# The initial creep of columnar-grained ice. Part I. Observed behavior. Part II. Analysis

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# THE INITIAL CREEP OF COLUMNAR-GRAINED ICE.
# PART I: OBSERVED BEHAVIOR
# PART II: ANALYSIS

by

**L. W. Gold**



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## PART I. OBSERVED BEHAVIOR

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N.R.C. No. 8475

# THE INITIAL CREEP OF COLUMNAR-GRAINED ICE

## PART I. OBSERVED BEHAVIOR[1]

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Received March 24, 1965

ABSTRACT

Previously undeformed columnar-grained ice exhibits a period of increasing or constant creep rate during the transient creep stage when loaded in simple compression perpendicular to the long axis of the columns. It is shown that this behavior is associated with the formation of small-angle boundaries and internal cracks. Creep strain beyond 0.25% for first load tends to a power-law dependence on time. On reload, specimens exhibit a normal transient creep behavior and have, initially, a lower resistance to deformation than for first load. With deformation, this resistance increases so that the reload creep curves cross the first load curves at about 0.2% creep. For reload, creep strain less than about 0.025% and greater than about 0.2% appears to have a power-law dependence on time with exponent about equal to that for first load.

Observations by Gold (1960) suggested that columnar-grained ice that has not undergone prior deformation may have a period of increasing creep rate during transient creep under a constant compressive load applied perpendicular to the long axis of the grains. Further information on this behavior was obtained by Krausz (1963), who observed a period of constant or increasing deflection rate during transient creep of columnar-grained ice beams subject to constant bending moment. He found, as well, that if the load time exceeded 8 to 10 hours, those beams that had a constant or increasing deflection rate during transient creep for first load had a normal creep behavior in subsequent tests on the same beam at the same load. Microscopic examination of the surface of beams during deformation indicated that structural changes occurred during the transient creep stage of first load.

Further observations have been made on the creep of columnar-grained ice under constant compressive load perpendicular to the long axis of the columns to obtain additional information on the dependence of creep behavior on stress and time. Observations were made on the structural changes that accompany the deformation. The results of these observations are presented in this paper.

### PREPARATION OF ICE

Ice was made from deaerated tap water using the technique described by Gold (1960, 1962). This technique produces a columnar-grained ice with grain sizes between 1 and 7 mm. A bias in crystallographic orientation develops during freezing such that within 2 cm of the ice–air interface there is a marked preference for the crystallographic axis of hexagonal symmetry to be perpendicular to the direction of growth, and thus for the basal plane to be parallel to the long direction of the columnar grains. Rectangular specimens about 5 by 10 by 25 cm were machined from the ice, the long axis of the grains being perpendicular to the 10- by 25-cm face (Fig. 1).

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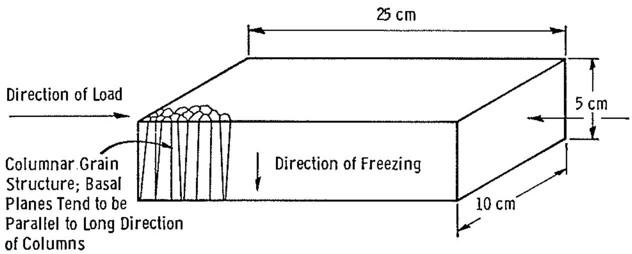


FIG. 1. Grain structure and load direction for specimens.

## LOADING PROCEDURE

Compressive loads were applied to the 5- by 10-cm face of the specimen with a simple lever system having a load magnification of 10.2:1. The loading plates in contact with the ice were made of steel with surfaces ground to a flat "mirror" finish.

The weights used for applying the load, which could be applied or removed within about 2 seconds, were supported on a jack. Just prior to application of the load the specimen supported the lever arm. This preload of about 1.8 kg/cm$^2$ was applied between 2 and 4 seconds before the full load.

Creep was measured over a 15.25-cm gauge length with an extensometer of adjustable working range from 0 to 4% strain and variable sensitivity, the maximum being about $3 \times 10^{-4}$%. The extensometer was supported on the lower loading plate and attached by thumbscrews to two metal collars frozen to the specimen. The design of the extensometer and its mounting was such that the deformation measured was not influenced by possible slight bending of the specimen during creep. Because the extensometer was mounted on the base, tilting of the specimen during deformation could affect the measurements. This influence was reduced to a negligible amount by making the distance between the specimen and the lever arm about 45 cm and suitably constraining the arm through which the load was applied. A linear differential transformer was used to transform creep strain into an electric signal for recording. The extensometer is shown mounted on a specimen in Fig. 2.

The experiments were conducted in a cold room maintained at a temperature of $-9.5 \pm 0.5$ °C. Constant compressive loads of between 4 and 14 kg/cm$^2$ were applied. Creep strain was recorded on a strip chart recorder located outside the cold room.

Twenty-four first-load tests were conducted, the load being applied for 6 hours in most cases. Six of the specimens were loaded a second time and one specimen was loaded a third time. The same compressive load was applied in the reload tests as was applied during first load.

On completion of the loading program for each specimen, thin sections were cut from each of the 10- by 25-cm faces and average grain size determined by the linear intercept method. The average grain size in the plane perpendicular to the long axis of the grains was found to be between 1 and 5 mm for all specimens. The measured grain size for each thin section is given in Table I. It may be noticed that for each specimen the average grain size for one face

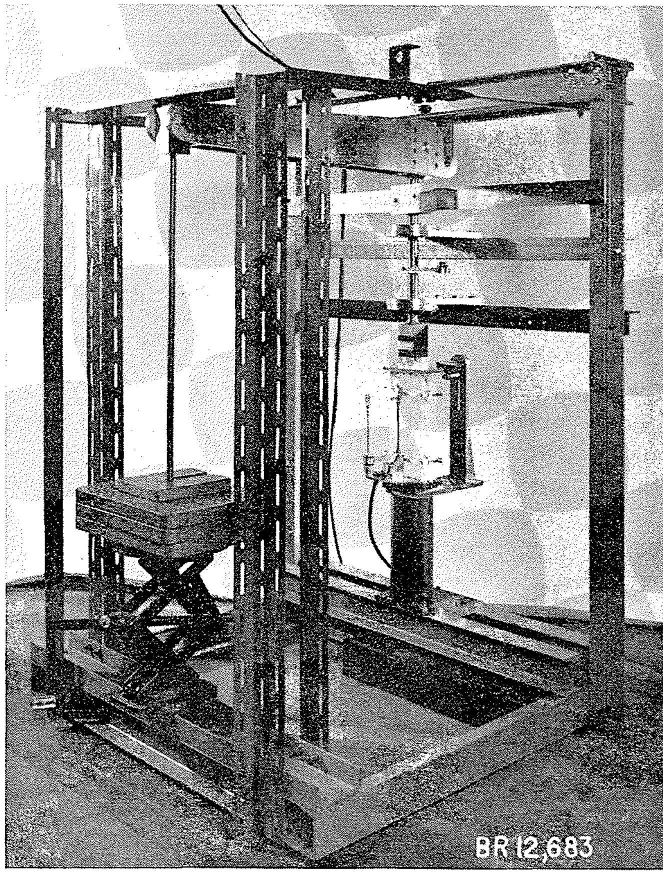


FIG. 2. Loading apparatus with ice and extensometer in place.

is always larger than the average grain size for the other face. This is due to a gradual increase in average grain size in the direction of freezing because of preferred growth directions in ice crystals.

One face of the center section that remained after the two thin sections had been prepared was machined and further smoothed by holding it briefly on a warm plate. This piece of ice was placed in a plastic box and thermally etched using the technique described by Krausz (1961).

## RESULTS

Figure 3 gives examples of the changing deformation behavior observed when specimens were subjected to various periods of load and recovery time. The first time load was applied specimens usually had a relatively high resistance to deformation. The creep rate usually decreased rapidly within the first 10 minutes to a minimum or relatively constant value. In some cases the creep rate appeared to start at a low value and increase with time (Fig. 3(*e*)). On reloading, the specimens displayed a reduced resistance to deformation.

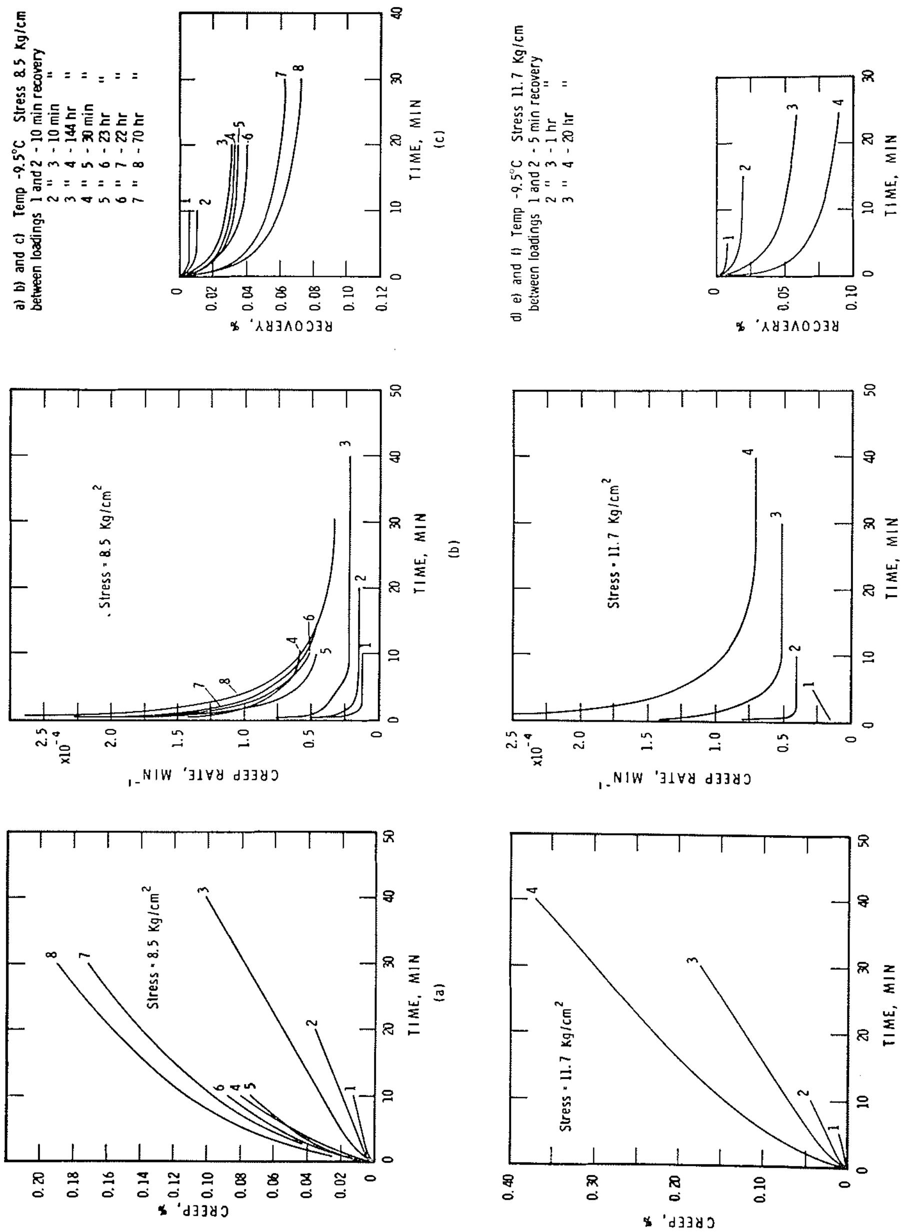


FIG. 3. Creep and creep rates observed for repetitive loading of two specimens.

TABLE I

Grain size and observed values of the exponent $q$ for first load and reload

| Load, kg/cm² | Average grain size, mm | | $q$ | | |
|---|---|---|---|---|---|
| | | | | Reload | |
| | Side 1 | Side 2 | First load, $t$ = 300 min | $t$ = 0.3 min | $t$ = 300 min |
| 5.9 | 2.4 | 3.4 | 0.80* | | |
| 4.0 | 2.1 | 3.6 | 0.88* | | |
| 7.8 | 2.4 | 3.1 | 0.60* | | |
| 10.1 | 1.7 | 3.6 | 0.57 | | |
| 11.9 | 2.4 | 3.3 | 0.80* | | |
| 14.1 | 2.0 | 3.0 | 0.69 | | |
| 7.0 | 3.4 | 4.1 | 0.63* | | |
| 4.1 | 2.3 | 4.6 | 0.85* | | |
| 10.8 | 1.2 | 2.3 | 0.62 | | |
| 7.1 | 1.5 | 2.3 | 0.51* | | |
| 8.9 | 1.5 | 2.2 | 0.48 | | |
| 12.7 | 2.4 | 3.8 | 1.25 (tertiary creep) | | |
| 5.0 | 3.1 | 4.7 | 0.73* | | |
| 8.0 | 3.1 | 3.5 | 0.56 | | |
| 8.0 | | | 0.56 | 0.48 | 0.53* |
| 13.9 | 2.9 | 4.1 | 0.57 | | |
| 5.1 | 3.3 | 4.3 | 0.65* | 0.61 | 0.54* |
| 8.0 | 1.6 | 3.8 | 0.58 | | |
| 8.0 | 1.6 | 2.6 | 0.52 | 0.51 | 0.54* |
| 6.1 | 3.1 | 4.1 | 0.58 | 0.57 | 0.48* |
| 8.0 | 1.7 | 2.5 | 0.53 | 0.65 | 0.42* |
| | | | (2nd reload) | 0.60 | 0.55 |
| 11.9 | 1.7 | 2.8 | 0.52* | | |
| 12.1 | 2.8 | 3.5 | 0.56 | 0.46 | 0.64* |
| 12.0 | 2.9 | 3.5 | 0.78* | | |

*For first load, value for $q$ still decreasing.
For reload, value for $q$ still increasing.

With continued reload and deformation, the deformation behavior developed the reproducible characteristics usually associated with normal creep behavior. These observations are in agreement with those made on ice beams by Krausz (1963).

Figure 4 is a log–log plot showing typical examples of the deformation behavior observed for longer periods of first load and reload. When first loaded, the specimens did not exhibit a consistent pattern of deformation behavior for creep strain less than about 0.01%. Between 0.01% and 0.25% creep strain the creep rate did not decrease continuously in the way associated with normal transient creep behavior; with some specimens it appeared to go through a minimum and subsequently a maximum, while in others it exhibited a temporary leveling off (Fig. 3). The consequence of this behavior was an inflection in the log creep strain – log time curves for first load (Fig. 4). For creep strain greater than about 0.25% the observations showed that creep strain tended to a power-law dependence on time of the form

$$e = kt^q,$$

where $e$ is the creep strain, $t$ is the time, and $k$ and $q$ are constant for a given stress.

On reload after the recovery times used in this study (usually 20 hours or more), the initial creep rate was considerably greater than that associated

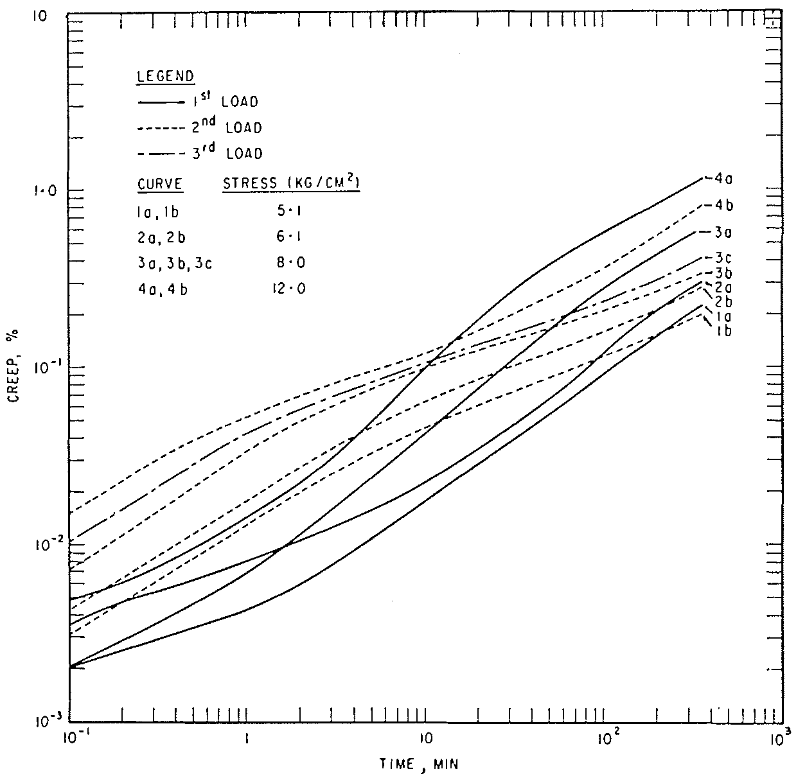


FIG. 4. Time-dependence of creep of multigrained, columnar ice during first load and reload.

with first load. Up to about 0.025% creep strain, the creep appeared to have a power-law dependence on stress with values of $q$ about the same as those for strain beyond 0.25% during the first load, the log creep strain – log time curves again exhibiting an inflection point, but in a sense opposite that observed for first load. For each specimen the creep strain – time curve for reload crossed the corresponding curve for first load, indicating a higher resistance to deformation for reload than occurred at the same time during first load. The crossover occurred at about 0.2% strain. When the creep strain exceeded about 0.3% it again tended to a power-law dependence on stress, the value of $q$ again being about equal to that for first load. For loads greater than 10 kg/cm² the observations were sometimes complicated by the onset of tertiary creep. Values of $q$ for time equal to 300 minutes for first load and 0.3 and 300 minutes for reload are given in Table I. An asterisk beside a value indicates that the linear dependence of log creep strain on log time had not yet been established; for first load $q$ was still decreasing and for reload, increasing.

## STRUCTURAL CHANGES

Thermal etching gave definite evidence of the formation of small-angle boundaries during deformation; examples are shown in Fig. 5. Etching observations on 58 specimens subjected to constant compressive stress between 3 and

15 kg/cm$^2$ indicated that these boundaries did not form until the strain exceeded about 0.1%. The degree of small-angle boundary development generally increased with increasing deformation. In even the most severely deformed specimens, however, there were some grains in which the etching process showed that small-angle boundaries had apparently not formed.

Two types of small-angle boundaries were observed; one that etched as a sharp line (Fig. 5(*a*)) and one that etched as a band (Fig. 5(*b*)). The band-type feature was observed to be broad and shallow. Sometimes a line indicating a more sharply defined boundary was observed at the bottom of the band. The bands appeared to be associated with grains with a crystallographic prismatic plane parallel to the grain surface, whereas the more sharply defined boundaries appeared to occur in grains in which the crystallographic $c$ axis is not parallel to the surface. The surfaces upon which the band features occurred were generally rough (Fig. 5(*b*)), whereas those associated with sharp boundaries were smooth. The small-angle boundaries appeared to be parallel or very nearly parallel to the plane containing the crystallographic $c$ axis. They were sometimes continuous from one grain to the next (Fig. 5(*a*)), the boundary changing direction abruptly at the grain boundary.

In addition to small-angle boundary formation, internal crack formation occurred during deformation. These cracks formed during about the first hour after application of the load, and their rate of formation was dependent on the applied stress (Gold 1960). Small-angle boundaries were associated with the cracks (Fig. 5(*c*)). They were usually perpendicular to the crack, indicating that it was parallel to the basal plane. When a crack terminated within a grain, the tip was associated with a small-angle boundary on one side of the crack only (Fig. 5(*c*)).

## DISCUSSION

The ice from which the specimens were cut was at least 3 cm below the original ice–air interface. The average rate of growth of this ice was about $10^{-2}$ mm/hour and the temperature gradient at the ice–water interface decreased progressively from about 1/3 °C/mm to about 1/10 °C/mm. The ice was annealed for several days at −10 °C before the specimens were machined and stored in kerosene for at least four days at the same temperature before the first load was applied. The ice was therefore subjected to very little thermal stress during and subsequent to its formation. Thermal etching of the undeformed ice showed that no small-angle boundaries were present.

For ice at −10 °C, the basal plane is the plane upon which slip occurs most readily (Glen 1958). In the present experiments the basal plane tended to be parallel to the long direction of the grains. It would be expected that resistance to deformation of such columnar-grained material, with each grain initially having only one easy mode of deformation, would increase rapidly under the constraints imposed by grains upon their neighbors. Stress would build up at grain boundaries and initiate new modes of deformation. This appears to have occurred in the present experiments. The observations indicate that previously undeformed columnar ice of average grain size between 1 and 5 mm perpendicu-

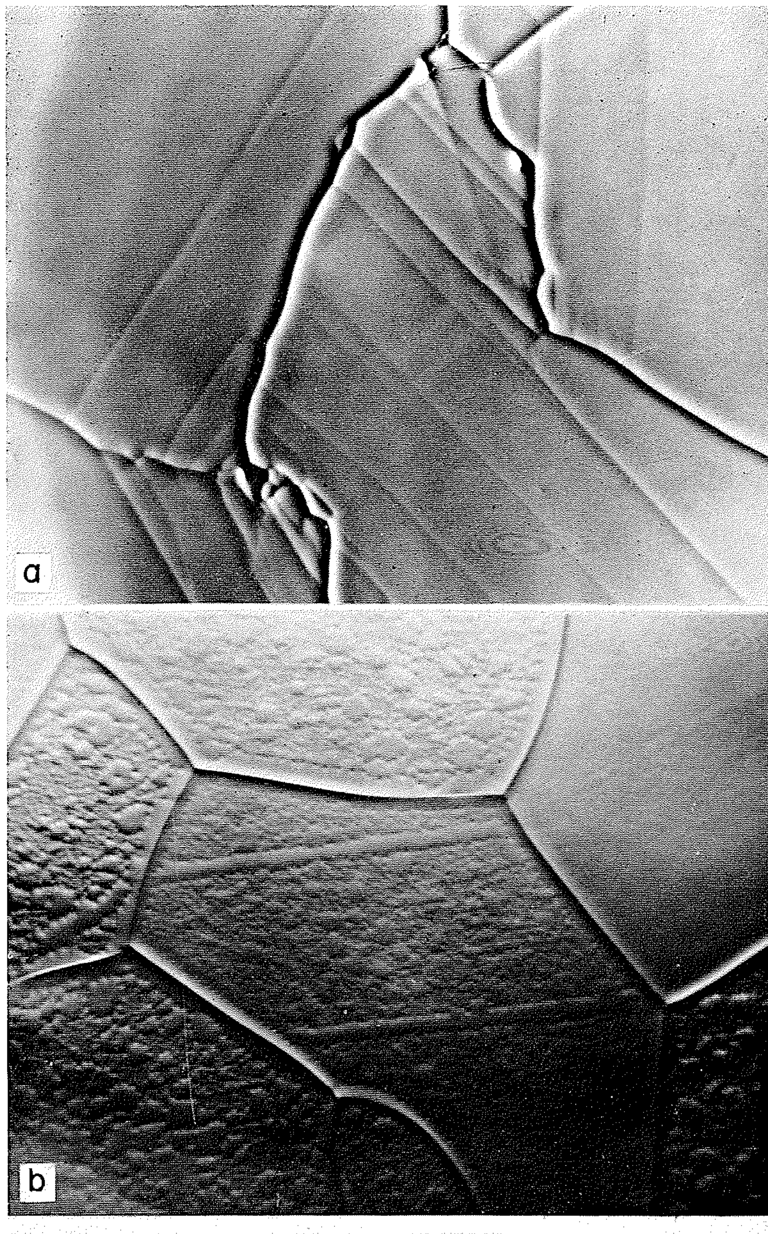


FIG. 5. (*a*) Small-angle boundary formation in columnar-grained ice. (*b*) Band-type small-angle boundary formation.

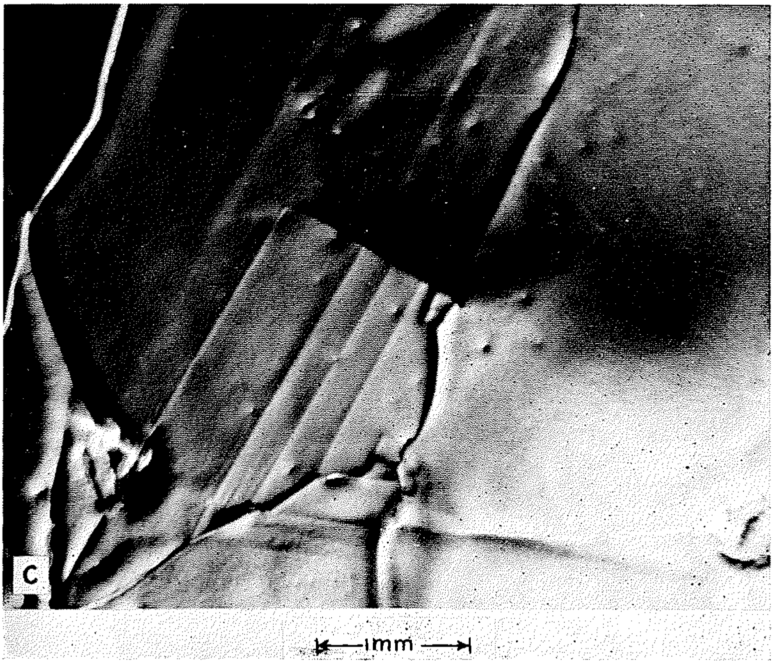


FIG. 5. (*c*) Small-angle boundaries associated with crack formation.

lar to the long axis of the grains can undergo only about 0.1% creep strain before small-angle boundaries are induced in the grains. If the stress is sufficiently high, crack formation occurs as well. As the small-angle boundaries appear to be almost perpendicular to the basal plane, the plane of easy slip, they probably form in the same manner as do the small-angle boundaries observed by Orowan (1942), Gervais *et al.* (1953), and Gilman (1954). The changing deformation behavior shown in Fig. 3 suggests that the processes responsible for the structural changes were probably initiated for creep strain as small as 0.01%.

Many of the structural changes induced in grains during first load were essentially irreversible, although there was probably some modification of them during recovery after the load was removed. On reload, the modes of deformation developed in association with the structural changes appeared to participate immediately in the creep process and the ice exhibited initially a lower resistance to deformation than when first loaded. This stage was confined to creep strain less than 0.2%, about the amount of strain observed during recovery. Thereafter the creep behavior appeared to approach that of the final period of first load, a state that was achieved in less total creep strain than was required for first load.

The dependence of the deformation behavior of ice on its past history of deformation has interesting consequences for some problems encountered with natural ice formations. Natural ice may be subjected in its lifetime to total creep of a fraction of a percent, as with some ice covers on lakes, to several hundred percent as in glaciers and ice caps. The results of the present work are pertinent to the deformation of short-lived ice such as that on frozen lakes or rivers. The present observations indicate that previous deformation will introduce an ambiguity into calculations of stresses or deflections for natural ice covers because there is no convenient method of determining what that deformation has been. This ambiguity would apply during about the first 0.25% creep strain, the range of particular interest for many practical problems associated with natural ice covers. It would be expected, however, that during the first part of the winter the characteristics of the deformation behavior would approximate those for first load. As the winter progresses, and depending on the stress to which the ice is subjected (e.g. vehicle movement, temperature cycles), the characteristics would tend to those associated with reload.

The present experiments do provide some information that can be applied to such problems. In Part II of this paper the observations are further analyzed to put this information into a convenient form for calculations. The results of the analyses are then applied to some of the results obtained by Krausz (1963) from his experiments on ice beams.

## CONCLUSIONS

Previously undeformed columnar-grained ice at −10 °C shows a relatively high resistance to deformation when loaded in simple compression perpendicular to the long axis of the grains. In the creep strain range 0.01 to 0.25% irreversible changes occur in the structure of the ice, two of these changes being the development of small-angle boundaries and the formation of internal cracks.

For creep strain beyond 0.25% the creep for a given stress tends to a power-law dependence on time.

On reload, the columnar-grained ice has a relatively much lower resistance to creep initially than it has during first load. This resistance increases with continued deformation so that reload creep curves cross their respective first-load curves at about 0.2% creep strain. For reload and for creep strain less than 0.025%, or exceeding 0.2%, the creep for a given stress tends to a power-law dependence on time.

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# THE INITIAL CREEP OF COLUMNAR-GRAINED ICE

## PART II. ANALYSIS[1]

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ABSTRACT

Observations on the initial creep behavior of columnar-grained ice are analyzed by assuming that the creep strain at a given time has a power-law dependence on the applied constant compressive stress. The exponent for the stress was time-dependent during transient creep. For first load it started at a low value, increased to a maximum of about 2.23 approximately 75 minutes after the application of the load, and decreased thereafter. For reload it started at a high value and decreased continuously to a constant value of 1.46 by 100 minutes after the application of the load. Creep rates at a given time, calculated from the observed power-law dependence of the creep strain on stress, also had a power-law dependence on stress for time greater than about 25 minutes after the application of the load. The observations are shown to be in agreement with observations by Krausz (1963) on the deflection rate of ice beams and by Steinemann (1954) and Glen (1958) on the stress-dependence of the minimum creep rate during secondary creep. The observations indicate that the creep rate during secondary creep varies approximately as $t^{-0.5}$.

Part I (Gold 1965) presented information on the difference in creep behavior, due to constant compressive load, between columnar-grained ice previously undeformed and the same ice after it has been subjected to deformation. It was shown that structural changes occurred in the ice during first load in association with an unusual behavior of the creep rate during transient creep. Evidence of the formation of small-angle boundaries after the creep strain exceeded about 0.1% was obtained by thermal etching. The formation of internal cracks during the transient creep stage was visually evident.

In the present paper the observations are analyzed by assuming a power-law dependence of creep on stress. From the empirical relationships obtained, the relationship between creep rate and stress and its change with time is determined. This relationship is used to interpret the creep behavior of ice beams reported by Krausz (1963). The results of the present observations are compared with observations by Steinemann (1954) and Glen (1958) on the stress-dependence of the minimum creep rate of ice during secondary creep.

### STRESS-DEPENDENCE OF THE CREEP

In Fig. 1($a$) log creep strain for first load is plotted against log stress for times $t = 1$ minute, 100 minutes, and 350 minutes. The corresponding observations for reload are given in Fig. 1($b$) for times $t = 1$ minute, 100 minutes, and 300 minutes. From these figures it is apparent that the relationship between the creep strain and the stress at a given time is given approximately by an equation of the form

(1) $$(e)_t = A(t)\sigma^{n(t)},$$

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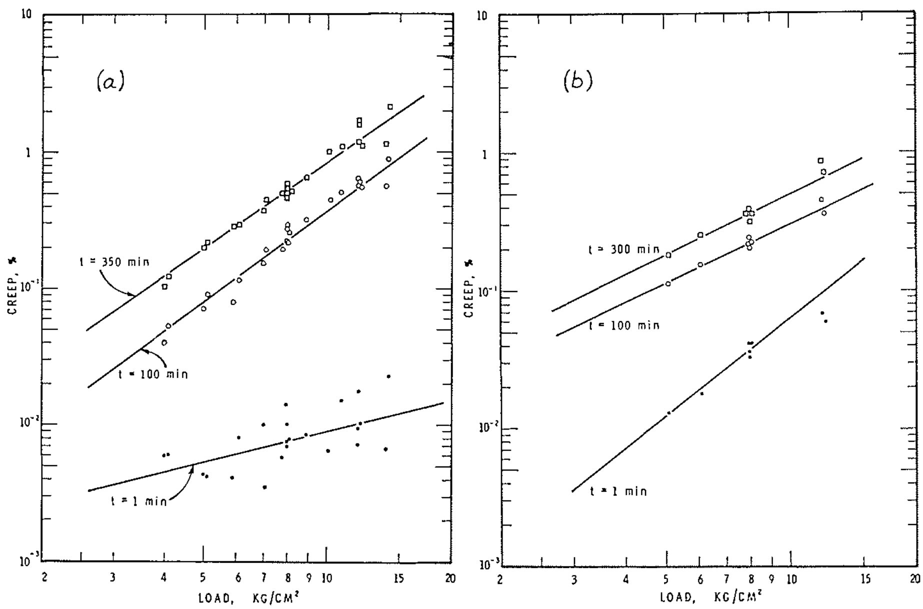


FIG. 1(*a*). Dependence of creep on constant compressive stress for first loading (time = 1, 100, and 350 minutes; temp. = −9.5 ± 0.5 °C). (*b*) Dependence of creep on constant compressive stress for reloading (time = 1, 100, and 300 minutes; temp. = −9.5 ± 0.5 °C).

where $(e)_t$ is the creep strain at time $t$, $\sigma$ is the constant compressive stress, and $A(t)$ and $n(t)$ are constants for given time $t$. For first load there is considerable scatter for time $t = 1$ minute, due in part to the erratic initial creep behavior of ice, as was mentioned in Part I. The relative scatter was reduced appreciably by $t = 10$ minutes. For reload, creep observed for high stresses ($\sim$14kg/cm$^2$) appears to deviate from a power-law dependence on stress, being less than expected for $t$ less than 100 minutes and greater than expected for $t$ greater than 100 minutes as may be seen in Fig. 1(*b*).

The least-squares fit for given times was calculated for first load using stress as the independent variable, and values for $n$ were obtained. Although only eight reload tests were carried out, it was decided to analyze these along with the first-load tests to show the marked difference in deformation behavior between the two loading situations. The line through reload observations was located by eye. Calculated values for $n$ for first load and reload are plotted against time in Fig. 2; subscript 1 refers to first load and 2 to reload.

The marked difference in the characteristics of deformation for first load and reload is clearly evident. For example, the initial value of $n$ for the first load is quite small, increases to a maximum at $t$ equal to approximately 75 minutes, then gradually decreases. Unfortunately, the load times were not long enough to establish the existence of constant $n_1$. For reload, the value for $n$ is relatively large initially and decreases continuously to a constant value of 1.46 for $t$ greater than 100 minutes.

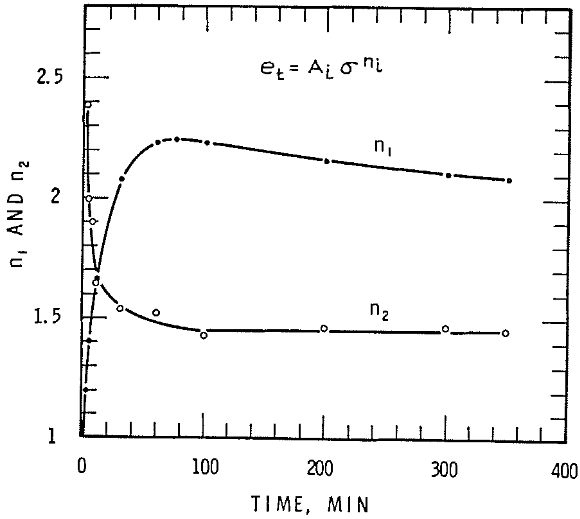


FIG. 2. Time-dependence of $n_1$ (first load) and $n_2$ (reload).

## STRESS-DEPENDENCE OF CREEP RATE

The dependence of creep rate on stress at given times can be determined from the results of the foregoing analysis. Before proceeding with the calculations, however, it will be useful to modify equation (1). For given time

$$A = e_0/\sigma_0^n,$$

where $e_0$ is the creep strain at time $t$ due to stress $\sigma_0$. Substituting this expression into equation (1) gives

$$e(\sigma, t) = e_0(\sigma/\sigma_0)^n, \tag{2}$$

an equation that is more satisfactory dimensionally, particularly under conditions of varying $n$. Differentiating this expression with respect to time gives

$$\frac{\partial e(\sigma, t)}{\partial t} = \left(\frac{\sigma}{\sigma_0}\right)^n \left[\frac{\partial e_0}{\partial t} + e_0 \frac{\partial n}{\partial t} \ln \frac{\sigma}{\sigma_0}\right]. \tag{3}$$

In Fig. 3, log $e$, obtained from the least-squares fit to the first-load observations, is plotted against log $t$ for various values of $\sigma$. It may be seen that because the value of $n_1$ is time-dependent, the shape of the log $e$ vs. log $t$ curve is not independent of stress, as it would be if $n$ were constant. It is of interest that extrapolation of the experimental results to $\sigma = 1.3$ kg/cm$^2$ gives $e$ equal to a constant for about the first 10 minutes of loading. Extrapolation of the observations to loads greater than 25 kg/cm$^2$ gives $e \propto t^{0.55}$ for times greater than 100 minutes after application of the load.

It was possible, for given time periods and stress, $\sigma$, to determine simple functions giving the time-dependence of $n$ and $e$. The derivatives of $n$ and $e_0$ with respect to time were determined from these functions and $\partial e/\partial t$ calculated

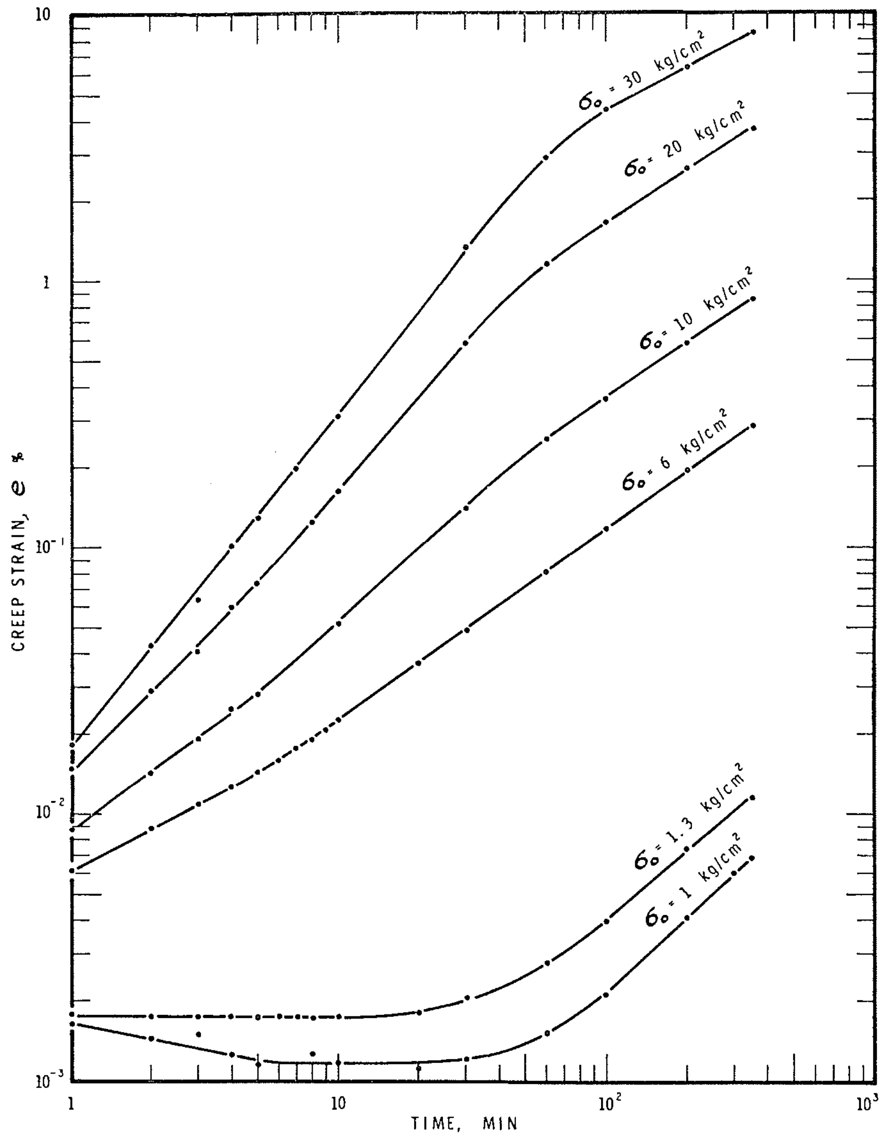


FIG. 3. Creep curves for given stresses obtained from least-squares fit to first-load observations (temp. = $-9.5 \pm 0.5$ °C).

for given times and $\sigma = 4, 6, 8, 10, 12$, and $14$ kg/cm$^2$, by the use of equation (3). Calculated values are given in Table I. The values for $\sigma = 4$ and $14$ kg/cm$^2$ are plotted against time in Fig. 4. It may be seen that for first load the creep rate has a high value initially, decreases to a minimum within the first 10 minutes of loading, then rises to a maximum for $t$ between 10 to 20 minutes. There is evidence of a plateau in the creep rate between 20 to 40 minutes after the application of the load, following a fairly rapid decrease from the maximum.

TABLE I

Calculated creep rate for given time and load (in kg/cm²) for first load (temp. −9.5 ± 0.5 °C)

| Time (min) | Creep rate (%/min × 10⁴) for a load of: 4 | 6 | 8 | 10 | 12 | 14 |
|---|---|---|---|---|---|---|
| 1 | 18.0 | 32.8 | 48.2 | 63.7 | 79.1 | 95.1 |
| 3 | 9.9 | 21.7 | 36.3 | 53.0 | 71.2 | 92.4 |
| 5 | 7.6 | 18.0 | 32.0 | 49.1 | 68.9 | 91.9 |
| 7 | 6.2 | 15.7 | 29.1 | 45.7 | 65.8 | 90.2 |
| 10 | 5.9 | 15.3 | 28.9 | 47.1 | 69.1 | 95.3 |
| 15 | 5.2 | 14.4 | 28.5 | 47.9 | 72.3 | 102.0 |
| 20 | 4.8 | 13.3 | 26.8 | 45.7 | 70.0 | 100.0 |
| 25 | 4.7 | 12.6 | 24.8 | 42.0 | 64.0 | 91.3 |
| 30 | 4.4 | 11.9 | 23.8 | 40.8 | 62.9 | 90.5 |
| 40 | 4.0 | 11.0 | 22.5 | 39.0 | 60.8 | 88.4 |
| 50 | 3.8 | 10.4 | 21.0 | 36.3 | 56.0 | 81.9 |
| 60 | 3.8 | 9.9 | 19.6 | 33.2 | 51.1 | 73.5 |
| 70 | 3.7 | 9.5 | 18.4 | 30.8 | 46.8 | 66.7 |
| 80 | 3.7 | 9.2 | 17.4 | 28.7 | 43.2 | 61.0 |
| 100 | 3.6 | 8.6 | 16.0 | 25.7 | 37.8 | 52.2 |
| 150 | 3.3 | 7.7 | 13.8 | 21.4 | 30.8 | 41.9 |
| 200 | 3.1 | 7.1 | 12.5 | 19.3 | 27.6 | 37.6 |
| 250 | 3.0 | 6.7 | 11.7 | 18.0 | 25.6 | 34.7 |
| 300 | 2.9 | 6.4 | 11.2 | 17.0 | 24.2 | 32.5 |
| 350 | 2.8 | 6.1 | 10.7 | 16.1 | 22.3 | 30.7 |

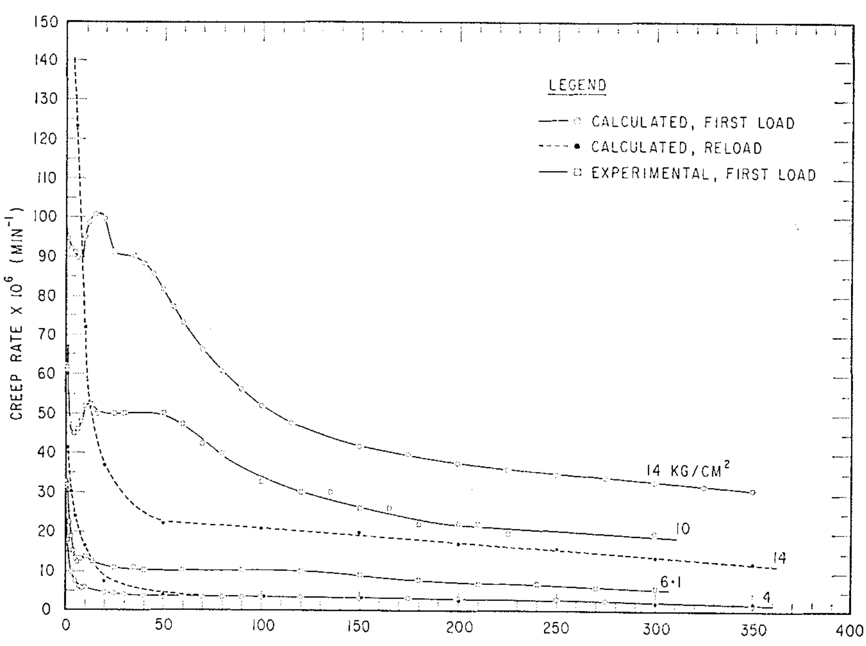


FIG. 4. Time-dependence of creep rate for first load and reload (temp. = −9.5 ± 0.5 °C).

Thereafter the creep rate decreases smoothly and continuously. Creep rates calculated directly from observations show the same features; two examples are presented in Fig. 4.

Creep rates were determined also from the reload observations. The calculated values for stress equal to 4 and 14 kg/cm² are shown in Fig. 4. In contrast with first load, the creep rate has a high initial value and decreases continuously with time, most of the decrease occurring within the first 50 minutes.

The logarithm of the calculated creep rate at given times is plotted against the logarithm of the stress for first load in Fig. 5(*a*) and reload in Fig. 5(*b*). For $t$ greater than about 25 minutes the dependence of creep rate on stress for both first load and reload is closely approximated by

$$\partial e(\sigma, t)/\partial t = B(t)\sigma^{m(t)}, \tag{4}$$

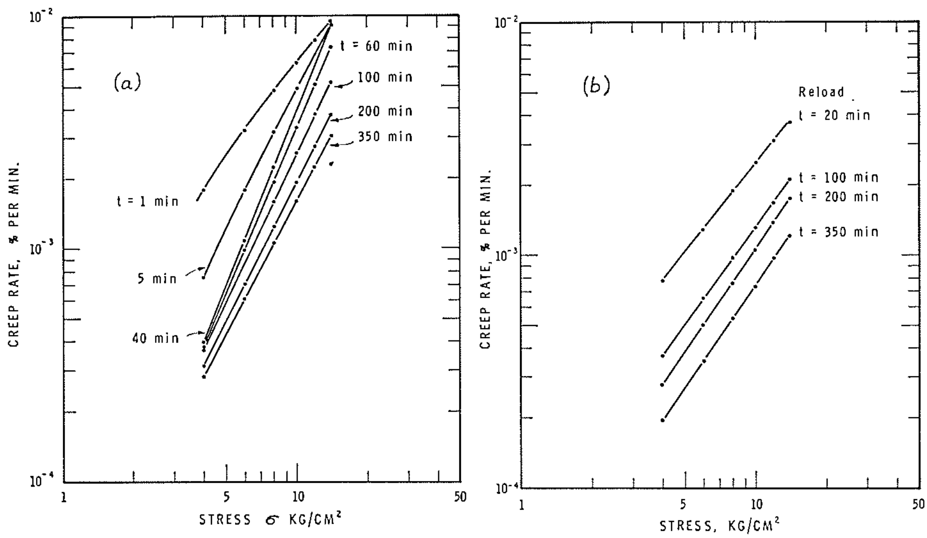


FIG. 5(*a*). Stress-dependence of creep rate at given times for first load (temp. = −9.5 ± 0.5 °C). (*b*) Stress-dependence of creep rate at given times (temp. = −9.5 ± 0.5 °C).

where $B$ and $m$ are constants for given $t$. For time less than 25 minutes the plot of logarithm of the creep rate against logarithm of the stress was not linear, as would be anticipated from equation (3) in view of the dependence of $n$ on time.

Values of $m$ and $B$ were determined for $t$ greater than 20 minutes and plotted against time in Figs. 6 and 7, respectively. From equations (2) and (3) it may be seen that $m$ should equal $n$ if $n$ is constant, as it was for reload for $t$ greater than about 150 minutes. For first load, $n_1$ was still changing at the time the load was removed; correspondingly, $m_1$ also was still decreasing.

Figure 7 shows that for $100 < t < 350$ minutes, $B_2$ has a power dependence on time. The exponent of this dependence was found to be equal to −0.53, in

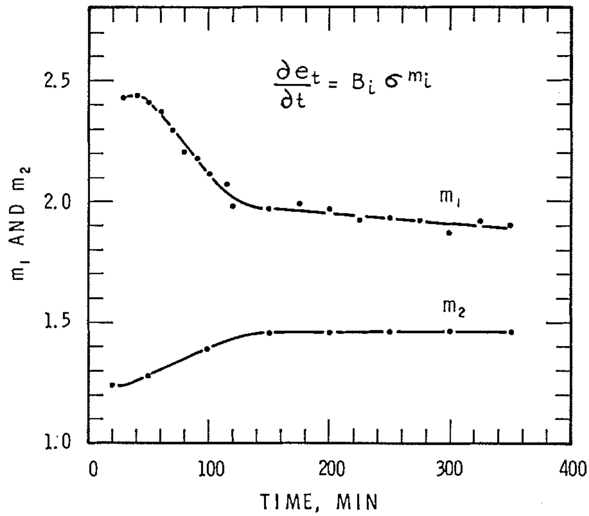


FIG. 6. Time-dependence of $m_1$ (first load) and $m_2$ (reload).

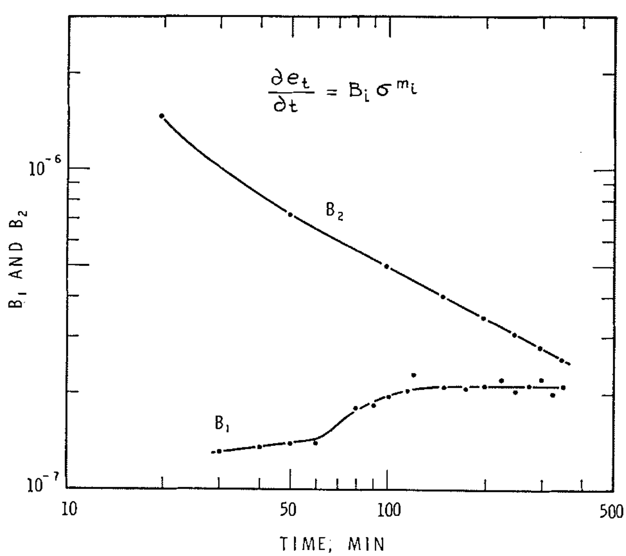


FIG. 7. Time-dependence of $B_1$ (first load) and $B_2$ (reload).

good agreement with the exponent for the time-dependence of the creep for $t$ equal to 300 minutes given in Table I, Part I. For $150 < t < 350$ minutes, therefore, the analysis gives for the creep rate for reload

(5) $$\partial e(\sigma, t)/\partial t = 5.6 \times 10^{-4} t^{-0.53} \sigma^{1.46} \% \text{ per minute.}$$

An empirical equation can be obtained as well for first load for $150 < t < 350$ minutes, namely,

$$\partial e(\sigma, t)/\partial t = 2.13 \times 10^{-5} \sigma^{2.04 - 4.5 \times 10^{-4} t};$$

but this equation is not in a form that can be readily related to creep theory as is equation (5). It is of interest that if the time between first load and reload (about 20 hours) is substituted in the above equation, the exponent obtained for the stress is about equal to that observed for the reload tests.

## COMPARISON WITH PREVIOUS WORK

Steinemann (1954) and Glen (1958) have presented information on the stress-dependence of the minimum creep rate of granular ice in the secondary creep stage. The present investigations were not continued until that creep rate was attained. Furthermore, the stress-dependence was established for given time, whereas the condition of minimum creep rate used by Glen and Steinemann would be more closely associated with given creep strain. The dependence of the creep rate on stress and creep strain can be calculated from equation (1). Assuming that $n$ is constant and $A = Dt^p$ ($D$ and $p$ constants),

$$e(\sigma, t) = Dt^p\sigma^n. \tag{6}$$

Differentiating with respect to time,

$$\partial e(\sigma, t)/\partial t = pDt^{p-1}\sigma^n. \tag{7}$$

Solving for $t$ in equation (6) and substituting in equation (7) gives

$$\partial e(\sigma, e)/\partial t = pD^{1/p}e^{p-1/p}\sigma^{n/p}. \tag{8}$$

Assuming that with time the values for $p$ and $n$ for first load approach those for reload, i.e. $p = 0.47$, $n = 1.46$,

$$\partial e(\sigma, e)/\partial t = pD^{1/p}e^{-1.13}\sigma^{3.11}.$$

The exponent for the stress, 3.11, is in good agreement with that obtained by Glen (1958) ($3.17 \pm 0.1$ for a temperature of $-1.5$ °C). Steinemann (1954) did not obtain a linear dependence of log creep rate on log stress, but found that the exponent increased from about 1.80 at $\sigma = 1$ kg/cm$^2$ to about 4.16 at $\sigma = 14$ kg/cm$^2$, being equal to about 3.0 for $4 < \sigma < 8$ kg/cm$^2$. If the creep rate of ice is, in fact, approximately inversely proportional to the creep strain, as indicated by equation (8), a possible explanation for the curvature in the dependence of log creep rate on log stress obtained by Steinemann is that the minimum creep rates occur for different amounts of creep strain for the stresses used in his experiments. Steinemann's observations did indicate a tendency for the onset of tertiary creep to occur at a smaller creep strain for large loads (15 kg/cm$^2$) than for small loads (4 kg/cm$^2$). A similar effect may exist for the minimum creep rate. Glen's observations were for loads less than 10 kg/cm$^2$.

Glen (1958) found that for granular ice near the melting point the observed creep strain could be approximated reasonably well by the Andrade law:

$$e = \beta t^{\frac{1}{3}} + kt.$$

Application of the Andrade law to the present observations yielded poor fit for first load and only fair fit for reload. No attempt was made to fit the observations with polynomials containing powers of $t$ other than $\frac{1}{3}$ and 1.

As the dependence of creep strain on stress was usefully approximated by a power-law function, it may be of interest to point out certain features of the exponent, $n$. The value of $n$ was found to be primarily dependent on time and independent of stress or the amount of deformation at a given time. Taking into consideration the evidence of formation of small-angle boundaries and internal cracks, and the consequences of recovery, it would appear that $n$ is in some way related to the formation and rate of operation of the microscopic elements responsible for creep. The characteristics of these elements to which $n$ is related must, however, be dependent primarily on time and only to a minor degree, if at all, on applied stress. It may be of interest that for first load and $t$ between 1 and 20 minutes $n$ was proportional to $\ln t$. For first load and $t$ between 150 and 350 minutes, and reload and $t$ between 0.5 and 100 minutes, $n$ appears to be proportional to $-\ln t$. It is unfortunate that the first-load observations were not continued long enough to establish whether, with continued deformation, the value for $n_1$ approaches the constant value observed for $n_2$.

Readey and Kingery (1964) observed for single crystals of ice deformed in tension (under conditions of constant strain rate) a power-law dependence of the creep rate on stress. They found that the exponent $n$ decreased with strain from about 2.5 to about 1.5. The range in creep strain over which they observed this change was much greater than that in the present experiments. Higashi, Koinuma, and Mae (1964) made observations on single crystals of ice similar to those of Readey and Kingery, giving particular attention to a "yield drop" phenomenon that occurs for creep strain about 1%. They observed a power-law relationship between the applied constant strain rate and the maximum stress that occurred, the exponent for the stress being 1.53. The strain rates applied were such that the maximum stress did not exceed about 4.5 kg/cm$^2$. These observations suggest that the influence of stress on the creep rate of ice may be basically the same for both single crystals and polycrystals.

## APPLICATION OF RESULTS TO BEAM EXPERIMENTS

Krausz (1963) conducted first-load and reload experiments on ice beams made from columnar-grained ice, and observed deformation behavior similar to that recorded in the present work. The ice beams were subjected to a constant bending moment of 11.4 kg-cm/cm (25 in.-lb/in.). The long axis of the columnar grains was perpendicular to the face to which the load was applied, so that the principal stresses had the same orientation with respect to the grain boundaries as in the compression tests.

The load distribution associated with the applied bending moment, if we assume that the beam deforms elastically, is shown in Fig. 8 for a beam 2.54 cm thick. As the ice creeps, it would be expected that the load distribution would change. It was assumed that after a sufficient length of time following the application of the load the creep rate perpendicular to the plane upon which the bending moment acts would be linearly proportional to the perpendicular distance from the neutral plane. Krausz's observations indicated that the

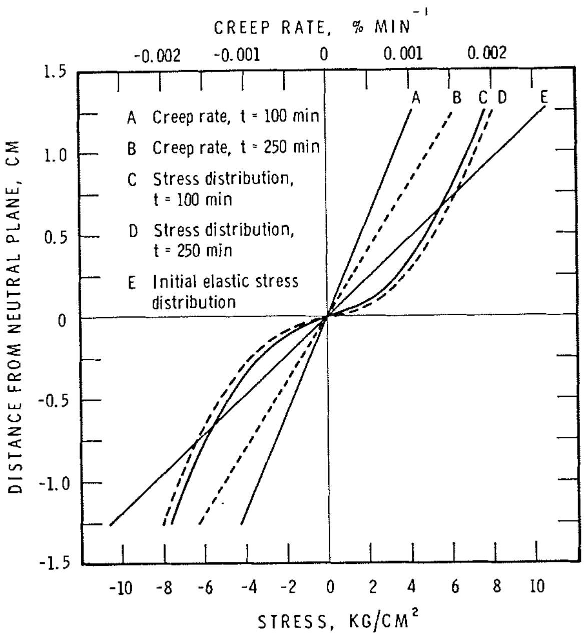


FIG. 8. Creep rate and stress associated with constant bending moment of 11.4 kg-cm/cm (temp. = $-9.5 \pm 0.5$ °C).

neutral plane remained at the center of the beam throughout deformation. By the use of the above assumption, creep rates were determined from observed deflection rates of a beam 2.54 cm thick at times $t$ equal to 60, 100, and 250 minutes. From these creep rates, stresses were determined by using the dependence of creep rate on stress for first load obtained from the simple compression experiments.

Assumed creep rates and resulting load distribution through the beams for $t$ equal to 100 and 250 minutes are given in Fig. 8. The calculation indicates that the stress near the surface of the beam has relaxed considerably and that near the center it has increased from the initial elastic distribution; but there is very little difference between the distributions obtained for either time. The stress calculated for $t = 60$ minutes was a little less than that for $t = 100$ minutes within 0.63 cm of the surface, and a little greater over the remaining center section of the beam. The bending moment calculated graphically from the mean of the stress distributions for $t$ equal to 100 minutes and 250 minutes was found to be 10.5 kg-cm/cm$^2$ (23 in.-lb/in.), within 10% of that applied. For $t = 60$ minutes the bending moment was somewhat less.

Maximum deflection rate in the above beam test occurred at $t$ between 60 and 100 minutes after the application of the constant bending moment. Analyses of the deflection rates for a second beam 3.05 cm thick showed similar agreement between calculated and applied bending moments for time equal to or greater than that associated with the maximum deflection rate. For times less than that at which the maximum deflection rate occurred, the bending moments calculated according to the foregoing assumption were smaller than that applied. These observations indicate that during first load of columnar-grained ice beams not only does the unusual creep behavior affect the deformation, but there is also an influence associated with the transition from the initial elastic to the final plastic condition. It would be expected that when the load is first applied the stress distribution would tend to be that associated with elastic deformation. According to the results of the compression tests, therefore, the creep rate near the surface would be considerably larger than the linear assumption would predict from the observed deflection rates.

The observed deflection rates indicate that the middle section of the beam, where the stresses initially are less than for the fully plastic condition, largely determines its initial deformation behavior. As the stress near the surface relaxes, the center section of the beam must carry more of the load. Because of the characteristics of the dependence of creep rate on stress for first load, the increase in stress over the middle section results in an increased creep rate and associated deflection rate. On the basis of this interpretation, Krausz's observations indicate that transition from the elastic to the plastic behavior requires from 1 to 5 hours for ice loaded under the conditions of the experiments. The transition for the reload condition appears to take place in less time and without a maximum in the deflection rate, although an inflection might be present (see Krausz 1963, Fig. 3).

Krausz's observations were made on beams between 2.28 and 3.05 cm thick. The deflection rate was very sensitive to beam thickness. Two beams could not be considered geometrically equivalent unless they were machined to a given thickness to a tolerance smaller than $\pm 0.5\%$. Beams 2.29 cm thick failed within half an hour of the application of the load.

The reason for the very sensitive dependence of deflection rate on thickness for beams about 2.54 cm thick subjected to a bending moment of 11.4 kg-cm/cm becomes clear when the power-law dependence of creep rate on stress is taken into consideration. The stress at the surface of a beam for a given bending moment varies inversely as the square of the beam thickness. For beams 2.29 to 3.05 cm thick subjected to a bending moment of 11.4 kg-cm/cm, the range in maximum stress at the surface is about 6 kg/cm$^2$ to 11 kg/cm$^2$. The corresponding creep rates for first load and $t = 100$ minutes are $8.7 \times 10^{-4}$ and $3.7 \times 10^{-3}\%$/min, a 25% reduction in beam thickness causing an increase of over 300% in the creep rate at the surface. Failure of the beam 2.29 cm thick within half an hour of the application of the load is to be expected from the results of investigations now in progress on the stress-dependence of the time to formation of internal cracks in ice.

## CONCLUSIONS

Analysis of creep observations for columnar-grained ice subject to compressive stress between 4 and 14 kg/cm$^2$ perpendicular to the long axis of the grains showed that the stress-dependence of the creep at a given time for first load and reload can be usefully approximated by a power-law function between $t$ equal to 1 and 360 minutes. The value for the exponent of the stress varies continuously with time during transient creep. For reload it tends to a constant value of 1.46 for $t$ greater than 100 minutes. The stress-dependence of the creep rate at a given time can be usefully approximated by a power-law function only for $t$ greater than about 25 minutes. The flow law obtained for first load agrees with observations on the deflection rate of beams for times greater than that associated with the maximum deflection rate, if it is assumed that the creep rate of the beam varies linearly with perpendicular distance from the neutral plane. Extrapolation of the present results to give creep rates for a given amount of creep yields an exponent for the stress in the flow law in good agreement with the values obtained by Steinemann (1954) and Glen (1958). Observations during reload indicate that the creep rate during secondary creep is approximately proportional to $t^{-0.5}$.

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