount of transmitted, absorbed and reflected irradiances, and similar amount of thermal heat losses/gains as the domed surface.
The optical and thermal characteristics of the dome-equivalent planar surface are calculated as follows:

Equivalent Optical Properties

The equivalent transmittance (τ_{eq}), absoptance (α_{eq}) and reflectance (ρ_{eq}) are expressed as:
For beam radiation,

$$\tau_{eq} = \tau_{dome} \cdot \varepsilon ; \qquad \alpha_{eq} = \alpha_{dome} \cdot \varepsilon ; \qquad \rho_{eq} = \rho_{dome} \cdot \varepsilon \qquad \textbf{(20)}$$

For diffuse radiation,

$$\tau_{d,eq} = \frac{F_{12}\tau_d}{1-\rho_d F_{11}} \cdot \varepsilon_d ; \quad \alpha_{d,eq} = \left\{ \alpha_d + \frac{F_{11}\tau_d \alpha_d}{1-\rho_d F_{11}} \right\} \cdot \varepsilon_d ; \quad \rho_{d,eq} = \left\{ \rho_d + \frac{F_{11}(\tau_d)^2}{1-\rho_d F_{11}} \right\} \cdot \varepsilon_d \qquad \textbf{(21)}$$

Conservation of solar radiative heat flux on the dome surface yields the following relationship:

$$\tau_{eq} + \alpha_{eq} + \rho_{eq} = \varepsilon \qquad \textbf{(22)}$$

where ε is the ratio of the incident irradiance on the domed surface to that incident on the dome-equivalent planar surface. ε_d is for diffuse radiation. For horizontal domed surfaces, ε and ε_d are given by:

$$\varepsilon = \frac{1}{A_h} \int_{A_1+A_2} \frac{\cos\theta}{\cos\theta_z} ds ; \qquad \varepsilon_d = \frac{1}{A_h} \int_{A_{dome}} \frac{I_{d,t}}{I_d} ds; \qquad \textbf{(23)}$$

Equation (23) for beam radiation can be expressed as:

$$\varepsilon = \frac{3}{4} + \frac{1}{\pi \cos^2 \sigma_0} \left\{ F_s + F_c \tan\theta_z \right\} \qquad \textbf{(24)}$$

with:

$$F_s = (\varphi_0(\sigma_2) - \pi/2)\cos\sigma_2 - \frac{1}{2}\cos^2\sigma_0 \sin^{-1}\left(\frac{(1+\sin^2\sigma_0)\sin^2\sigma_2 - 2\sin^2\sigma_0}{\cos^2\sigma_0 \sin^2\sigma_2}\right)$$

(25)

$$F_c = (\pi/2 - \varphi_0(\sigma_2))\sin\sigma_2 - \sin\sigma_0 \cos\varphi_0(\sigma_2)$$

where σ_2 is the angle that delimits the surface A_2 , and φ_0 is given by:

$$\varphi_0(\sigma_2) = \sin^{-1}(\sin\sigma_0 / \sin\sigma_2) \quad \textbf{(26)}$$

For isotopic diffuse overcast skies, Equation (23) for diffuse radiation reduces to:

$$\varepsilon_d = \frac{1/2}{F_{12}}\{1 + \rho_g + (1 - \rho_g)F_{12}\} \quad \textbf{(27)}$$

where ρ_g is the ground reflectance (albedo).

<u>Equivalent Solar Heat Gain Coefficient and U-Value</u>

SHGC and U-value for the dome-equivalent planar surface are given as follows:
For beam radiation,

$$SHGC_{eq} = \tau_{eq} + N_i\ \alpha_{eq} \quad \textbf{(28)}$$

For diffuse radiation,

$$SHGC_{d,eq} = \tau_{d,eq} + N_i\ \alpha_{d,eq} \quad \textbf{(29)}$$

$$U_{eq} = U_{dome} / F_{12} \quad \textbf{(30)}$$

The daily solar heat gain of a horizontal domed surface (or, the dome-equivalent horizontal planar surface) is calculated as follows:

$$SHG_{dome} = \int_{day} \{SHGC_{eq} \cdot I_b \cos\theta_z + SHGC_{d,eq} \cdot I_d \cos\theta_d\} A_h dt \quad \textbf{(31)}$$

Similarly, the daily solar heat gain of a horizontal planar surface, having the same surface area as the dome base surface, is given by:

$$SHG_h = \int_{day} \{SHGC_h \cdot I_b \cos\theta_z + SHGC_{d,h} \cdot I_d \cos\theta_d\} A_h dt \quad \textbf{(32)}$$

where I_d the diffuse radiation intensity on a horizontal surface, and θ_d the incidence angle for diffuse radiation.
The daily solar heat gain of horizontal domed surfaces is an important parameter to compare the thermal performance of different dome shapes. Likewise, the ratio of the daily solar heat gain of a horizontal domed surface to that of a horizontal planar surface - SHG_{dome}/SHG_h - is an important parameter to compare the monthly/seasonal solar heat gains of horizontal domed surfaces with those of horizontal planar surfaces.

<u>RESULTS AND DISCUSSION</u>

The models developed above are applied to predict the optical and thermal performance of a single-glazed hemispherical domed skylight. A comparison is made between the optical and thermal characteristics of a dome-equivalent planar skylight and those of a horizontal planar skylight. A clear float glass is used as the dome glazing. The solar transmittance and absorptance of the clear float glass at normal incidence angle are τ = 0.78 and α = 0.15 (Pilkington 1988). The solar transmittance and absorptance at other incidence angles are calculated and then fitted using five order polynomial series with argument $\cos(\theta_z)$, similar to those used by ASHRAE (1997). The U-value of the skylight is 5.84 W/m^2K in summer and 6.19 W/m^2K in winter, based on the ASHRAE design conditions (ASHRAE 1997). The double integral in Equations (2), (3), (4) and (9) is evaluated using Simpson's rule for numerical integration.

Figure 3 shows the profile of the equivalent solar transmittance of a single-glazed dome as a function of the sun zenith angle for a number of dome shapes. Contrary to planar surfaces, the equivalent solar transmittance of single-glazed domes increases with the increase in the sun zenith angle, especially for domes with truncation angles up to 45°. The equivalent solar transmittance of single-glazed domes with truncation angles $\sigma_0 < 30°$ is slightly lower at near normal zenith angles ($\theta_z < 40°$), and significantly higher at high zenith angles and around the horizon than that of single-glazed planar surfaces. The equivalent solar transmittance of single-glazed domes with truncation angles greater than 30° is approximately the same as that of single-glazed planar surfaces for zenith angles up to 55°, and much higher at high zenith angles and around the horizon. The equivalent solar transmittance increases slightly with the dome truncation angle at near normal zenith angles. However, at high zenith angles and around the horizon, the equivalent solar transmittance decreases with the increase of the dome truncation angle. Single-glazed fully hemispherical domes have the lowest equivalent solar transmittance at near normal zenith angles (about 10% lower than that of single-glazed planar surfaces) and the highest equivalent solar transmittance at high zenith angles and around the horizon. As a result, depending on the site latitude and the day of the year, single-glazed domes may yield lower or higher equivalent solar transmittance than that of single-glazed planar surfaces. For tropical regions (latitude lower than 24°), single-glazed domes yield lower equivalent solar transmittance at noontime than that of single-glazed planar surfaces in both winter and summer season. For higher latitudes, single-glazed domes yield lower equivalent solar transmittance at noontime in summer, and higher equivalent solar transmittance in winter than that of single-glazed planar surfaces. This is a very important feature of domed surfaces, particularly for daylighting in summer and winter season.

Figure 4 shows the profile of the equivalent solar heat gain coefficient (SHGC) of a single-glazed dome for beam radiation as a function of the sun zenith angle for a number of dome shapes. The equivalent SHGC is calculated based on summer design conditions (N_i=0.2573). The equivalent SHGC follows the same trend as the equivalent solar transmittance (Figure 3). At near normal zenith angles, the equivalent SHGC is slightly higher than the equivalent solar transmittance. However, at high zenith angles and around the horizon, the equivalent SHGC is much higher than the equivalent solar transmittance. This is because the equivalent solar absorptance of single-glazed domes is much higher at high zenith angles and around the horizon than that at near normal zenith angles ($\theta_z < 40°$). At near normal zenith angles, the equivalent SHGC of single-glazed domes is slightly lower than that of single-glazed planar surfaces. However, at high zenith angles and around the horizon, the equivalent SHGC of single-glazed domes is substantially higher than that of single-glazed planar surfaces. The equivalent SHGC increases slightly at near normal zenith angles, and decreases significantly at high zenith angles and around the horizon with the increase of the dome truncation angle. Fully hemispherical single-glazed domes have the lowest equivalent SHGC at near normal zenith angle (about 9% lower than that of single-glazed planar surfaces) and the highest equivalent SHGC at high zenith angles and around the horizon. For tropical regions, single-glazed domes yield lower equivalent SHGC at noontime than that of single-glazed planar surfaces in both winter and summer season. For high latitude countries, single-glazed domes yield lower equivalent SHGC at noontime in summer, and higher equivalent SHGC in winter than that of single-glazed planar surfaces.

Figure 5 shows the profiles of the equivalent solar transmittance, SHGC and U-value of a single-glazed dome as a function of the dome shape under isotropic diffuse skies. The equivalent SHGC and U-value are calculated based on summer design conditions (N_i = 0.2573). Single-glazed domes under isotopic diffuse skies have lower equivalent solar transmittance and SHGC than those of single-glazed planar surfaces. Fully hemispherical single-glazed domes have the lowest equivalent solar transmittance and

SHGC. The equivalent solar transmittance of a fully hemispherical single-glazed dome is 14% lower than that of a single-glazed planar surface while the equivalent SHGC is 7% lower than that of a single-glazed planar surface. However, the equivalent U-value of single-glazed domes is significantly higher than that of single-glazed planar surfaces. Fully hemispherical single-glazed domes have the highest equivalent U-value, due to the fact that fully hemispherical domes have the largest surface area. As a result, under isotropic skies, single-glazed domes are susceptible to lower solar heat gains and higher thermal heat losses/gains than those of single-glazed planar surfaces.

Figure 6 shows the profile of the ratio of the daily solar heat gain of a single-glazed dome to that of a horizontal single-glazed planar surface as a function of the dome shape for the 21st of December and June. Several site latitudes with zero longitude difference are covered. The beam (I_b) and horizontal sky diffuse (I_d) radiation are calculated using the ASHRAE standard method for solar radiation calculation (ASHRAE 1997). The ratio of beam-to-diffuse radiation is 0.134 and 0.057 for December and June, respectively. Perez et al. model (Duffie and Beckman 1991) is used to estimate the total diffuse radiation on a sloped surface for the non-isotropic sky diffuse radiation.

The Figure shows that single-glazed domes result in larger daily solar heat gains than those of horizontal single-glazed planar surfaces during winter and summer season. Fully hemispherical single-glazed domes have the largest solar heat gains. In summer, the solar heat gain ratio for fully hemispherical single-glazed domes is between 1.03 and 1.09 for latitudes varying between 0° and 55° (north/south). However, in winter, the solar heat gain ratio increases significantly with the site latitude. In tropical countries (latitudes lower than 24°), the solar heat gain ratio can reach 1.19. In subtropical countries (latitudes between 24° and 35°), the solar heat gain ratio can reach 1.33. In midlatitude countries (latitudes between 35° and 55°), the solar heat gain ratio can reach 2.32.

CONCLUSIONS

The optical and thermal characteristics of a clear single-glazed hemispherical domed skylight are evaluated under sun and sky light. Since energy computer programs deal with planar surfaces, a simple method was proposed to replace single-glazed hemispherical domed skylights by optically and thermally equivalent single-glazed planar skylights. Single-glazed hemispherical domed skylights have the following important features:

1. Under sunlight (direct beam), single-glazed domed skylights have low equivalent solar transmittance and SHGC at near normal zenith angles (noontime) and substantially high equivalent solar transmittance and SHGC at high zenith angles and around the horizon. Nearly fully-hemispherical single-glazed domed skylights ($\sigma_0 < 30°$) have slightly lower equivalent solar transmittance and SHGC at near normal zenith angles, and significantly higher equivalent solar transmittance and SHGC at high zenith angles and around the horizon than those of single-glazed planar skylights. Single-glazed domed skylights with truncation angles greater than 30° yield approximately the same equivalent solar transmittance and SHGC as single-glazed planar skylights for zenith angles up to 55°, and yield much higher equivalent solar transmittance and SHGC at high zenith angles and around the horizon.
2. Under isotropic diffuse overcast skies, single-glazed domed skylights have slightly lower equivalent solar transmittance and SHGC than single-glazed planar skylights. Fully hemispherical single-glazed domed skylights have the lowest equivalent solar transmittance (about 14% lower than that of single-glazed planar skylights) and SHGC (about 7% lower than that of single-glazed planar skylights).
3. Under combined sunlight and non-isotropic sky light, single-glazed domed skylights yield higher daily solar heat gains than those of single-glazed horizontal planar skylights. Fully hemispherical domed skylights yield the largest daily solar heat gains in both winter and summer season. In summer, the solar heat gain of fully hemispherical single-glazed domed skylights is 3% to 9% higher than that of horizontal single-glazed planar skylights for latitudes varying between 0° and 55° (north/south). In winter, however, the solar heat gain of single-glazed domed skylights increases significantly with the site latitude. In tropical countries (latitudes lower than 24°), the

solar heat gain of fully hemispherical single-glazed domed skylights can reach 19% higher than that of horizontal single-glazed planar skylights. In subtropical countries (latitudes between 24° and 35°), the solar heat gain of fully hemispherical single-glazed domed skylights can reach 33% higher than that of horizontal single-glazed planar skylights. In midlatitude countries (latitudes between 35° and 55°), the solar heat gain of fully hemispherical single-glazed domed skylights can reach 232% higher than that of horizontal single-glazed planar skylights.

4. Single-glazed domed skylights have higher equivalent U-value than that of single-glazed planar skylights. Fully hemispherical single-glazed domed skylights have the largest equivalent U-value (twice higher than that of single-glazed planar skylights). Single-glazed domed skylights are, thus, susceptible to higher thermal heat gains/losses than those of horizontal single-glazed planar skylights. Therefore, the shape of the skylight dome should be chosen according to the site latitude and the prevailing climate to compromise solar heat gains with thermal losses.

NOMENCLATURE

A_1 : surface for the directly-transmitted beam radiation through a domed surface.
A_2 : surface for the transmitted-reflected beam radiation through a domed surface
A_{dome} : area of the dome surface.
A_h : area of the dome base surface.
F_c, F_s : functions, Equation (25).
F_{11} : view factor of the dome interior surface to itself.
F_{12} : view factor of the dome interior surface to its base surface.
h_i : combined indoor radiation and convection coefficient.
h_0 : combined outdoor radiation and convection coefficient.
I : incident irradiance on a surface (W).
I_b : beam solar radiation (W/m^2).
I_d : sky diffuse solar radiation on a horizontal surface (W/m^2).
$I_{d,t}$: total diffuse radiation on an inclined surface (W/m^2).
IT : transmitted irradiance (W).
IA : absorbed irradiance (W).
IR : reflected irradiance (W).
L : site latitude.
R : dome radius.
R_c : glazing conductive resistance (m^2K/W).
SHG : daily solar heat gain (J).
SHGC : solar heat gain coefficient.
t : time.
U_{dome} : U-value of the dome structure (W/m^2K).
U_{eq} : U-value of the dome-equivalent planar surface (W/m^2K).

Geek Symbols

α, ρ, τ : absorptance, reflectance and transmittance of a flat glazing, respectively.
δ : sun declination angle.
ε : ratio of dome irradiance to planar surface irradiance, Equation (23)
φ : equivalent angle to φ' in the plane of the point P.
φ' : the relative azimuth angle of the elementary surface (ds).
θ : incidence angle on the elementary surface (ds).
θ_d : incidence angle for diffuse radiation.
θ_z : sun zenith angle.
ρ_g : ground reflectance (albedo).
σ : inclination angle of a plane perpendicular to the plane of the sun's rays.
σ_0 : dome truncation angle.
σ_1, σ_2 : angles that delimit the surfaces A_1 ($\sigma_0 \le \sigma \le \sigma_1$) and A_2 ($\sigma_1 \le \sigma \le \sigma_2$), respectively.
ω : hour angle.
ξ : elevation angle of the point P on the dome surface with respect to the dome base surface plane.

Subscripts

b : beam radiation.
d : diffuse radiation.
dome : dome surface.
eq : dome-equivalent planar surface.
h : horizontal surface.
I : indoor.
0 : outdoor.

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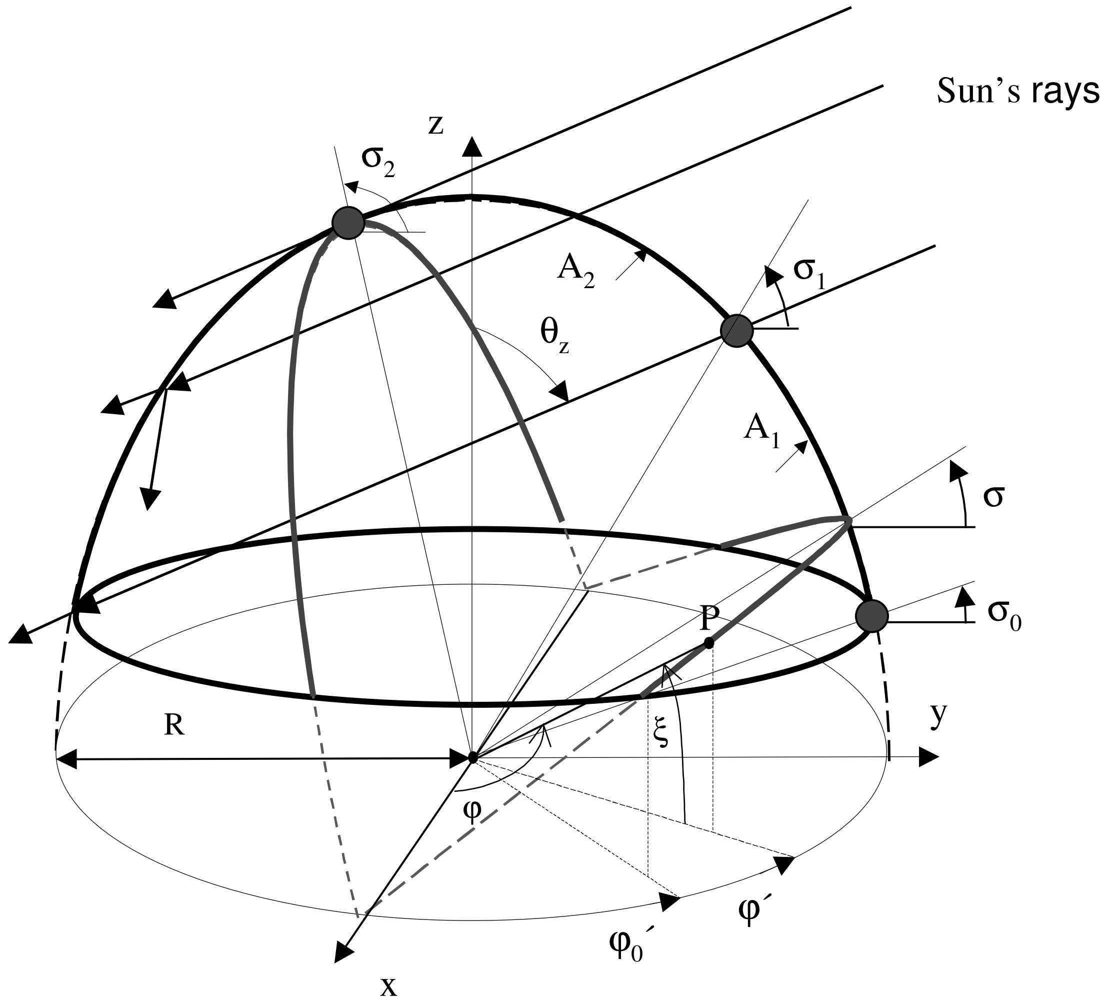
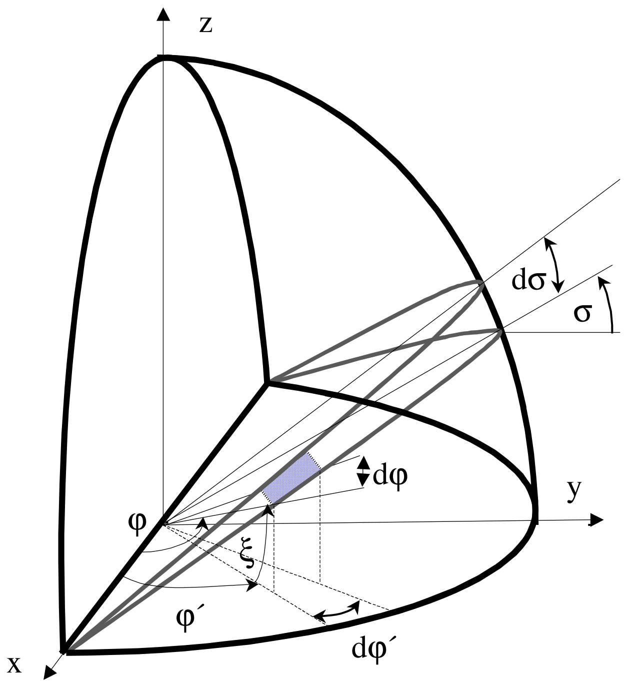
FIGURE CAPTIONS

Figure 1 Beam radiation transmission process through a horizontal domed surface.
Figure 2 Coordinates of the elementary surface (ds) on the dome.
Figure 3 Profile of the equivalent solar transmittance of a single-glazed dome as a function of the sun zenith angle.
Figure 4 Profile of the equivalent solar heat gain coefficient (SHGC) of a single-glazed dome as a function of the sun zenith angle.
Figure 5 Profiles of the diffuse equivalent solar transmittance $\tau_{d,eq}$, $SHGC_{d,eq}$ and U_{eq} of a single-glazed dome as a function of the dome shape under isotropic diffuse skies.
Figure 6 Profile of the ratio of the daily solar heat gain of a single-glazed dome to that of a horizontal single-glazed planar surface as a function of the dome shape.



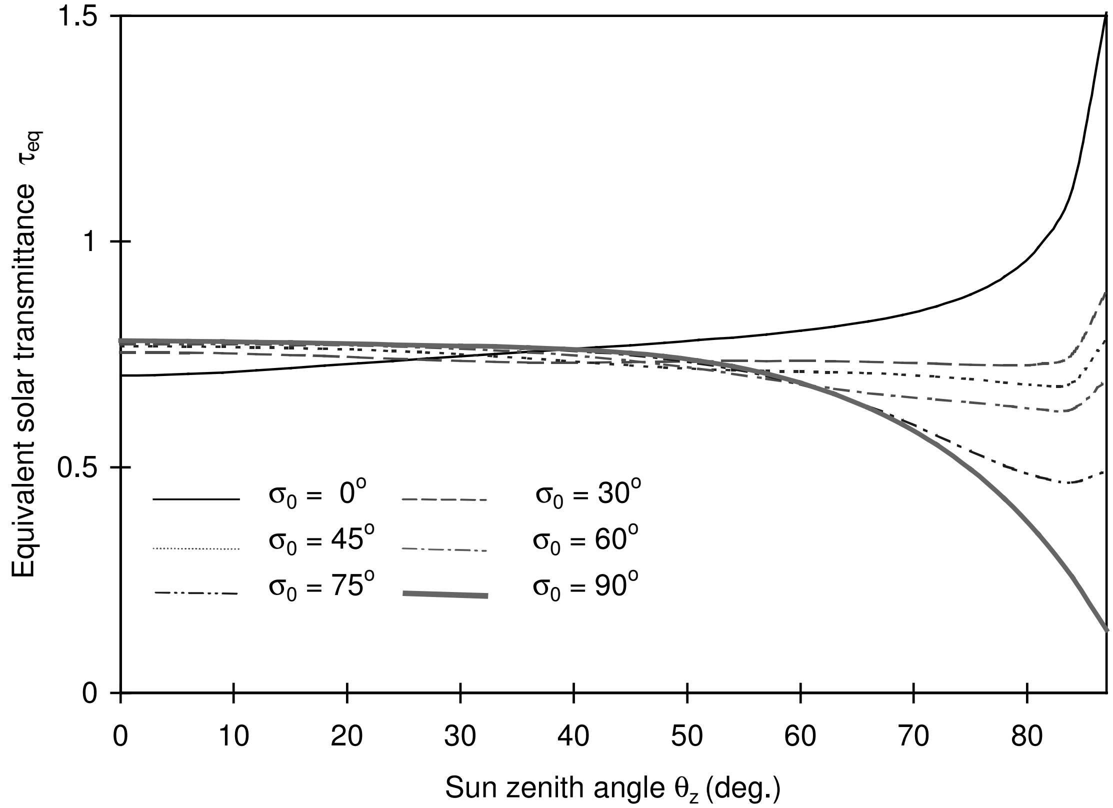
Sun's rays
z
σ_2
A_2
σ_1
θ_z
A_1
σ
σ_0
P
R
y
ξ
φ
φ'
φ_0'
x

Figure 1



$$ds = R^2 \sin\varphi \, d\varphi \, d\sigma$$

Figure 2



1.5
1
0.5
0
0
10
20
30
40
50
60
70
80
Equivalent solar transmittance τeq
Sun zenith angle θz (deg.)
σ0 = 0o
σ0 = 30o
σ0 = 45o
σ0 = 60o
σ0 = 75o
σ0 = 90o

Figure 3

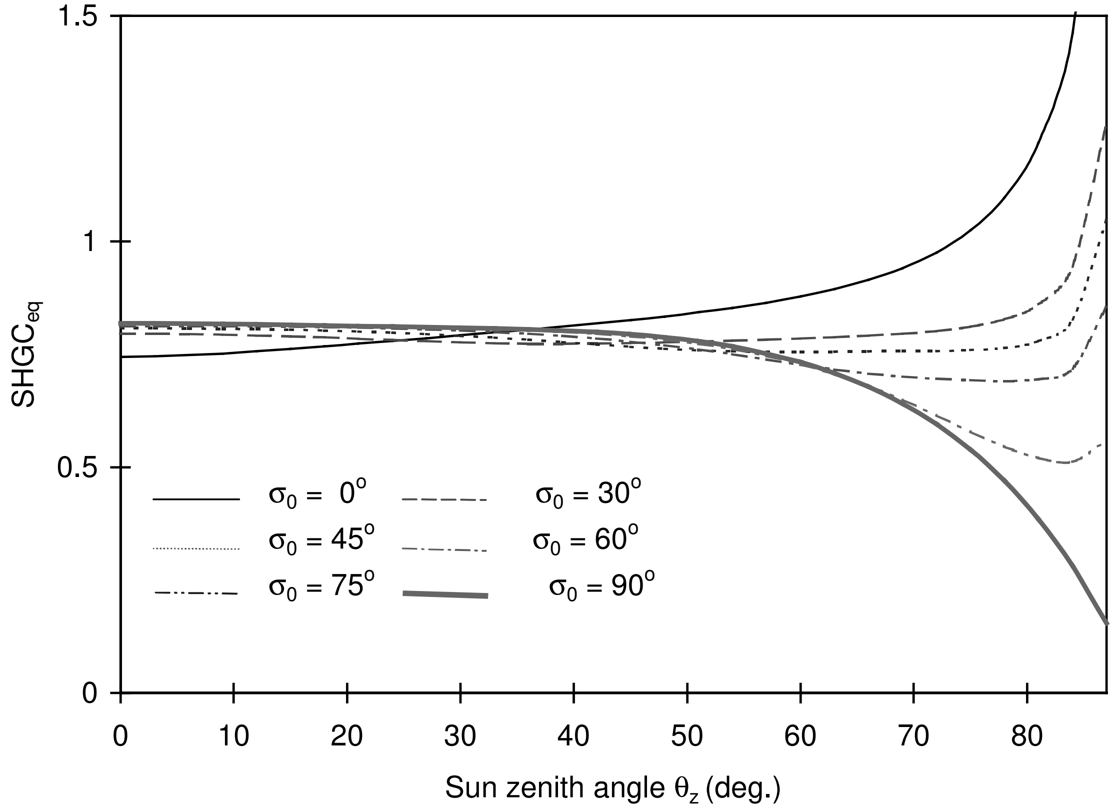
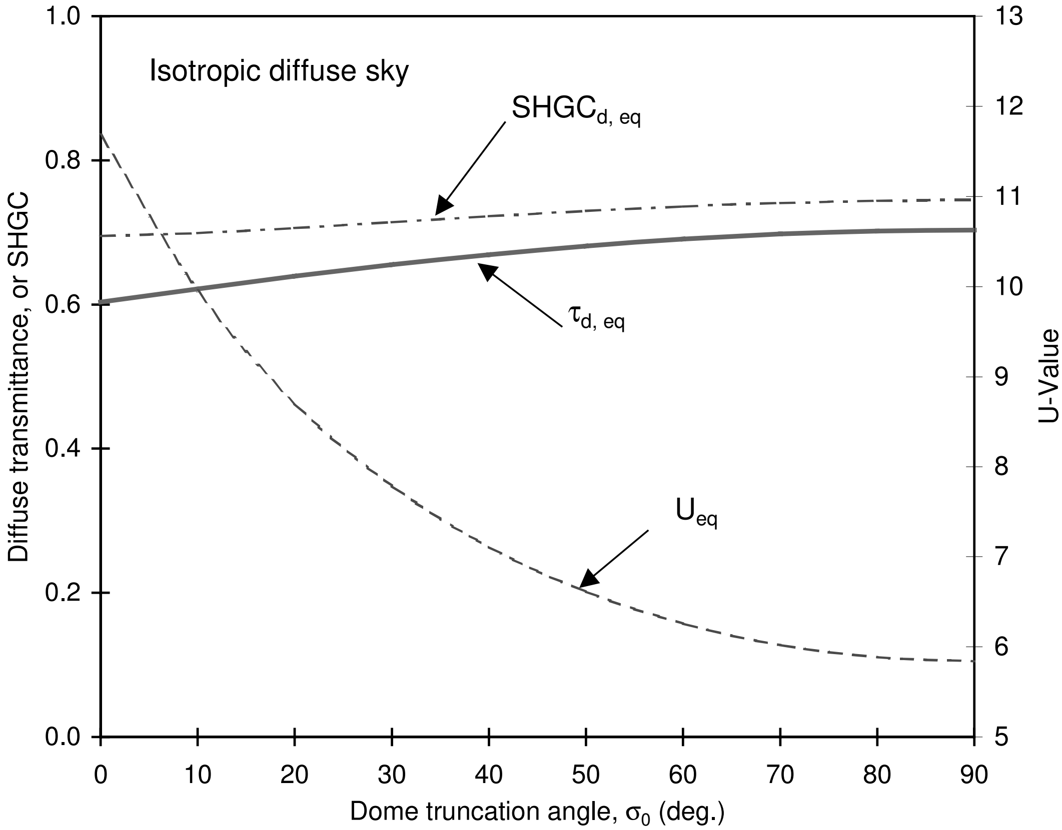
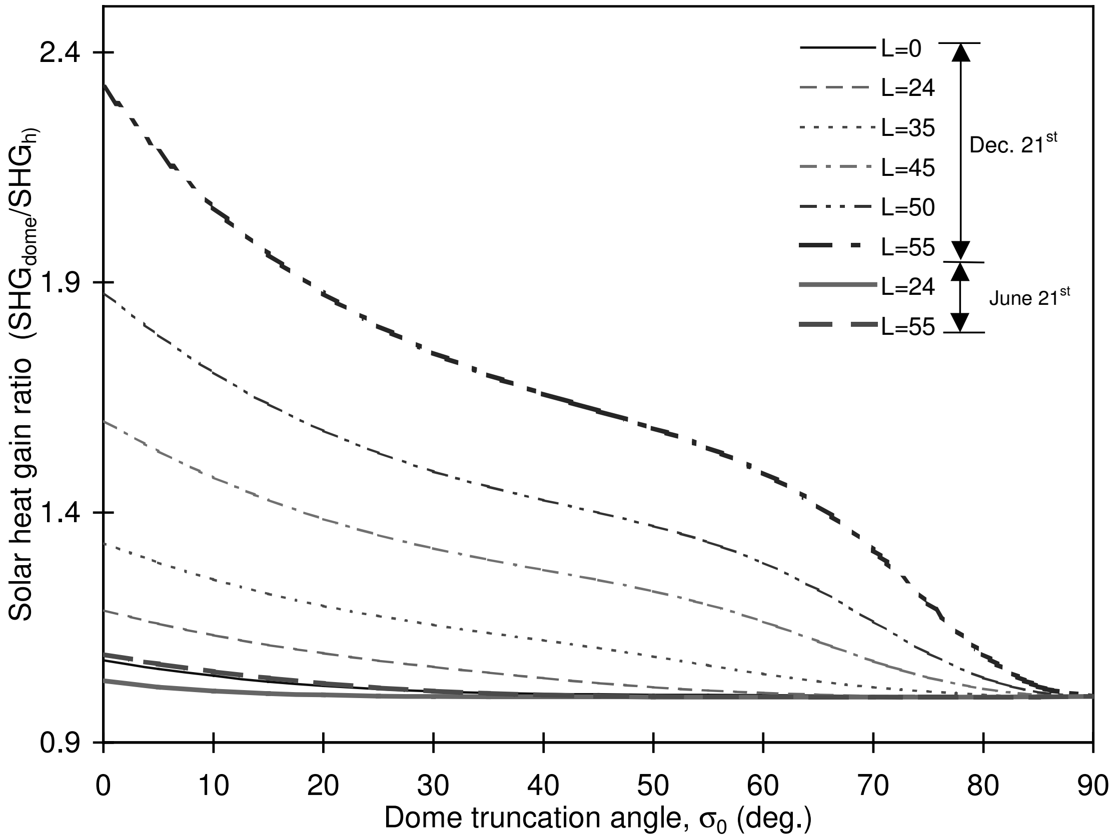

1.5
1
0.5
0
SHGC$_{eq}$
σ_0 = 0^o
σ_0 = 30^o
σ_0 = 45^o
σ_0 = 60^o
σ_0 = 75^o
σ_0 = 90^o
0
10
20
30
40
50
60
70
80
Sun zenith angle θ_z (deg.)

Figure 4



1.0
0.8
0.6
0.4
0.2
0.0
13
12
11
10
9
8
7
6
5
Isotropic diffuse sky
SHGC$_{d, eq}$
$\tau_{d, eq}$
U$_{eq}$
Diffuse transmittance, or SHGC
U-Value
0
10
20
30
40
50
60
70
80
90
Dome truncation angle, σ_0 (deg.)

Figure 5



Solar heat gain ratio (SHGdome/SHGh)
2.4
1.9
1.4
0.9
0
10
20
30
40
50
60
70
80
90
Dome truncation angle, σ0 (deg.)
L=0
L=24
L=35
L=45
L=50
L=55
L=24
L=55
Dec. 21st
June 21st

Figure 6