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BY

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Absorption of Diffuse Sound by a Strip or Rectangular Patch of Absorptive Material*

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(Received 28 March 1963)

Previous calculations of the effective absorption coefficient of a rectangular patch of absorptive material have been extended in several respects. The new results are applicable to locally reacting materials for any size of patch and for a wide range of complex admittances. Specific use of the results is made in an examination of the reverberation-room method of measuring sound-absorption coefficients.

A PREVIOUS paper[1] presented calculations of the absorption of diffuse sound by a narrow strip of absorptive material set in an otherwise reflecting surface. Although the formulas used were developed in the first instance for an infinite strip, it was found experimentally that they could be modified to apply to a rectangular patch. The results have been used in recent reverberation-room studies with sufficient success to warrant more-complete calculations. The expanded results are reported here.

The principal extension of the calculation has been from real to complex admittances. In addition, following experience with the earlier results, the values of the various parameters were selected to permit more-accurate interpolations in some regions. Finally, the numerical-integration procedure was refined somewhat, so that the new results are more accurate than those reported previously.

With the extensions the complete results are rather voluminous for brief presentation. In this paper, the more important functional relations are demonstrated and discussed, particularly with reference to the reverberation-room measurement of sound absorption. A manuscript containing full details is available from the author.

NOTATION

When the results were extended to include complex admittances, it was realized that the previous expression for the admittance ratio $\eta=\delta+i\xi$ gave the wrong sign for the susceptance ratio as compared to the conventions used by most writers. As the absorption of a small patch depends on the sign of the susceptance, the notation in this paper has been revised to agree with usual practice; the admittance ratio is now denoted by $y=g-ib$. The mathematical expressions, recapitulated here, incorporate this change and incidentally correct one or two typographical errors in the earlier work.

BASIC EQUATIONS

The absorption coefficient for diffuse sound incident on a long absorptive strip set in a reflecting plane is given by

$$\bar{\alpha}=\frac{16g}{\pi}\int_0^{\pi/2}\int_0^{\pi/2}\frac{\sin^3\phi d\phi d\theta}{(g^2+b^2)(A^2+B^2)+2\sin\phi(gA+bB)+\sin^2\phi}, \tag{1}$$

where

$$A=\int_0^{ka\sin\phi}\cos(x\sin\theta)J_0(x)dx-\frac{1}{ka\sin\phi}\int_0^{ka\sin\phi}x\cos(x\sin\theta)J_0(x)dx, \tag{2}$$

$$B=\int_0^{ka\sin\phi}\cos(x\sin\theta)N_0(x)dx-\frac{1}{ka\sin\phi}\int_0^{ka\sin\phi}x\cos(x\sin\theta)N_0(x)dx,$$

* This paper is a contribution of the Division of Building Research, National Research Council, Canada, and is published with the approval of the Director of the Division.

[1] T. D. Northwood, M. T. Grisaru, and M. A. Medcof, J. Acoust. Soc. Am. 31, 595 (1959).

a is the width of the strip, λ is the wavelength (in same units as a), and $k=2\pi/\lambda$.

Numerical integrations were done for eight values of g between 0.1 and 1.0, for eleven values of b from -0.5 to $+0.5$, and for nine values of ka between 2 and 50. These permit accurate determinations of effective absorption coefficients for any size of rectangular patch and for the range of admittances commonly encountered. For limiting values of ka, Eq. (1) reduces to simpler forms: when ka is very large, it reverts to Morse's formula for the statistical absorption coefficient of an infinite area; when ka approaches zero, the effective absorption coefficient becomes $8g$.

The results are adapted to a rectangular patch by putting $a=cd/(c+d)$, where c and d are the dimensions of the patch. This expression, which reverts to the infinite-strip form when either dimension becomes very large, is found to fit experimental data for squares and long strips.[1] It is similar also to the parameters used by Kuhl[2] and others for experimental studies.

RESULTS

Figure 1 shows the variation of absorption coefficient with the parameter $1/(ka)$. The reciprocal parameter is chosen in order to show the transition to infinite area ($1/(ka)=0$). Particular values of $1/(ka)$, corresponding to the standard sample size used in absorption tests and the standard test frequencies,[3] are marked off along the abscissa. It will be noted that there is a substantial difference between the coefficients obtained for the standard sample and for an infinite area at all but the highest test frequencies. The two broken lines are for complex admittances; the principal point to observe is that, although these curves are similar in form to the real-admittance curves over most of the range, they deviate substantially in the transition to infinite area.

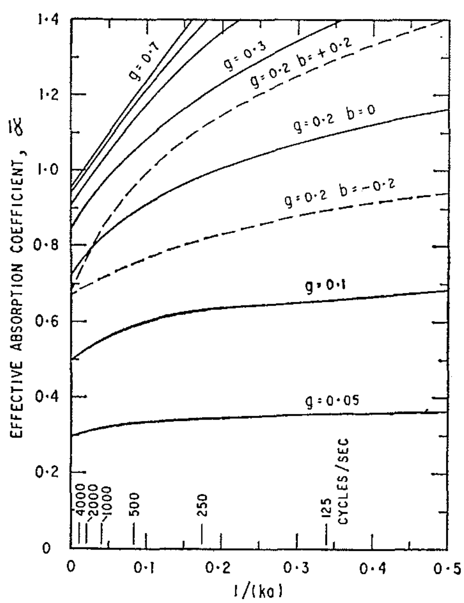

FIG. 1. Effective absorption as a function of sample dimensions for a range of admittance ratios. Solid lines are for real-admittance ratios ($b=0$); broken lines show range of susceptance ratios for one value of conductance ratio.

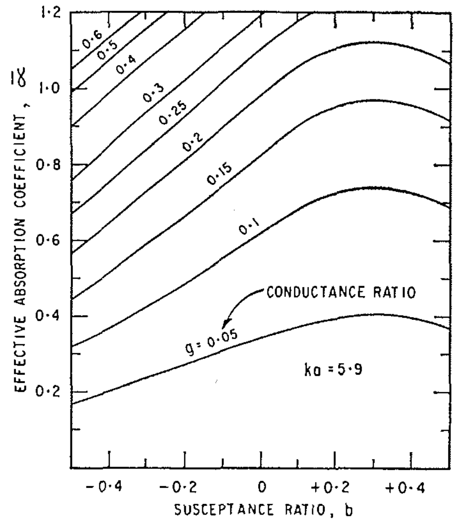

FIG. 2. Effective absorption coefficient of standard 9- ×8-ft sample at 250 cps.

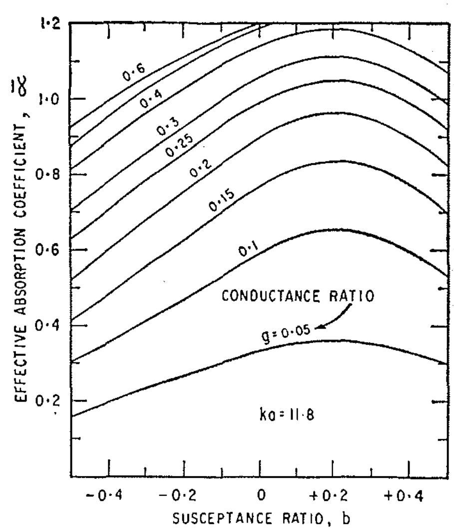

FIG. 3. Effective absorption coefficient of standard 9- ×8-ft sample at 500 cps.

Further information concerning the imaginary component is shown in Figs. 2–5, which include most of the results for four of the standard test conditions. Whereas for an infinite area (Fig. 6) the absorption is maximum for zero susceptance, the peak shifts toward positive susceptance as the sample size, relative to wavelength, decreases.

[2] W. Kuhl, Acustica 10, 264 (1960).

[3] "Tentative Method of Practice for Sound Absorption of Acoustical Materials in Reverberation Rooms," Am. Soc. Testing Mater. ASTM C423-60T (1960).

COMPARISON WITH REVERBERATION-ROOM RESULTS

The most important application of the foregoing calculations is to provide an independent check on the reverberation-room method for measuring sound-absorption coefficients. The reverberation-room method purports to measure the random-incidence absorption coefficient of a sample of material, but the intricacies of the method, based more on empirical attempts to obtain repeatable results than on theoretical considerations, leave some doubt regarding the quantity that finally emerges. For a special category of materials, it is now possible to calculate from impedance-tube measurements the absorption coefficients that should be obtained in the reverberation room.

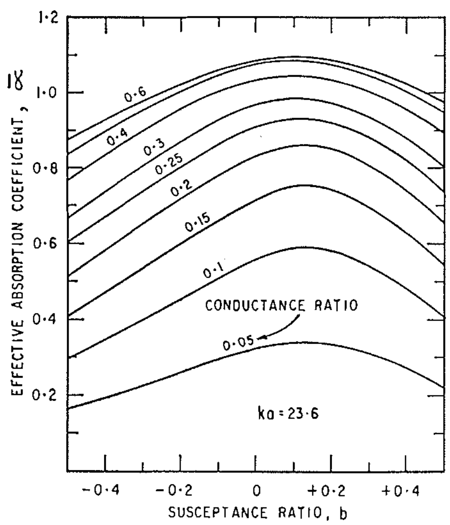

FIG. 4. Effective absorption coefficient of standard 9- ×8-ft sample at 1000 cps.

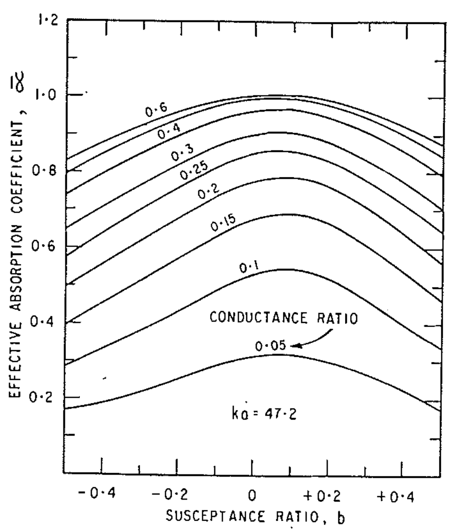

FIG. 5. Effective absorption coefficient of standard 9- ×8-ft sample at 2000 cps.

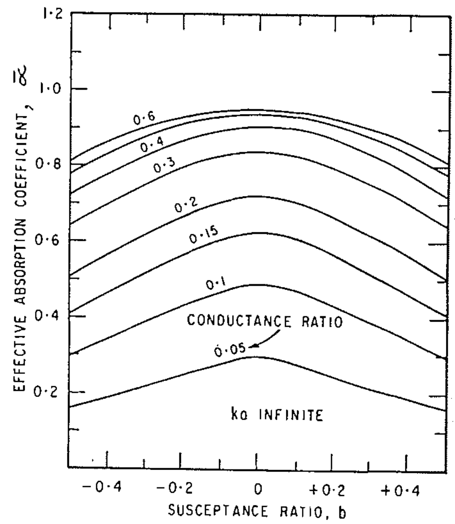

FIG. 6. Absorption coefficient of infinite area of material.

The materials for which the comparison can be made must fulfill three requirements: (1) They must be of the locally reacting type; i.e., obliquely incident waves must not be propagated obliquely within the material. (2) The mounting condition must be the same in reverberation-room and impedance-tube tests, and must not introduce deviations from requirement (1). (3) The material should not be too thick, unless it can be set in a cavity with its face flush with the surrounding surface, since the diffraction calculation is based on the interaction of the absorbing surface and an adjacent reflecting surface. These requirements seem very stringent, but they do not rule out all materials. Any reasonably dense porous material is approximately locally reacting, as attenuation and refraction reduce oblique transmission in the material to negligible importance. Perforations or fissures part way through such materials will not rule them out, except at high frequencies where the perforation spacing relative to a wavelength may be important. Ruled out completely, of course, are all types of mountings involving a space behind the surface material. The thickness of the material when mounted on, rather than set into, the surrounding surface may become important at the higher frequencies where it becomes comparable to a wavelength.

It is a necessary preliminary to note that neither the current ASTM recommended practice nor the proposed ISO standard will ensure that any two laboratories will obtain the same results. Results of the recent round robin reported by Kosten[4] illustrate this point; the spread in results obtained by eight laboratories that conform to the proposed ISO method is shown in Figs. 7–9. Reasons for this spread are currently being sought by several laboratories. In the meantime, it is of interest to compare the *average* of the round-robin

[4] C. W. Kosten, Acustica **10**, 400 (1960).

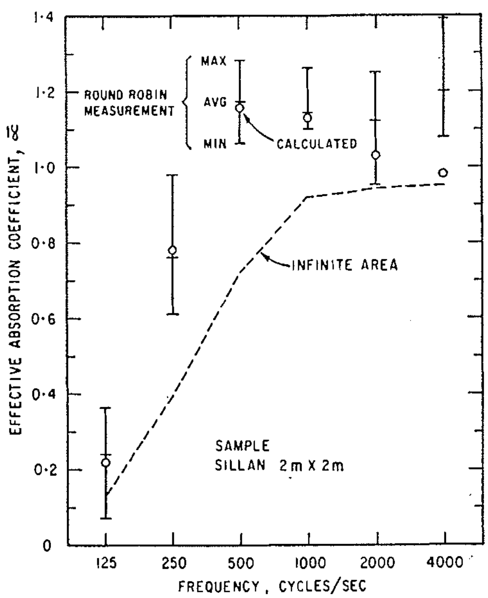

Fig. 7. Calculated and measured absorption coefficients for 2- ×2-m sample of Sillan in reverberation room. (See Ref. 4.)

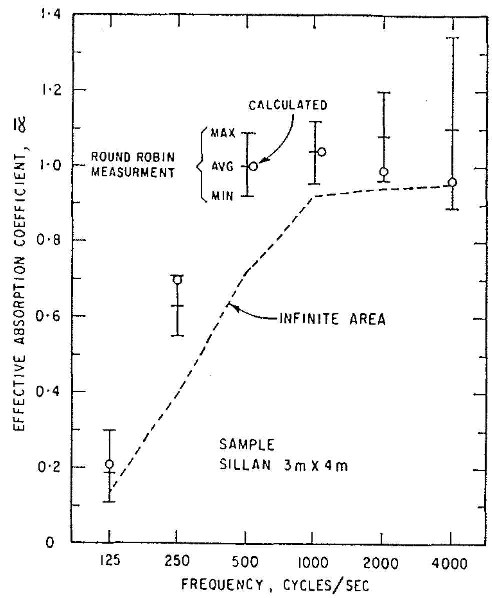

Fig. 9. Calculated and measured absorption coefficients for 3- ×4-m sample of Sillan in reverberation room. (See Ref. 4.)

results with the values predicted from impedance measurements and calculations for the finite sample sizes. These quantities are also shown on Figs. 7–9. Impedance-tube measurements were made by the author on several samples of the material tested in his laboratory (one of the eight in Kosten's report). The average results are given in Table I.

It appears that there is a slight discrepancy at the top frequencies, where reverberation-room results are systematically higher than calculated values. Apart from this, there is substantial agreement, and it appears that the average of results for several reverberation rooms is indeed the value that would be predicted from acoustical-impedance data. Thus, it is possible that the reverberation-room method, with suitable refinements to reduce the present scatter in results, may yield results that have a definite physical significance.

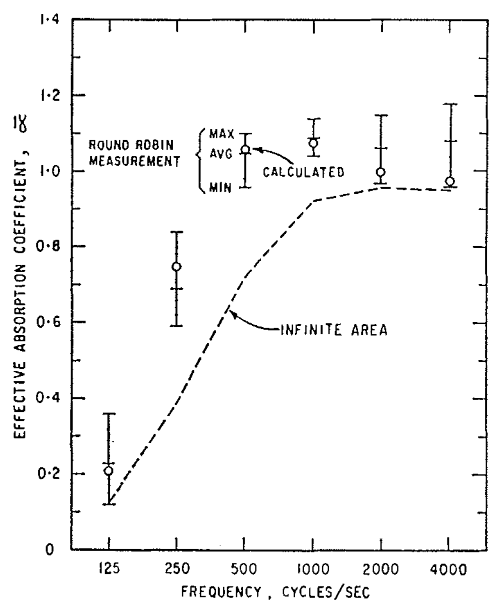

Fig. 8. Calculated and measured absorption coefficients for 3- ×2.5-m sample of Sillan in reverberation room. (See Ref. 4.)

Application of Reverberation-Room Measurements

Assuming that the remaining difficulties in the reverberation-room method can be disposed of, so that all laboratories can consistently measure the random-incidence absorption of a standard size of sample, it is pertinent to consider what uses can be made of such measurements. Should the effective coefficients for a standard sample be the final quoted quantities? These data involve the minimum of processing and form an adequate basis for comparing materials; but the more common applications of materials involve areas larger than the standard sample, for which the coefficients should be smaller. Moreover, there is the awkward problem that when the reverberation method is cor-

Table I. Admittance and absorption calculations for Sillan samples.

Frequency	125	250	500	1000	2000	4000
Conductance ratio	0.025	0.10	0.33	0.58	0.58	0.59
Susceptance ratio	0.15	0.29	0.42	0.20	0.09	0.00
Absorption coefficients						
4 sq m	0.22	0.78	1.15	1.13	1.03	0.98
7.5 sq m	0.21	0.75	1.05	1.08	1.00	0.97
12 sq m	0.21	0.71	0.99	1.04	0.98	0.96
infinite area	0.13	0.39	0.72	0.92	0.94	0.95

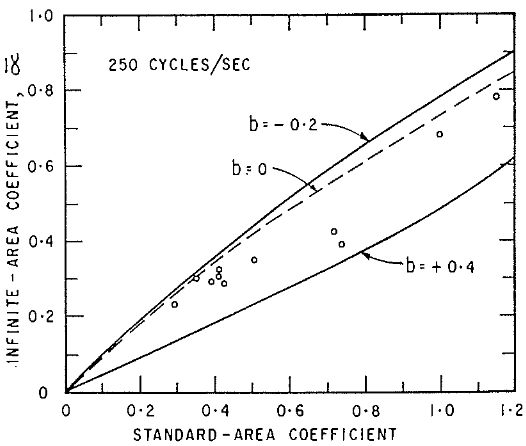

FIG. 10. Relation between absorption coefficients of standard-sample area and infinite area for a range of susceptances, at 250 cps; circles are for typical acoustical materials.

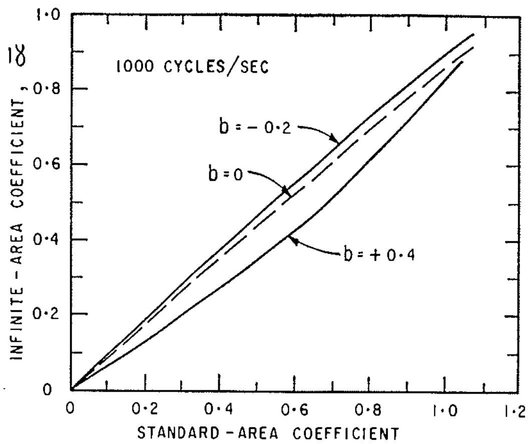

FIG. 11. Relation between absorption coefficients of standard-sample area and infinite area for a range of susceptances, at 1000 cps.

rectly used many absorption coefficients, for the standard sample, are greater than unity. The layman might be puzzled by the implication that materials can absorb more energy than they receive.

An attractive alternative is to correct the standard-sample data to make them applicable to an infinite area. Unfortunately, however, there is no precise general relation between standard-sample absorption and infinite-area absorption. Figures 10 and 11 illustrate, for the category of materials considered here, the order of indeterminacy that exists for a range of acoustical properties of the materials involved. Actually, most materials fall within a narrower range; the points in Fig. 10, which correspond to a random selection of test samples, indicate the trend. Possibly, one might adopt an arbitrary set of correction curves, similar to those shown but representing an average case, that would be adequately close for most design purposes. It seems philosophically undesirable, however, to go to great lengths to make accurate measurements (accurate at least for a standard area) and then to quote "corrected" data that are accurate only to within, say, 10%.

A more satisfactory procedure might be to quote standard-sample data and accompany them with a statement regarding the applicability of the results to other areas. The statement could deal not only with the size of absorbing patches but also with the other variables, such as location of the material relative to other surfaces, that enter into field applications. Such a statement, suitably documented with quantitative information, is badly needed in any case.

ACKNOWLEDGMENTS

The computations on which these results are based, including extensive checking and programming, were largely the work of two summer assistants, André Dupras and Robert Brunet, to whom the author is most grateful.