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Publisher's version / Version de l'éditeur:

Journal of Geophysical Research, 64, 9, pp. 1293-1298, 1959-11-01

NATIONAL RESEARCH COUNCIL
CANADA

DIVISION OF BUILDING RESEARCH

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BY

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REPRINTED FROM

JOURNAL OF GEOPHYSICAL RESEARCH
VOL. 64, NO. 9, SEPTEMBER 1959, P.1293 - 1298

RESEARCH PAPER NO. 86
OF THE
DIVISION OF BUILDING RESEARCH

OTTAWA

NOVEMBER 1959

PRICE 10 CENTS

NRC 5304

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JOURNAL OF GEOPHYSICAL RESEARCH VOLUME 64, NO. 9 SEPTEMBER, 1959

Observations of Ground Temperature and Heat Flow at Ottawa, Canada

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Abstract—Observations were made at a site in Ottawa, Canada, on the thermal regime of the ground and the heat flow 10 cm below the ground surface. Close agreement was found between the thermal diffusivities calculated from the depth dependence of the temperature amplitude and the depth dependence of the phase angle. It was also found that the annual component of the rate of heat flow was out of phase with the annual component of the ground temperature by 45.8 degrees. These observations showed that for the Ottawa site the annual wave of ground temperature and the annual component of the rate of heat flow are consistent with the simple theory of periodic heat flow in a one-dimensional, semi-infinite, homogeneous medium. In accordance with this theory, thermal constants for the soil at the test site were calculated from the observations

Introduction—Since its inception, many of the problems encountered by the Division of Building Research of the National Research Council have required for their solution a knowledge of the thermal regime of the ground. Consequently, this has been a subject of extensive study. Several reports describing previous work have been published [*Crawford* and *Legget*, 1957; *Pearce*, 1958; *Gold*, 1958]. Late in 1956, the instrumentation of the test area on the Montreal Road property at Ottawa was considerably expanded. This paper deals with observations made at this site on ground temperature and heat flow during the period from January 1, 1957 to January 1, 1958.

Instrumentation—The area under investigation was situated in an open grass-covered field. The soil in this region is a Leda clay which extends to the bedrock at a depth of about thirty feet. The instrumentation was designed to measure both soil temperatures and the heat flow through the soil.

For temperature measurements, sealed thermometer units were fabricated, each containing a glass-enclosed bead-type thermister and a copper constantan thermocouple. The thermometer elements were sealed in ¼-inch o.d. aluminum tubing. Ten such units were installed at depths of 5, 10, 15, 20, 30, 40, 50, 60, 75, and 90 cm below the ground surface. The units were installed horizontally from the vertical wall of a trench. Each thermometer was located about 3 feet from the trench wall. After installation of the thermometers, the trench was backfilled with the original material and tamped.

The electrical leads were brought to the ground surface through a wax-filled conduit and were then carried to an adjacent service tunnel about 150 feet from the test area by an elevated surface conduit. Temperatures were recorded with Leeds and Northrup recording equipment located in the tunnel.

The heat flow was measured with heat meters of the Gier and Dunkle type which had been calibrated in a hot-plate apparatus. For the observations in the spring of 1957, two adjacent heat meters at a depth of about 5 cm were employed. The heat flow was determined from the output of the two meters. The installation was modified somewhat in the early autumn of 1957. Thereafter, four heat meters were used in two separate units of two meters in series at the 10-cm depth. This gave an average heat flow over a larger area of ground than did the original installation. The leads for the heat meters were carried to the service tunnel through the elevated conduit, and the outputs of the heat meters were recorded with automatic equipment.

Shortly after installation, faults developed in the thermister circuits. The source of the difficulty was discovered to be the development of

N.R.C. 5304

appreciable leakage conductance between the thermister leads and ground. The magnitude of the leakage resistance was sufficient to render the previous thermister calibrations useless. As recalibration was impossible, the thermister temperature observations were discontinued. All the temperature observations discussed in this paper were obtained from thermocouple measurements.

Method of analysis—The object of the analysis was to see if the observations on the ground temperature and heat flow were consistent with the simple theory of periodic heat flow in a homogeneous medium. Since variations of ground temperature are essentially periodic phenomena, the results were analyzed by the techniques of Fourier analysis. A small digital computer was used to determine the Fourier components of the observed dependence of temperature on time. The computer program allowed only about 50 measurements per year to be used in the analyses. Consequently, the strip charts were processed to obtain the average weekly temperatures at each depth. From the weekly averages it was possible to obtain a reliable value for the annual mean and the first Fourier component of the temperature at each depth. The first Fourier component has a period of one year and will be called the annual component throughout the remainder of this paper.

The observations from the heat meter were processed in a slightly different manner. This modified procedure was developed to smooth some of the fluctuation in the rate of heat flow. The first step of the calculation was to determine by graphical integration the total heat flow for each day. Since the heat flow at the 10-cm level is very nearly equal to that at the 5-cm level, it was assumed that the observations during the first part of 1957 gave the heat flow at the 10-cm level within the error of measurement. The daily totals were then summed, and the accumulated heat flow was tabulated as a function of time. The annual component of the accumulated heat flow was then computed, and, finally, the annual component of the rate of heat flow was determined by differentiation of the annual component of the accumulated heat flow.

Previous studies [*Pearce*, 1958] have indicated that at the Ottawa site the annual component of the temperature at each depth in the soil is remarkably consisent with simple heat conduction theory. It is therefore possible to calculate from this component meaningful thermal properties of the soil. It must be pointed out that the values of the thermal properties of the soil, which were determined by the technique of Fourier analysis and are given herein, should be considered as a type of average which would be valid only for the same frequency in the temperature and heat-flow variation as that from which they were calculated. It is possible that different values for the thermal properties of soil might be obtained if the Fourier component with a period of one day were used instead of the annual component.

At Ottawa, the ground under a natural, undisturbed snow cover can freeze to a depth of about 50 cm, depending on the depth of the snow. During 1957 the frost line penetrated to 53 cm. Some of the heat flow which was observed at the 10-cm level, must, therefore, have been associated with the freezing of the soil water. This contribution to the measured heat flow was neglected in the calculation of the mean and annual components of the ground temperature and accumulated heat flow. Justification for this will be given in the discussion.

Results—The mean and annual components of the ground temperature at each depth were computed with January 1 being taken as the zero of time (Table 1).

TABLE 1—*Mean and annual component of ground temperature*

Depth cm	Ground temperature °C
10	$6.61 + 11.40 \sin(\omega t - 2.05)$
15	$7.54 + 11.48 \sin(\omega t - 2.10)$
20	$7.70 + 11.41 \sin(\omega t - 2.14)$
30	$7.82 + 11.1 \sin(\omega t - 2.15)$
40	$7.87 + 10.59 \sin(\omega t - 2.22)$
50	$7.98 + 9.82 \sin(\omega t - 2.28)$
60	$7.86 + 9.32 \sin(\omega t - 2.34)$
75	$7.86 + 8.42 \sin(\omega t - 2.42)$
90	$7.96 + 7.76 \sin(\omega t - 2.51)$

The computed annual component of the accumulated heat flow at the 10-cm depth with the zero of time taken as January 1 is $q(t) = -335.1 \cos \omega t - 1025.2 \sin \omega t$ cal/cm². By differentiation of the annual component of the

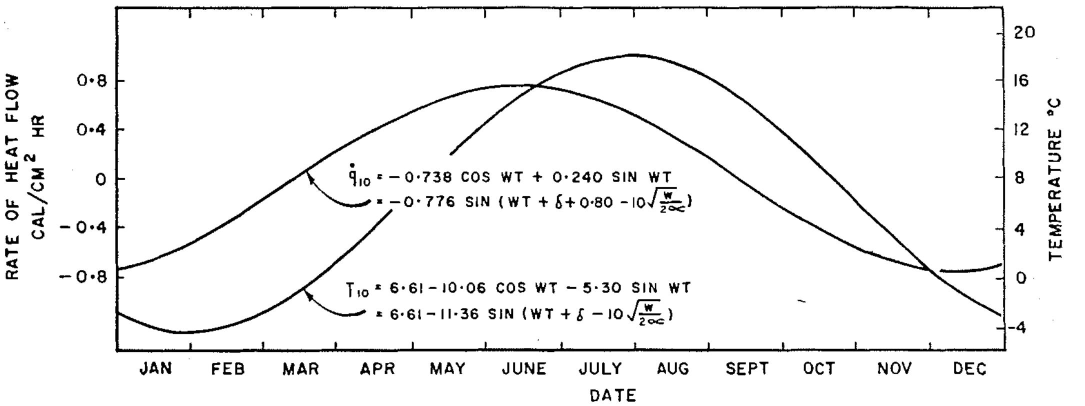

FIG. 1—Annual component of the rate of heat flow and of the ground temperature at the 10 cm level

accumulated heat flow the rate of heat flow at the 10-cm depth is $\dot{q}(t) = -0.738 \cos \omega t + 0.240 \sin \omega t$ cal/cm²hr. The annual component of the rate of heat flow and the annual component of the ground temperature are plotted in Figure 1. The observed accumulated heat flow and its annual component are given in Figure 2.

To obtain thermal constants for the soil, these observations were compared with the theory of heat flow in a homogeneous semi-infinite medium. The theoretical temperature distribution is given by the equation

$$U(x, t) = A_0 + A_1 \exp\left(-x\sqrt{\frac{\omega}{2\alpha}}\right) \cdot \sin\left(\omega t + \delta - x\sqrt{\frac{\omega}{2\alpha}}\right) \quad (1)$$

where

$U(x, t)$ is the temperature at depth x and time t
A_0 is the mean temperature of the medium
ω is the angular frequency of the temperature variation
$A_1 \sin(\omega t + \delta)$ is the surface temperature variation
α is the thermal diffusivity of the medium.

The theoretical rate of heat flow at depth x and time t is given by the equation

$$\dot{q}(x, t) = \sqrt{\frac{\omega}{\alpha}}\, K A_1 \exp\left(-x\sqrt{\frac{\omega}{2\alpha}}\right) \cdot \sin\left(\omega t + \delta + \frac{\pi}{4} - x\sqrt{\frac{\omega}{2\alpha}}\right) \quad (2)$$

where K is the thermal conductivity of the medium.

Figures 3 and 4 are plots of the amplitude and the phase of the observed annual-temperature-wave at various depths. For the selected coordinates, the curves should be linear if the temperature obeys simple conduction theory. It can be seen that the observations are consistent with this theory, except near the ground surface. Both curves give essentially the same value for the thermal diffusivity, 11 cm²/hr. Extrapolation of the linear curves to zero depth yields the values 13.1°C and 2.007 respectively for the constants A_1 and δ.

From equations (1) and (2) it is seen that at any depth the annual component of the rate of heat flow should be 45° out of phase with the annual component of the ground temperature. From the observations it was found that the calculated annual components were out of phase by 45.8° at the 10-cm level—very close to the theoretically expected value. This suggests that the annual component of the heat flow can be described by simple conduction theory; that is, the process can be characterized by a unique, real value of the thermal conductivity of the soil which can be calculated from the observations. This was done and the resulting value of K was found to be equal to 8.45 (cal)/(cm hr °C). Using this value for K and $\alpha = 11$ (cm²)/(hr) gives for the volumetric heat capacity of the soil a value of 0.77 (cal)/(cm³ °C). This is not unreasonable for the soil at the test site, which has a dry density of about 1.40 gm/cm³ at a water content of 30 to 40 per cent by dry weight.

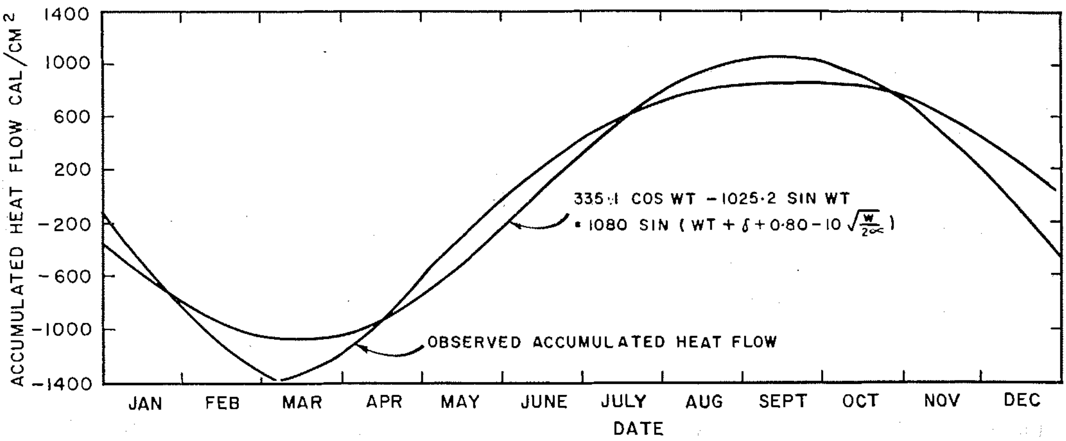

FIG. 2—The observed accumulated heat flow and its annual component

The values of the thermal properties of the soil for the test area, computed from the annual component of the rate of heat flow, and the observations of the ground temperature are given in Table 2.

Discussion—This analysis has demonstrated that the annual temperature variation in the soil at the Ottawa test site is remarkably consistent with the theory of heat flow in a one-dimensional, homogeneous, semi-infinite medium. Furthermore, the computed value of the thermal diffusivity is very close to the values obtained for clay soils in the Ottawa area in other years [*Pearce*, 1958]. This indicates that in the Ottawa area the thermal diffusivity of the soil as determined from annual temperature variations does not vary appreciably from year to year.

Only near the air-soil interface do significant deviations from this theory occur. The observations suggest that the thermal conductivity increases near the ground surface. This interpretation should, however, be employed with caution. Because the observed phase relationships between the annual components of the ground temperature and the rate of heat flow and the observed dependence on depth of the amplitude of the annual component of the ground temperature was consistent with theory, it was assumed that unique thermal constants existed for the soil. In the Ottawa area, however, the upper part of the soil is always frozen for a part of the year, and, since the thermal constants of frozen and unfrozen soil are quite different [*Kersten*, 1952], it is possible that the thermal behavior of the upper soil layers cannot be described satisfactorily by these average values. This could account for the deviation in the observed ground temperatures near the surface, which were neglected in the calculations. Nevertheless, it is concluded that the simple conduction model does yield a satisfactory description of the thermal behavior of the soil at the test site for depths greater than about 30 cm.

Other studies by the Division of Building Research have demonstrated difficulties in the use of heat meters. Problems with contact resistance are particularly troublesome. Consequently, the observed heat flows are estimated to be accurate only to about ±10 per cent.

TABLE 2—*Thermal properties of soil*

Thermal diffusivity		Volumetric heat capacity		K thermal conductivity	
cm^2/hr	ft^2/hr	cal/cm^3 °C	Btu/ft^3 °F	cal/cm °C hr	Btu/ft^2 °F hr
11.0	1.2×10^{-2}	0.77	48	8.45	0.57

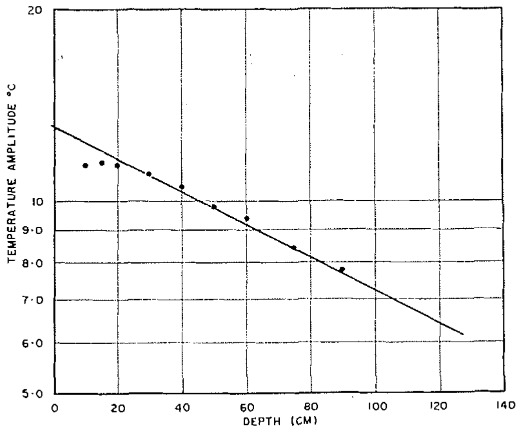

FIG. 3—Amplitude of the annual ground temperature change as a function of depth

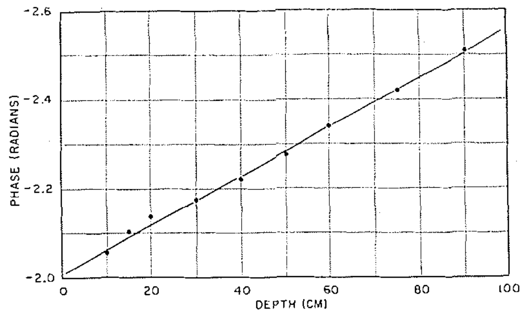

FIG. 4—Phase of the annual ground temperature as a function of depth

Freezing and thawing of the soil and heat transfer by vapor and water movement occur below the 10-cm level. Since the values for the thermal properties which were calculated from the annual components of the ground temperature and the rate of heat flow are reasonable for the soil in question, it would appear that the influence of such effects on the annual component is not very great at the Ottawa site. Some justification for this conclusion was observed for the winter period during which the maximum difference of about 400 cal/cm^2 between the observed accumulated heat flow and the computed annual component, as shown in Figure 1, was in fact about equal to the estimated heat associated with the freezing of the soil water. The following method was used to estimate the ice content per unit volume of the frozen soil. First, an estimate based on the work of *Lovell* [1957] was made of the amount of water still unfrozen at the time of observation. This amount was then subtracted from the measured total water content per unit volume to yield the ice content.

The recorded results in this paper indicate that a calculation of the depth of the 32° isotherm might be used to estimate the depth of frost penetration. Once, in the spring of 1958, the actual depth of freezing was determined by probing. This depth was almost a foot less than the measured depth of the 32° isotherm. This may mean only that the soil freezes at a temperature below 32°F, a not unexpected result, but it illustrates that caution is necessary when trying to predict the depth of the freezing plane with simple theories.

The fact that the thermal diffusivity which was obtained from this analysis is in agreement with the results of previous studies, including those of areas with and without snow cover, indicates that a snow cover has very little influence on the thermal properties of the soil. Thus the effect of snow cover can be treated purely as a surface boundary condition. This assumption allows the prediction of certain general thermal relationships at the soil-air interface.

This analysis indicates that approximate thermal properties of the ground, and an estimate of the annual component of the rate of heat flow from the ground, can be determined from a measurement of ground temperature and the volumetric heat capacity of the soil. We believe that approximate values which are obtained in this way may have numerous practical applications.

Acknowledgments—The authors wish to acknowledge the able assistance of R. Armour and R. Jaekel in the instrumentation and the analysis of the results. They also wish to express their appreciation to members of the Division of Building Research for their contribution through discussion. This is a contribution from the Division of Building Research of the National Research Council of Canada and is published with the approval of the Director of the Division.

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(Manuscript received March 12, 1959; revised June 2, 1959.)

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