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by
T. Z. Harmathy

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LE COMPORTEMENT THERMIQUE AU FEU DES MURS EN MACONNERIE DE BÉTON

SOMMAIRE

Mille cent quatre-vingt calculs par ordinateur ont été établis pour étudier les courants de chaleur dans le feu à traverse les murs construits d'unité de maconnerie de béton. Cas calculs ont couvert une grande sélection de quatre variables géométriques et quatre bétons qui pourraient etre considérés comme "matériaux limités" dans les groupes de bétons ordinaires et de bétons légers. Il est possible d'exprimer la résistance au feu des unités de maconnerie dans une condition sèche à l'aide de trois équations empériques. Ces équations peuvent servir à exprimer la résistance au feu de variables géométriques et des propriétés matérielles seulement dans le cas de bétons ayant des agrégats chimiquement stables. Leur utilité réelle est de tracer le chemin vers une planification économique et, comme formules d'extrapolations, d'agrandir le champ des renseignements d'essais aux géométries et aux matériaux qui ne sont pas couverts par les essais conte le feu.

T. Z. Harmathy[1]

Thermal Performance of Concrete Masonry Walls in Fire

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ABSTRACT: Eleven hundred eighty computer calculations have been performed to study the heat flow in fire through concrete masonry unit walls. They covered wide ranges of the four geometric variables, and four concretes which could be regarded as "limiting materials" in the normal weight and lightweight groups. It was possible to express the thermal fire endurances of the masonry units in dry condition with the aid of three empirical equations. These equations can be used to estimate the fire endurance from the geometric variables and material properties only in the case of concretes made with chemically stable aggregates. Their real usefulness lies in showing the way to economical design and, as extrapolation formulas, in extending test information to geometries and materials not covered by fire tests.

KEY WORDS: computer prediction, concrete, design criteria, walls, fire endurance, lightweight concrete, masonry units, masonry walls, thermal performance, thermal properties, evaluation, tests

Nomenclature

Notations

a	Web thickness, ft
A	Empirical constant
b	Web spacing, ft
B	Empirical constant
c	Specific heat at constant pressure, Btu/lb R
C	Surface of cavity
e	Error
E	"Exposed" surface
F	Function

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h	Coefficient of heat transfer; without subscript: that on the unexposed side, Btu/h ft^2 R
j	$= 1, 2, 3, \ldots$
k	Thermal conductivity, Btu/h ft R
l	Face shell thickness, ft
L	Over-all thickness, ft
m	Empirical constant
M	Length of masonry unit, ft
n	Distance along the outwardly directed normal vector, ft
n	$= 1, 2, 3, \ldots$
N	Number of webs in a masonry unit; number of surface elements along the cavity; number of data in error estimation
p	Empirical constant
q	Empirical constant
r	$= 1, 2, 3, \ldots$
s	$= 1, 2, 3, \ldots$
S	Region containing all interior points
t	Time, h
T	Temperature, R (if not stated otherwise)
U	"Unexposed" surface
x	Coordinate, ft
y	Coordinate, ft

Greek Letters

β	Empirical constant, Btu/h ft^2 R$^{5/4}$
Δ	Increment or difference
$\Delta\xi$	Mesh width, ft
ϵ	Emissivity of surface, dimensionless
κ	Thermal diffusivity, ft^2/h
λ	Equivalent thickness, ft
ρ	Density, lb/ft^3
σ	Stefan-Boltzmann constant, 0.1713×10^{-8} Btu/h ft^2 R^4
τ	Thermal fire endurance, h
$\bar{\tau}$	Thermal fire endurance of solid unit, h
$\bar{\bar{\tau}}$	Thermal fire endurance of double-layer configuration, h

Subscripts

a	For the case of change in a
a	Absolute
av	Average
c	Computed
c	Pertaining to a point for which $x = c\Delta\xi/\sqrt{2}$
d	Pertaining to a point for which $x = d\Delta\xi/\sqrt{2}$
e	Estimated with the aid of the formulas

E	On the "exposed" side
f	Of the "furnace"
i	Of the surface at $x = 0$
k	Of the k-th surface element
l	Of the l-th surface element
l	For the case of change in l
L	Of the surface at $x = L$
max	Maximum
o	At $t = 0$; of the surroundings on the "unexposed" side
r	Pertaining to a point for which $x = r\Delta\xi\sqrt{2}$
s	Pertaining to a point for which $y = s\Delta\xi\sqrt{2}$
160	Based on 160 F rise in temperature
250	Based on 250 F rise in temperature

Superscripts

j	At $t = j\,\Delta t$
o	At $t = 0$
*	Denoting a known fire endurance value and all information pertaining to this value

Concrete masonry units are undoubtedly among the most popular simple components of modern buildings. Because of their great importance in the design of industrial, commercial, and residential buildings, during the past few decades there has been an intensive investigation into their behavior both in normal service and under unusual circumstances, especially during fire exposure. Because meeting certain requirements with respect to their thermal insulation, dimensional stability, and weathering characteristics rarely presents great practical problems, nowadays probably more money is spent on obtaining information on their fire endurance characteristics than on all of their other design aspects combined.

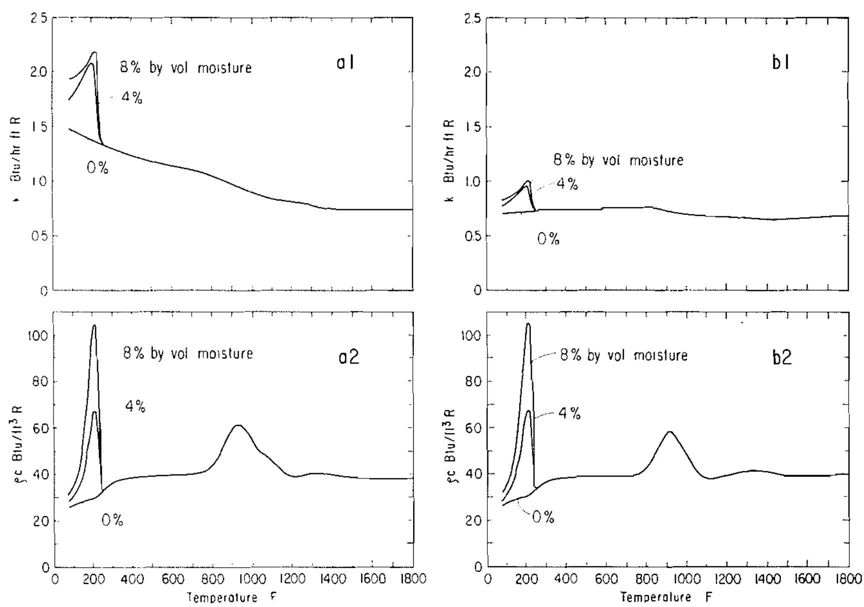
Because of the great expenses involved in fire tests of building elements, it is quite natural that over the years certain shortcut methods have evolved which make possible the extension of experimental fire endurance information to supposedly similar concrete masonry unit constructions. Unfortunately, most of these methods have been based on rather superficial conclusions obtained usually from log-log plots representing scattered experimental data, without respect to some fundamental similarity requirements. Therefore, they may result in completely erroneous conclusions.

To obtain a large amount of information about the thermal performance of concrete masonry units in fire under a number of strictly maintained similarity conditions, well over a thousand computations were performed. In these numerical studies the complex geometry of the units and the true nature of the temperature dependence of the thermal properties of the material were taken into account, and great care was exercised to simulate the proper mechanisms of heat transport to the last detail.

The primary purpose of these studies was to determine the effect of certain variables or groups of variables, concerning partly the material and partly the geometry of the units, on their thermal performance in fire. Because of the well-known difficulties connected with the standard fire testing procedure, the fire endurance values reported here do not necessarily correspond to the true interpretation of ASTM Methods of Fire Tests of Building Construction and Materials (E 119-67); nevertheless, under certain conditions the semiempirical formulas developed in this paper may prove useful for the "estimation" of the "standard" fire endurance of concrete masonry unit walls.

Thermal Properties of Concrete

The large differences in the thermal properties of various concretes, unfortunately, are still often disregarded in connection with fire endurance problems. Many research workers are satisfied with dividing all



(*a*) Concrete 1 (quartz aggregate).
(*b*) Concrete 2 (anorthosite aggregate).
FIG. 1—*Thermal properties of two normal weight concretes.*

concretes into two groups: (1) the normal weight and (2) the lightweight group. Those engaged in more refined research work sometimes further divide the normal weight concretes into "siliceous" and "calcareous" groups.

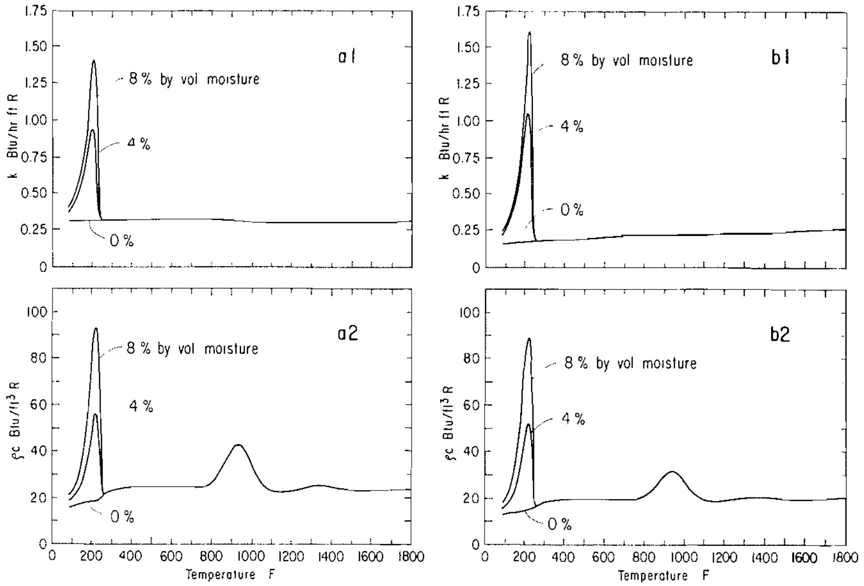
The Boulder Canyon Report [*1*][2] was the first to call attention to the large differences that may exist in the thermal properties of normal weight concretes; differences of similar magnitude also exist in the lightweight group. This problem is complicated further by the fact that both the thermal conductivity, k, and the "volume specific heat," that is, the ρc product, may undergo substantial changes with a rise of temperature. Because of the lack of physicochemical stability of concrete, the "normal" property changes that occur in any solid material upon heating are supplemented by other changes, often much more substantial, brought about by the evolving decomposition, transition, and other reactions.

To be able to draw conclusions that are applicable to larger groups of concretes, it is necessary to study the thermal performance of at least those concretes which represent, with respect to their insulating characteristics, the two limiting cases in the groups considered. In the computer studies carried out in this laboratory, four concretes, two normal weight and two lightweight, were examined. The k versus T and ρc versus T relations for these four concretes (hereafter referred to as Concretes 1, 2, 3, and 4) were derived in Ref *2* from experimental data with the aid of some theoretical considerations; these relations are reproduced in Figs. 1 and 2. The room-temperature values of k, ρc, and κ for these concretes at 0 percent moisture are listed in Table 1.

In Figs. 1 and 2 the k versus T and ρc versus T curves pertaining to 4 and 8 percent (by volume) moisture also are shown. The effect of moisture on the thermal fire endurance of building elements, however, is fairly well known [*3, 4, 5*]. It has been decided, therefore, that the present fire endurance studies would relate to dry masonry units only, and any correction for the presence of moisture would be left to the reader. It must be borne in mind, therefore, that, *before applying the formulas of this paper to any experimental fire endurance values, these values must be converted to correspond to dry condition, and after the application of the formulas the effect of moisture should be taken into account, as described in Refs* 3, 4, *and* 5. The usual procedure is illustrated in a numerical example in the section "Utilization of the Results."

Concretes 1 to 4 were conceived to represent the poorest and best concretes in the normal weight and lightweight groups, respectively, from a fire endurance point of view. As seen from Fig. 3, all experimental

[2] The italic numbers in brackets refer to the list of references appended to this paper.



(*a*) Concrete 3 ("expanded shale A" aggregate).
(*b*) Concrete 4 ("expanded shale B" aggregate).
FIG. 2—*Thermal properties of two lightweight concretes.*

TABLE 1—*Some thermal properties of Concretes 1 and 2 (normal weight) and Concretes 3 and 4 (lightweight) at room temperature, in dry condition.*

Concrete	k Btu/h ft R	c Btu/ft^3 R	κ ft^2/h
1	1.491	26.10	0.05713
2	0.718	26.80	0.02679
3	0.317	16.35	0.01939
4	0.159	13.45	0.01182

thermal conductivity values obtained in this laboratory for normal weight concretes in dry condition (0 percent moisture) fall between the values for Concretes 1 and 2. The thermal conductivities of a number of lightweight concretes also are seen to fall between those for Concretes 3 and 4, although some exhibited conductivities slightly higher than Concrete 4. (For further details see Ref 2.)

The conventional grouping of the normal weight concretes did not seem to be entirely justifiable. The thermal conductivity of "siliceous" concretes is not necessarily higher than that of "calcareous" concretes. This finding is understandable from the well-known fact that, in addition

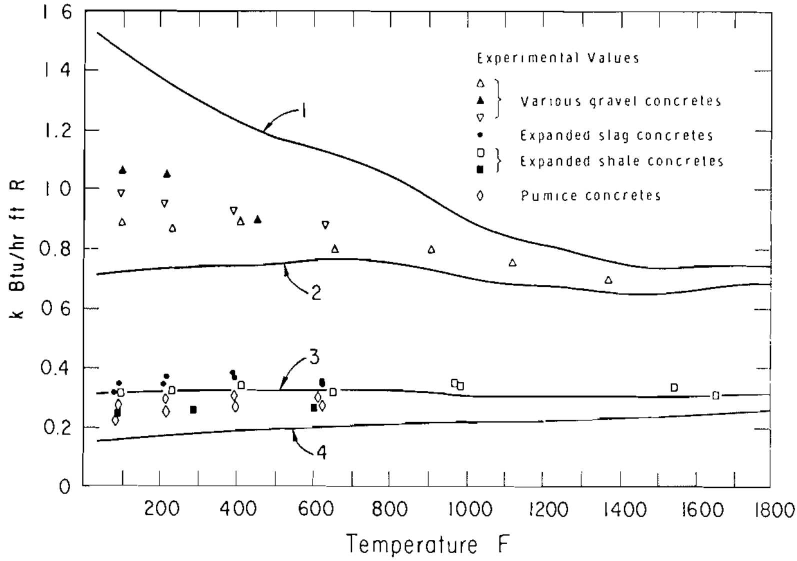


FIG. 3—*Thermal conductivity of various concretes (full lines: Concretes 1 to 4).*

to the mineralogical composition, the degree of crystallinity of natural rocks is also an important factor in their thermal conductivity.

The problem of whether knowledge of the performance in fire of some concretes enables one to draw conclusions concerning the performance of other concretes, is a fundamental one and must be resolved before any generalized statement can be made. To illustrate the essence of this

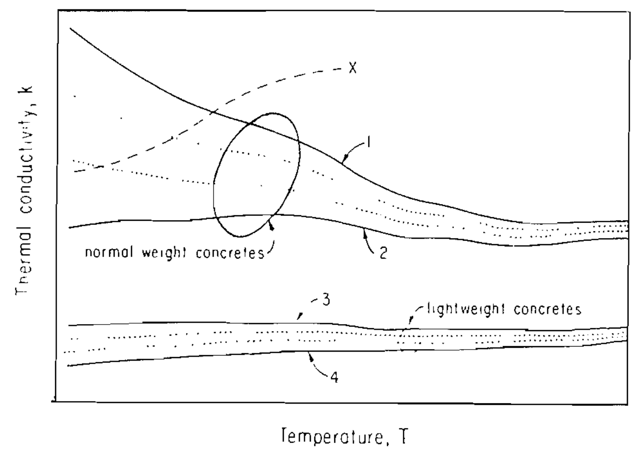


FIG. 4—*Illustration to show the quasi-congruence of the* k *versus* T *relations for concretes.*

problem, attention is directed to curve X in Fig. 4 which is assumed to represent the k versus T relation for a particular Concrete X. Because curve X is in no way related to the curves representing the two "limiting concretes" (that is, Concretes 1 and 2) in the normal weight group, one cannot expect that the performance of Concrete X in fire can be predicted, by interpolation or by some other means, from the fire performance data available for the limiting concretes. If it is found that the k versus T and ρc versus T relations for various concretes can assume entirely diverse courses, there would be no other way to obtain thermal performance information for any concrete masonry unit but by separate tests or by separate numerical analyses based on rather laborious experimental investigations.

The condition of the extensibility to other concretes, of experimental results or results obtained by numerical analyses is, therefore, that the k versus T and ρc versus T relations for all concretes within a certain group must form a quasi-congruous system; in other words, that these relations must fit into families of nonintersecting curves.

There is sufficient evidence to claim that the quasi-congruence of the k versus T relations is satisfied, at least roughly, in both the normal weight and lightweight groups. For reasons discussed in Refs 2, 6, and 7, concretes that exhibit high thermal conductivities at room temperature (that is, those made with highly crystalline aggregates) can be expected to exhibit lower conductivity values at elevated temperatures, and those exhibiting low conductivities at room temperature (that is, those made with amorphous or highly porous aggregates) most probably will slightly increase their conductivities with the increase of temperature.

The most likely forms of the k versus T curves for various concretes in both the normal weight and lightweight groups are shown in dotted lines in Fig. 4. Although there may be a few odd exceptions to this pattern of variation, it is entirely inconceivable that any concrete could exhibit such an incongruous curve as curve X.

The quasi-congruence of the ρc versus T curves for curves for Concretes 1 to 4 is obvious from Figs. 1 and 2. It must be emphasized, however, that, as shown in Ref 2, in these concretes the aggregates were regarded as physicochemically more or less stable constituents. The only aggregate of imperfect stability was the quartz in Concrete 1. The relatively small heat of the α-β transformation, however, was taken into account in the development of the ρc versus T curve.

Unfortunately, recent thermomogravimetric studies indicated that many so-called "calcareous" (and also some "siliceous") aggregates, used especially in normal weight concrete masonry units, undergo substantial decomposition upon heating. In extreme cases, some "calcareous" aggregates may exhibit more than 50 percent weight loss at 1650 F. Because the heat of decomposition of inorganic materials is usually between 1500 and 2000 Btu/lb weight loss, the average value of the ρc product for strongly decomposing aggregates in the 75 to 1650 F temperature range may become twice or three times higher (consequently, the thermal diffusivity κ becomes lower by a factor of two or three) than that for stable aggregates.[3] There is experimental evidence that the fire endurance of masonry units made with such decomposing aggregates may be twice as high as reported in this paper.

Obviously, the use of some empirical formulas introduced in this paper must be restricted to the calculation of the thermal fire endurances of masonry units made with nondecomposing aggregates. For reasons to be discussed later, it seems permissible, however, to use without restriction the procedure described in the section "Utilization of the Results" to find the fire endurance of a masonry unit if the fire endurance of another unit of different geometry but the same material is known.

Besides justifying the extensibility of certain experimental or theoretical results to materials not covered in the studies, the quasi-congruences of the k versus T and ρc versus T relations also offer an important convenience which will be used in subsequent discussions. If such congruences exist, it is immaterial whether one uses the average values of k and ρc, or their values at any particular temperature in developing empirical relationships related to the process of heat transport. As a rule, it is most convenient to use the room temperature values of these properties.

Geometry of Masonry Units

Much care was taken to ensure that all geometric variables within sufficiently large domains would be covered in the numerical studies. Figure 5*a* shows that for conventional concrete masonry units there are four geometric variables:

Overall thickness	L
Face shell thickness	l
Web thickness	a
Web spacing	b

It seemed convenient to group these variables as follows:

$$L, \frac{l}{L}, \frac{a}{l}, \frac{b-a}{L-2l}$$

The values used in the computer studies are given in the column and row headings of Table 2.

[3] Figures 1—a2 and 1—b2 show that the average value of the ρc product is about 40 Btu/ft^3 R in the case of aggregates of high physicochemical stability.

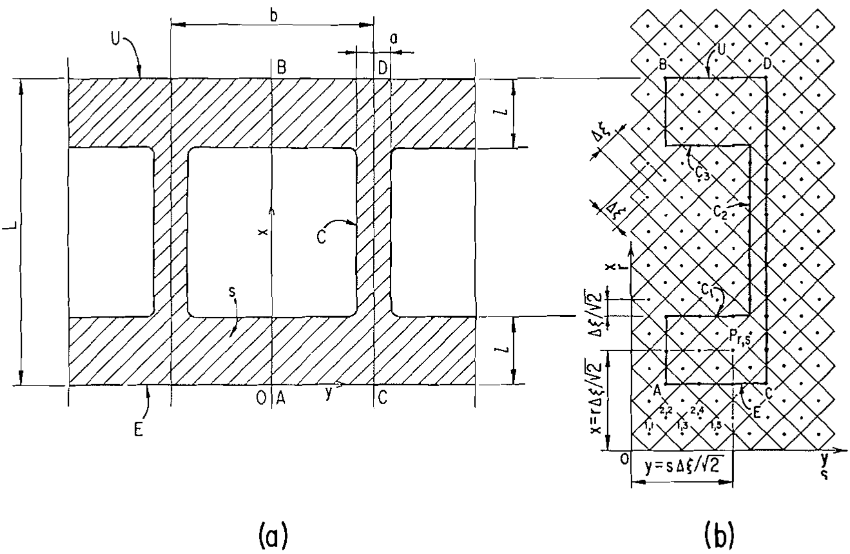


FIG. 5—*Illustration to show* (a) *the geometry of a masonry unit and* (b) *arrangement of the principal portion of the unit on the diagonal mesh.*

The ranges of the variables were wide enough to cover all practical cases. In addition, a number of hypothetical cases were examined, such as the masonry units without webs. These "double-layer units" and the solid units are obviously the two limiting cases with regard to the geometry of the units.

Heat Transport Through the Units

As mentioned earlier, efforts have been made to carefully simulate in the computer studies the true mechanisms of heat transport. When simulating the "standard fire exposure," however, some difficulties arose in connection with the interpretation of a few vague terms in ASTM Methods E 119.

The "furnace temperature" is one such term. The transmission of heat from the burning fuel to the surface of the fire test specimen is a very complicated process. In it, the average of the temperature readings obtained from the "standard" furnace thermocouples rarely have any special significance. Nevertheless, as this "furnace temperature" is the only information available from inside the furnace, there is no choice but to refer the heat transfer coefficient at the exposed side of the test specimen to this temperature, no matter what the actual mechanism of heat transfer may be.

After evaluating a large number of temperature measurements taken from the exposed surface of different test specimens, it was found that

TABLE 2—*Results of computer calculations concerning the thermal fire endurance of concrete masonry units in dry conditions for solid units and for* $(b - a)/(L - 2l) = 0.25$.
(Upper number: 160 F thermal fire endurance, h;
lower number: 250 F thermal fire endurance, h).

$(b-a)/(L-2l)$ →				0.25												
l/L →			Solid	0.100			0.167			0.250			0.333			
a/L →				.200	.400	.600	.167	.333	.500	.125	.250	.500	.083	.167	.250	.333
L = 1.000	Concrete	1	5.00 8.06	1.65 2.42	2.09 3.28	2.59 3.87	2.45 3.56	2.98 4.29	3.36 4.83	3.37 4.79	3.80 5.55	4.20 6.31	4.23 6.23	4.48 6.75	4.61 7.04	4.69 7.22
		2	8.30 12.8	1.99 3.01	2.84 4.20	3.45 5.11	3.24 4.47	4.06 5.70	4.64 6.54	4.70 6.46	5.46 7.59	6.29 9.00	6.19 8.73	6.78 9.68	7.10 > 10	7.30 > 10
		3	16.8 33.3	2.61 4.16	3.82 6.35	4.87 8.34	4.41 6.93	5.64 9.38	6.97 > 10	6.98 > 10	8.43 > 10	> 10 > 10	> 10 > 10	> 10 > 10	> 10 > 10	> 10 > 10
		4	23.8 52.5	3.21 5.28	4.84 8.32	6.33 > 10	5.61 9.45	7.58 > 10	9.23 > 10	9.23 > 10	> 10 > 10	> 10 > 10	> 10 > 10	> 10 > 10	> 10 > 10	> 10 > 10
L = 0.833	Concrete	1	3.44 5.38	1.30 1.85	1.68 2.44	1.94 2.85	1.85 2.57	2.21 3.13	2.42 3.48	2.47 3.47	2.75 3.96	2.99 4.42	3.03 4.39	3.17 4.70	3.24 4.86	3.28 4.92
		2	5.67 8.52	1.60 2.27	2.15 3.12	2.57 3.75	2.41 3.30	2.99 4.15	3.38 4.72	3.42 4.66	3.95 5.47	4.47 6.32	4.43 6.15	4.79 6.77	4.98 7.12	5.11 7.33
		3	11.0 21.2	1.91 2.98	2.74 4.47	3.38 5.77	3.14 4.79	4.12 6.45	4.87 7.74	4.87 7.78	5.87 9.60	7.10 > 10	6.88 > 10	7.74 > 10	8.26 > 10	8.65 > 10
		4	15.9 32.5	1.95 3.65	3.40 5.72	4.41 7.61	3.92 6.39	5.26 8.62	6.36 10.6	6.32 > 10	7.70 > 10	9.57 > 10	9.27 > 10	> 10 > 10	> 10 > 10	> 10 > 10
L = 0.667	Concrete	1	2.21 3.35	0.96 1.35	1.22 1.73	1.38 1.98	1.33 1.81	1.54 2.16	1.67 2.37	1.71 2.37	1.87 2.65	1.99 2.89	2.03 2.90	2.10 3.06	2.13 3.13	2.15 3.17
		2	3.60 5.26	1.17 1.64	1.54 2.20	1.82 2.60	1.71 2.30	2.08 2.85	2.32 3.20	2.35 3.15	2.67 3.65	2.97 4.14	2.95 4.05	3.16 4.40	3.27 4.59	3.33 4.71
		3	6.56 12.1	1.33 2.02	1.86 2.96	2.31 3.74	2.11 3.12	2.73 4.14	3.18 4.91	3.18 4.83	3.79 5.92	4.50 7.26	4.38 7.04	4.87 7.99	5.19 8.63	5.39 9.09
		4	9.61 18.3	1.55 2.39	2.24 3.65	2.87 4.81	2.58 3.96	3.41 5.37	4.08 6.49	4.05 6.58	4.90 8.09	6.02 10.2	5.80 9.84	6.52 > 10	7.01 > 10	7.37 > 10
L = 0.500	Concrete	1	1.30 1.89	0.68 0.92	0.82 1.14	0.91 1.27	0.88 1.18	1.00 1.37	1.06 1.47	1.09 1.48	1.16 1.61	1.21 1.72	1.25 1.74	1.27 1.80	1.27 1.82	1.28 1.83
		2	2.05 2.91	0.81 1.11	1.03 1.43	1.19 1.65	1.12 1.49	1.33 1.79	1.46 1.98	1.48 1.95	1.64 2.21	1.79 2.44	1.79 2.42	1.89 2.57	1.93 2.65	1.95 2.69
		3	3.45 5.92	0.86 1.27	0.98 1.79	1.41 2.20	1.39 1.87	1.64 2.42	1.88 2.81	1.88 2.73	2.21 3.29	2.56 3.94	2.50 3.79	2.75 4.25	2.89 4.54	2.99 4.75
		4	5.03 9.10	0.97 1.44	1.36 2.13	1.70 2.72	1.57 2.25	2.00 2.99	2.35 3.57	2.33 3.53	2.79 4.36	3.36 5.36	3.24 5.18	3.62 5.88	3.86 6.36	4.03 6.71
L = 0.333	Concrete	1	0.65 0.91	0.43 0.55	0.49 0.66	0.53 0.71	0.52 0.68	0.56 0.75	0.58 0.79	0.60 0.80	0.62 0.84	0.63 0.87	0.65 0.89	0.65 0.90	0.65 0.90	0.65 0.90
		2	0.98 1.34	0.51 0.67	0.61 0.82	0.68 0.91	0.65 0.84	0.73 0.97	0.79 1.04	0.80 1.04	0.86 1.13	0.91 1.21	0.92 1.22	0.95 1.26	0.96 1.28	0.96 1.30
		3	1.48 2.30	0.50 0.70	0.64 0.94	0.75 1.11	0.70 0.97	0.85 1.20	0.95 1.35	0.95 1.32	1.08 1.55	1.21 1.76	1.19 1.71	1.28 1.87	1.33 1.96	1.35 2.02
		4	2.09 3.38	0.53 0.77	0.72 1.07	0.87 1.31	0.79 1.10	1.00 1.42	1.14 1.64	1.13 1.59	1.33 1.91	1.54 2.28	1.50 2.19	1.65 2.45	1.74 2.61	1.79 2.73

TABLE 2—*(Continued) Results of computer calculations concerning the thermal fire endurance of concrete masonry units in dry conditions for* (b − a)/(L − 2*l*) = *0.50.*
(Upper number: 160 F thermal fire endurance, h;
lower number: 250 F thermal fire endurance, h).

(b-a)/(L-2l) →			0.50												
l/L →			0.100			0.167			0.250			0.333			
a/L →			.200	.400	.600	.167	.333	.500	.125	.250	.500	.083	.167	.250	.333
L=1.000	Concrete	1	1.11 1.54	1.39 1.98	1.64 2.34	1.87 2.54	2.23 3.20	2.51 3.67	2.85 3.96	3.23 4.60	3.66 5.37	3.86 5.55	4.14 6.06	4.30 6.39	4.41 6.63
		2	1.36 1.90	1.73 2.46	2.09 2.97	2.44 3.35	2.97 4.24	3.41 4.95	3.93 5.34	4.54 6.32	5.32 7.58	5.56 7.73	6.09 8.56	6.43 9.14	6.67 9.57
		3	1.67 2.58	2.28 3.67	2.88 4.71	3.31 5.16	4.26 7.02	5.07 8.74	5.81 9.64	6.95 > 10	8.60 > 10	8.94 > 10	9.99 > 10	> 10 > 10	> 10 > 10
		4	2.09 3.33	2.93 4.95	3.81 6.60	4.29 7.22	5.65 9.76	6.88 > 10	7.78 > 10	9.38 > 10	> 10 > 10	> 10 > 10	> 10 > 10	> 10 > 10	> 10 > 10
L=0.833	Concrete	1	0.88 1.21	1.09 1.52	1.28 1.78	1.43 1.89	1.69 2.38	1.88 2.69	2.12 2.90	2.38 3.33	2.65 3.82	2.80 3.96	2.97 4.28	3.06 4.48	3.13 4.61
		2	1.06 1.45	1.33 1.87	1.59 2.23	1.83 2.48	2.21 3.11	2.52 3.59	2.88 3.86	3.30 4.52	3.82 5.36	3.99 5.47	4.34 6.02	4.56 6.39	4.72 6.68
		3	1.23 1.87	1.65 2.60	2.07 3.29	2.36 3.57	3.00 4.79	3.54 5.88	4.05 6.36	4.82 7.78	5.89 9.96	6.10 10.3	6.82 > 10	7.33 > 10	7.73 > 10
		4	1.50 2.31	2.07 3.37	2.65 4.45	3.00 4.82	3.90 6.58	4.71 8.15	5.33 8.95	6.39 > 10	8.02 > 10	8.29 > 10	9.27 > 10	10.0 > 10	> 10 > 10
L=0.667	Concrete	1	0.68 0.91	0.83 1.13	0.95 1.30	1.05 1.40	1.22 1.68	1.34 1.88	1.49 2.01	1.65 2.27	1.81 2.45	1.91 2.66	2.00 2.84	2.05 2.94	2.08 2.97
		2	0.79 1.07	0.99 1.35	1.16 1.60	1.31 1.67	1.57 2.16	1.76 2.47	1.99 2.63	2.26 3.05	2.58 3.55	2.69 3.63	2.90 3.96	3.03 4.17	3.11 4.33
		3	0.87 1.29	1.15 1.75	1.41 2.17	1.59 2.34	1.99 3.08	2.33 3.71	2.65 3.95	3.12 4.79	3.76 6.03	3.89 6.18	4.32 6.95	4.62 7.54	4.85 8.02
		4	1.03 1.53	1.38 2.17	1.74 2.79	1.97 2.98	2.52 4.05	3.00 5.03	3.40 5.45	4.05 6.65	5.02 8.52	5.17 8.74	5.77 9.83	6.23 10.7	6.59 > 10
L=0.500	Concrete	1	0.50 0.66	0.59 0.79	0.67 0.89	0.72 0.95	0.82 1.10	0.88 1.21	0.97 1.29	1.05 1.43	1.13 1.56	1.20 1.64	1.23 1.71	1.25 1.74	1.26 1.76
		2	0.57 0.75	0.69 0.93	0.80 1.07	0.81 1.15	1.03 1.39	1.15 1.56	1.27 1.65	1.42 1.88	1.59 2.14	1.66 2.20	1.76 2.36	1.82 2.46	1.86 2.53
		3	0.74 0.83	0.84 1.09	0.90 1.32	0.99 1.41	1.22 1.81	1.40 2.12	1.58 2.25	1.84 2.68	2.16 3.29	2.24 3.33	2.46 3.72	2.61 4.00	2.72 4.22
		4	0.75 0.94	0.86 1.28	1.06 1.61	1.18 1.70	1.48 2.25	1.74 2.74	1.98 2.90	2.31 3.53	2.81 4.48	2.89 4.56	3.21 5.14	3.44 5.58	3.62 5.93
L=0.333	Concrete	1	0.34 0.43	0.39 0.50	0.42 0.55	0.44 0.57	0.49 0.64	0.52 0.69	0.56 0.73	0.59 0.78	0.61 0.82	0.65 0.87	0.65 0.88	0.65 0.89	0.65 0.89
		2	0.38 0.48	0.45 0.58	0.50 0.65	0.53 0.68	0.61 0.79	0.66 0.87	0.71 0.91	0.78 1.01	0.84 1.10	0.88 1.14	0.91 1.20	0.93 1.23	0.94 1.25
		3	0.35 0.48	0.43 0.61	0.51 0.71	0.57 0.75	0.65 0.92	0.73 1.05	0.81 1.10	0.92 1.28	1.05 1.51	1.09 1.53	1.17 1.67	1.23 1.77	1.26 1.84
		4	0.37 0.52	0.47 0.67	0.57 0.81	0.62 0.85	0.75 1.08	0.86 1.27	0.96 1.32	1.11 1.57	1.31 1.92	1.34 1.93	1.48 2.15	1.57 2.31	1.63 2.43

TABLE 2–*(Continued) Results of computer calculations concerning the thermal fire endurance of concrete masonry units in dry conditions where* (b − a)/(L − 2*l*) = *1.* (Upper number: 160 F thermal fire endurance, h; lower number: 250 F thermal fire endurance, h).

(b-a)/(L-2l) →			1											
l/L →			0.100			0.167			0.250			0.333		
a/L →			.200	.400	.600	.167	.333	.500	.125	.250	.500	.083	.167	.250	.333
L=1.000	Concrete	1	0.77 1.05	0.90 1.26	1.01 1.40	1.44 1.95	1.63 2.31	1.78 2.60	2.38 3.29	2.64 3.73	2.96 4.36	3.47 4.89	3.69 5.32	3.86 5.62	3.98 5.85
		2	0.97 1.30	1.10 1.49	1.24 1.76	1.86 2.54	2.13 3.05	2.36 3.58	3.29 4.47	3.69 5.14	4.19 6.15	4.96 6.90	5.36 7.51	5.67 8.01	5.90 8.41
		3	1.11 1.71	1.32 2.23	1.56 2.79	2.53 4.00	3.04 5.25	3.51 6.49	4.92 8.32	5.72 10.1	6.84 > 10	8.08 > 10	8.86 > 10	9.50 > 10	10.1 > 10
		4	1.37 2.27	1.69 2.74	2.03 4.12	3.34 6.00	4.17 7.98	4.96 10.1	6.77 > 10	8.01 > 10	9.89 > 10	> 10 > 10	> 10 > 10	> 10 > 10	> 10 > 10
L=0.833	Concrete	1	0.61 0.83	0.71 0.99	0.80 1.07	1.11 1.48	1.25 1.73	1.36 1.93	1.78 2.42	1.96 2.72	2.17 3.13	2.53 3.53	2.68 3.78	2.80 3.98	2.86 4.12
		2	0.74 1.01	0.86 1.17	0.96 1.59	1.41 1.89	1.59 2.24	1.76 2.43	2.41 3.22	2.69 3.68	3.05 4.35	3.57 4.87	3.84 5.28	4.04 5.61	4.18 5.87
		3	0.83 1.25	0.97 1.58	1.14 1.94	1.81 2.74	2.14 3.53	2.45 4.29	3.41 5.41	3.94 6.46	4.65 8.34	5.49 9.28	6.01 10.4	6.45 > 10	6.80 > 10
		4	0.99 1.58	1.21 2.12	1.44 2.69	2.32 3.81	2.85 5.13	3.34 6.45	4.58 8.18	5.38 10.1	6.55 > 10	7.61 > 10	8.39 > 10	9.05 > 10	9.57 > 10
L=0.667	Concrete	1	0.48 0.64	0.56 0.76	0.62 0.84	0.82 1.08	0.92 1.25	0.99 1.38	1.27 1.69	1.38 1.88	1.51 2.13	1.75 2.38	1.84 2.54	1.90 2.65	1.93 2.73
		2	0.57 0.75	0.66 0.87	0.72 0.97	1.02 1.34	1.14 1.57	1.25 1.76	1.67 2.18	1.85 2.49	2.07 2.90	2.42 3.24	2.59 3.45	2.71 3.70	2.79 3.84
		3	0.60 0.86	0.70 1.07	0.80 1.29	1.22 1.79	1.43 2.24	1.61 2.67	2.25 3.33	2.54 3.93	3.02 4.94	3.50 5.53	3.80 6.12	4.05 6.63	4.26 7.06
		4	0.70 1.05	0.82 1.35	0.97 1.67	1.52 2.33	1.83 3.07	2.11 3.80	2.90 4.75	3.37 5.75	4.13 7.51	4.67 8.26	5.16 9.20	5.55 > 10	5.84 > 10
L=0.500	Concrete	1	0.37 0.47	0.42 0.55	0.46 0.60	0.58 0.75	0.64 0.84	0.68 0.92	0.84 1.10	0.91 1.22	0.98 1.34	1.12 1.50	1.16 1.57	1.18 1.62	1.20 1.65
		2	0.42 0.54	0.47 0.61	0.52 0.67	0.69 0.89	0.77 1.03	0.84 1.14	1.07 1.39	1.18 1.59	1.31 1.77	1.51 1.97	1.58 2.11	1.66 2.21	1.73 2.29
		3	0.40 0.57	0.47 0.68	0.53 0.80	0.77 1.08	0.88 1.32	0.99 1.53	1.32 1.89	1.49 2.19	1.71 2.67	2.00 2.97	2.16 3.25	2.29 3.49	2.39 3.74
		4	0.45 0.65	0.53 0.81	0.60 0.97	0.92 1.32	1.08 1.68	1.22 2.02	1.61 2.47	1.90 2.94	2.29 3.74	2.60 4.13	2.84 4.60	3.04 5.00	3.20 5.33
L=0.333	Concrete	1	0.25 0.32	0.29 0.36	0.30 0.39	0.37 0.47	0.40 0.52	0.43 0.55	0.50 0.65	0.53 0.69	0.56 0.74	0.63 0.82	0.64 0.84	0.64 0.86	0.65 0.87
		2	0.28 0.35	0.31 0.40	0.34 0.44	0.43 0.53	0.47 0.61	0.50 0.66	0.62 0.78	0.67 0.86	0.72 0.95	0.82 1.04	0.85 1.10	0.87 1.13	0.89 1.16
		3	0.25 0.34	0.29 0.40	0.32 0.45	0.43 0.58	0.49 0.68	0.53 0.77	0.68 0.93	0.76 1.06	0.85 1.23	0.98 1.36	1.05 1.47	1.10 1.56	1.13 1.63
		4	0.27 0.36	0.31 0.44	0.34 0.51	0.48 0.66	0.55 0.80	0.62 0.93	0.81 1.12	0.91 1.30	1.05 1.57	1.21 1.73	1.31 1.89	1.39 2.02	1.45 2.14

TABLE 2—*(Continued) Results of computer calculations concerning the thermal fire endurance of concrete masonry units in dry conditions for* $(b - a)/(L - 2l) = 2$ *and* 4.
(Upper number: 160 F thermal fire endurance, h;
lower number: 250 F thermal fire endurance, h).

(b-a)/(L-2l)→			2							4	
l/L →			0.167		0.250		0.333			0.167	
a/L →			.333	.667	250	500	167	.333	.500	.333	.667
L=1.000	Concrete	1	1.26 1.69	1.41 1.91	2.27 3.17	2.47 3.55	3.42 4.90	3.62 5.28	3.76 5.56	1.10 1.60	1.17 1.74
		2	1.64 2.21	1.81 2.54	3.14 4.36	3.47 4.99	4.92 6.98	5.29 7.55	5.57 8.04	1.51 2.04	1.61 2.19
		3	2.14 3.57	2.55 4.69	4.74 8.34	5.48 10.2	8.13 > 10	8.92 > 10	9.61 > 10	1.84 3.02	2.00 3.53
		4	2.85 5.57	3.55 7.23	6.62 > 10	7.90 > 10	> 10 > 10	> 10 > 10	> 10 > 10	2.37 4.56	2.63 5.46
L=0.833	Concrete	1	0.99 1.30	1.09 1.45	1.69 2.33	1.84 2.58	2.49 3.51	2.62 3.75	2.71 3.92	0.87 1.19	0.93 1.33
		2	1.19 1.55	1.39 1.88	2.30 3.14	2.52 3.55	3.53 4.87	3.78 5.28	3.96 5.60	1.15 1.52	1.23 1.65
		3	1.55 2.43	1.81 3.06	3.28 5.41	3.75 6.60	5.43 9.42	6.03 > 10	6.47 > 10	1.34 2.05	1.45 2.35
		4	1.99 3.60	2.41 4.80	4.46 7.83	5.25 9.58	7.60 > 10	8.43 > 10	9.13 > 10	1.66 2.93	1.84 4.53
L=0.667	Concrete	1	0.74 0.96	0.81 1.06	1.21 1.63	1.30 1.79	1.73 2.38	1.80 2.51	1.85 2.61	0.66 0.88	0.73 0.98
		2	0.92 1.19	1.01 1.33	1.60 2.14	1.75 2.39	2.39 3.23	2.54 3.48	2.65 3.66	0.84 1.09	0.90 1.18
		3	1.06 1.58	1.23 1.92	2.14 3.30	2.42 3.95	3.48 5.60	3.79 6.24	4.04 6.78	0.91 1.34	1.00 1.52
		4	1.31 2.14	1.56 2.81	2.81 4.72	3.26 5.75	4.70 8.17	5.20 9.14	5.60 10.0	1.12 1.77	1.22 2.08
L=0.500	Concrete	1	0.52 0.67	0.57 0.73	0.81 1.07	0.86 1.15	1.11 1.49	1.14 1.55	1.16 1.59	0.45 0.61	0.49 0.65
		2	0.63 0.80	0.69 0.88	1.04 1.35	1.12 1.49	1.49 1.96	1.56 2.09	1.67 2.18	0.56 0.73	0.60 0.81
		3	0.69 0.96	0.77 1.12	1.28 1.86	1.42 2.16	1.99 2.99	2.15 3.29	2.27 3.54	0.60 0.85	0.64 0.92
		4	0.80 1.20	0.93 1.48	1.61 2.48	1.83 3.00	2.61 4.17	2.85 4.66	3.05 5.07	0.69 1.00	0.75 1.15
L=0.333	Concrete	1	0.35 0.43	0.42 0.46	0.49 0.63	0.51 0.66	0.63 0.80	0.63 0.81	0.64 0.85	0.32 0.40	0.34 0.42
		2	0.40 0.49	0.43 0.54	0.60 0.76	0.63 0.82	0.81 1.04	0.82 1.09	0.85 1.12	0.35 0.45	0.38 0.49
		3	0.39 0.52	0.44 0.59	0.66 0.91	0.72 1.02	0.97 1.37	1.03 1.48	1.08 1.56	0.35 0.46	0.37 0.50
		4	0.43 0.60	0.49 0.70	0.79 1.11	0.87 1.29	1.20 1.74	1.30 1.92	1.38 2.06	0.38 0.53	0.40 0.57

TABLE 2–*(Continued) Results of computer calculations concerning the thermal fire endurance of concrete masonry units in dry conditions for* $(b - a)/(L - 2l) = 4$ *and double layer.*
(Upper number: 160 F thermal fire endurance, h; lower number: 250 F thermal fire endurance, h).

(b-a)/(L-2l) →			4						double-layer			
l/L →			0.250			0.333			.100	.167	.250	.333
a/L →			.250	.500	.750	.167	.333	.500	0	0	0	0
L=1.000	Concrete	1	2.14 2.94	2.22 3.14	2.32 3.31	3.24 4.64	3.38 4.88	3.48 5.07	0.42 0.66	1.01 1.33	1.86 2.55	3.01 4.28
		2	2.87 3.96	3.08 4.29	3.20 4.60	4.64 6.51	4.86 6.80	5.04 7.23	0.50 0.78	1.31 1.70	2.56 3.47	4.30 6.02
		3	4.27 7.40	4.66 8.45	5.02 9.38	7.60 10.5	8.11 > 10	8.55 > 10	0.68 0.90	1.68 2.52	3.81 6.29	7.05 > 10
		4	5.89 > 10	6.55 > 10	7.20 > 10	> 10 > 10	> 10 > 10	> 10 > 10	0.77 1.10	2.18 3.55	5.23 9.34	9.86 > 10
L=0.833	Concrete	1	1.56 2.17	1.64 2.30	1.72 2.42	2.38 3.33	2.46 3.48	2.52 3.60	0.35 0.54	0.79 1.01	1.40 1.88	2.21 3.07
		2	2.11 2.87	2.23 3.08	2.35 3.27	3.33 4.59	3.48 4.85	3.60 5.05	0.40 0.63	0.99 1.28	1.88 2.51	3.09 4.24
		3	2.96 4.83	3.21 5.49	3.44 6.12	5.16 8.81	5.48 9.50	5.77 10.2	0.53 0.68	1.22 1.76	2.64 4.15	4.77 8.07
		4	3.98 6.99	4.39 8.01	4.78 9.04	7.14 > 10	7.64 > 10	8.10 > 10	0.58 0.80	1.54 2.35	3.55 6.01	6.62 > 10
L=0.667	Concrete	1	1.12 1.52	1.18 1.61	1.22 1.68	1.66 2.26	1.70 2.35	1.74 2.42	0.27 0.42	0.59 0.75	1.01 1.33	1.54 2.09
		2	1.49 1.95	1.55 2.10	1.62 2.22	2.25 3.05	2.35 3.20	2.42 3.32	0.31 0.48	0.72 0.92	1.32 1.72	2.10 2.82
		3	1.93 2.96	2.08 3.31	2.21 3.65	3.26 5.24	3.45 5.63	3.61 5.99	0.39 0.49	0.83 1.17	1.72 2.58	3.02 4.81
		4	2.52 4.20	2.74 4.79	2.96 5.34	4.42 7.66	4.60 8.28	4.97 8.87	0.43 0.56	1.02 1.48	2.26 3.56	4.10 7.04
L=0.500	Concrete	1	0.76 1.00	0.79 1.05	0.81 1.09	1.07 1.42	1.09 1.46	1.10 1.50	0.21 0.32	0.43 0.53	0.68 0.88	1.00 1.33
		2	0.96 1.24	1.00 1.31	1.04 1.37	1.41 1.86	1.46 1.94	1.50 2.00	0.25 0.35	0.50 0.63	0.86 1.10	1.32 1.73
		3	1.16 1.68	1.23 1.84	1.30 1.98	1.87 2.81	1.97 2.99	2.04 3.16	0.27 0.34	0.54 0.72	1.03 1.47	1.73 2.59
		4	1.45 2.21	1.56 2.49	1.67 2.77	2.45 3.89	2.60 4.20	2.72 4.48	0.29 0.37	0.63 0.87	1.30 1.91	2.27 3.57
L=0.333	Concrete	1	0.46 0.64	0.47 0.70	0.48 0.63	0.61 0.79	0.62 0.81	0.63 0.82	0.19 0.22	0.28 0.35	0.42 0.53	0.59 0.76
		2	0.55 0.71	0.58 0.74	0.60 0.77	0.74 0.99	0.79 1.02	0.81 1.05	0.20 0.24	0.31 0.39	0.50 0.63	0.73 0.93
		3	0.59 0.82	0.64 0.89	0.66 0.94	0.91 1.29	0.95 1.35	0.98 1.41	0.18 0.22	0.31 0.40	0.54 0.73	0.85 1.19
		4	0.71 1.00	0.76 1.10	0.80 1.18	1.13 1.64	1.20 1.74	1.24 1.84	0.18 0.23	0.34 0.45	0.64 0.88	1.06 1.51

the conditions during a fire endurance test can be approximated by taking a "standard fire exposure" to be equivalent to the transmission of radiant heat to the specimen surface from a black body whose temperature varies according to the prescribed furnace temperature versus time curve of ASTM Methods E 119. (It must be emphasized that accepting this model does not imply that radiant heat transmission is the only effective transport mechanism. Nevertheless, one need not be overly pedantic about the actual values of h_E, since if $h_E > 15$ Btu/h ft^2 R, the heat flow into the specimen is generally controlled not by h_E but by the thermal conductivity of the specimen (see, for example, Ref *8*).

To facilitate the computer calculations the standard furnace temperature versus time curve of ASTM Methods E 119 was replaced by the following analytical expression:

$$T_f = 530 + 1350\,[1 - \exp(-3.79553t^{1/2})] + 306.74t^{1/2} \qquad (1)$$

This is only slightly different from the function developed by the Centre Scientifique et Technique du Bâtiment [9], and it approximates the standard furnace temperature curve within $\pm$ 11 R in the 15-min to 8-h interval. (In special studies this "standard" curve can be replaced by a more realistic fire exposure curve.)

According to the above adopted model for heat transport to the exposed surface (surface E in Fig. 5*a*) and with the aid of the Stefan-Boltzmann law and the Fourier law one obtains

$$k\frac{\partial T}{\partial x} + \sigma\epsilon(T_f^4 - T^4) = 0 \quad \text{at } x = 0 \qquad (2)$$

Within the solid (region S) the heat is transferred essentially by two-dimensional heat conduction, described by the well-known equation

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) = \rho c\frac{\partial T}{\partial t} \quad x, y \text{ in } S \qquad (3)$$

At the so-called "unexposed surface" (surface U), that is, the surface away from the fire, the heat transport takes place to nonreflecting surroundings of constant temperature, partly by radiation and partly by free convection. The radiant heat transmission is again calculable from the Stefan-Boltzmann law. The convective contribution can be described satisfactorily by an empirical expression given by McAdams [*10*]. Therefore, the combined convective-radiant heat transport can be written:

$$k\frac{\partial T}{\partial x} + \sigma\epsilon(T^4 - T_o^4) + \beta(T - T_o)^{5/4} = 0 \quad \text{at } x = L \qquad (4)$$

Along the surface of the cavity (surface C in Fig. 5*a*), the primary heat transport mechanism is radiant heat interchange between gray surface elements (see Ref *10* for the definition of gray surfaces). Again, some heat is transferred by free convection. By using the method described by Gebhart [*11*] for calculating the radiation exchange and the previously mentioned empirical formula by McAdams, the following N equations are obtained:

$$\sigma \epsilon \sum_{k=1}^{N} B_{lk} T_k^4 + \beta \left| T_{av} - T_l \right|^{1/4} (T_{av} - T_l) + k \frac{\partial T}{\partial n} = 0$$

$$\ell = 1, 2, 3, \ldots N \qquad (5)$$

where the subscripts l and k refer to the surface elements of the cavity (that is, to the elements of surface C), n is a distance along the outwardly directed normal vector to the l-th surface element, and B_{lk} are the absorption factors for which

$$\sum_{k=1}^{N} B_{lk} = 0 \qquad (6)$$

T_{av}, the average temperature of air inside the cavity, is expressed as

$$T_{av} = \frac{1}{N} \sum_{k=1}^{N} T_k \qquad (7)$$

Although ϵ in Eqs 2, 4, and 5, strictly speaking, is a material and temperature-dependent quantity, it is sufficiently accurate to treat it as a constant in the present studies and to take $\epsilon = 0.9$. For vertical surfaces $\beta = 0.27$ [*10*].

The initial condition is

$$T = T_o \quad \text{in } S \text{ and along } E,\ U, \text{ and } C \qquad (8)$$

T_o was always chosen as 530 R (70 F).

The finite-difference equivalents of Eqs 2 to 5, which were actually utilized in the computer studies, are introduced in Appendix I.

Results of Computer Calculations

The computed "thermal fire endurance" values for the various masonry units are listed in Table 2. (There are certain indications that the thermal fire endurance values for $l/L = 0.1$ are somewhat low; this is probably due to the crudeness of the network selected.)

The interpretation of the point of thermal failure in a computer simulation of standard fire tests is not quite straightforward. According to ASTM Methods E 119, failure occurs when the average temperature of the surface opposite to the fire exposure (the "unexposed surface") exceeds the initial temperature by 250 F. The standard prescribes, however, that the temperature readings should be taken from under 6-in.-square, 0.4-in.-thick asbestos pads of a specified density. The temperature readings thus obtained, the so-called "standard surface temperatures," are considerably higher than the true temperature of the unexposed surface.

Because the heat flow pattern in the vicinity of the asbestos pads is three dimensional and may be substantially different from the prevailing pattern, it is difficult to correlate the standard surface temperature with the true surface temperature, especially in such a transient process as a simulated fire exposure.

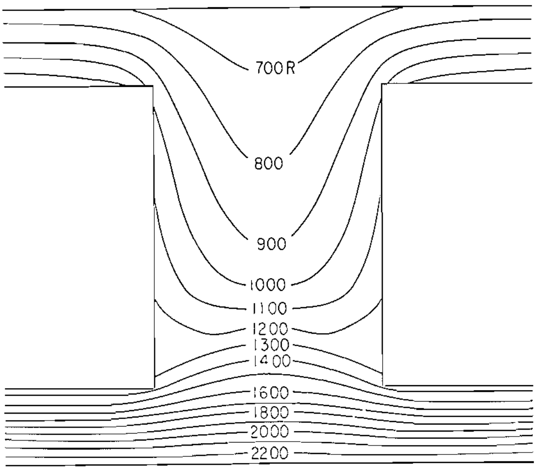
In connection with a series of tests undertaken to study the effect of moisture on the fire endurance [3] simultaneous measurements were taken of the true surface temperature and the standard surface temperature. It was found that at the time of "standard failure" (that is, when the standard surface temperature reached initial temperature plus 250 F) the true surface temperature was, on an average, only 160 F above the initial value. This finding was based on badly spreading data which did not permit a definite conclusion as to whether the material properties had any influence on this 90 F temperature difference. On the basis of theoretical considerations it is believed, however, that at least within the usual ranges of variation of the material properties this influence is not too significant.

Each box in Table 2 contains two numbers. The upper number is the time in which the true temperature of the unexposed surface rises 160 F above the initial value. According to the previously described findings this time probably is comparable to the thermal fire endurance derivable from standard fire tests. It will be referred to as the "160 F thermal fire endurance," and will be denoted by τ_{160}. The lower number represents the time that the true surface temperature attains a level 250 F higher than the initial temperature. This will be referred to as the "250 F thermal fire endurance," and will be denoted by τ_{250}. In general, the term "thermal fire endurance" (without temperature denotation) should be interpreted as the 160 F thermal fire endurance.

The method of using Table 2 for finding fire endurance values by interpolation will be discussed later in the section "Utilization of the Results."

These computer studies have made available not only the thermal fire endurance of the various concrete masonry units but also their entire temperature history in a standard fire exposure. This information may be utilized in a number of ways, for example in the calculation of the stress-strain history of these products in fire.

Figure 6 shows the temperature distribution in one particular masonry unit at the time of its 160 F thermal failure and is given here as an example. It is impossible to publish this kind of information for all 1180 computed cases, but it can be made available upon request.



Material: Concrete 4, $L = 0.667$ ft, $(b - a)/(L - 2l) = 1$, $l/L = 0.167$, $a/l = 0.5$.
FIG. 6—*Temperature distribution in a masonry unit at the time of its thermal failure* ($\tau_{160} = 2.11$ h).

Theoretical Considerations

It has been shown [8] that the temperature history on the $x = L$ surface of a *solid* wall which on the $x = 0$ side is suddenly exposed to a constant temperature T_i, can be expressed as follows:

$$\frac{T_L - T_o}{T_i - T_o} = \frac{1 - F_1\left(\dfrac{\kappa t}{L^2}, \dfrac{hL}{k}\right)}{1 + \dfrac{hL}{k}} \quad (9)$$

provided that the heat transfer coefficient on the side $x = 0$ is much larger than h. Since at the time of thermal failure (that is, when t equals either τ_{160} or τ_{250}), $T_L - T_o =$ constant (either 160 or 250 R), the time of fire endurance of solid walls can be expressed from a relationship of the following form:

$$F_2\left(\frac{hL}{k}, \frac{\kappa\bar{\tau}}{L^2}\right) = 0 \quad (10)$$

It was assumed here that T_i, k, h, and κ are all constants. In reality they are dependent directly or indirectly on time. Since the functions $T_i(t)$ and $h(t)$ are determined by the test procedure, it seems reasonable to regard k and κ in Eqs 9 or 10 as the only true variables (apart from the geometric variable L). Furthermore, one may attempt to use the room temperature values of these material properties[4] in the dimensionless groups $\kappa\bar{\tau}/L^2$ and hL/k, and take care of all inaccuracies due to the simplifying assumptions by introducing $\bar{\tau}$ as an additional variable. In this way one obtains the expression

$$F_3(\bar{\tau}, L/k, \kappa\bar{\tau}/L^2) = 0 \qquad (11)$$

An examination of the thermal fire endurance values obtained from the computer runs indicated that Eq 11 could be simplified further as

$$\bar{\tau} = F_4(L/k, \kappa/L^2) \qquad (12)$$

and, furthermore, that the right-hand side of this equation could be approximated as a power product,

$$\bar{\tau} = A\left(\frac{L}{k}\right)^m \left(\frac{\kappa}{L^2}\right)^n \qquad (13)$$

Because of the several simplifying assumptions used, one can expect that at a certain set of values for A, m, and n, the validity of this equation has to be restricted to groups of materials as characterized by certain ranges of the two material properties k and κ.

By graphically correlating the computer results, the A, m, and n constants have been evaluated and are listed in Table 3. In this table the average error, the average absolute error, and maximum error made by using Eq 13 also are given. (see Appendix II for the definition of these errors).

The solid concrete unit represents a limiting case among masonry units. Another limiting case is a hypothetical masonry unit for which $a/L = 0$; that is, which consists of two parallel layers of identical thickness, l. The heat flow pattern in this so called "double-layer configuration" is also one dimensional and, as pointed out elsewhere (Refs *12* and *13*), the heat flux is practically independent of the distance between the solid layers.

The heat transmission process here is more complicated than in the case of solid masonry units. Another heat transfer coefficient enters the problem: that describing the radiant and convective heat transfer between the two solid layers. This heat transfer coefficient, however, is determined also by the test procedure, and thus can be dismissed as a true variable. It seems logical, therefore, to attempt to find the relation between $\bar{\bar{\tau}}$, that is, the fire endurance of this double-layer configuration, and the material and geometric variables in the following form:

$$\bar{\bar{\tau}} = B\left(\frac{\ell}{k}\right)^p \left(\frac{\kappa}{\ell^2}\right)^q \qquad (14)$$

[4] The permissibility of this practice in the case of concretes made with aggregates of high chemical stability was discussed at the end of the section "Thermal Properties of Concrete."

TABLE 3—*Constants in Eqs 13 and 14.*

$$\bar{\tau} = A\left(\frac{L}{k}\right)^m \left(\frac{\kappa}{L^2}\right)^n \quad (13)^a$$

$$\bar{\bar{\tau}} = B\left(\frac{\ell}{k}\right)^p \left(\frac{\kappa}{\ell^2}\right)^q \quad (14)^b$$

Upper numbers (in each box): for calculating τ_{160} (in dry condition).

Lower number (in each box): for calculating τ_{250} (in dry condition).

Eq	Constant	For Normal Weight Concretes $0.718 \leqslant k \leqslant 1.491$ $0.027 \leqslant \kappa \leqslant 0.057$	For Lightweight Concretes $0.159 \leqslant k \leqslant 0.317$ $0.012 \leqslant \kappa \leqslant 0.019$
(13)............................	A	0.092 0.138	0.152 0.210
	m	−0.67 −0.67	−0.40 −0.40
	n	−1.3 −1.3	−1.3 −1.4
(14)............................	B	0.385 0.520	0.860 1.250
	p	−0.85 −0.85	−0.50 −0.50
	q	−1.2 −1.2	−1.2 −1.3

[a] Errors committed by using Eq 13:

(1) for the calculation of $\bar{\tau}_{160}$: $e_{av} = -0.35\%$, $(e_a)_{av} = 2.04\%$, $e_{max} = -8.91\%$;

(2) for the calculation of $\bar{\tau}_{250}$: $e_{av} = +1.85\%$, $(e_a)_{av} = 4.01\%$, $e_{max} = +10.10\%$.

[b] The validity of Eq 14 is restricted to the calculation of fire endurance $\bar{\bar{\tau}}_{160} \geqslant 0.7$ h or $\bar{\bar{\tau}}_{250} \geqslant 1.0$ h.

Errors committed by using Eq 14:

(1) for the calculation of $\bar{\bar{\tau}}_{160}$: $e_{av} = +2.58\%$, $(e_a)_{av} = 4.38\%$, $e_{max} = +15.34\%$;

(2) for the calculation of $\bar{\bar{\tau}}_{250}$: $e_{av} = +3.74\%$, $(e_a)_{av} = 5.39\%$, $e_{max} = +15.27\%$.

An analysis of the computer results indicated at least a partial success. The values of B, p, and q to be used in various cases of practical interest are listed in Table 3. Unfortunately, owing to the numerous simplifying assumptions used, Eq 14 yields consistently low values when estimating lower fire endurance values. Its use, therefore, must be restricted to $\bar{\bar{\tau}}_{160} > 0.7$ h or $\bar{\bar{\tau}}_{250} > 1.0$ h.

The errors committed by using Eq 14 are given also in Table 3.

According to Table 3, the formulas for the 160 and 250 F fire endurances are very similar; consequently, one may venture to conclude that the thermal fire endurances obtained from standard fire tests can also be described by an equation of essentially the same form.

With the introduction of the variable l/L, Eq 14 may be written in the following form:

$$\bar{\bar{\tau}} = B\left(\frac{\ell}{L}\right)^{(p-2q)}\left(\frac{L}{k}\right)^{p}\left(\frac{\kappa}{L^2}\right)^{q} \qquad (14a)$$

To find a suitable formula for the calculation of the fire endurance of hollow masonry units of conventional shapes it seemed worthwhile to explore the applicability of the following simple assumptions:

1. Hollow masonry units may be regarded as consisting of solid and double-layer sections, the relative magnitudes of which are[5]

$$\frac{a}{b} \text{ and } 1 - \frac{a}{b}$$

respectively.

2. The fire endurance of hollow masonry units is a function of four variables only; of two geometric variables, a/b and $(1 - a/b)$, and of two fire endurance values pertaining to the solid and double-layer sections, $\bar{\tau}$ and $\bar{\bar{\tau}}$.

From among the several expressions examined the following was selected for its simplicity:

$$\frac{1}{\tau^{1/2}} = \frac{a}{b}\,\frac{1}{\bar{\tau}^{1/2}} + \left(1 - \frac{a}{b}\right)\frac{1}{\bar{\bar{\tau}}^{1/2}} \quad \text{if } \tau \geq 1\,\text{h} \qquad (15)$$

The limitation imposed on the validity of this equation is not entirely a result of the restrictions concerning Eq 14, and the limitation should be respected even if $\bar{\bar{\tau}}$ is obtained by experiment.

The errors committed by using Eq 15, with values of $\bar{\tau}$ and $\bar{\bar{\tau}}$ from Eqs 13 and 14, for the calculation of the 160 F thermal fire endurances are listed in Table 4. It is seen that there is reasonable accuracy for the ranges $1 \leqslant (b - a)/(L - 2l) \leqslant 2$ and $0.167 \leqslant l/L \leqslant 0.333$, which represent geometries most commonly met in practice. For lower values of $(b - a)/(L - 2l)$ or of l/L or of both, the thermal fire endurance values obtained with the use of Eqs 13, 14, and 15 are generally lower than the computed values. The accuracy may be somewhat improved by using more complicated expressions instead of Eq 15. It must be kept in mind, however, that the ultimate accuracy is still subject to the previously described two assumptions.

Utilization of the Results

The primary purpose of these studies was not to produce "prediction"

[5] In practice, concrete masonry units are made generally with two different web spacings. b should, therefore, be interpreted as M/N where M = length of the masonry unit, N = number of full webs.

TABLE 4—*Errors committed by using Eq 15 with values of $\bar{\tau}$ and $\bar{\bar{\tau}}$ from Eqs 13 and 14, for the calculation of the 160 F thermal fire endurance (in dry condition).*

Average error: upper number; average absolute error: middle number; maximum error: lower number.

Table values: percent

l/L	$(b-a)/L-2l)$						
	Solid	0.25	0.5	1	2	4	Double Layer
0.1	. . .[a]	−22.80	−19.65	−5.52	. . .[b]	. . .[b]	+15.3
	. . .	22.80	19.65	6.72	. . .	. . .	15.3
	. . .	−41.0	−32.5	−16.0	. . .	. . .	+15.3
0.167	. . .[a]	−9.33	−9.62	−4.20	+8.72	+13.08	+4.34
	. . .	10.77	9.62	4.97	9.40	13.08	4.40
	. . .	−25.1	−20.7	−15.7	+18.7	+21.2	+7.7
0.25	. . .[a]	−3.62	−4.29	−1.85	+1.40	+3.06	+3.59
	. . .	6.80	5.62	3.19	2.89	4.01	4.01
	. . .	−22.0	−16.0	−11.0	−7.3	+8.9	+6.0
0.333	. . .[a]	−2.02	−2.53	−1.42	−0.58	+0.29	+1.16
	. . .	4.02	3.98	2.98	2.19	2.43	4.07
	. . .	−9.5	−10.1	−10.4	−6.7	−6.2	−11.3
Solid	−0.35	. . .[a]	. . .[a]	. . .[a]	. . .[a]	. . .[a]	. . .[a]
	2.04	. . .	. . .	. . .	. . .	. . .	. . .
	−8.9	. . .	. . .	. . .	. . .	. . .	. . .

[a] Not applicable.
[b] No results available.

formulas but to find basic rules for the correct design of concrete masonry units.

For some time it has been common practice to correlate the fire endurance of concrete masonry units with their "equivalent thicknesses." The equivalent thickness is defined as

$$\lambda = 2\ell + \frac{a}{b}(L - 2\ell) \qquad (16)$$

For the two limiting configurations, that is, for solid units and double-layer configurations, $\lambda = L$ and $\lambda = 2l$, respectively.

In Fig. 7 the 160 F thermal fire endurances for these two limiting configurations are plotted against λ for Concretes 1 to 4, based on the computer results listed in Table 2. As expected, at identical values of λ the double-layer configurations are seen to offer substantially higher fire endurances than the solid units, especially in the group of normal weight concretes. The equivalent thickness is, therefore, not a satisfactory correlating factor.

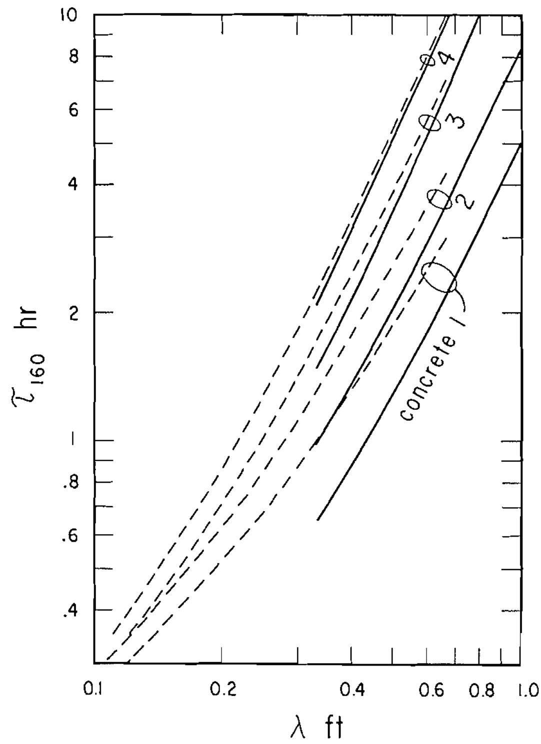


FIG. 7—*The τ_{160} versus λ plot for solid masonry units and double-layer configurations made from Concretes 1 to 4.*

TABLE 5—*Values of (L — 2l) and l/L, at which, for a given L, hollow and solid masonry units offer identical performances.*

(L — 2l) upper number
l/L lower number

Concrete	*(L — 2l)* (upper values) and *l/L* (lower values) at			
	$L = 0.333$	$L = 0.5$	$L = 0.667$	$L = 0.833$
1	0.092	0.105	0.112	0.118
	0.362	0.395	0.416	0.429
2	0.059	0.065	0.067	0.063
	0.411	0.435	0.450	0.462
3	0.024	0.025	0.025	0.024
	0.464	0.475	0.481	0.486
4	0.013	0.011	0.011	0.011
	0.480	0.489	0.492	0.493

Another interesting conclusion that one may draw from the curves in Fig. 7 is that at a given value of the overall thickness, L, it is possible to include in the masonry unit air cavities up to a certain thickness, and thus to reduce its weight and save on the cost of material without any loss in the fire endurance of the unit. The values of the air layer thickness, $(L - 2l)$, at which hollow and solid units offer identical performances, are obtained from Fig. 7 as the differences in the values of λ for solid and double layer configuration. In Table 5 these values of $(L - 2l)$ and the corresponding values of l/L are listed for Concretes 1 to 4.

By expressing the total increment of τ with the aid of Eqs 13, 14, and 15 and eliminating $(\tau/\bar{\bar{\tau}})$ from the resulting expression with the aid of Eq 15, finally the following formula is obtained:

$$\frac{\Delta\tau}{\tau} = \frac{2\frac{a}{b}}{1 - \frac{a}{b}}\left[1 - \left(\frac{\tau}{\bar{\bar{\tau}}}\right)^{1/2}\right]\left(\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$$
$$+ \frac{a}{b}\left(\frac{\tau}{\bar{\bar{\tau}}}\right)^{1/2}\left[(m - 2n)\frac{\Delta L}{L} + m\frac{\Delta k}{k} + n\frac{\Delta\kappa}{\kappa}\right]$$
$$+ \left[1 - \frac{a}{b}\left(\frac{\tau}{\bar{\bar{\tau}}}\right)^{1/2}\right]\left[(p - 2q)\frac{\Delta \ell}{\ell} + p\frac{\Delta k}{k} + q\frac{\Delta\kappa}{\kappa}\right] \quad (17)$$

This formula may be utilized either directly as an extrapolation formula, or indirectly as a starting point from which useful design principles may be derived.

The latter use of Eq 17 is illustrated by examining the following rather typical problem: when attempting to improve the performance of concrete masonry units in fire without changing their overall thickness, L, is it more economical to increase the web thickness, a, or the face shell thickness, l?

To answer this Eq 17 is applied to the following two cases. First case: the face shell thickness is increased by Δl, while $\Delta a = \Delta b = \Delta L = \Delta k = \Delta\kappa = 0$. The resulting increment in $\Delta\tau$ is then

$$(\Delta\tau)_l = \tau\left[1 - \frac{a}{b}\left(\frac{\tau}{\bar{\bar{\tau}}}\right)^{1/2}\right](p - 2q)\frac{\Delta\ell}{\ell} \quad (18)$$

Second case: the web thickness is increased by Δa, while $\Delta b = \Delta L = \Delta l = \Delta k = \Delta\kappa = 0$. For this case

$$(\Delta\tau)_a = \tau\frac{2\frac{a}{b}}{1 - \frac{a}{b}}\left[1 - \left(\frac{\tau}{\bar{\bar{\tau}}}\right)^{1/2}\right]\frac{\Delta a}{a} \quad (19)$$

The question is now which is larger, $(\Delta\tau)_l$ or $(\Delta\tau)_a$ under the condition that

$$a(L - 2\ell) = 2\Delta\ell(b - a) \qquad (20)$$

that is, that identical amounts of material are used to increase a and l.

Combining Eqs 15, 18, 19, and 20 one finds that an increase in the face shell thickness is more advantageous as long as

$$\frac{\ell}{L} < \frac{1}{2} \frac{\frac{p - 2q}{2}}{\frac{p - 2q}{2} + 1 - \left(\frac{\bar{\bar{\tau}}}{\bar{\tau}}\right)^{1/2}} \qquad (21)$$

A closer examination will show that this condition always is fulfilled; therefore, it is always more advantageous to increase the face shell thickness than the web thickness.

In a similar way the effect of any set of changes in the six variables (a, b, l, L, k, κ) can be compared with that of any other set of changes. In practice one need not always try to develop general criteria, such as inequality [*21*]. It is easier to compare the numerical values of $\Delta\tau$ pertaining to various specified sets of changes. As an example, the following problem will be examined:

A wall built of lightweight concrete masonry units (made from Concrete 3) yielded 1.63 h fire endurance, when tested at 5.5 percent by volume moisture content (in equilibrium with 70 percent relative humidity). Using the method described in Refs *4* and *5* the thermal fire endurance of this wall in dry condition is $\tau^* = 1.22$ h. The information concerning the masonry units is as follows:[6]

$L^* = 0.66667$ ft
$l^* = 0.11111$ ft
$a^* = 0.11111$ ft
$b^* = 0.55556$ ft
$\lambda^* = 0.31111$ ft (from Eq 16)
$k^* = 0.317$ Btu/h ft R
$\kappa^* = 0.01939$ ft^2/h

The manufacturer would like to have the fire endurance increased to 2.0 h, without changing the overall sizes of the units and with a minimum increase in their weights. He considers a 7 percent weight increase permissible, and wonders whether he should increase correspondingly the face shell thickness to 0.12472 ft or the web thickness to 0.13833 ft.

To use Eq 17 and any of its applied forms, for example, Eqs 18 and

[6] The asterisk is used hereafter to denote values of the fire endurance (from experiments or from Table 2) and all information pertaining to these known values.

19, one has first to find the value of $\bar{\tau}^*$, for which generally there is no experimental result available. It can be calculated with the aid of the following equations:

$$\bar{\tau}^* = \tau^* \left[\frac{a^*}{b^*} + \left(1 - \frac{a^*}{b^*}\right) \Big/ \left(\frac{\bar{\bar{\tau}}^*}{\bar{\tau}^*}\right)^{1/2} \right]^2 \qquad (22)$$

where

$$\frac{\bar{\bar{\tau}}^*}{\bar{\tau}^*} = \frac{B}{A} \frac{(\ell^*)^{p-2q}}{(L^*)^{m-2n}} (k^*)^{p-m} (\kappa^*)^{q-n} \qquad (23)$$

Equation 22 is a rearranged form of Eq 15, and Eq 23 is obtained by dividing Eq 14 by Eq 13.

It may be noted that, since $|p - m|$ and $|q - n|$ in Eq 23 are very small numbers, even estimated values of k and κ cannot yield serious errors in the calculation of $\bar{\bar{\tau}}^*$. For the same reason, one may apply this calculation procedure even if it is known that the concrete was made with decomposing aggregates. In such cases it is advisable to use relatively low, estimated values for κ.

With the information given earlier it is found that $\bar{\tau}^* = 5.89$ h. Then, using Eqs 18 and 19 one finally obtains that $(\Delta \tau)_l = 0.26$ h and $(\Delta \tau)_a = 0.08$ h. (These results confirm the conclusion previously arrived at, according to which it is more advantageous to increase the face shell thickness than the web thickness.) Thus, by increasing the face shell thickness to 0.12472 ft the new value of the fire endurance becomes $1.22 + 0.26 = 1.48$ h in dry condition and (after applying the method described in Refs *4* and *5*) 1.97 h at 5.5 percent by volume moisture content (which represents the "standard" amount of moisture for this kind of lightweight concrete). It is seen now that with a mere 7 percent increase in weight, it is not possible to increase the fire endurance of the units to 2.0 h.

In this example it also was shown how one can utilize Eq 17 directly as an extrapolation formula. It must be emphasized, however, that in a strict sense neither Eq 17 nor any of its applied forms (such as Eqs 18 and 19) is valid when $\Delta \tau/\tau$, $\Delta a/a$, etc. are larger than about 0.1. There remains some doubt, therefore, about the accuracy of the numerical results reached above.

The general procedure used to calculate the fire endurance of a masonry unit, if the fire endurance of another masonry unit of the same material but of different geometry is known, is as follows:

Step 1—Using Eqs 22 and 23 calculate $\bar{\tau}^*$, then again using Eq 23 calculate $\bar{\bar{\tau}}^*$. (As mentioned earlier, these are values corresponding to the geometry and material to which τ^* relates.) If τ^* happens to relate to a solid unit, then $\tau^* \equiv \tau^*$, and only $\bar{\bar{\tau}}^*$ needs to be calculated with the aid of Eq 23.

Step 2—With the aid of the following two equations (obtained from Eqs 13 and 14, respectively)

$$\frac{\bar{\tau}}{\bar{\tau}^*} = \left(\frac{L}{L^*}\right)^{m-2n} \left[\left(\frac{k}{k^*}\right)^{-m} \left(\frac{\kappa}{\kappa^*}\right)^{n}\right] \tag{24}$$

$$\frac{\bar{\bar{\tau}}}{\bar{\bar{\tau}}^*} = \left(\frac{\ell}{\ell^*}\right)^{p-2q} \left[\left(\frac{k}{k^*}\right)^{-p} \left(\frac{\kappa}{\kappa^*}\right)^{q}\right] \tag{25}$$

calculate $\bar{\tau}$ and $\bar{\bar{\tau}}$ corresponding to the new geometry. If there is no change in the material, the terms in the square brackets are naturally equal to 1.

Step 3—Finally, again using Eq 15, calculate τ for the new geometry.

Since the material properties do not enter this calculation procedure, one may expect that it is applicable to any kind of concrete irrespective of whether the aggregates are chemically stable or are liable to undergo decomposition reactions.

With this more accurate calculation procedure one finds that, by increasing the face shell thickness of the masonry unit discussed in the example from 0.11111 to 0.12472 ft, the fire endurance of the dry masonry units increases from 1.22 to 1.49 h or, at 5.5 percent moisture, from 1.63 to 1.98 h, which still falls somewhat short of the requirement.

It may be useful to illustrate the calculation procedure through another example.

A manufacturer intends to introduce three kinds of new masonry units all made from the same lightweight concrete. One of them, Unit 1, is a solid unit. The following information is available:

	Unit 1	Unit 2	Unit 3
L ft	0.30417	0.47083	0.47500
l ft	. . .	0.12833	0.19792
a ft	. . .	0.15486	0.14514
b ft	. . .	0.44375	0.44375
k Btu/h ft R		0.242	
ρ lb/ft^3		75.5	
c Btu/lb R		0.199	
κ ft^2/h		0.0161	

Of the three units only No. 3 has been subjected to standard fire test. The test yielded 4.05-h fire endurance at 6.5 percent (by volume) moisture content. With the aid of the nomogram in Ref *4* the fire endurance in dry condition is obtained as 3.13 h. Question: What fire endurances can the manufacturer expect for Units 1 and 2?

From Eq 23 for a lightweight concrete

$$\frac{\bar{\bar{\tau}}^*}{\bar{\tau}^*} = \frac{0.860}{0.152} \times \frac{0.19792^{1.9}}{0.47500^{2.2}} \times 0.242^{-0.1} \times 0.0161^{0.1} = 1.02233$$

and thus from Eq 22

$$\bar{\tau}^* = 3.13 \left[\frac{0.14514}{0.44375} + \left(1 - \frac{0.14514}{0.44375}\right) \Big/ 1.02233\right]^2 = 3.0387\,\text{h}$$

and again from Eq 23

$$\bar{\bar{\tau}}^* = 1.02233 \times 3.0387 = 3.1065\,\text{h}$$

For Unit 1 (solid unit!) the fire endurance in dry condition is now obtained by using Eq 24:

$$\tau \equiv \bar{\tau} = 3.0387 \left(\frac{0.30417}{0.47500}\right)^{2.2} \times 1 = 1.1398\,\text{h}$$

For Unit 2, using first Eq 24,

$$\bar{\tau} = 3.0387 \left(\frac{0.47083}{0.47500}\right)^{2.2} \times 1 = 2.9802\,\text{h}$$

then using Eq 25

$$\bar{\bar{\tau}} = 3.1065 \left(\frac{0.12833}{0.19792}\right)^{1.9} \times 1 = 1.3639\,\text{h}$$

Finally the fire endurance in dry condition is calculated with the aid of Eq 15:

$$\tau = 1 \Big/ \left[\frac{0.15486}{0.44375} \times \frac{1}{2.9802^{1/2}} + \left(1 - \frac{0.15486}{0.44375}\right)\frac{1}{1.3639^{1/2}}\right]^2 = 1.7331\,\text{h}$$

These values ($\tau = 1.14$ and 1.73) have to be corrected according to Ref *4* to take the beneficial effect of moisture (usually about 6 percent by volume for this type of materials) into account.

Essentially the same procedure can be used to find, with the aid of the information given in Table 2, the fire endurance for masonry units of geometries and material properties different from those covered in Table 2. In such calculations τ^* means some tabulated value. There is no need here to execute the calculations described in Step 1, because the values of $\bar{\tau}^*$ and $\bar{\bar{\tau}}^*$ corresponding to τ^* also are listed in Table 2. (They are found in the same row as τ^*; $\bar{\tau}^*$ in the column "Solid," and $\bar{\bar{\tau}}^*$ under the "Double Layer" heading, in the column of the appropriate value of l/L.) When the fire endurance of a masonry unit made from a concrete different from those dealt with in Table 2 is sought, the terms in the square brackets in Eqs 24 and 25 are not equal to 1.

This procedure of using information in Table 2 to develop new thermal fire endurance information, is applicable only if the masonry unit is made from a concrete of chemically stable aggregates.

Conclusions

There have been 1180 computer calculations performed. They covered large ranges of the four geometric variables, a, b, l, and L (Table 2) and four different concretes which can be regarded as "limiting cases" in the normal weight and lightweight groups (see Figs. 1 and 2). Special effort was made to simulate in the computer program the true mechanisms of heat transport in dry concretes in every detail.

It was possible to express the thermal fire endurances of the masonry units, which is probably the most important information deduced from the computer calculations (Table 2), with the aid of three empirical equations (Eqs 13, 14, and 15). These equations yield the 160 and 250 F fire endurances in dry condition of "solid units," "double-layer configurations," and conventional hollow units, respectively, as functions of two material properties at room temperature, k and κ, and four geometric variables, a, b, l, and L.

In applying these formulas to the prediction of fire endurance from known values of k, κ, a, b, l, and L some caution must be exercised for the following two reasons:

(*a*) The relation between the actual temperature of the unexposed surface, (which is obtained from the computations) and the temperature under asbestos pad covers (which is obtained from standard fire endurance tests) is not known accurately. From a large number of tests it appears, however, that the temperature of thermal failure can be interpreted approximately as a 160 F rise in the true temperature of the unexposed surface above the initial level.

(*b*) The thermal properties of Concretes 1 to 4, which were used in all computations, are typical of concretes that are made with aggregates of high physicochemical stability.[7] Some more recent investigations revealed that it is customary to use, especially in normal weight masonry units, physicochemically highly unstable aggregates. Units made with decomposing aggregates may have thermal fire endurances up to twice as high as those made with stable aggregates.

[7] The enthalpy change accompanying the $\alpha-\beta$ quartz transformation was taken into account when calculating the ρc versus T relation for Concrete 1. In general, concretes that do not show more than 5 percent weight loss when heated from 221 to 1600 F can be regarded as made with physicochemically stable aggregates.

The formulas derived probably will prove most useful when applied to studies concerning the economy of masonry unit design, or in the prediction of the performance in fire of masonry units from the performance of other units of different geometry but made from the same or just slightly different material.

As an example of the first of these applications, it has been shown that the fire endurance increases more rapidly by adding material to the face shell rather than to the web of masonry units.

By rearrangement of the formulas (see Eqs 22 to 25) special expressions have been obtained which can be used to extrapolate to the performance of masonry units from some known fire endurance information (from standard fire tests or from Table 2). All procedures described relate to dry masonry units. The way of finding the fire endurance in dry condition from experimental fire endurance data, and of correcting the fire endurance for any moisture level, has been described elsewhere (see Refs *3*, *4*, and *5*).

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APPENDIX I

Numerical Procedure

The first step in preparing the computer program for numerical studies is to choose a network of points at which the variation of temperature is to be examined. From practical consideration a diagonal mesh has been selected (see Fig. 5*b*) with equal mesh widths, $\Delta \xi$, in both directions. In an x-y coordinate system a point $P_{r,s}$ has the coordinates $x = r \Delta \xi/\sqrt{2}$ and $y = s \Delta \xi/\sqrt{2}$. Figure 5*b* makes it clear that only those points of the x-y plane are defined for which $(r + s)$ is an even number.

Since the planes *AB* and *CD* (see Fig. 5*a*) are planes of symmetry, it is possible to study that portion of the masonry unit which lies between these two planes, instead of the whole unit. Figure 5*b* shows how this portion of the unit is arranged over the diagonal mesh. Naturally, the $\Delta \xi$ dimension has to be selected to fit the dimensions of the unit.

Convenient expressions for the numerical studies can be obtained either by mechanically replacing all derivatives in Eqs 2 to 5 by their finite difference equivalents (see, for example, Refs *11* or *14*), or by writing heat balance equations for a few typical network points (see, for example, Refs *15* or *13*). In the present work the second procedure was chosen.

The heat balance for some point $P_{r,s}$ inside the solid (that is, in the region S) can be expressed as follows:

$$\frac{k^j_{(r-1),(s-1)} + k^j_{r,s}}{2} \times \frac{T^j_{(r-1),(s-1)} - T^j_{r,s}}{\Delta\xi} + \frac{k^j_{(r+1),(s-1)} + k^j_{r,s}}{2} \times \frac{T^j_{(r+1),(s-1)} - T^j_{r,s}}{\Delta\xi}$$

$$+ \frac{k^j_{(r-1),(s+1)} + k^j_{r,s}}{2} \times \frac{T^j_{(r-1),(s+1)} - T^j_{r,s}}{\Delta\xi} + \frac{k^j_{(r+1),(s+1)} + k^j_{r,s}}{2} \times \frac{T^j_{(r+1),(s+1)} - T^j_{r,s}}{\Delta\xi}$$

$$= (\rho c)^j_{r,s}\, \Delta\xi\, \frac{T^{j+1}_{r,s} - T^j_{r,s}}{\Delta t} \quad (26)$$

This equation is the difference equation that in numerical studies replaces Eq 3. Since the thermal conductivity and volume specific heat of the material are functions of the temperature, their values, as a rule, vary along the network. In Eq 26

$$k^j_{r,s} = k(T^j_{r,s}) \quad (27)$$

$$(\rho c)^j_{r,s} = \rho c(T^j_{r,s}) \quad (28)$$

in other words, $k^j_{r,s}$ means the value of k at the temperature that prevails at the point $P_{r,s}$ at $t = j\,\Delta t$.

Equation 26 can be rearranged into the form

$$T^{j+1}_{r,s} = T^j_{r,s} + \frac{\Delta t}{2(\rho c)^j_{r,s}(\Delta\xi)^2}\{(k^j_{(r-1),(s-1)} + k^j_{r,s})(T^j_{(r-1),(s-1)} - T^j_{r,s})$$
$$+ (k^j_{(r+1),(s-1)} + k^j_{r,s})(T^j_{(r+1),(s-1)} - T^j_{r,s})$$
$$+ (k^j_{(r-1),(s+1)} + k^j_{r,s})(T^j_{(r-1),(s+1)} - T^j_{r,s})$$
$$+ (k^j_{(r+1),(s+1)} + k^j_{r,s})(T^j_{(r+1),(s+1)} - T^j_{r,s})\}$$

$$\text{for region } S \quad (29)$$

With this expression, $T^{j+1}_{r,s}$, that is, the temperature at $x = r\Delta\xi/\sqrt{2}$, $y = s\Delta\xi/\sqrt{2}$ and at $t = (j+1)\,\Delta t$, can be calculated if the temperatures $T^j_{r,s}$, $T^j_{(r-1),(s-1)}$, $T^j_{(r+1),(s-1)}$, $T^j_{(r-1),(s+1)}$, and $T^j_{(r+1),(s+1)}$, (that is, the temperatures at the points $P_{r,s}$, $P_{(r-1),(s-1)}$, $P_{(r+1),(s-1)}$, $P_{(r-1),(s+1)}$ and $P_{(r+1),(s+1)}$ at $t = j\,\Delta t$) are known.

With similar reasoning the following equations can be obtained:

$$T^{j+1}_{r,s} = T^j_{r,s} + \frac{\Delta t}{(\rho c)^j_{r,s}(\Delta\xi)^2}\{(k^j_{(r+1),(s-1)} + k^j_{r,s})(T^j_{(r+1),(s-1)} - T^j_{r,s})$$
$$+ (k^j_{(r+1),(s+1)} + k^j_{r,s})(T^j_{(r+1),(s+1)} - T^j_{r,s})$$
$$+ \sqrt{2}\sigma\epsilon\Delta\xi[(T^j_f)^4 - (T^j_{r,s})^4]\}$$

$$\text{for boundary } E \quad (30)$$

$$
\begin{aligned}
T_{r,s}^{j+1} = T_{r,s}^{j} + \frac{\Delta t}{(\rho c)_{r,s}^{j} (\Delta \xi)^2} \{ & (k_{(r-1),(s-1)}^{j} + k_{r,s}^{j})(T_{(r-1),(s-1)}^{j} - T_{r,s}^{j}) \\
& + (k_{(r-1),(s+1)}^{j} + k_{r,s}^{j})(T_{(r-1),(s+1)}^{j} - T_{r,s}^{j}) \\
& + \sqrt{2}\beta \Delta \xi \, | T_o - T_{r,s}^{j} |^{1/4} (T_o - T_{r,s}^{j}) \\
& + \sqrt{2}\sigma \epsilon \Delta \xi \, [T_o^4 - (T_{r,s}^{j})^4] \}
\end{aligned}
$$

for boundary U ....(31)

$$
\begin{aligned}
T_{r,s}^{j+1} = T_{r,s}^{j} + \frac{\Delta t}{(\rho c)_{r,s}^{j} (\Delta \xi)^2} \{ & (k_{(r-1),(s-1)}^{j} + k_{r,s}^{j})(T_{(r-1),(s-1)}^{j} - T_{r,s}^{j}) \\
& + (k_{(r-1),(s+1)}^{j} + k_{r,s}^{j})(T_{(r-1),(s+1)}^{j} - T_{r,s}^{j}) \\
& + \sqrt{2}\beta \Delta \xi \, | T_{\mathrm{av}}^{j} - T_{r,s}^{j} |^{1/4} (T_{\mathrm{av}}^{j} - T_{r,s}^{j}) \\
& + \sqrt{2}\sigma \epsilon \Delta \xi \sum_{k=1}^{N} (B_{r,s})_k (T_k^{j})^4 \}
\end{aligned}
$$

for boundary C_1 ....(32)

$$
\begin{aligned}
T_{r,s}^{j+1} = T_{r,s}^{j} + \frac{\Delta t}{(\rho c)_{r,s}^{j} (\Delta \xi)^2} \{ & (k_{(r-1),(s+1)}^{j} + k_{r,s}^{j})(T_{(r-1),(s+1)}^{j} - T_{r,s}^{j}) \\
& + (k_{(r+1),(s+1)}^{j} + k_{r,s}^{j})(T_{(r+1),(s+1)}^{j} - T_{r,s}^{j}) \\
& + \sqrt{2}\beta \Delta \xi \, | T_{\mathrm{av}}^{j} - T_{r,s}^{j} |^{1/4} (T_{\mathrm{av}}^{j} - T_{r,s}^{j}) \\
& + \sqrt{2}\sigma \epsilon \Delta \xi \sum_{k=1}^{N} (B_{r,s})_k (T_k^{j})^4 \}
\end{aligned}
$$

for boundary C_2 ....(33)

$$
\begin{aligned}
T_{r,s}^{j+1} = T_{r,s}^{j} + \frac{\Delta t}{(\rho c)_{r,s}^{j} (\Delta \xi)^2} \{ & (k_{(r+1),(s-1)}^{j} + k_{r,s}^{j})(T_{(r+1),(s-1)}^{j} - T_{r,s}^{j}) \\
& + (k_{(r+1),(s+1)}^{j} + k_{r,s}^{j})(T_{(r+1),(s+1)}^{j} - T_{r,s}^{j}) \\
& + \sqrt{2}\beta \Delta \xi \, | T_{\mathrm{av}}^{j} - T_{r,s}^{j} |^{1/4} (T_{\mathrm{av}}^{j} - T_{r,s}^{j}) \\
& + \sqrt{2}\sigma \epsilon \Delta \xi \sum_{k=1}^{N} (B_{r,s})_k (T_k^{j})^4 \}
\end{aligned}
$$

for boundary C_3 ....(34)

Here again, the index k refers to the network points all along the boundaries C_1, C_2, and C_3 on both sides of the symmetry line AB in Fig. 5*b*, and T_{av} has been defined by Eq 7. The absorption factors, $(B_{r,s})_k$, have been determined for the selected cavity shapes (see values of $(b - a)/(L - 2l)$ in Table 2) in a way described by Gebhart [*11*].

Equation 30 is the difference equation that replaces Eq 2 in the numerical work; Eq 31 replaces Eq 4, and Eqs 32, 33, and 34 replace Eq 5.

Equations 29 to 34 must be supplemented by the following equations:

$$T_{r,s} = 0 \quad \text{if } r + s \text{ is odd} \qquad (35)$$

$$T_{r,(c-1)} = T_{r,(c+1)} \qquad (36)$$

$$T_{r,(d-1)} = T_{r,(d+1)} \qquad (37)$$

Equation 35 follows from the fact that only those points of the x-y plane are defined for which $(r + s)$ is an even number. Equations 36 and 37 represent the conditions of symmetry about the planes AB and CD, respectively (for which $y = c \Delta \xi/\sqrt{2}$ and $y = d \Delta \xi/\sqrt{2}$, respectively).

The initial condition is

$$T^{o}_{r,s} = T_o \text{ for any point such that } (r + s) \text{ is even} \qquad (38)$$

The regions of applicability of Eqs 29 to 34 have to be defined among the input information (by specifying the points that make up the various regions). With these six equations and the two auxiliary equations, Eqs 36 and 37, it is then possible to calculate the temperature at any point in the region S or in the boundaries E, U, C_1, C_2, and C_3 for $t = (j + 1) \Delta t$, if the temperatures at all these points are known for $t = j \Delta t$. Thus with a set of values for $T^{o}_{r,s}$ defined in the initial conditions, one can follow the temperature history of the solid by repeated application of the equations.

In the numerical solution of parabolic differential equations this technique is called forward difference or explicit scheme (Refs *14* and *16*) to indicate that while all space derivatives are expressed at the time level $t = j \Delta t$, the time derivative refers forward to the $t = (j + 1) \Delta t$ level (see the last term in Eq 26. The values of $T^{j+1}_{r,s}$ at all network points thus can be calculated with the aid of explicit expressions. It is known that the solutions yielded by this scheme are not stable for all selections of $\Delta \xi$ and Δt. To ensure that any error existing in the solution at some time level will not be amplified in the subsequent calculations, a stability criterion has to be satisfied which, for a selected value of $\Delta \xi$ limits the maximum value of Δt.

In all of the cases studied the criterion of stability seemed to be the most restrictive along the surface E.

APPENDIX II

Definition of Errors

The equation defining the errors in thermal fire endurances, referred to in the section "Results of Computer Calculations," are as follows:

Average error:

$$e_{av} = \frac{1}{N} \sum_{n=1}^{N} \frac{\tau_e - \tau_c}{\tau_c} \qquad (39)$$

Average absolute error:

$$(e_a)_{av} = \frac{1}{N} \sum_{n=1}^{N} \left| \frac{\tau_e - \tau_c}{\tau_c} \right| \qquad (40)$$

Maximum error:

$$e_{max} = \frac{(\tau_e - \tau_c)_{max}}{\tau_c} \qquad (41)$$

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